

Dynamic Meteorology: lecture 12

12/1/2022 (Wednesday, 13:15)

Sections 1.32-1.33 (part G)

Balanced circular vortex (gradient wind balance)

Example: tropical cyclone (section 1.32)

Potential vorticity inversion (section 1.33)

Insights from PV-inversion on the structure of a balanced vortex and balanced flow

“Kleinschmidt’s principles”

Tutorial after this lecture (12) (online)

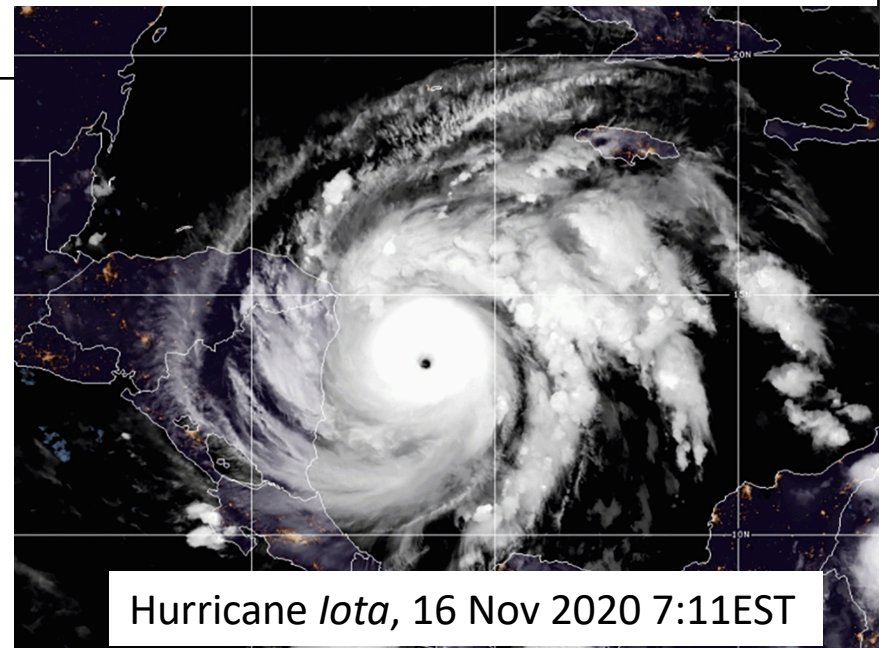
Project 5 (Principal Component Analysis)

Problems 1.29, 1.30, 1.33 (p. 224-227)

(...and problems 1.34 & 1.35)

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(<http://www.uu.nl/~nvdelden/dynmeteorology.htm>)



Hurricane *Iota*, 16 Nov 2020 7:11EST

Anatomy of a tropical cyclone

example of a balanced nearly axi-symmetric baroclinic vortex

p. 124 lecture notes

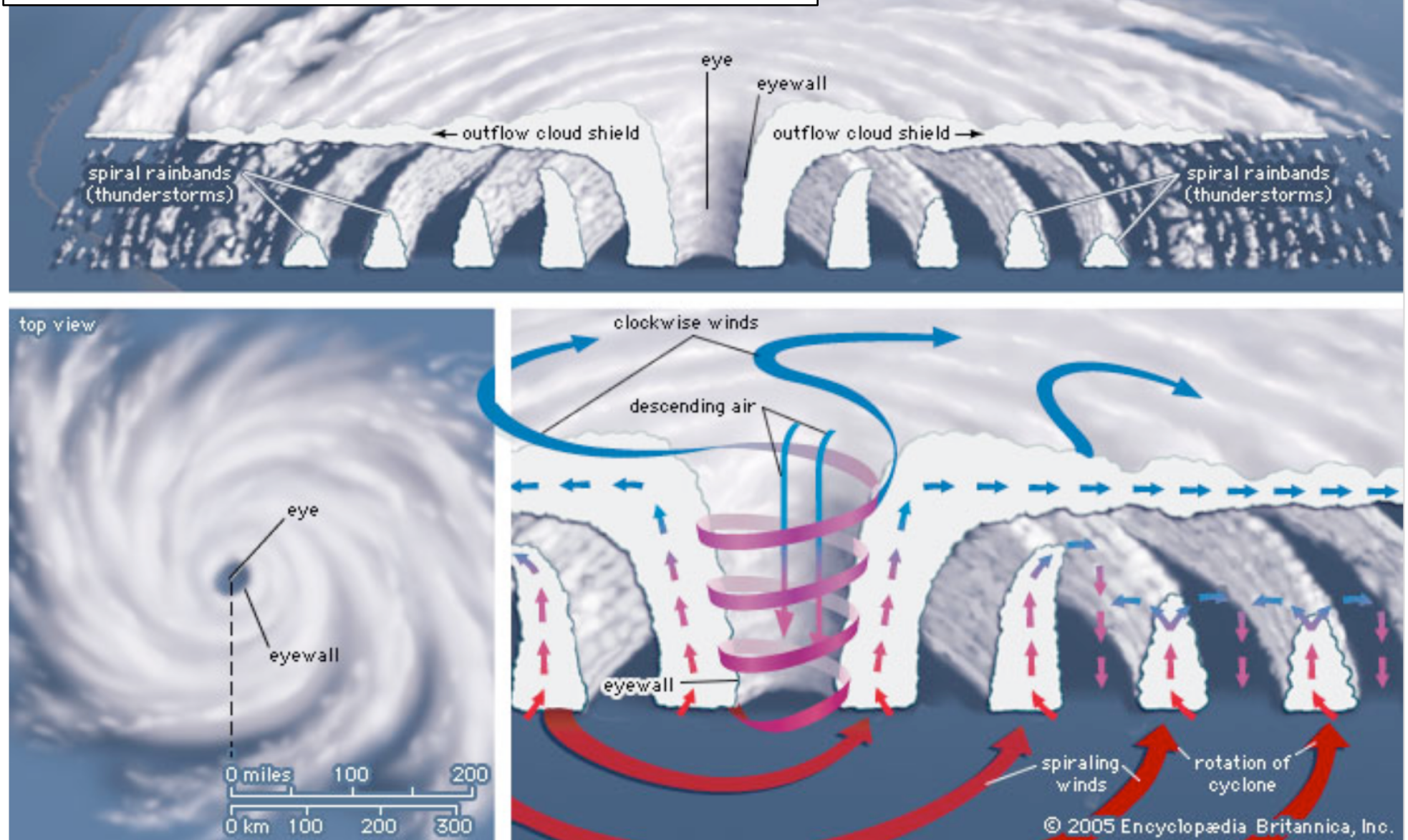


FIGURE 1 (Box 1.11). Anatomy of an ideal tropical cyclone in the northern hemisphere (source: Encyclopædia Britannica: <http://www.britannica.com/EBchecked/topic/606551/tropical-cyclone>).

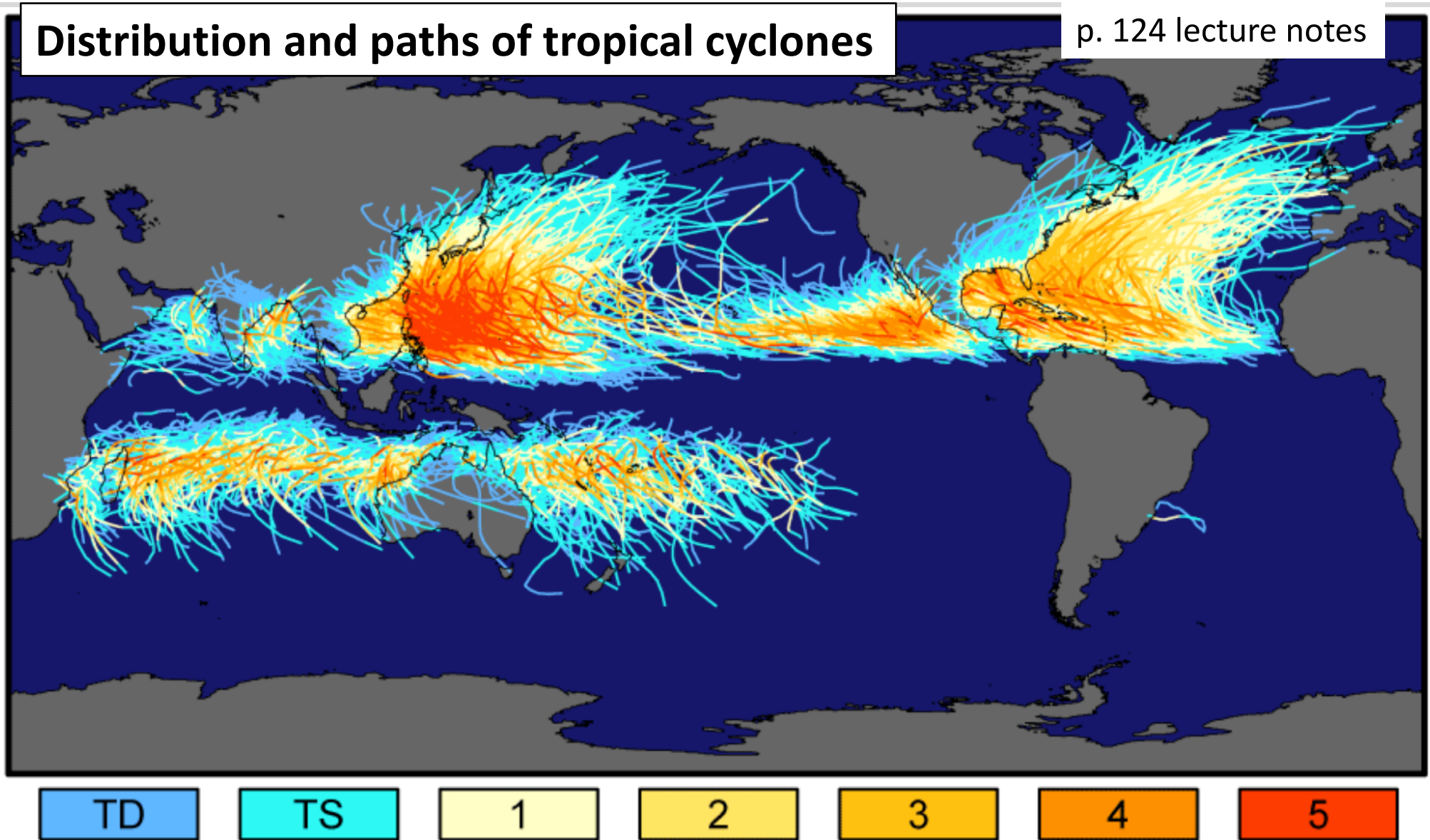


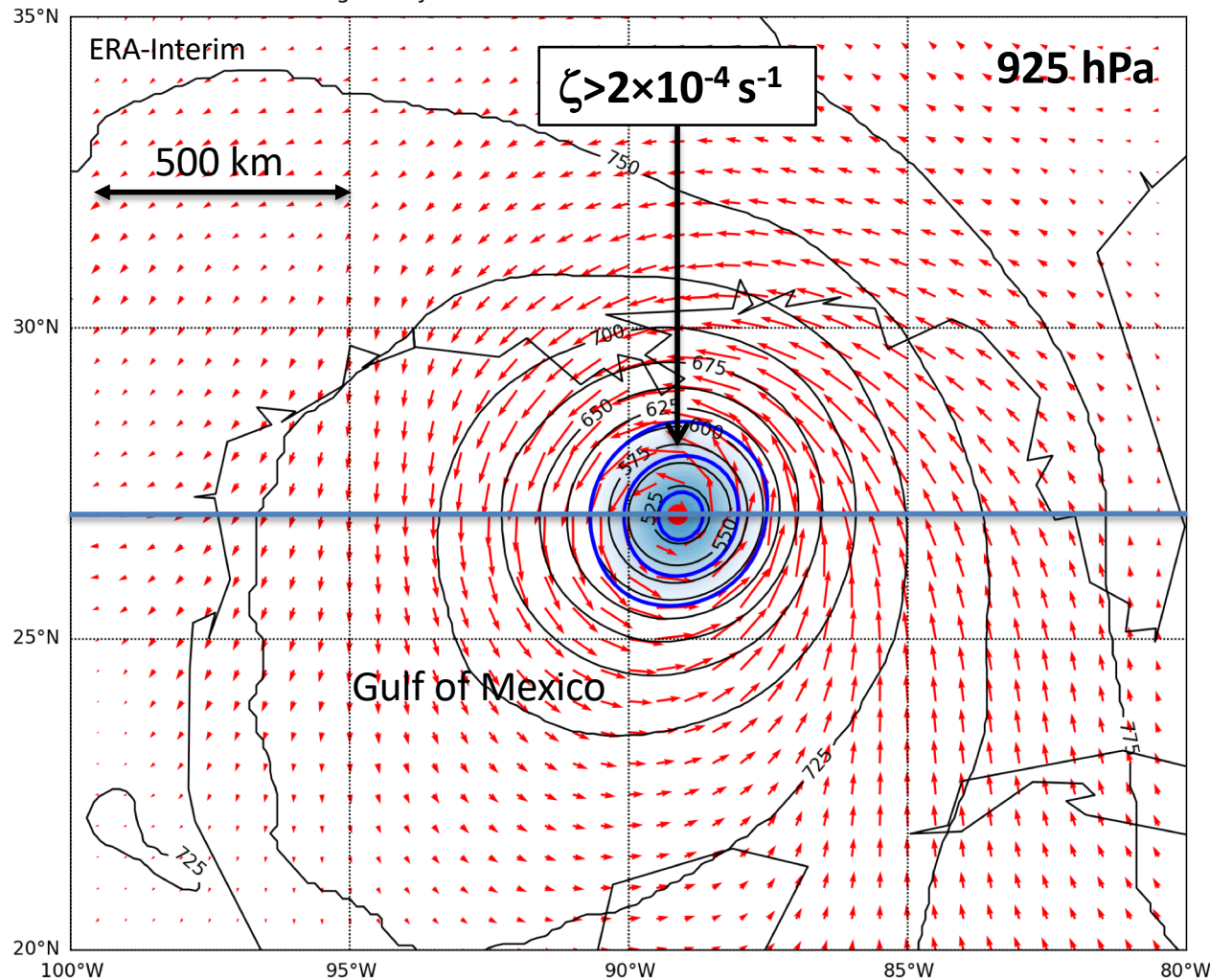
FIGURE 2 (Box 1.11). Tracks of tropical cyclones over nearly 150 years. The tracks are coloured according to the **Saffir-Simpson** tropical cyclone intensity scale: category 5: windspeed at 10 m height > 135 knots; category 4: windspeed: 114-135 knots; category 3: windspeed: 96-113 knots; category 2: windspeed: 84-95 knots; category 1: windspeed: 65-83 knots; tropical storm (TS): 34-64 knots; tropical depression (TS): <34 knots (1 knot ≈ 0.5 m/s).

Example: hurricane “Katrina” (2005)

which caused severe damage in New Orleans

925 hPa: WIND, REL.VORTICITY (blue shading $>2 \times 10^{-4} \text{ s}^{-1}$) and HEIGHT (interval: 25m)

Year: 2005 Month: Aug. Day: 28.0



Blue shading indicates area where $\zeta > 2 \times 10^{-4} \text{ s}^{-1}$.

Next slide:
cross-section along this latitude >

Figure 1.95: Hurricane Katrina (2005) at 925 hPa

→ $u = 33 \text{ m/s}$ (hurricane force)

Can we apply the QG-approximation to hurricane “Katrina”? Answer is “NO”

Vorticity, ζ , and geopotential, Φ , are related by eq. 1.253:

$$\nabla_h^2 \Phi = f_0 \zeta_g \quad (1.253)$$

(Poisson equation).
Qualitatively OK, but not always quantitatively, especially not in a tropical cyclone! (problem 1.31)

Assuming axisymmetry, eq. 1.253 in polar coordinates is:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) = f_0 \zeta_g$$

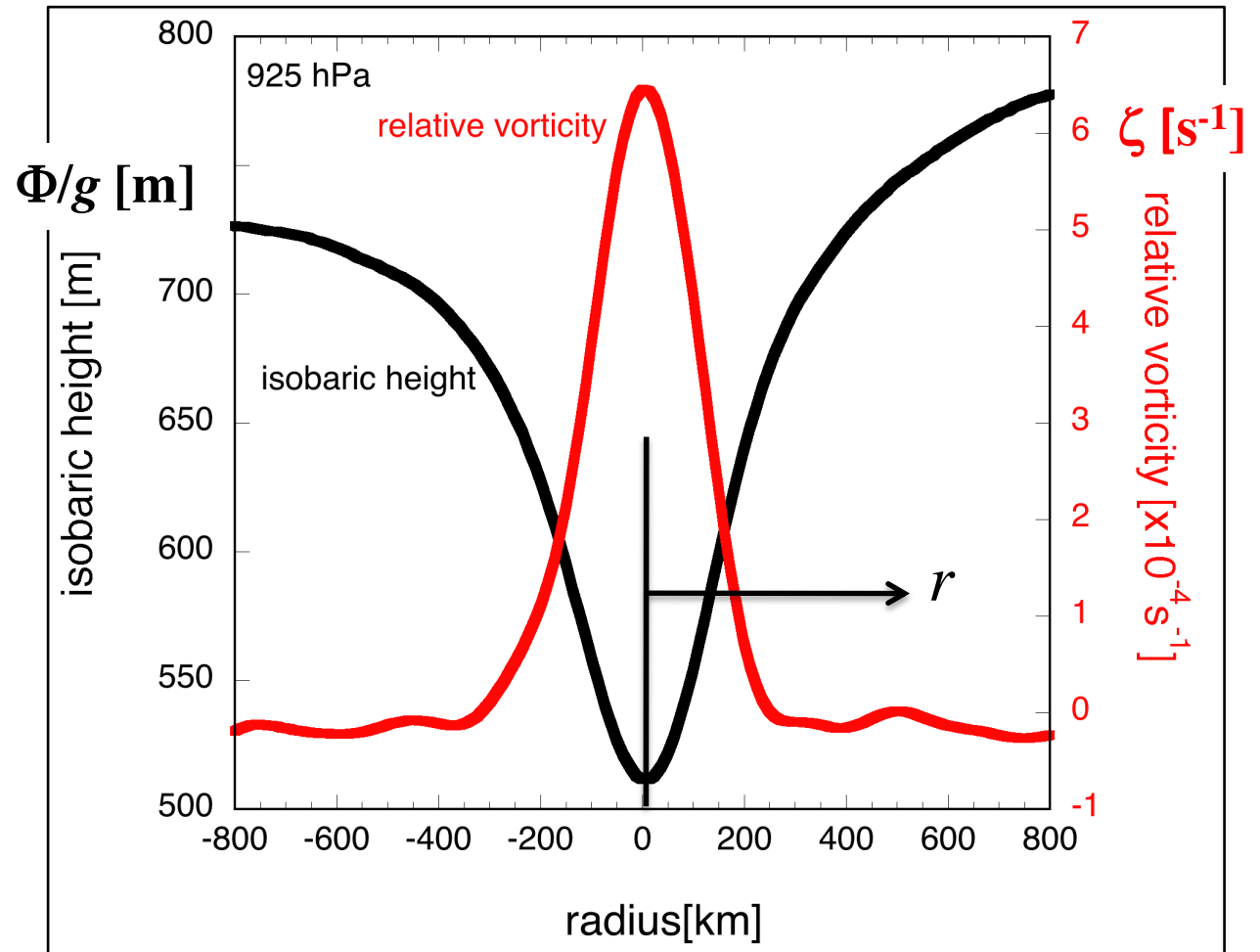


Figure 1.96: west-east cross section through centre of “Katrina”, at 925 hPa, showing relative vorticity and isobaric height (ERA-Interim).

Primary circulation (tangential wind): in gradient wind balance

(see problem 1.18)

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) = f_0 \zeta_g$$

assumes Geostrophic balance:

$$fu = \frac{\partial \Phi}{\partial r}$$

In an intense tropical cyclone:
Gradient wind balance:

$$\frac{u^2}{r} + fu - \frac{\partial \Phi}{\partial r} = 0$$

↑
centrifugal “effect”

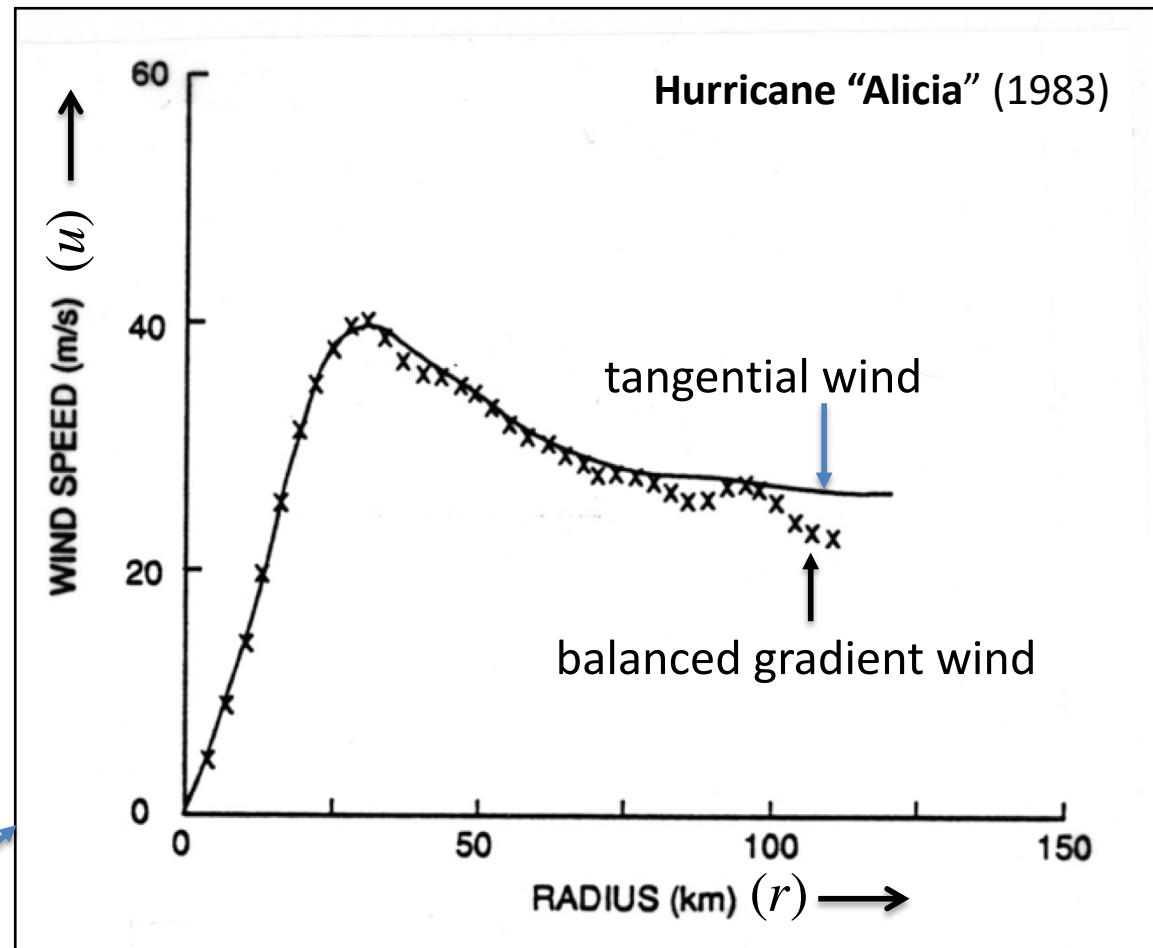


Figure 1.49. The time mean axisymmetric **tangential** wind (solid line) and the **gradient (balanced)** wind (crosses) (**explained in section 1.32**) at the 850 hPa pressure level, measured in tropical cyclone (hurricane) Alicia (28°N, 94°W) in the time interval 11:59 to 18:18 UTC, August 17, 1983.

Gradient wind balance of the *primary* circulation

Gradient wind balance in an axisymmetric vortex in ***pressure coordinates***:

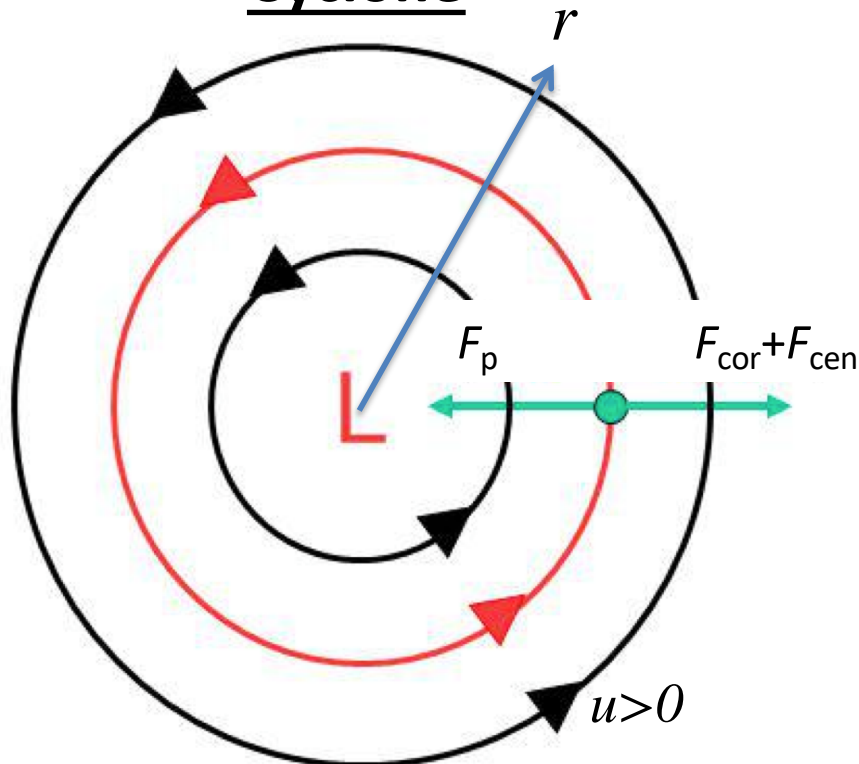
Swirling wind, u :

$$\frac{u^2}{r} + fu - \frac{\partial \Phi}{\partial r} = 0 \quad *$$

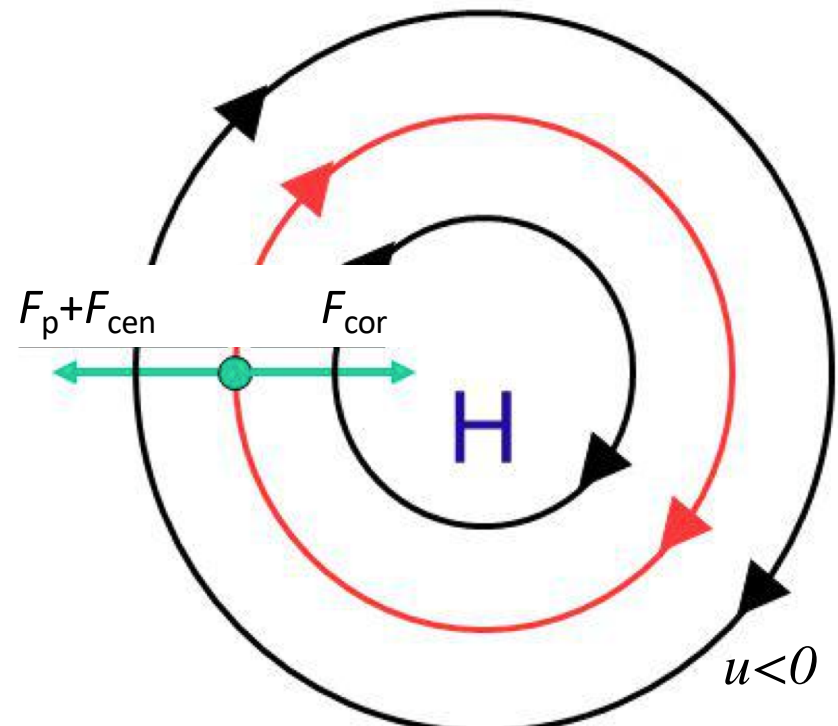
$\uparrow$ $\uparrow$ $\uparrow$
 F_{cen} F_{cor} F_p

Radial wind: $v = 0$

Cyclone



Anticyclone



* note: we have not answered the question why air parcels should flow in circular paths

Exercise on gradient wind balance

p. 224:

PROBLEM 1.29. Gradient wind balance in a circular vortex (see also problem 1.18)

Write down the solution of eq. 1.260 for an arbitrary distribution of the radial gradient of the geopotential, $\partial\Phi/\partial r = F_p$. Demonstrate the existence of four types of gradient flow. Which type will occur in reality and which type will probably not? Why? Show that the gradient wind speed in cyclonically (anticyclonically) curved flow is smaller (larger) than the corresponding geostrophic wind. Is this conclusion in agreement with **figure 1.94**?


"typo" in lecture notes

Vertical cross-section through a hurricane: the secondary circulation

The secondary circulation is needed to keep the vortex in gradient wind balance. It also drives the vortex-intensity changes, due to latent release in the updraught, forced by frictional convergence in the boundary layer.

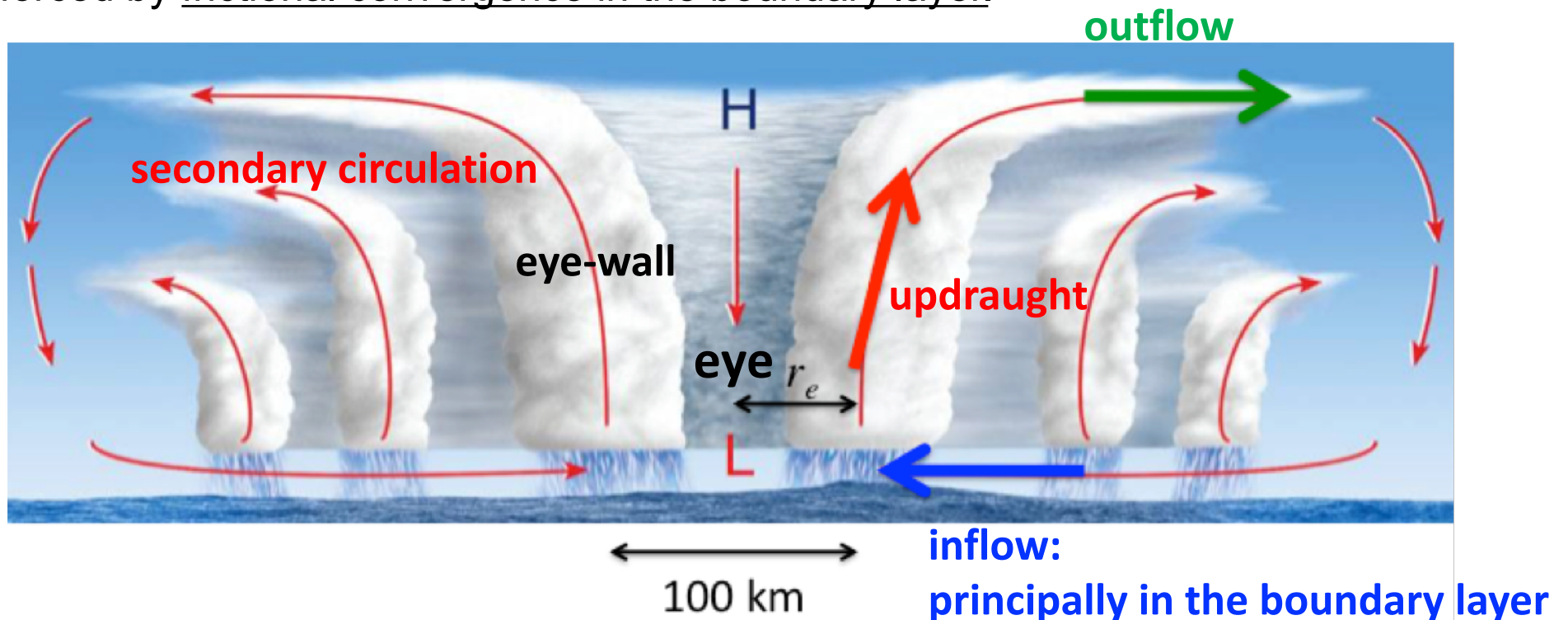


Figure 1.97

Gradient wind balance in isentropic coordinates

Gradient wind balance in an axisymmetric vortex in *isentropic coordinates*:

Swirling wind, u :

$$\frac{u^2}{r} + fu + \left(\frac{\partial \Psi}{\partial r} \right)_\theta = 0$$

Radial wind: $v = 0$

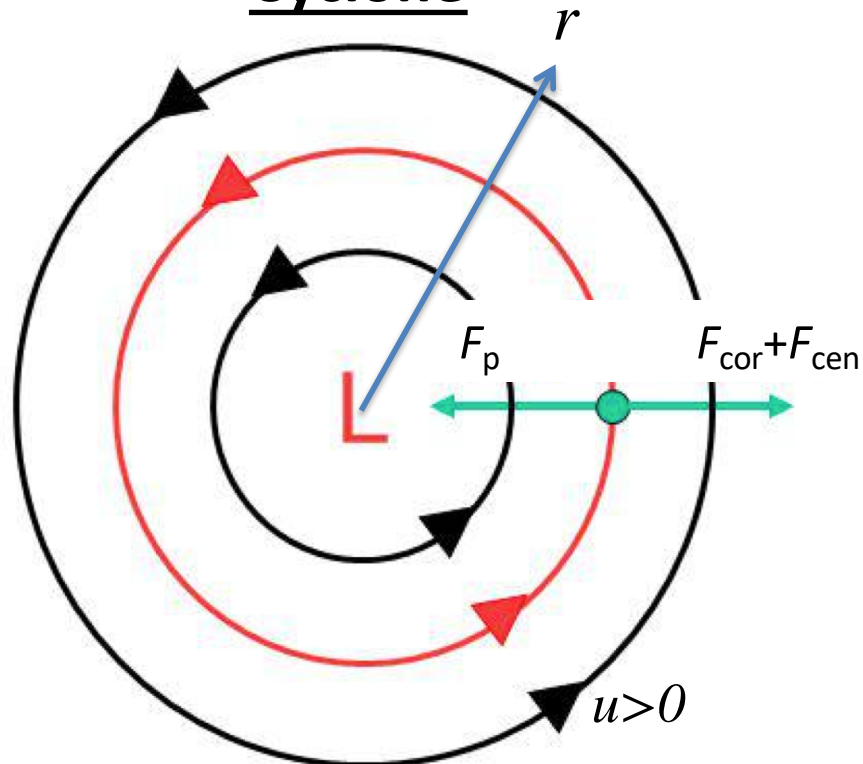
Isentropic streamfunction: $\Psi \equiv c_p T + gz$

F_{cen}

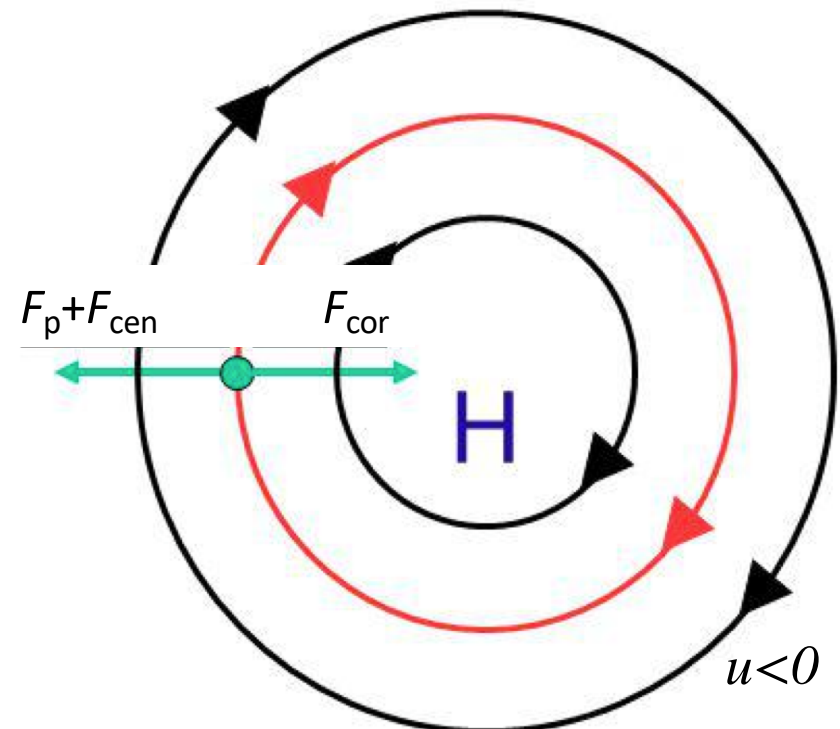
F_{cor}

F_p

Cyclone



Anticyclone



Vortex in thermal wind balance: potential vorticity inversion equation

The potential vorticity inversion equation is nothing more than an expression of thermal wind balance in terms of potential vorticity. Potential vorticity was introduced in isentropic coordinates. Therefore, we continue in **isentropic coordinates**:

Gradient wind balance:
(eq. 1.266)

$$u^2 + fru - r \frac{\partial \Psi}{\partial r} = 0$$

Hydrostatic balance:
(eq. 1.189 & 1.267)

$$\frac{\partial \Psi}{\partial \theta} = \Pi$$

}

eliminate Ψ
(isentropic stream function)



Thermal wind balance:

$$\frac{\partial \Pi}{\partial r} = \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta}$$

Vortex in thermal wind balance: potential vorticity inversion equation

Thermal wind balance, axisymmetric vortex:

$$\frac{\partial \Pi}{\partial r} = \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta}$$

Eliminate Exner function in favour of pressure:

$$\frac{\partial \Pi}{\partial r} = \frac{\partial}{\partial r} \left(c_p \left(\frac{p}{p_{ref}} \right)^\kappa \right) = \frac{c_p \kappa}{p} \left(\frac{p}{p_{ref}} \right)^\kappa \frac{\partial p}{\partial r} \stackrel{p = \rho R T}{=} \frac{1}{\rho \theta} \frac{\partial p}{\partial r}$$

$\theta = T \left(\frac{p_{ref}}{p} \right)^\kappa$

Thermal wind balance:

$$\frac{\partial p}{\partial r} = \rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta}$$

Isentropic density: $\sigma = -\frac{1}{g} \frac{\partial p}{\partial \theta}$

Insert thermal wind balance

$$\frac{\partial \sigma}{\partial r} = -\frac{1}{g} \frac{\partial}{\partial r} \frac{\partial p}{\partial \theta} = -\frac{1}{g} \frac{\partial}{\partial \theta} \frac{\partial p}{\partial r} = -\frac{1}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right)$$

Vortex in thermal wind balance: potential vorticity inversion equation

Potential vorticity: $Z_\theta \equiv \frac{\zeta + f}{\sigma}$

Omit the subscript, θ

$$\frac{\partial Z}{\partial r} \equiv \frac{\partial}{\partial r} \left(\frac{\zeta + f}{\sigma} \right) = \frac{1}{\sigma} \frac{\partial \zeta}{\partial r} - \frac{(\zeta + f)}{\sigma^2} \frac{\partial \sigma}{\partial r} = \frac{1}{\sigma} \frac{\partial \zeta}{\partial r} - \frac{Z}{\sigma} \frac{\partial \sigma}{\partial r}$$

$\uparrow$
 $f=\text{constant}$

$$\left. \begin{array}{l} \frac{\partial Z}{\partial r} = \frac{1}{\sigma} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) - \frac{Z}{\sigma} \frac{\partial \sigma}{\partial r} \\ \zeta \equiv \frac{1}{r} \frac{\partial u r}{\partial r} \end{array} \right\}$$

Previous slide:

$$\frac{\partial \sigma}{\partial r} = -\frac{1}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right)$$

Final result:

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right) = \sigma \frac{\partial Z}{\partial r}$$

PV-inversion equation

(for a vortex on the “ f -plane”)

Potential vorticity (PV) inversion equation:

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right) = \sigma \frac{\partial Z}{\partial r}$$

(a way of expressing thermal wind balance)

This is an elliptic partial differential equation if $(f + 2u/r)Z > 0$

Earlier: $\nabla_h^2 \Phi = f_0 \zeta_g$ (also elliptic partial differential equation)
(quasi-geostrophic vorticity equation)

An elliptic partial differential equation has a unique solution, if boundary conditions are specified.

Given the distribution of potential vorticity (Z) in the atmosphere and boundary conditions, the tangential (u) wind can be determined by solving the PV-inversion equation.

The tangential wind determines the vorticity

Vorticity and potential vorticity determines isentropic density.

Isentropic determines isentropic pressure

Reference state and anomaly

$$Z \equiv Z_{ref} + Z' \equiv \frac{f_0}{\sigma_{ref}} + Z'$$

"PV-anomaly"

"Reference-PV"

$$\sigma_{ref}(\theta) = \iint_A \sigma(x, y, \theta) dA$$

(A is area of isentropic surface on domain)

Note:

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right) = \sigma \frac{\partial Z}{\partial r}$$

$$\frac{\partial Z_{ref}}{\partial r} = 0$$

Therefore, if $Z = Z_{ref}$, the r.h.s. of the PV-inversion eq. is zero.

For homogeneous boundary conditions ($u=0$) the solution of

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right) = 0$$

is $u=0$ everywhere, i.e. the reference state is identified with the state of rest.

Boundary conditions

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right) = \sigma \frac{\partial Z}{\partial r}$$

(a way of expressing thermal wind balance)

The standard formulation of the **boundary condition for the solution of an elliptic second order partial differential equation** is to specify u on the boundary (*Dirichlet boundary condition*) or to specify the normal derivative of u on the boundary (*Neumann boundary condition*). In the problem at hand, we impose a Dirichlet boundary condition at the pole, at the upper boundary at large radius. At the lower boundary, which is not a smooth curve, we impose a numerical approximation of the Neumann boundary condition by prescribing $\partial u / \partial \theta$:

$$\frac{\partial u}{\partial \theta} = \left(f + \frac{2u}{r} \right)^{-1} \frac{\partial \Pi}{\partial r} \quad \text{at the lower boundary}$$

(thermal wind balance)

$$u = 0 \text{ at } r = 0 \text{ and at large } r; \quad u = 0 \text{ at large } \theta.$$

Boundary conditions

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right) = \sigma \frac{\partial Z}{\partial r}$$

(a way of expressing thermal wind balance)

Boundary conditions:

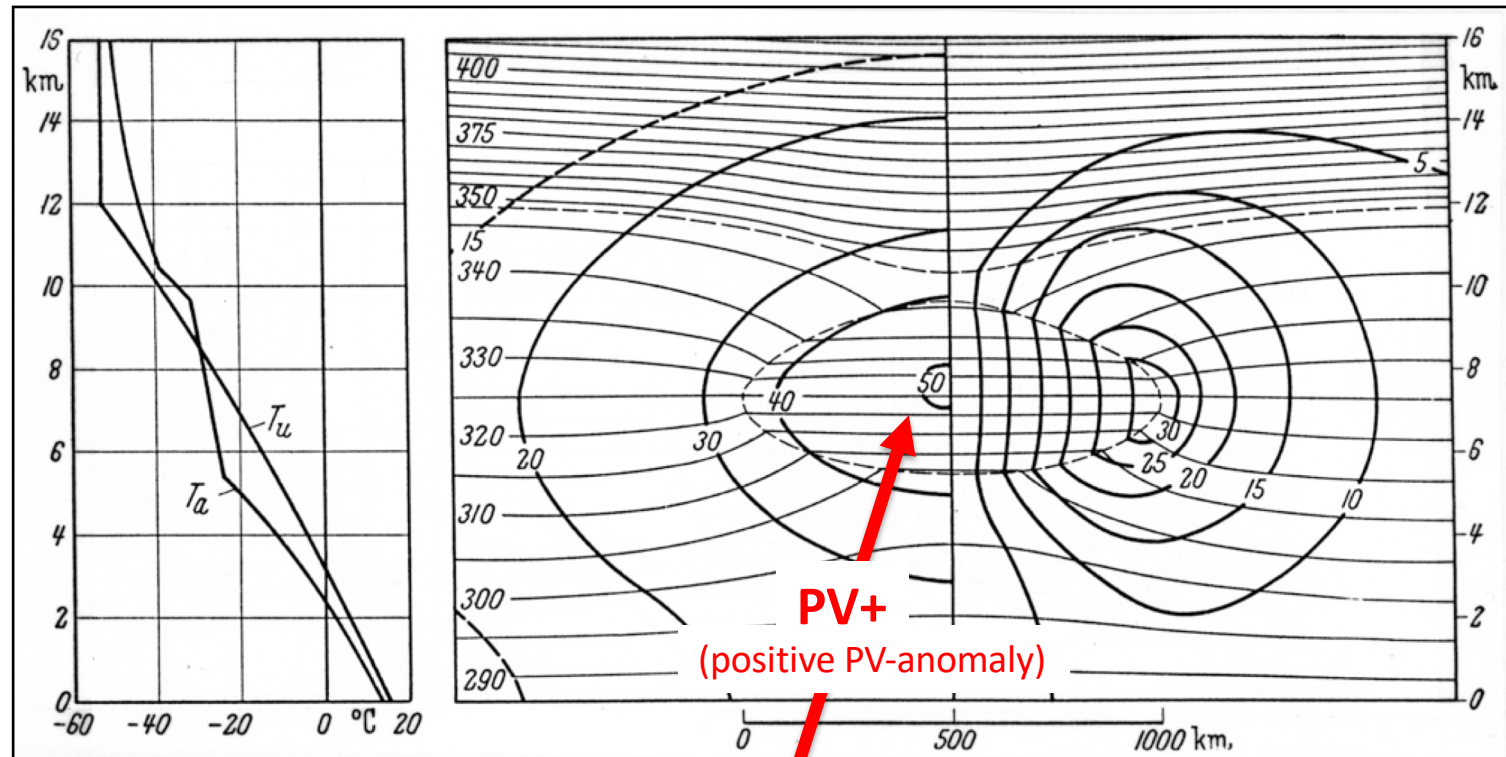
$$\frac{\partial u}{\partial \theta} = \left(f + \frac{2u}{r} \right)^{-1} \frac{\partial \Pi}{\partial r} \quad \text{at the lower boundary}$$

(thermal wind balance)

$$u = 0 \text{ at } r = 0 \text{ and at large } r; \quad u = 0 \text{ at large } \theta.$$

Because of the non-linearity of the PV-inversion eq. and because of *the complex **mixed boundary conditions***, the solution of this equation is far from a standard mathematical problem!

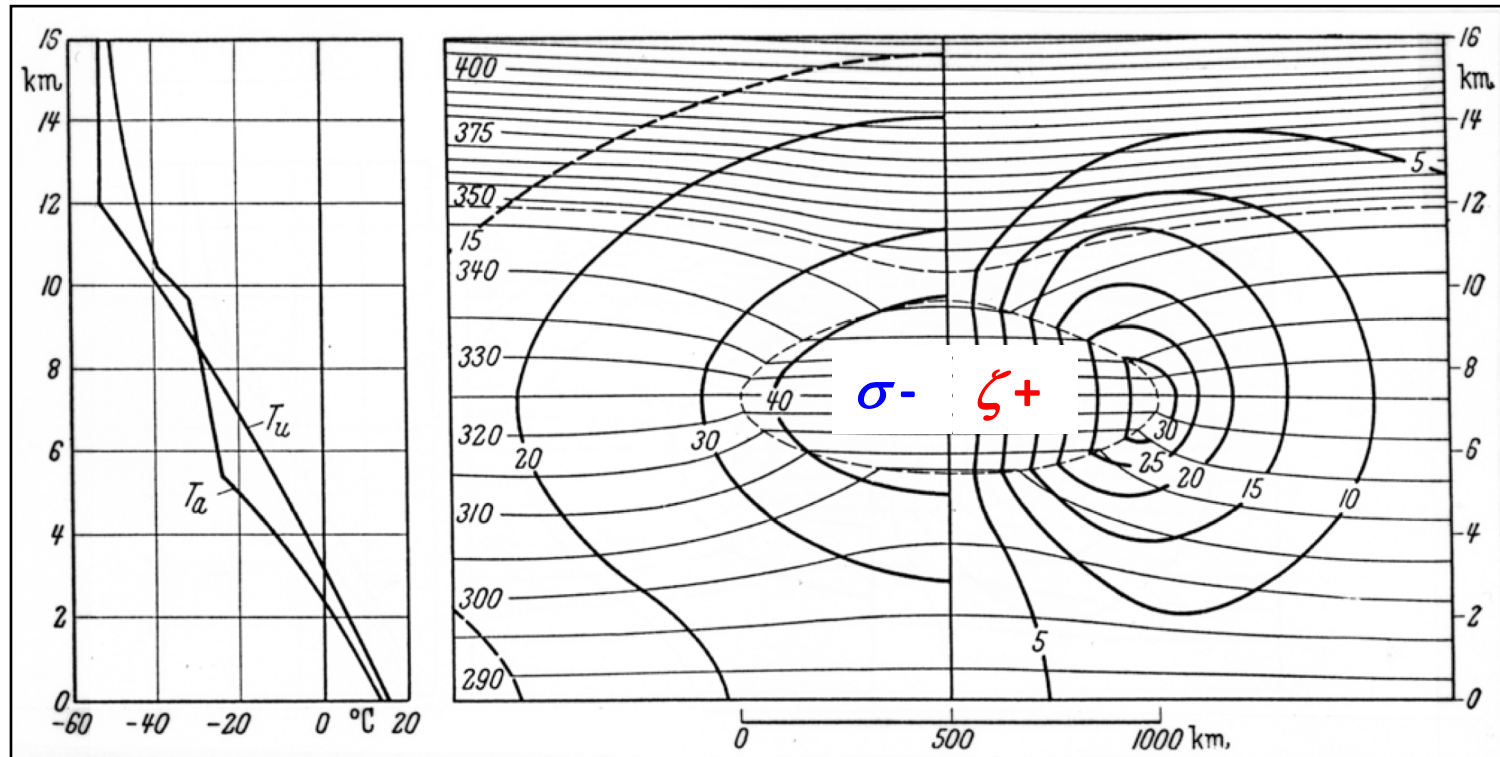
Kleinschmidt's* solution of the potential vorticity inversion equation for a positive PV-anomaly



* Reference (1957) in lecture notes, page 210, [figure 1.101](#).

A cyclone “produced” by a body of 6 times the normal potential vorticity. The undisturbed reference state consists of two layers with a constant temperature lapse rate: a troposphere with $dT/dz=5.8^\circ\text{C km}^{-1}$ and an isothermal stratosphere. The left-hand diagram shows the temperature on the axis (T_a) and in the undisturbed atmosphere (T_u) as a function height. In the cross-section (right) **thin lines are isentropes, labeled in K, heavy lines in the left half indicate the relative “depression” $(p_u-p)/p_u$ (per thousand) (p_u is the pressure in the undisturbed atmosphere). Heavy lines on the right are isopleths of the balanced tangential wind, labeled in m s^{-1} .**

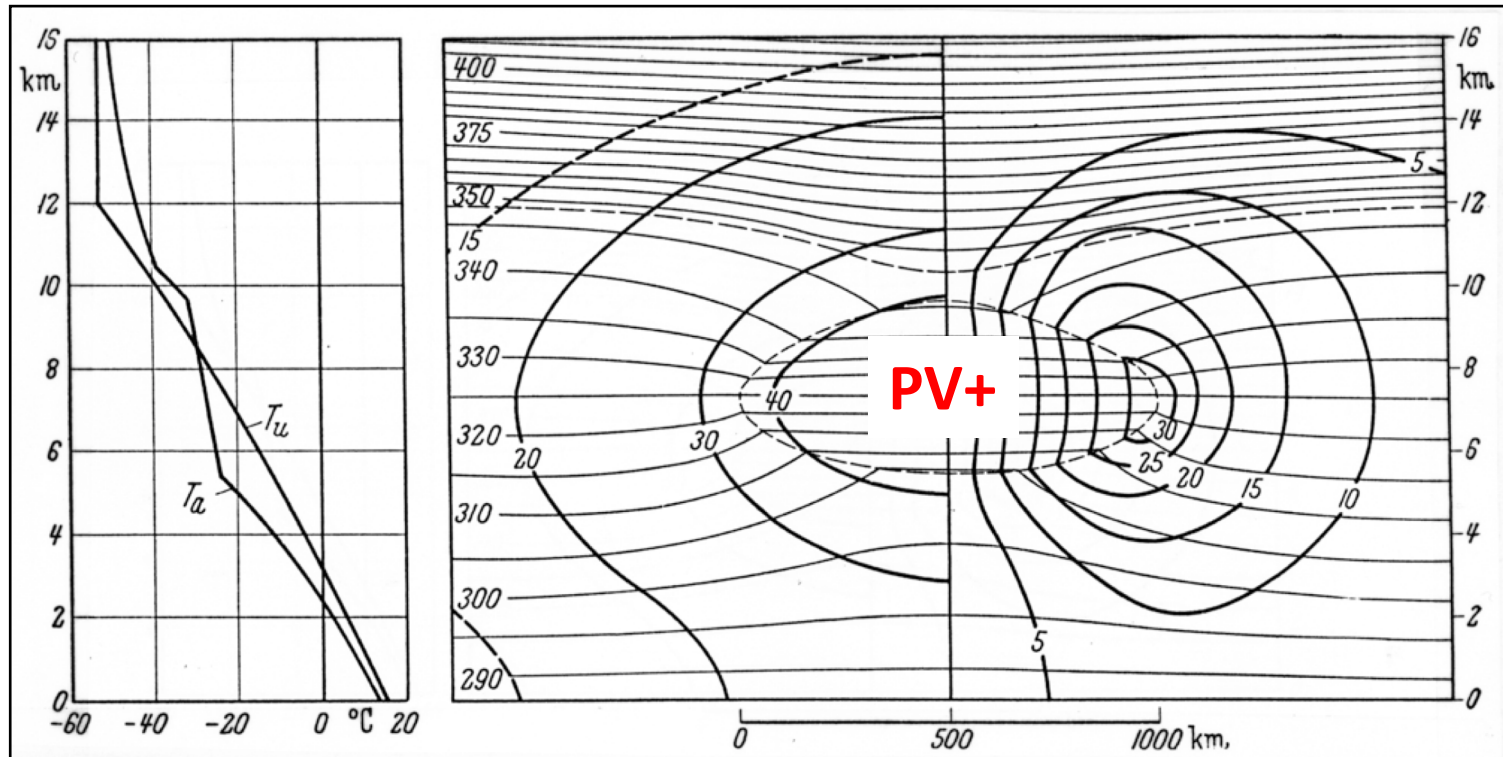
A body of 6 times the normal potential vorticity produces a cyclone



1a. Within an isolated air mass with **abnormal potential vorticity**, in thermal wind balance, the **absolute vorticity deviates from the normal in the same sense as the potential vorticity**.

1b. Within an isolated air mass with **abnormal potential vorticity**, in thermal wind balance, the **isentropic density deviates from the normal in the opposite sense as the potential vorticity**.

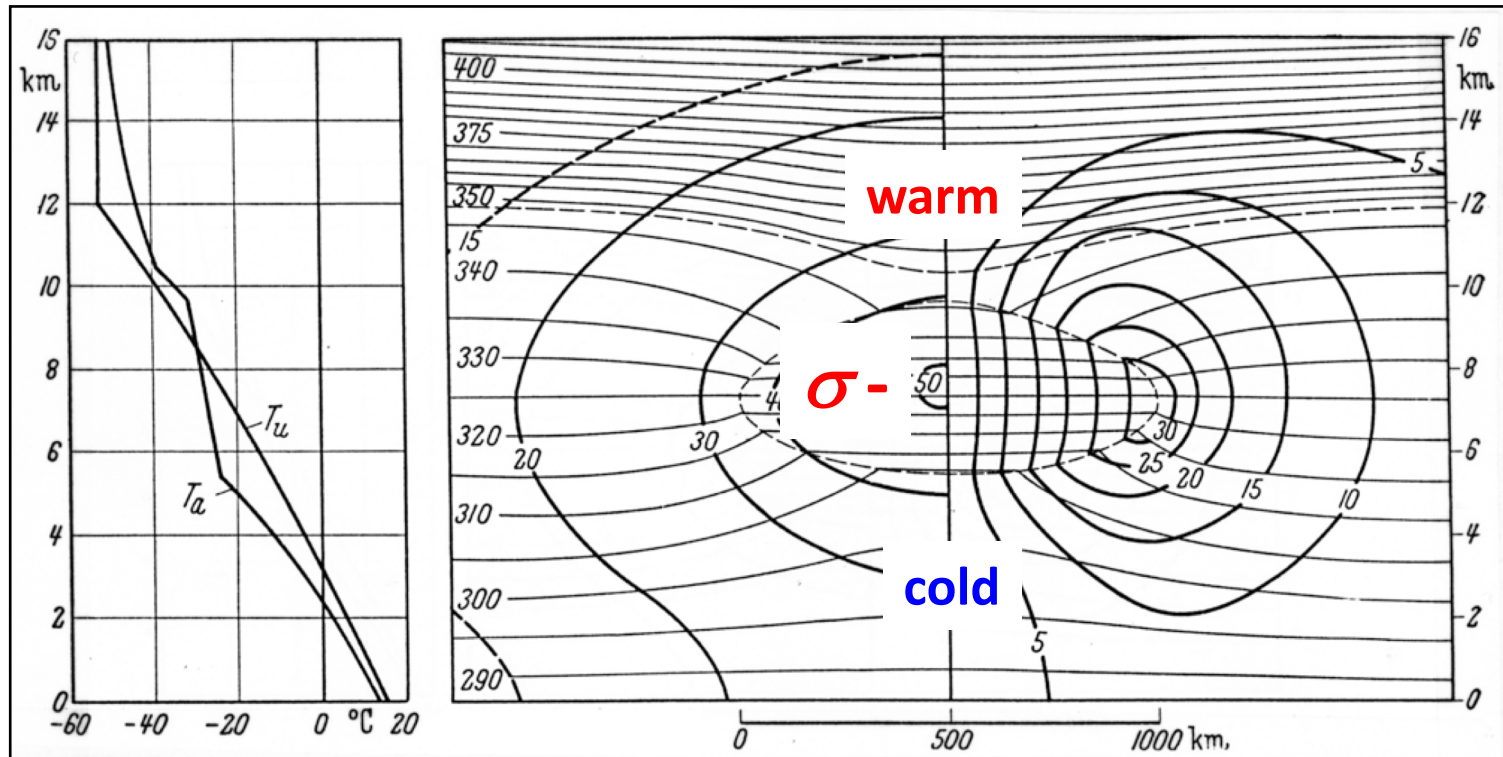
A positive potential vorticity anomaly produces a cyclone



2. An air mass of relatively **high (low) potential vorticity** gives rise to a **cyclone (an anticyclone)**.

(related to conclusion **1ab** on previous slide):

A positive potential vorticity anomaly
is associated with a **cold**(**warm**) anomaly **below**(**above**)



3. Isentropic surfaces are depressed above an air mass of relatively high potential vorticity in thermal wind balance. Isentropic surfaces are raised below an air mass of relatively high potential vorticity in thermal wind balance. Thus, an upper level cyclone has a cold core below and a warm core above, while an upper level anticyclone has a warm core below and a cold core above the potential vorticity anomaly

Character of air mass in relation to PV: “action at a distance”

Kleinschmidt's third Principle can be reversed as follows:

(3) A cold air mass, in thermal wind balance, only remains cold as long as there are masses of high potential vorticity above or masses of reduced potential vorticity below it. When this condition is no longer fulfilled, the air sinks down and loses the character of a cold air mass.

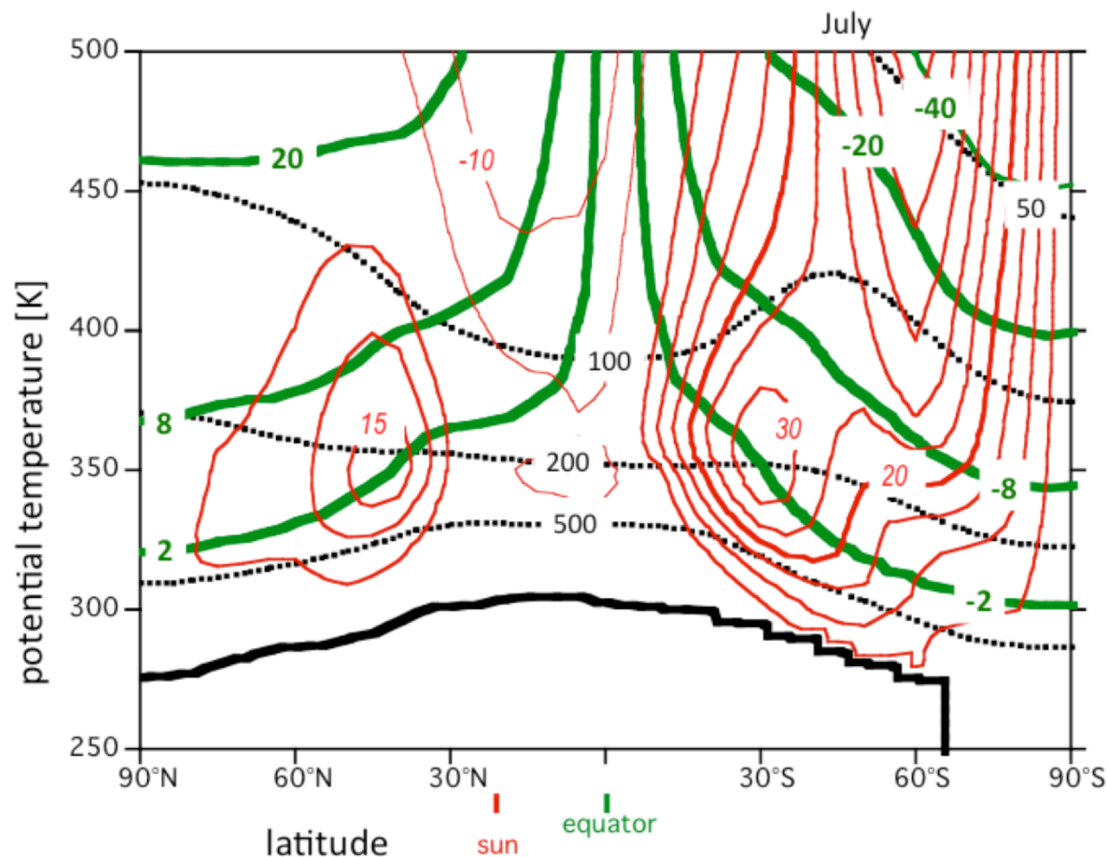
This is a manifestation of “action at a distance” !

Exercise on PV-inversion

p. 227:

PROBLEM 1.33. Consistency of polar vortex with potential vorticity inversion

Is the distribution of zonal mean pressure and zonal-mean zonal wind between $\theta=300$ K and $\theta=450$ K, in the summer in the Northern Hemisphere (lower panel of [figure 1.47](#)) qualitatively consistent with the existence, in an atmosphere in thermal wind balance, of a *positive* potential vorticity anomaly, centred at approximately $\theta=350$ K over the summer pole? Why?



red lines: zonal-mean zonal wind velocity (m/s)
dashed lines: pressure (hPa)
green lines: potential vorticity (PVU)
Black solid line: earth's surface

Numerical* solution of the PV-inversion equation

The PV-inversion equation for an balanced axisymmetric vortex:

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial u r}{\partial r} \right) + \frac{Z}{g} \frac{\partial}{\partial \theta} \left(\rho \theta \left(f + \frac{2u}{r} \right) \frac{\partial u}{\partial \theta} \right) = \sigma \frac{\partial Z}{\partial r}$$

Following Kleinschmidt we define a motionless reference state, determined by the reference isentropic density, and impose a PV-anomaly according to the formula,

$$Z(r, \theta) = Z_{ref} + Z_0 Z_{ref} \exp \left\{ - \left(\frac{r}{\Delta r} \right)^2 \right\} \exp \left\{ - \left(\frac{(\theta - \theta_0)}{\Delta \theta} \right)^2 \right\}$$

amplitude of the PV-anomaly

horizontal scale of the PV-anomaly

vertical scale of the PV-anomaly

vertical position of the PV-anomaly

Domain of the solution: $280 \text{ K} < \theta < 1250 \text{ K}$; $0 < r < 500 \text{ km}$
Upper and lower boundary condition: thermal wind=0
Side boundary conditions: $u=0$.

* Numerical method described in
Tellus A 2012, 64, 18710,
<http://dx.doi.org/10.3402/tellusa.v64i0.18710>

Numerical solution of the PV-inversion equation

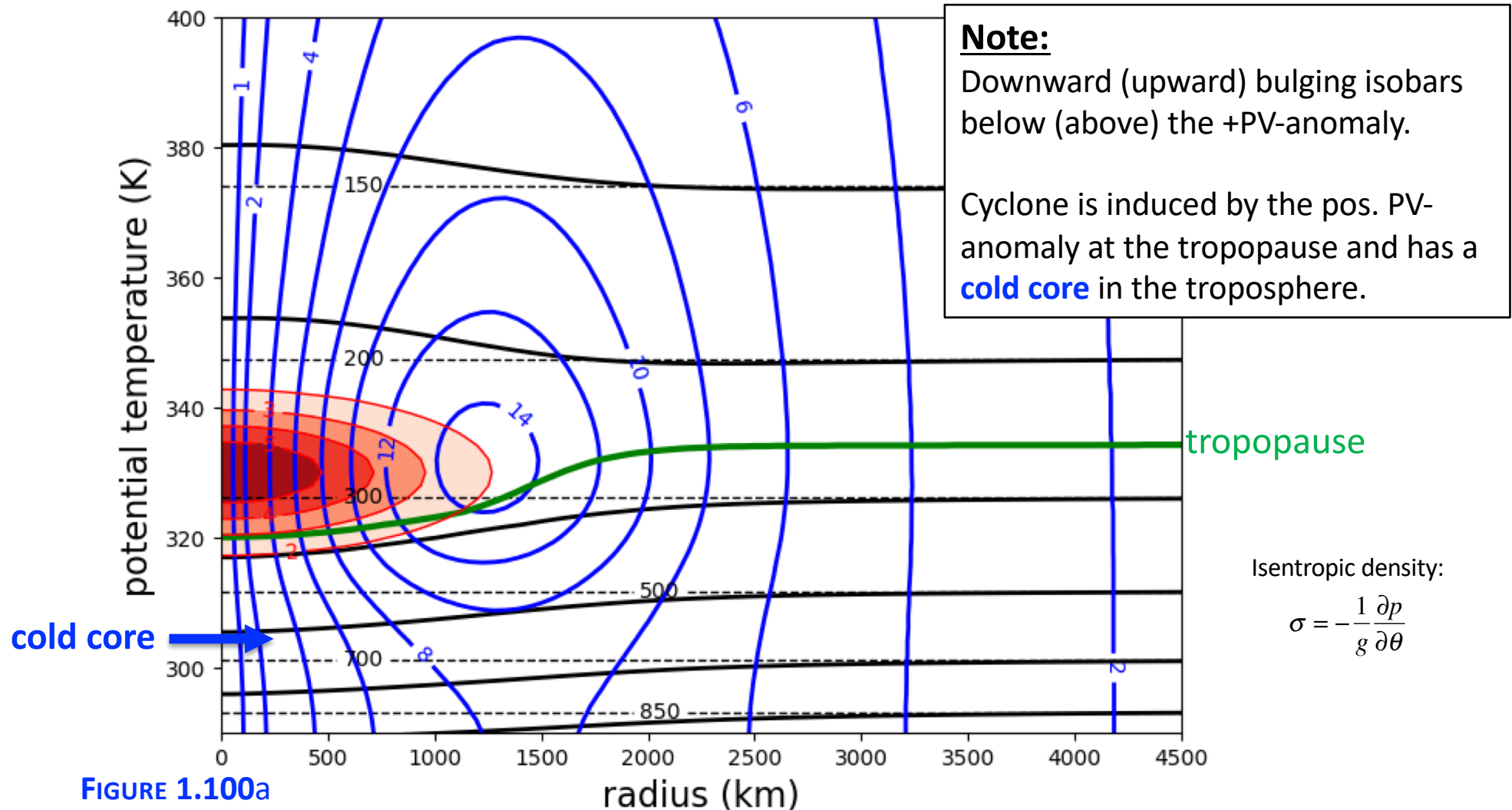


FIGURE 1.100a

The **balanced swirling wind**, u (blue contours), labeled in m s^{-1} , as a function of radius and potential temperature, associated with an axisymmetric **positive PV-anomaly** centred at $\theta_0=330$ K and $r=0$. The PV-anomaly has an amplitude, $Z_0=5$, a characteristic vertical scale, $\Delta\theta=10$ K, and a characteristic radial scale, $\Delta r=1000$ km.

Numerical solution of the PV-inversion equation

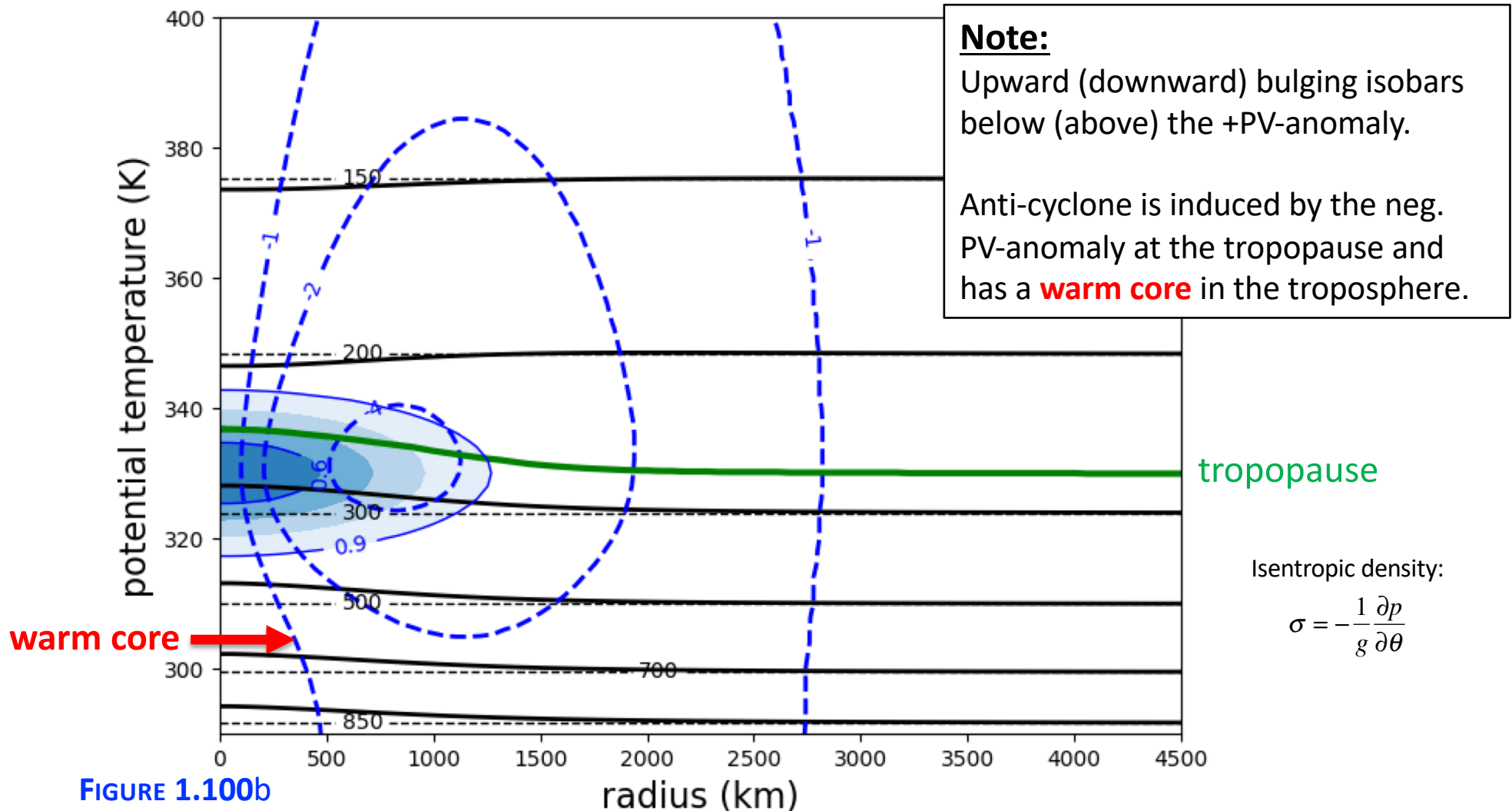
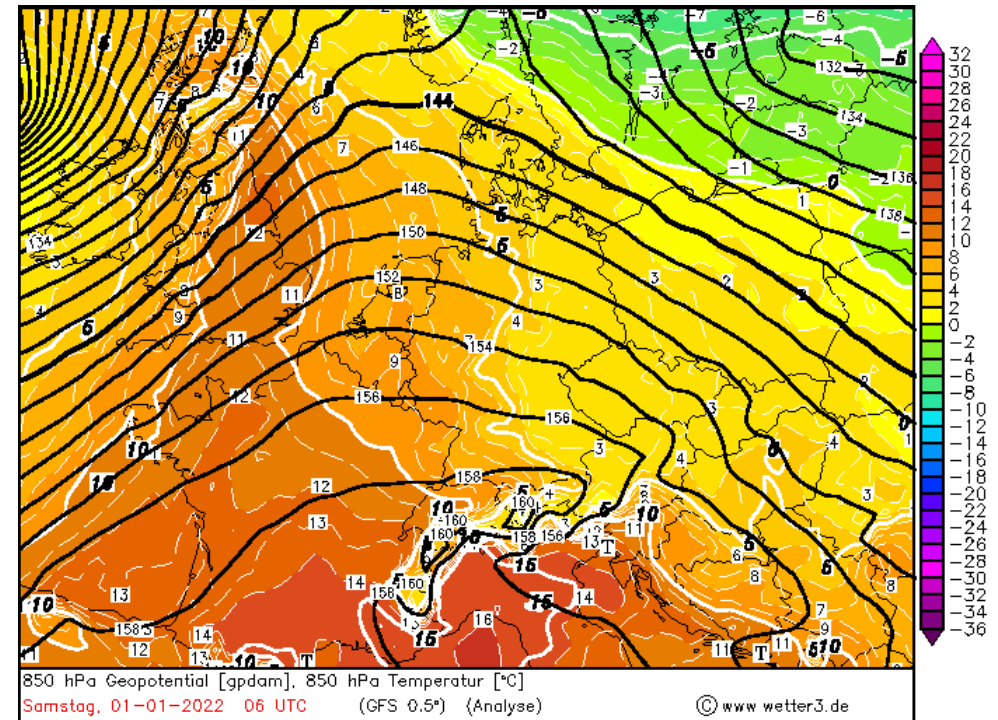
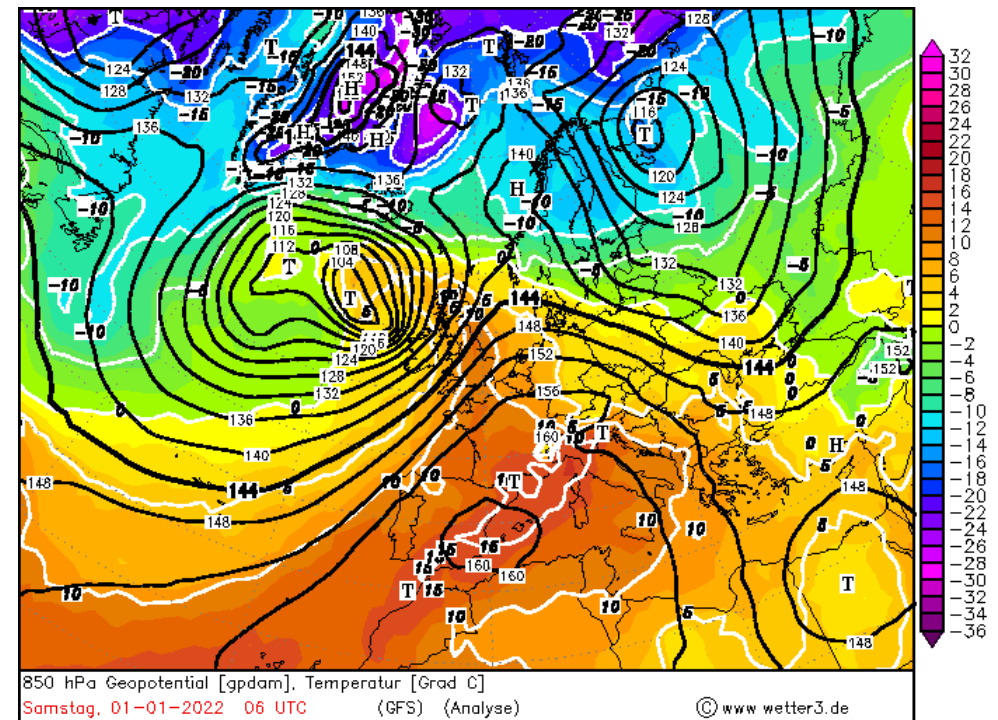
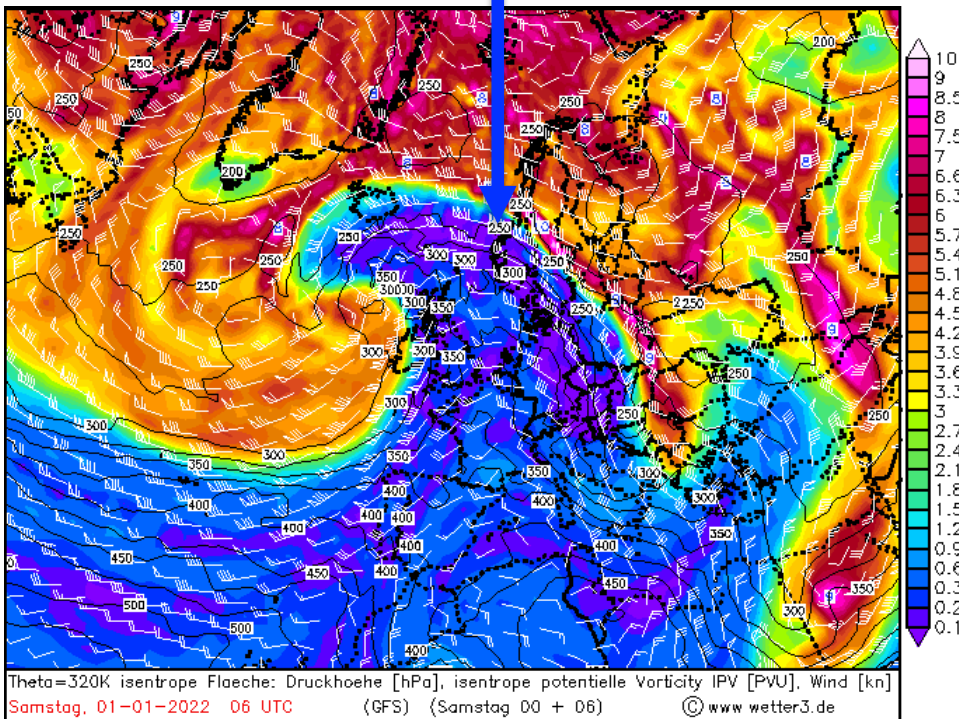


FIGURE 1.100b

The **balanced swirling wind**, u (blue contours), labeled in m s^{-1} , as a function of radius and potential temperature, associated with an axisymmetric **negative PV-anomaly** centred at $\theta_0=330 \text{ K}$ and $r=0$. The PV-anomaly has an amplitude, $Z_0=5$, a characteristic vertical scale, $\Delta\theta = 10 \text{ K}$, and a characteristic radial scale, $\Delta r=1000 \text{ km}$.

Due to presence of a mass of air with very low (tropical) PV-values

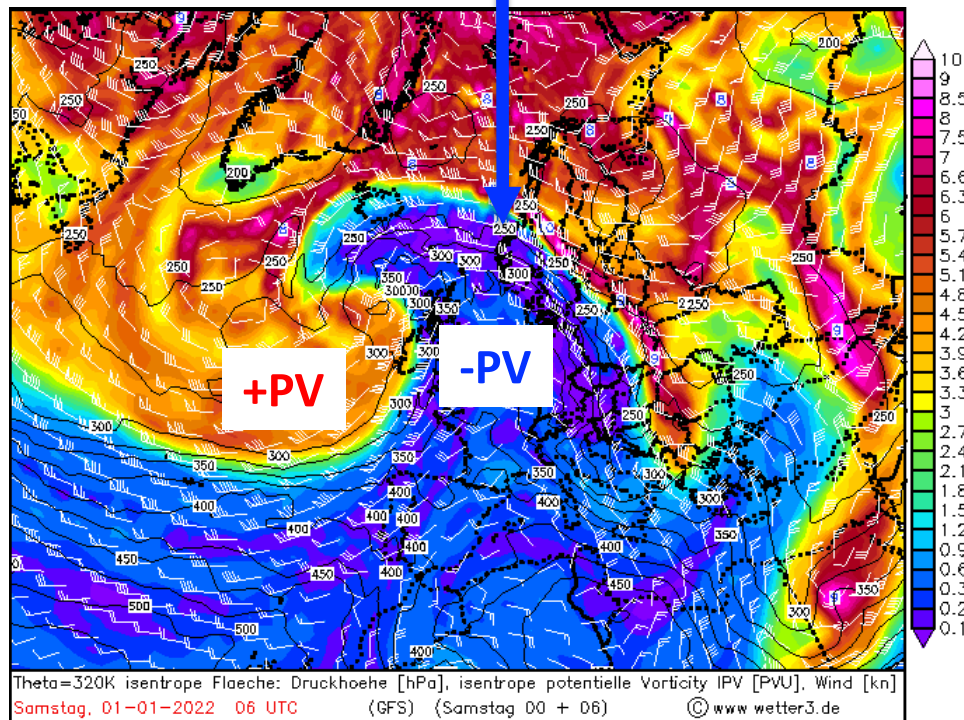
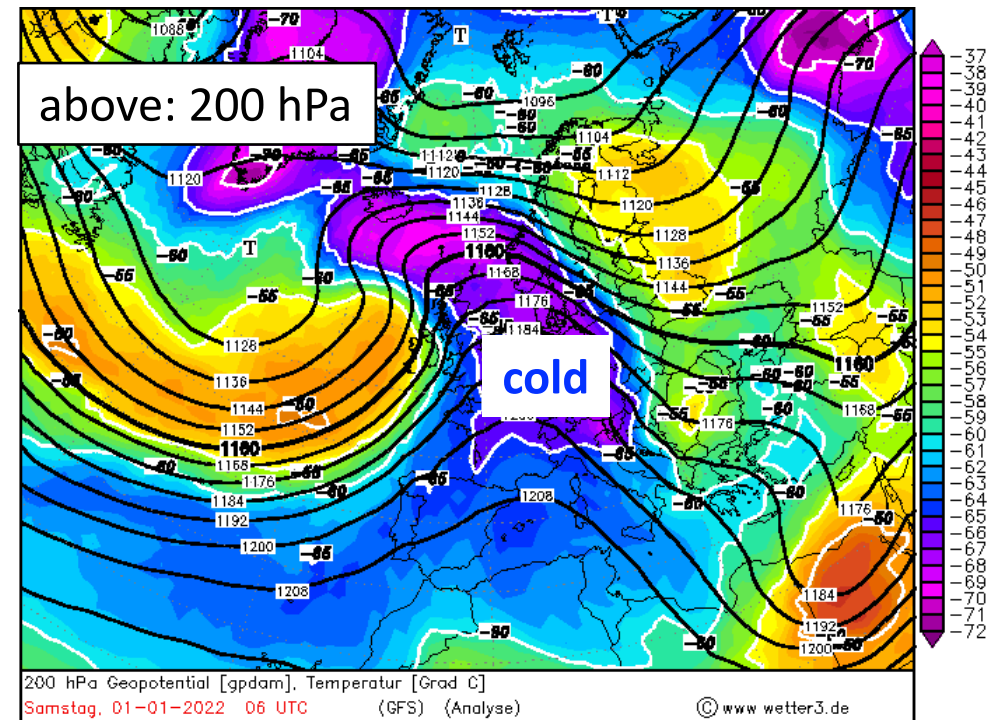
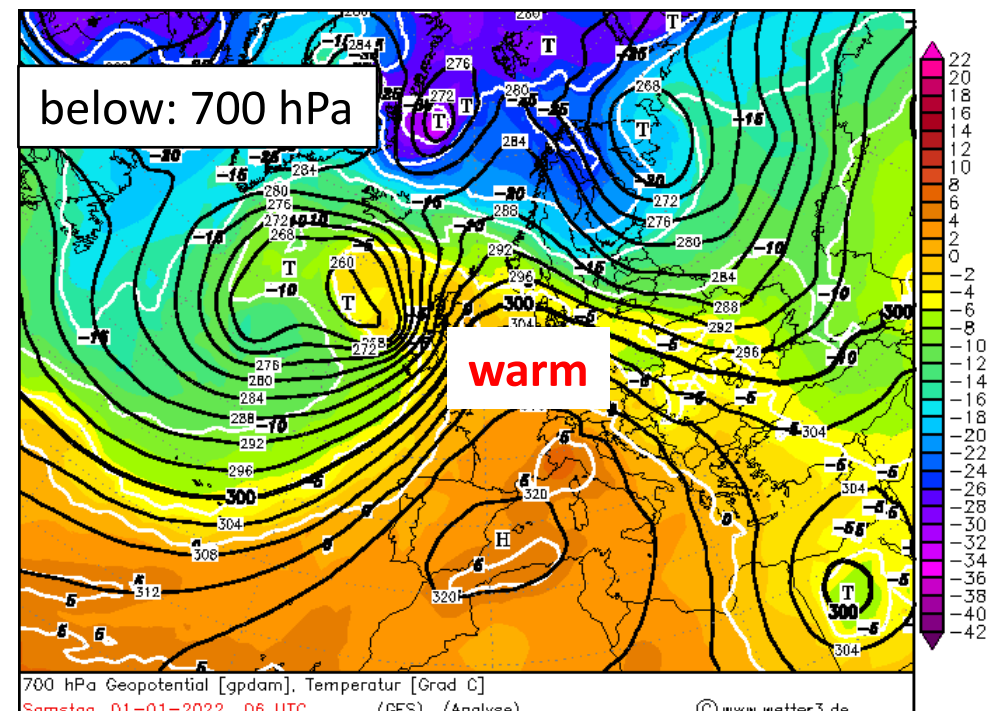
*De Bilt:
TX=13.2;
TN=10.9



Extreme warm first day of 2022

Due to presence of a mass of air with
very low (tropical) PV-values

Negative PV-anomaly at 320 K (≈ 400 hPa)
is associated with cold air mass above and warm
air mass below, consistent with PV-inversion.



Next:

Dynamic Meteorology: lecture 13

14/1/2022 (Friday, 9:00-10:45)

Sections 1.40-1.41 of the lecture notes

Different manifestations of **Rossby waves**

Potential vorticity mixing due to waves and eddies

Brewer-Dobson circulation: *poleward isentropic mass flux is due to isentropic mixing of potential vorticity and tendency to maintain zonal mean balance*

Polar vortex and Sub-Tropical Jet induced by PV-anomalies

PV-mixing and elimination of the polar vortex: **Sudden Stratospheric Warming**
(Piecewise PV-inversion of the zonal mean state)

Tutorial after this lecture (12) (online)

Project 5 (Principal Component Analysis)

Problems 1.29, 1.30, 1.33 (p. 224-227)

(...and problems 1.34 & 1.35)

(a.j.vandelden@uu.nl)

(<http://www.uu.nl/~nvdelden/dynmeteorology.htm>)

weather forecast (26 Dec. 2021) and (16 Jan. 2022)

TEAM	TX	TN	SP	RH	Points	TOTAL
Tropl Daddies	0	-	0	0	1	7
NoBadWeather	-	-	-	0	2	10
Cli(Mates)	-	0	0	+	1	8
Momentum	-	-	0	0	2	8
Weerhaan	0	-	0	+	1	9
We Trust	-	-	0	+	2	11
Weather Witch	-	0	0	+	1	8
Deep Weather	-	-	0	0	2	8
Teachers	-	0	0	+	1	8
KNMI	-	-	0	+	2	10
Weerplaza	-	0	0	+	1	11
Persistence	-	-	+	-	4	5
Climate	0	0	0	0	0	5
Observations*	-	-	+	-	4	16

range of "normal values" for 16 Jan. (De Bilt) (1992-2021) *

TX: 5.4 - 7.7 C 2022:

TN: -0.1 - 3.5 C 2022:

SP: 0.1 - 35 % 2022:

RH: 0.05 - 1.55 mm 2022:

percentage of days with no precipitation: 31

average precipitation 2.0 mm per day

* <https://www.daggegevens.knmi.nl/klimatologie/daggegevens>

range of "normal values" for 26 Dec. (De Bilt) (1991-2020) *

TX: 5.15 - 8.4 C 2021: 2.4 C

TN: -0.4 - 4.5 C 2021: -5.0 C

SP: 0.1 - 27 % 2021: 31 %

RH: 0.025 - 2.0 mm 2021: 0

percentage of days with no precipitation: 23

average precipitation 3.0 mm per day

* <https://www.daggegevens.knmi.nl/klimatologie/daggegevens>

KNMI	0	0	0	0
Weerplaza	0	0	0	0