

Dynamic Meteorology: lecture 13

14/1/2022 (Friday, 9:00)

Sections 1.36, 1.40, 1.41 of the lecture notes

Different manifestations of Rossby waves

Net poleward mass flux and **PV-mixing** due to waves and eddies

Balance of the zonal-mean zonal wind

PV-inversion of the zonal-mean state

Polar vortex and Sub-Tropical Jet induced by PV-anomalies

Maintaining zonal-mean balance requires a net poleward mass flux !

An example of PV-mixing: **Sudden Stratospheric Warming**

(Piecewise PV-inversion of the zonal mean state)

Tutorial after this lecture (13) (online)

Project 5 (Principal Component Analysis)

Problems 1.29, 1.30, 1.33 (p. 224-227) and 1.37 (piecewise PV-inversion)
(...and problems 1.34 & 1.35)

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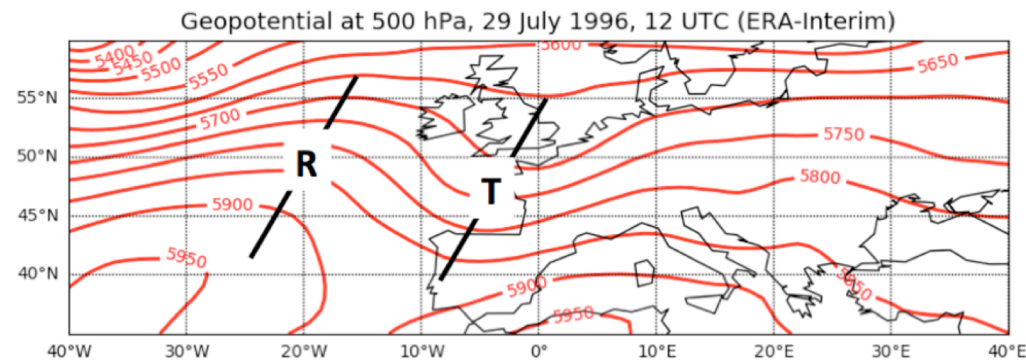
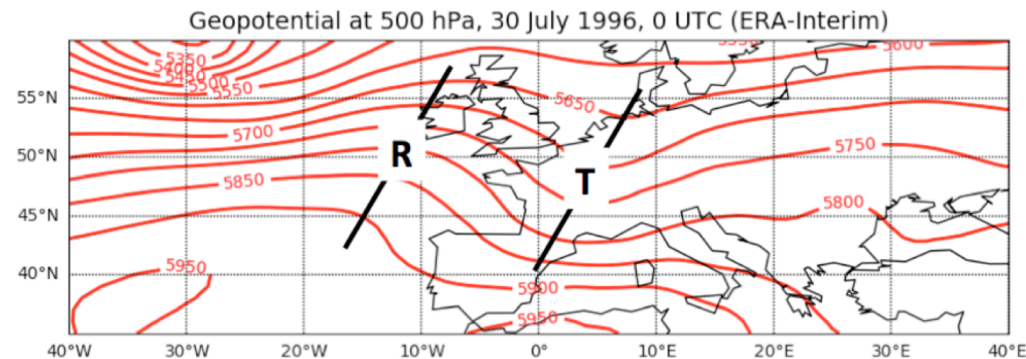
(<http://www.uu.nl/~nvdelden/dynmeteorology.htm>)

Planetary (Rossby) waves

Wave solution: $\Phi^* = \text{Re}\{A \exp[i(lx + my - \omega t)]\}$

Dispersion relation of barotropic Rossby waves:

$$c_x = \frac{\omega}{l} = U_g - \frac{\beta}{l^2 + m^2}$$



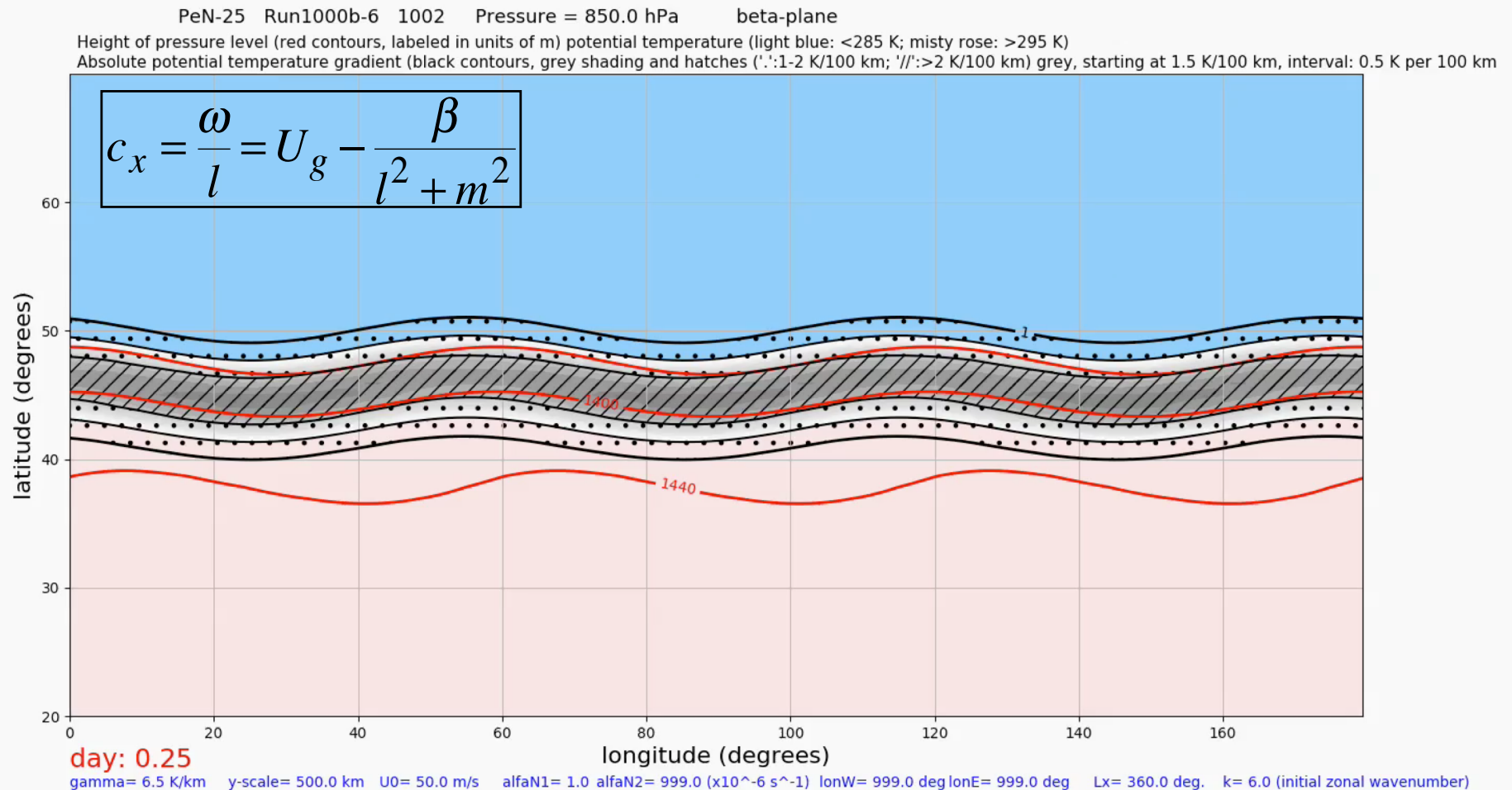
$$c_x = 16 \text{ m/s}$$

$$L_x \approx 3600 \text{ km}$$

Baroclinic instability of Rossby waves

First 6 days

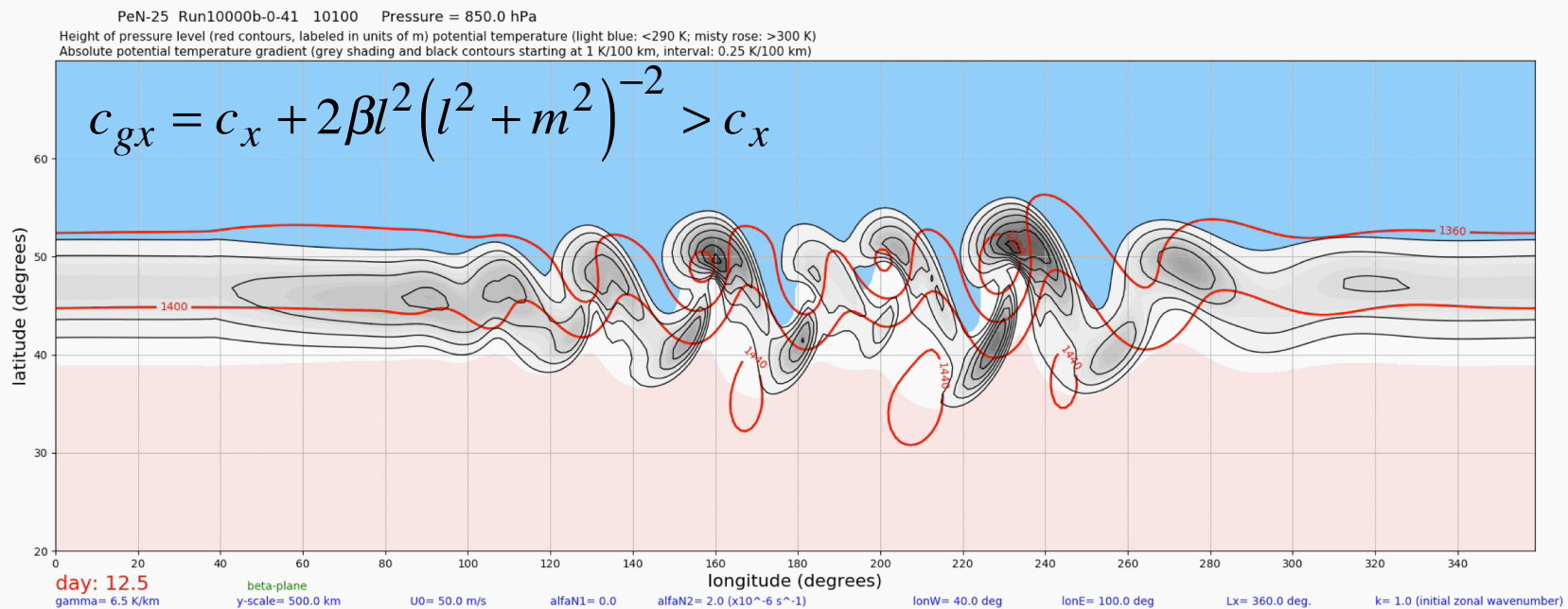
Initial state: perturbation with zonal wave-number=6



Fields at 850 hPa of temperature (light blue: <290 K; misty rose: >300 K), height (red contours, labeled in m) and absolute temperature gradient (shading and black contours starting at 1 K per 100 km, contour interval is 0.25 K per 100 km).

Rossby wave-dispersion manifest as “downstream development”

Initial state: perturbation in all zonal wave-numbers !



Group velocity is faster eastward than phase velocity, as expected from Rossby’s theory. The zonal-mean meridional temperature gradient is reduced by the eddy heat transport. Similarly, the zonal-mean meridional potential vorticity gradient is also reduced by the eddies, as we will see in a moment.

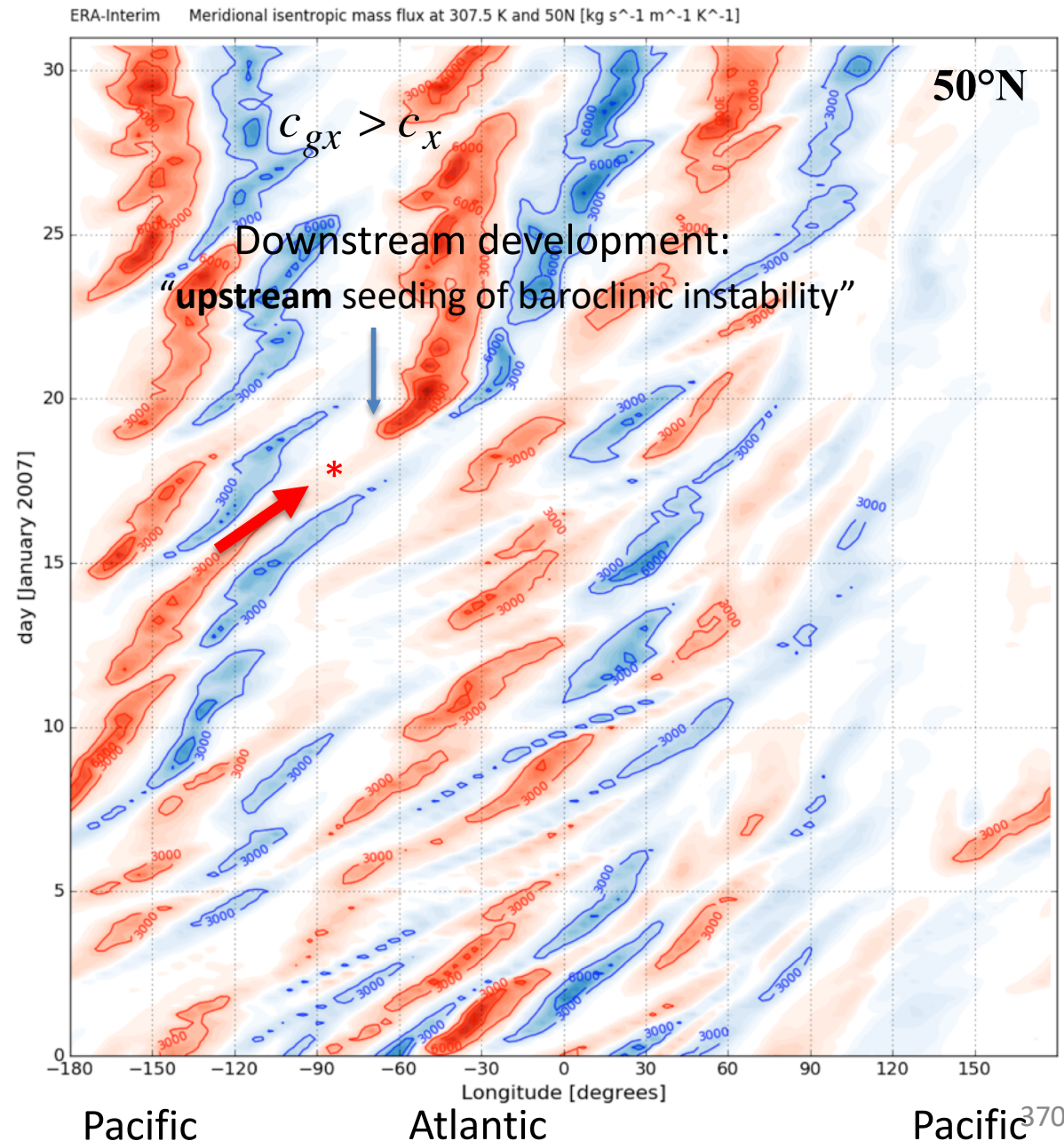
Propagating mass-flux pulses: upstream seeding

(“Hovmöller diagram”)

Rossby waves appear as eastward propagating pulses of meridional mass flux. Downstream development appears as upstream* seeding of baroclinic instability.

* seeded by disturbances entering from upstream

FIGURE 1.119. Meridional component of the isentropic mass flux at **50°N** as a function of time and longitude in the layer between $\theta=300$ K and $\theta=315$ K. **Red shading corresponds to northward mass flux, blue shading corresponds to southward mass flux.** Absolute values are labeled in units of $\text{kg s}^{-1}\text{m}^{-1}\text{K}^{-1}$.



Net meridional isentropic mass flux is poleward

This is a consequence of “meridional potential vorticity mixing”, while the zonal-mean zonal wind remains almost in **zonal-mean balance** (next slide)...

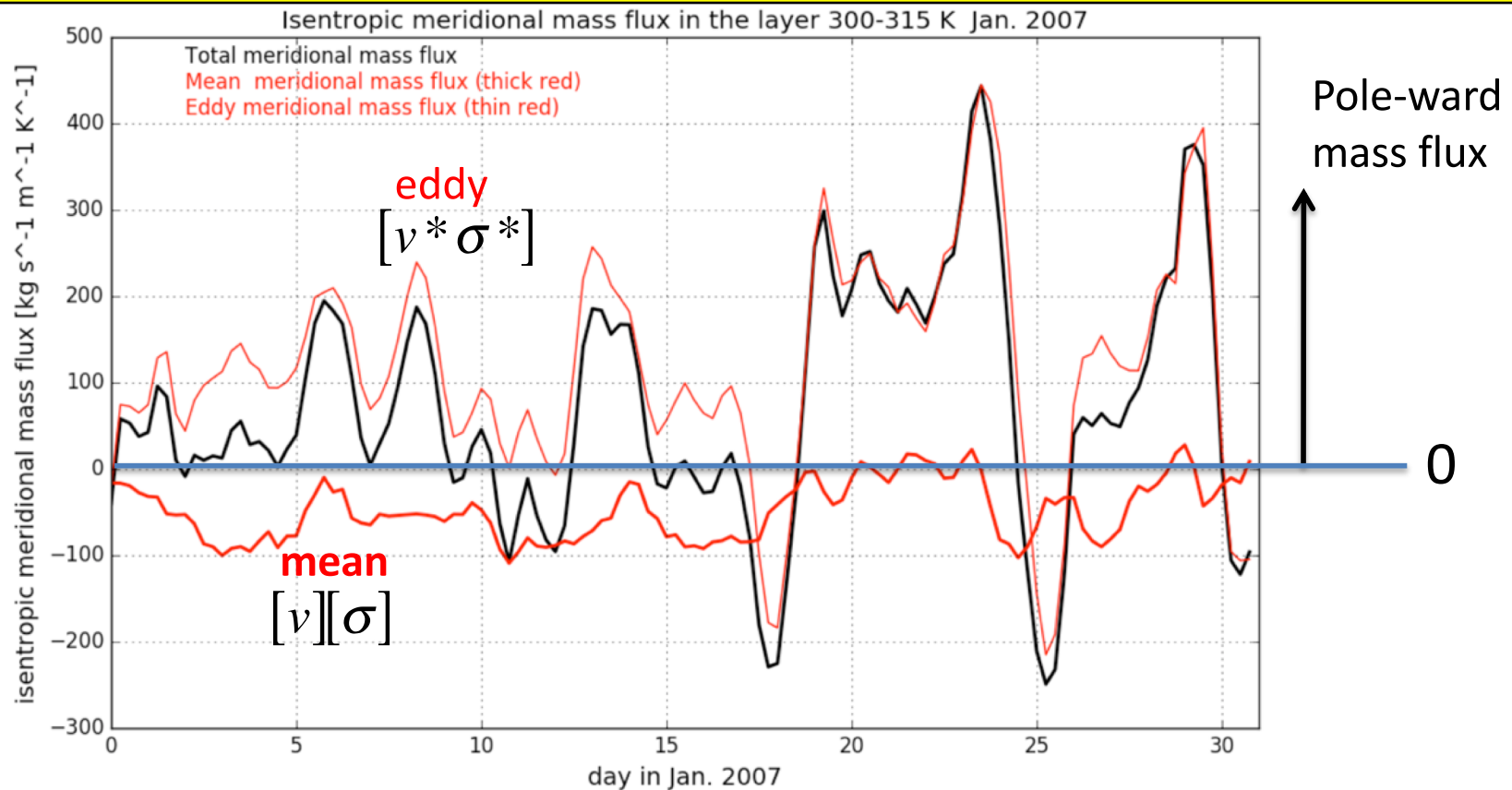


FIGURE 1.120. Meridional isentropic mass flux at 50°N in units of $\text{kg s}^{-1} \text{m}^{-1} \text{K}^{-1}$, as a function of time in the layer between $\theta=300 \text{ K}$ and $\theta=315 \text{ K}$. Positive is northward. The **black curve** represents the total mass flux; the **thick red curve** represents the meridional mass flux due to the zonal mean velocity (the “**mean mass flux**”); the **thin red curve** represents the isentropic mass flux due to eddies (the “**eddy mass flux**”).

Balance[#] of the zonal-mean zonal wind in the STJ* in January (2007 & 2010)

At 30°N and θ=350 K (see previous slide), based on 6-hourly data (124 points per month)

* STJ = subtropical jet

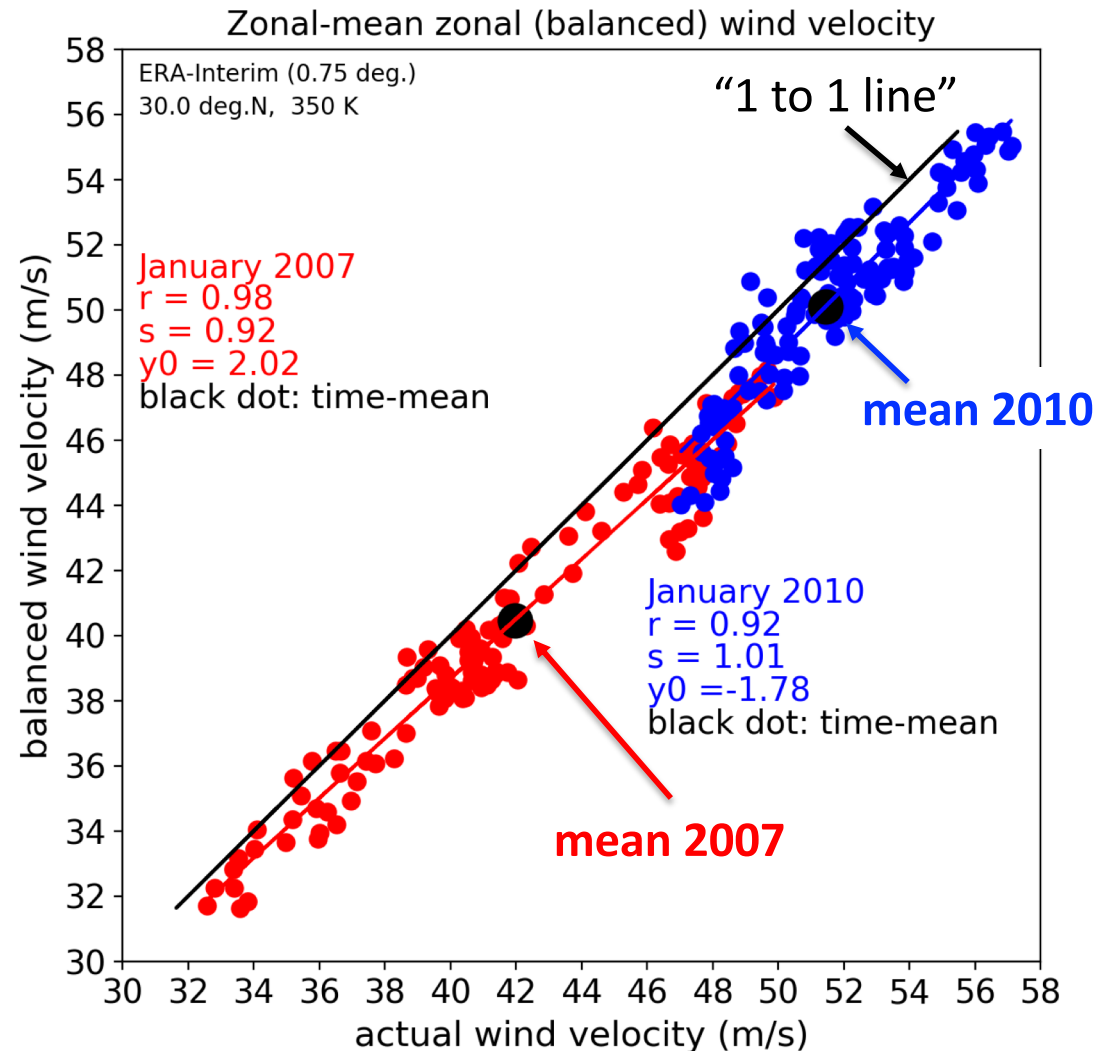
(see also lecture 11)

Zonal-mean gradient wind balance:

(eq. 1.190b) (p. 132)

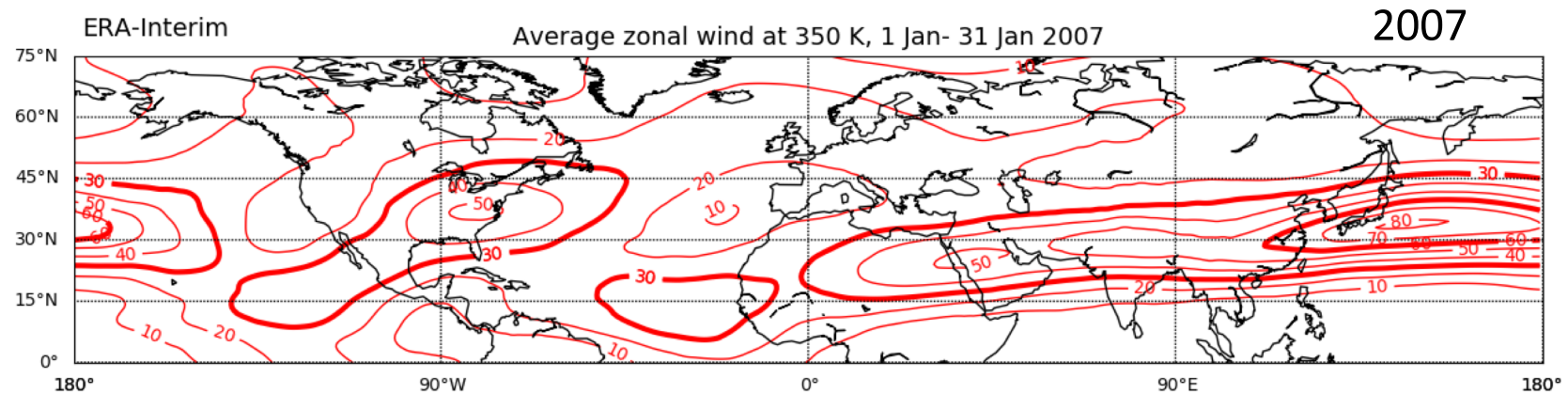
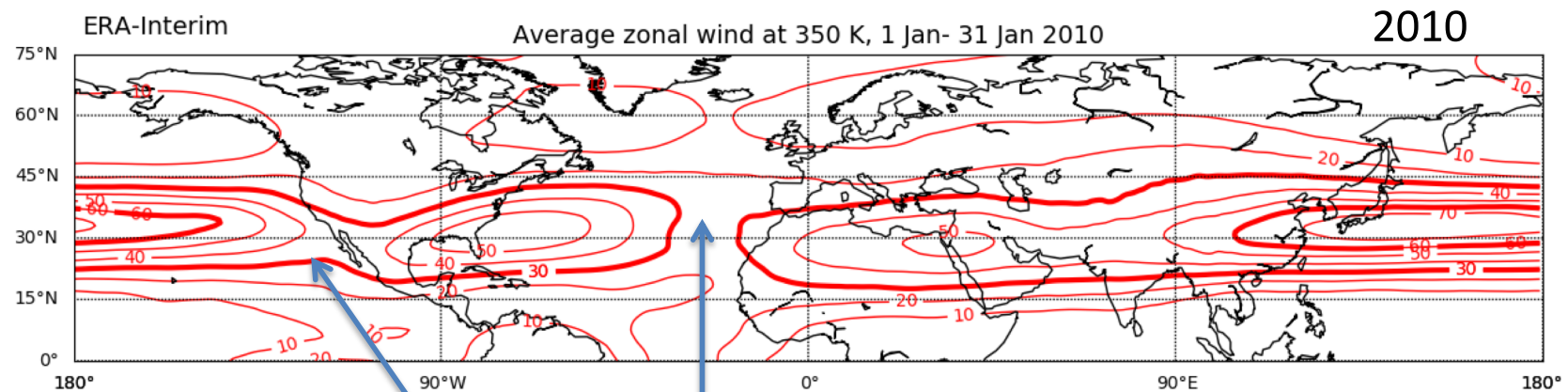
$$\frac{([u]^2 + [u \times u^*]) \tan \phi}{a} = -\frac{\partial[\Psi]}{\partial y} - f[u]$$

$$[u]^2 \gg [u^* u^*]$$



Zonal wind velocity at 350 K in January

Contour interval = 10 m/s



weaker zonal winds

This can be understood from PV-inversion

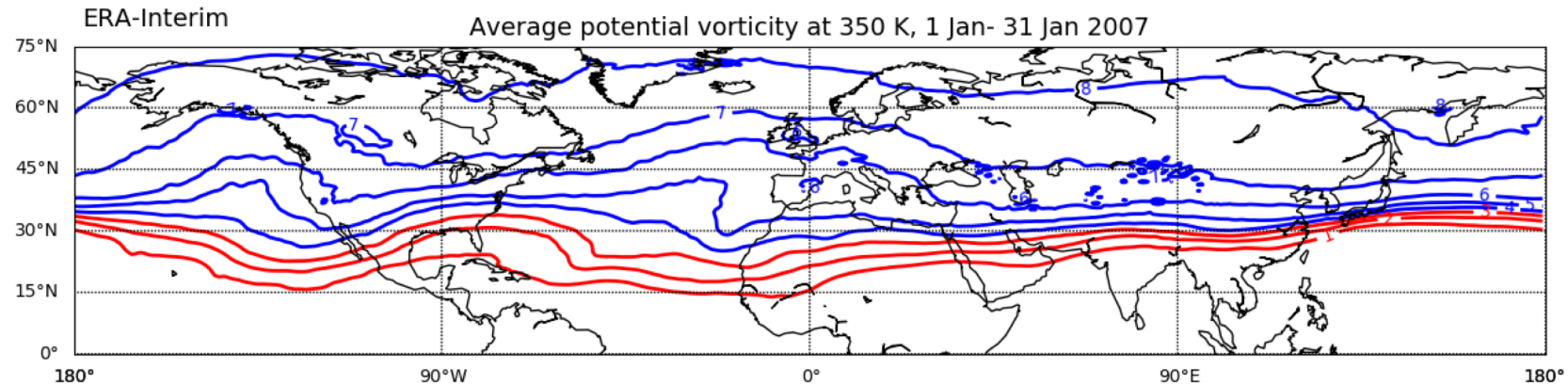
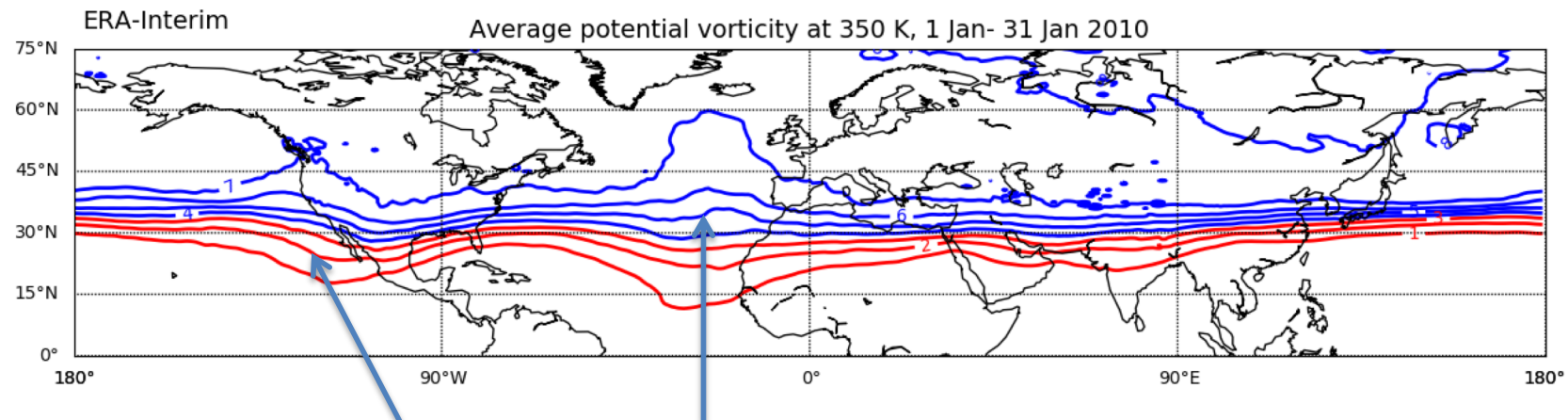
See figure 1.58

Potential vorticity at $\theta=350$ K

(monthly mean)

Blue: $PV > 4$ PVU; Red: $PV < 4$ PVU

Contour interval = 1 PVU

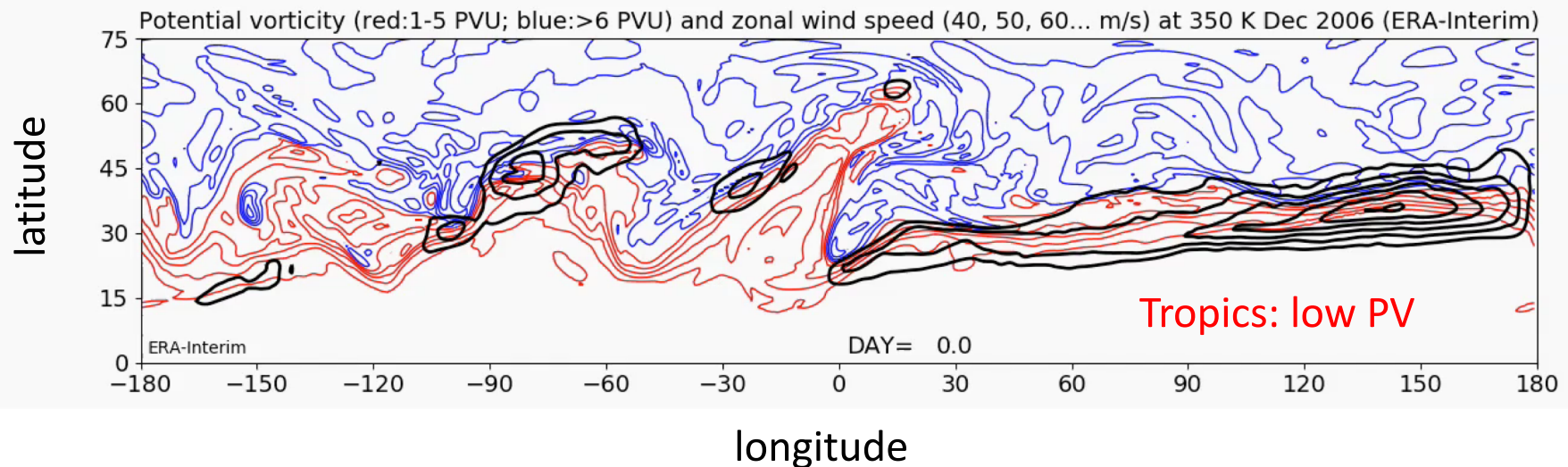


See [figure 1.58](#)

Reduced meridional gradient of PV due strong PV-mixing

Potential vorticity mixing

Preferred regions of wave-breaking: east Pacific and east Atlantic



ANIMATION OF FIGURE 1.57:

PV and zonal wind speed, u (contours starting at 30 m/s, interval 10 m/s).

Blue: $PV > 4$ PVU; Red: $PV < 4$ PVU

PV-inversion of the zonal mean state

Zonal-mean gradient wind balance: $\frac{[u]^2 \tan \phi}{a} = -\frac{\partial[\Psi]}{\partial y} - f[u]$
(eq. 1.190b) (p. 132)

Hydrostatic balance: $\frac{\partial[\Psi]}{\partial \theta} = [\Pi]$
(eq. 1.189) (p. 131)

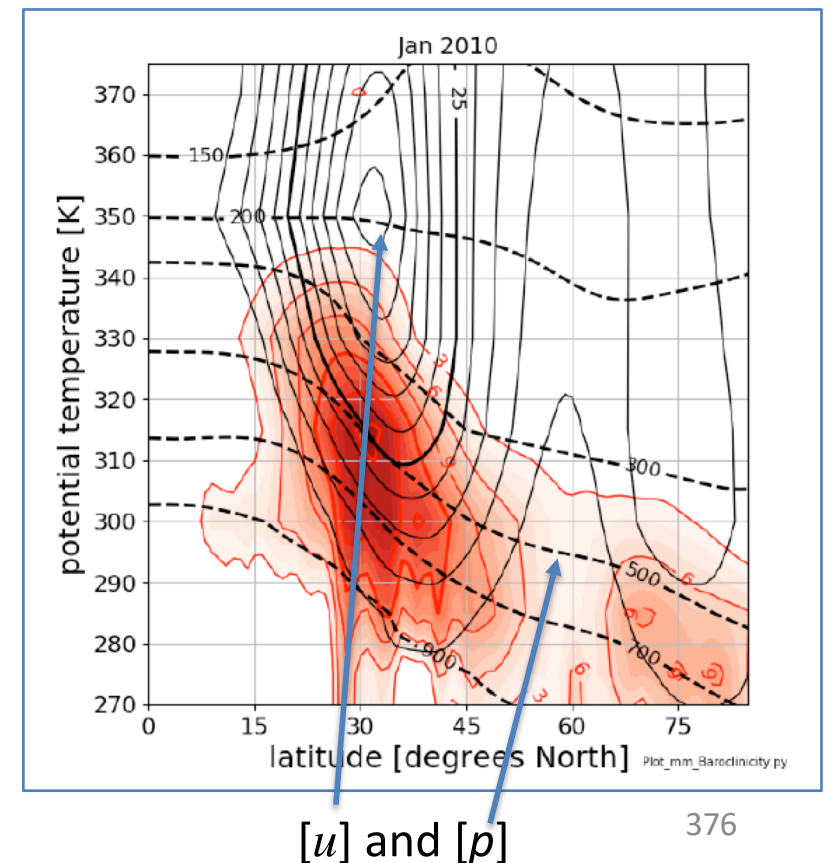
Combine:

Thermal wind balance: $f_{loc} \theta[\rho] \frac{\partial[u]}{\partial \theta} = -\frac{\partial[p]}{\partial y}$

at constant θ

$$f_{loc} = f + \frac{2[u] \tan \phi}{a}$$

Fig 1.65



PV-inversion of the zonal mean state

Thermal wind balance: $f_{loc} \theta[\rho] \frac{\partial[u]}{\partial \theta} = - \frac{\partial[p]}{\partial y}$

(previous slide)

$$f_{loc} = f + \frac{2[u] \tan \phi}{a}$$

Potential vorticity: $[Z] = \left[\frac{\zeta + f}{\sigma} \right] \approx \frac{[\zeta] + f}{[\sigma]}$

Isentropic density $[\sigma] = - \frac{1}{g} \frac{\partial[p]}{\partial \theta}$

Relative vorticity: $[\zeta] = \frac{[u] \tan \phi}{a} - \frac{\partial[u]}{\partial y}$

} substitute

$$[Z] = \left[\frac{\zeta + f}{\sigma} \right] \approx \frac{\frac{[u] \tan \phi}{a} - \frac{\partial[u]}{\partial y} + f}{[\sigma]}$$

PV-inversion of the zonal mean state

Thermal wind balance: $f_{loc} \theta[\rho] \frac{\partial[u]}{\partial \theta} = - \frac{\partial[p]}{\partial y}$

Potential vorticity: $[Z] \approx \frac{1}{[\sigma]} \left(\frac{[u] \tan \phi}{a} - \frac{\partial[u]}{\partial y} + f \right)$

Combine:

PV-inversion equation for the zonal-mean flow (thermal wind balance):

$$\frac{\partial}{\partial y} \left(\frac{\partial[u]}{\partial y} - \frac{[u] \tan \phi}{a} \right) + \frac{[Z]}{g} \frac{\partial}{\partial \theta} \left(f_{loc} \theta[\rho] \frac{\partial[u]}{\partial \theta} \right) = \frac{df}{dy} - [\sigma] \frac{\partial[Z]}{\partial y}$$

Solving (“inverting”) this equation implies: **given $[Z]$ >> determine $[u]$**

PV-inversion of the zonal mean state

PV-inversion equation for the zonal mean flow:

$$\frac{\partial}{\partial y} \left(\frac{\partial [u]}{\partial y} - \frac{[u] \tan \phi}{a} \right) + \frac{[Z]}{g} \frac{\partial}{\partial \theta} \left(f_{loc} \theta [\rho] \frac{\partial [u]}{\partial \theta} \right) = \frac{df}{dy} - [\sigma] \frac{\partial [Z]}{\partial y}$$

“Laplace of u ”

= “forcing”



$$\frac{df}{dy} - [\sigma] \frac{\partial [Z]}{\partial y}$$

“forcing” = 0 if

$$[\sigma] \frac{\partial [Z]}{\partial y} = \frac{df}{dy} \equiv \beta$$

Definition of the reference state on the sphere

Reference state: $[\sigma] \frac{\partial[Z]}{\partial y} = \frac{df}{dy} \equiv \beta$ so that forcing=0

anomaly



Define $[Z] = Z_{\text{ref}} + Z'$, such that $\beta - [\sigma_{\text{ref}}] \frac{\partial[Z_{\text{ref}}]}{\partial y} = 0$

with $\sigma_{\text{ref}}(\theta) = \frac{\int \sigma \cos \phi d\phi}{\int \cos \phi d\phi}$

Therefore: $Z_{\text{ref}} = \frac{f(y)}{\sigma_{\text{ref}}(\theta)}$

Reference state corresponds to state of rest

If $Z = Z_{\text{ref}}$ the PV-inversion equation for the zonal mean flow is:

$$\frac{\partial}{\partial y} \left(\frac{\partial [u]}{\partial y} - \frac{[u] \tan \phi}{a} \right) + \frac{[Z]}{g} \frac{\partial}{\partial \theta} \left(f_{loc} \theta [\rho] \frac{\partial [u]}{\partial \theta} \right) = 0$$

With $[u]=0$ at all boundaries the solution of the eq. above is $[u]=0$ everywhere.

Remember: $Z_{\text{ref}} = \frac{f(y)}{\sigma_{\text{ref}}(\theta)}$

The state of zonal-mean rest with respect to the earth's surface is associated with a y -dependent PV-distribution !

This fact invites the following thought-experiment:

A thought experiment:

perfect mixing of the reference state by waves and eddies
accelerates and decelerates the zonal-mean flow

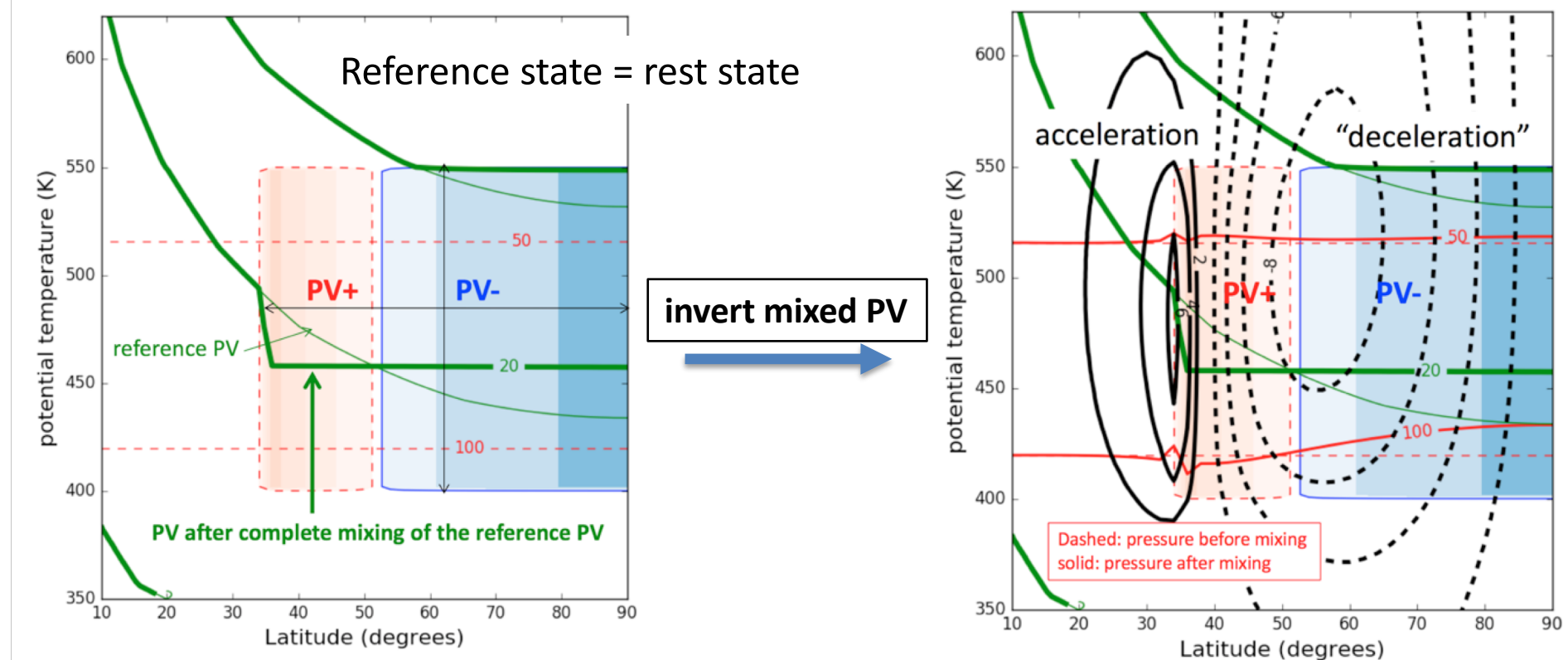
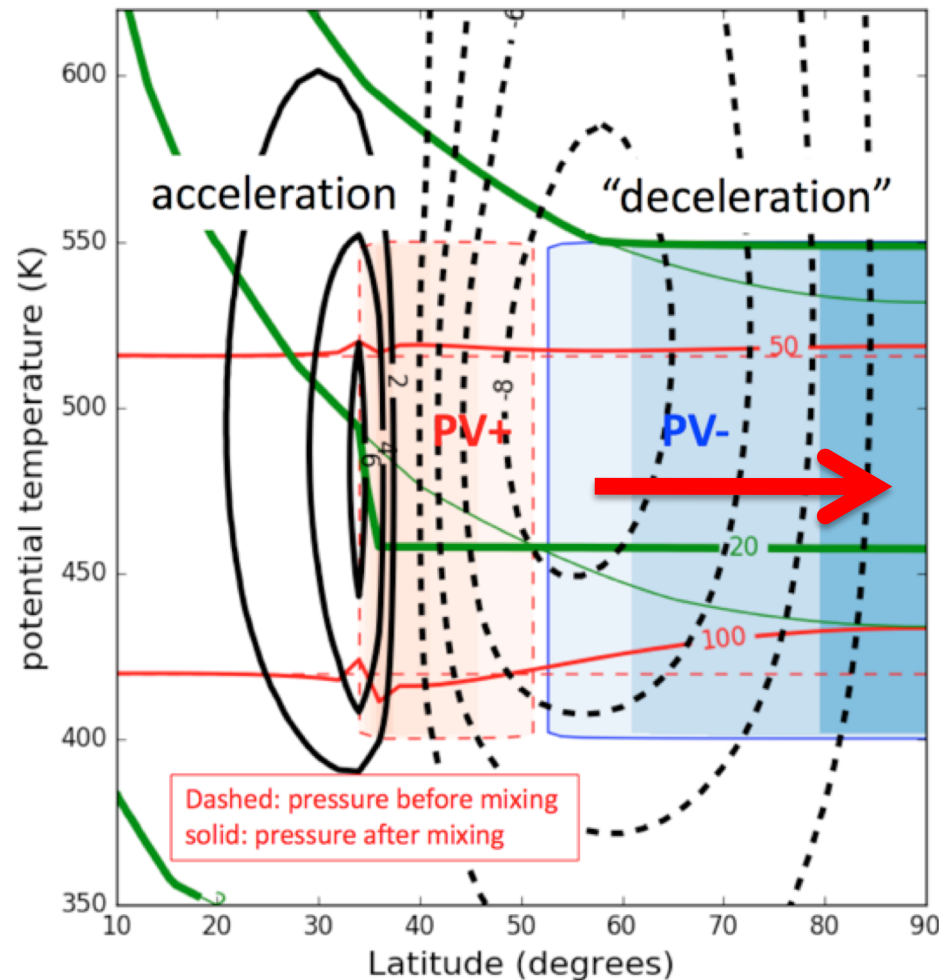


FIGURE 1.122 *. The latitude-dependent reference potential vorticity is “mixed completely” between 35°N and the Pole, in the layer, $\theta=400\text{--}550$ K creating a **negative PV-anomaly**, approximately between 51°N and the Pole, and a **positive PV-anomaly** between 35°N and approximately 51°N. **Thin green lines: reference potential vorticity. Thick green lines: potential vorticity after complete mixing.** Red horizontal dashed lines represent isobars in reference state of rest, labeled in units of hPa.

FIGURE 1.123 *. Balanced zonal mean zonal wind velocity (**solid black contours: positive; dashed black contours: negative**), which is created by complete mixing, between the 35°N and the North Pole, of the potential vorticity associated with the state of rest (the reference state)(contour interval: 2 m s^{-1}). **The solid red lines represent the isobars associated with the balanced state.**

A thought experiment:

PV-mixing and balance requirement explains poleward mass flux & the Brewer-Dobson circulation



Kleinschmidt's principle:

Isobars over the polar cap are squeezed in mixing region.

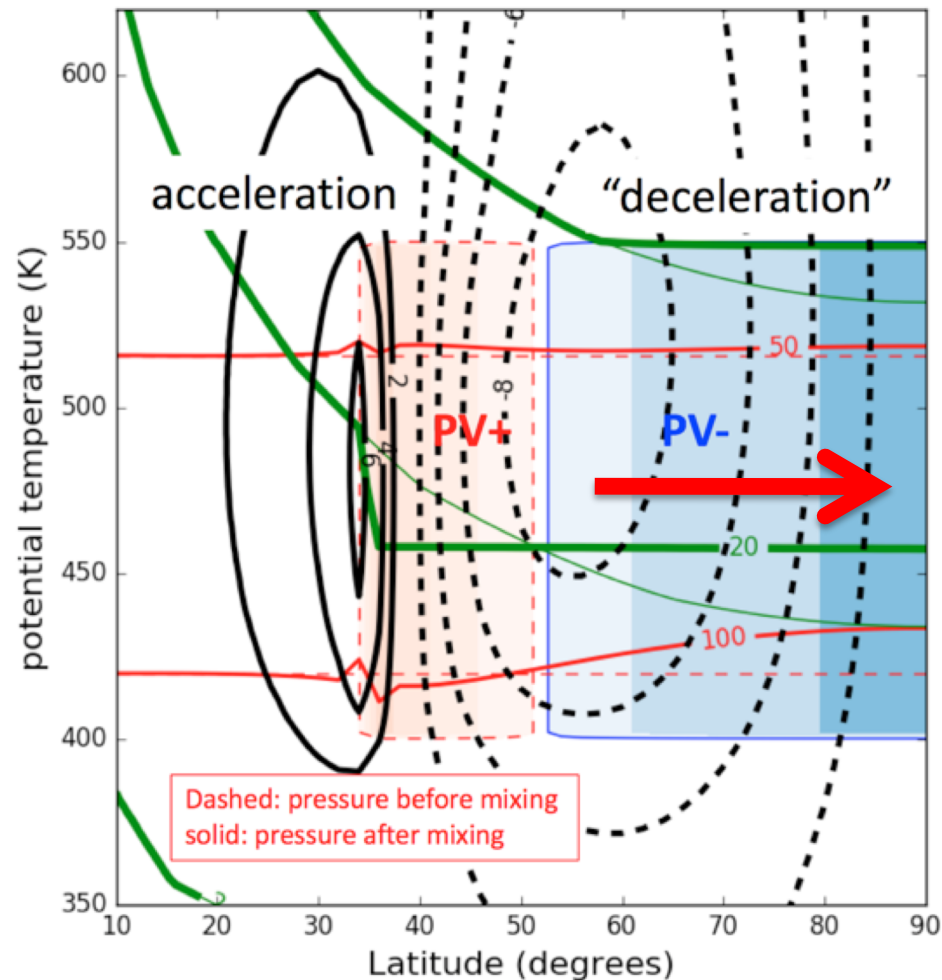
poleward mass flux (**arrow**)

* 2020-edition of lecture notes

FIGURE 1.123 * Balanced zonal mean zonal wind velocity (solid black contours: positive; dashed black contours: negative), which is created by complete mixing, between the 35°N and the North Pole, of the potential vorticity associated with the state of rest (the reference state)(contour interval: 2 m s⁻¹). The solid red lines represent the isobars associated with the balanced state.

A thought experiment:

PV-mixing and balance requirement explains poleward mass flux & the Brewer-Dobson circulation



In a balanced atmosphere, the *negative PV-anomaly over the pole*, which is caused by meridional PV-mixing by eddies and waves, must be associated with both a **negative vorticity anomaly** and a **positive isentropic density anomaly**.

Therefore, meridional PV-mixing and maintenance of zonal mean thermal wind balance must be associated with a net **equatorward PVS*-flux** and a **poleward mass-flux**.

The poleward mass-flux represents the upper branch of the Brewer-Dobson circulation

* PVS –absolute vorticity

Do we observe the consequences of PV-mixing?

Non-dimensional anomalies

$$Z_{\text{ref}} = \frac{f}{\sigma_{\text{ref}}}$$

$$\sigma_{\text{ref}}(\theta) = \frac{\int \sigma \cos \phi d\phi}{\int \cos \phi d\phi}$$

$$\zeta_{\text{ref}} = f$$

Non-dimensional anomalies:

$$Z^+ \equiv \frac{Z'}{Z_{\text{ref}}}; \quad \zeta^+ \equiv \frac{\zeta'}{f}; \quad \sigma^+ \equiv \frac{\sigma'}{\sigma_{\text{ref}}}$$

Next slide shows plot of Z^+ 

CIRA*-analysis of zonal-mean PV-anomaly

$$[Z] = \frac{[\zeta] + f}{[\sigma]}$$

$$[Z] = Z_{\text{ref}} + Z'$$

Thick green

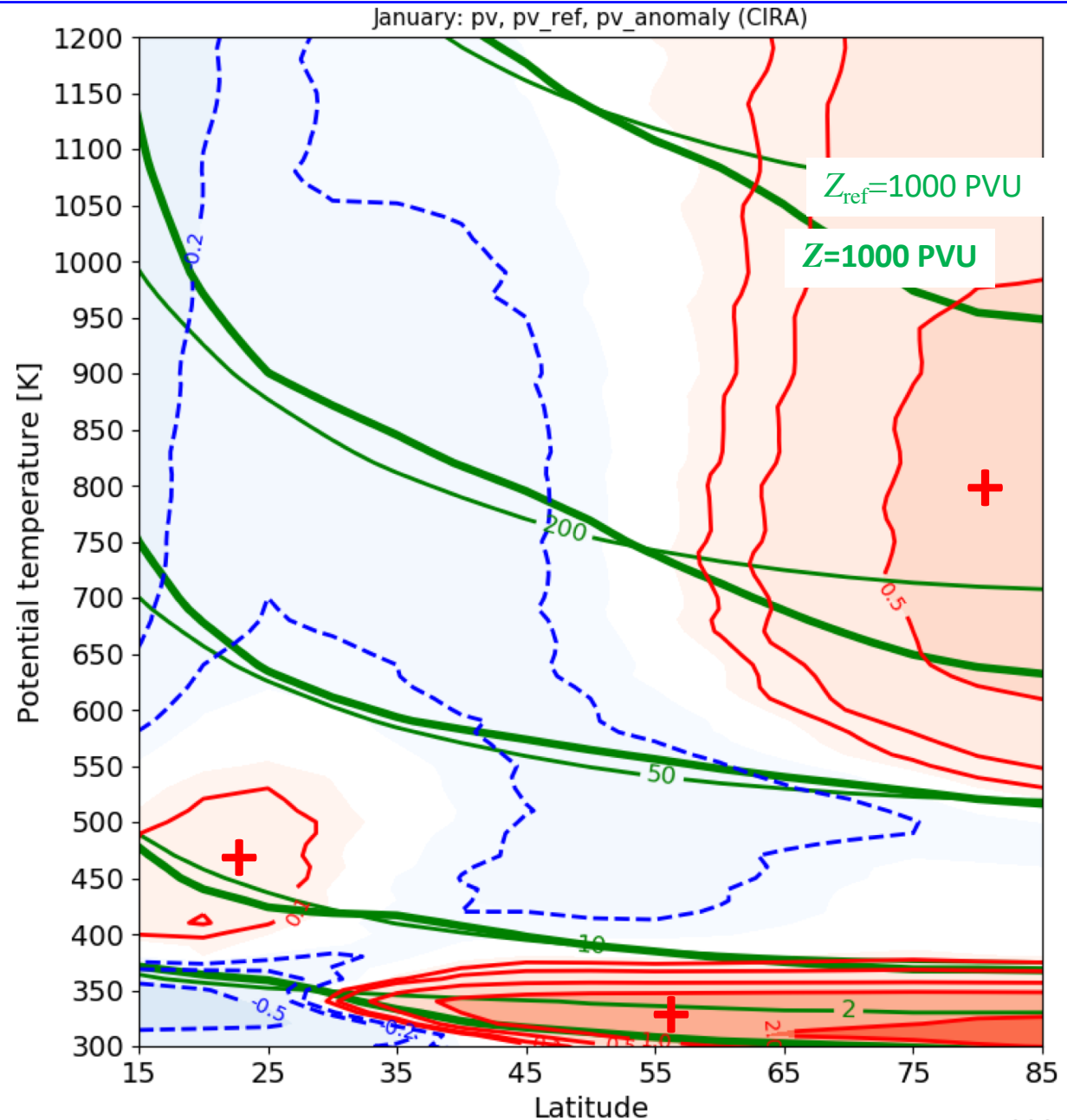
$$Z_{\text{ref}} = \frac{f(y)}{\sigma_{\text{ref}}(\theta)}$$

$$Z^+ = \frac{Z'}{Z_{\text{ref}}}$$

Red & blue shading

Isopleths of the January average zonal-mean zonal PV-distribution and the non-dimensional zonal mean PV-anomaly, Z^+ (**red – positive** - and **blue – negative** - shading and contours labeled in non-dimensional units) as a function of potential temperature and latitude, according to the

***COSPAR International Reference Atmosphere (CIRA) (Table 1.1).**



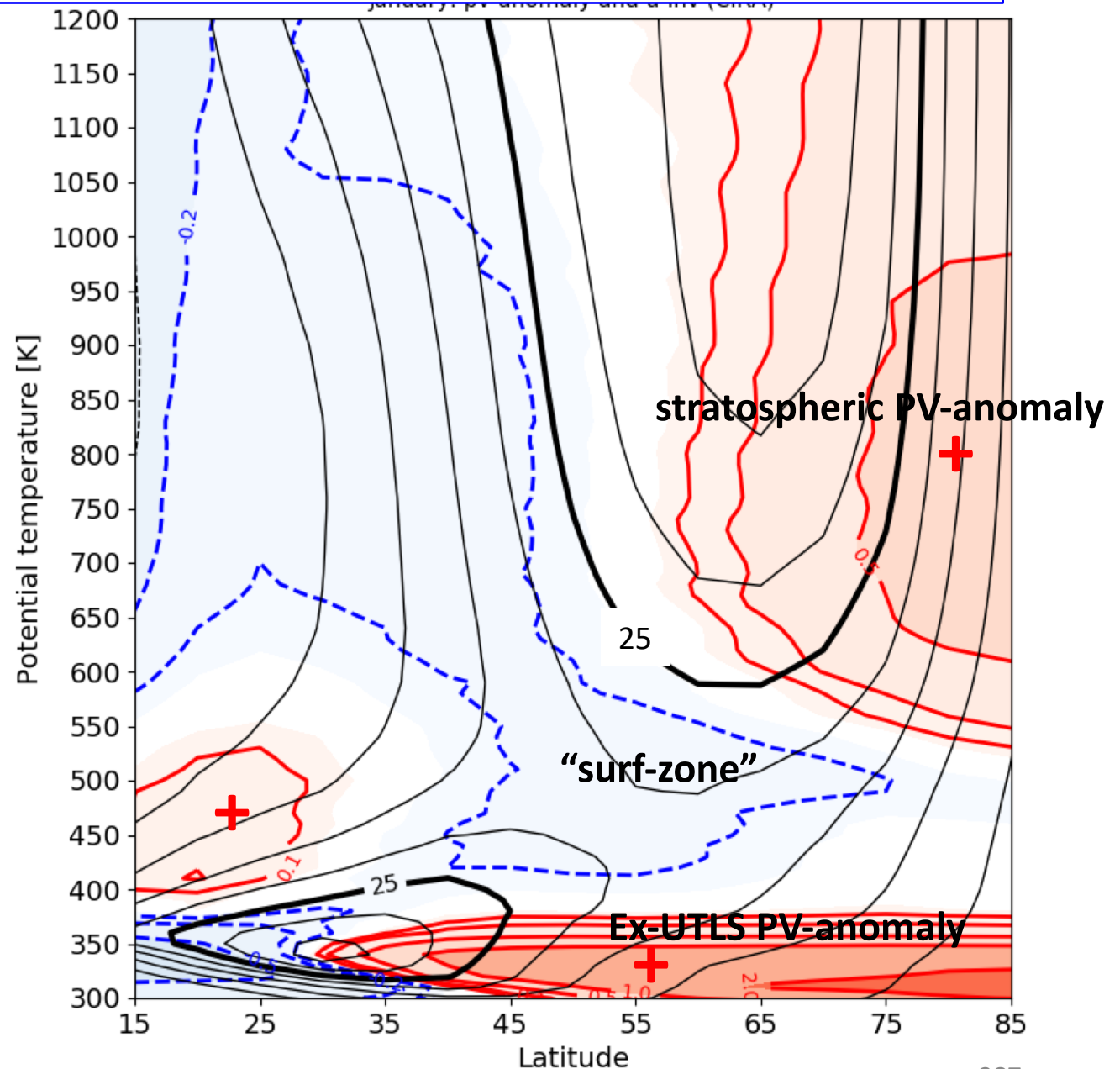
Zonal-mean PV-anomalies & Polar vortex

Black contours: $[u]$

$$Z^+ = \frac{Z'}{Z_{\text{ref}}}$$

Red & blue shading

Isopleths of the January average zonal-mean zonal wind velocity, $[u]$ (CIRA) (m/s), and the non-dimensional zonal-mean PV-anomaly, Z^+ (red – positive and blue – negative; shading and contours labeled in non-dimensional units) as a function of potential temperature and latitude.



Inversion zonal-mean PV-anomaly $\gg [u]_{\text{inverted}}$

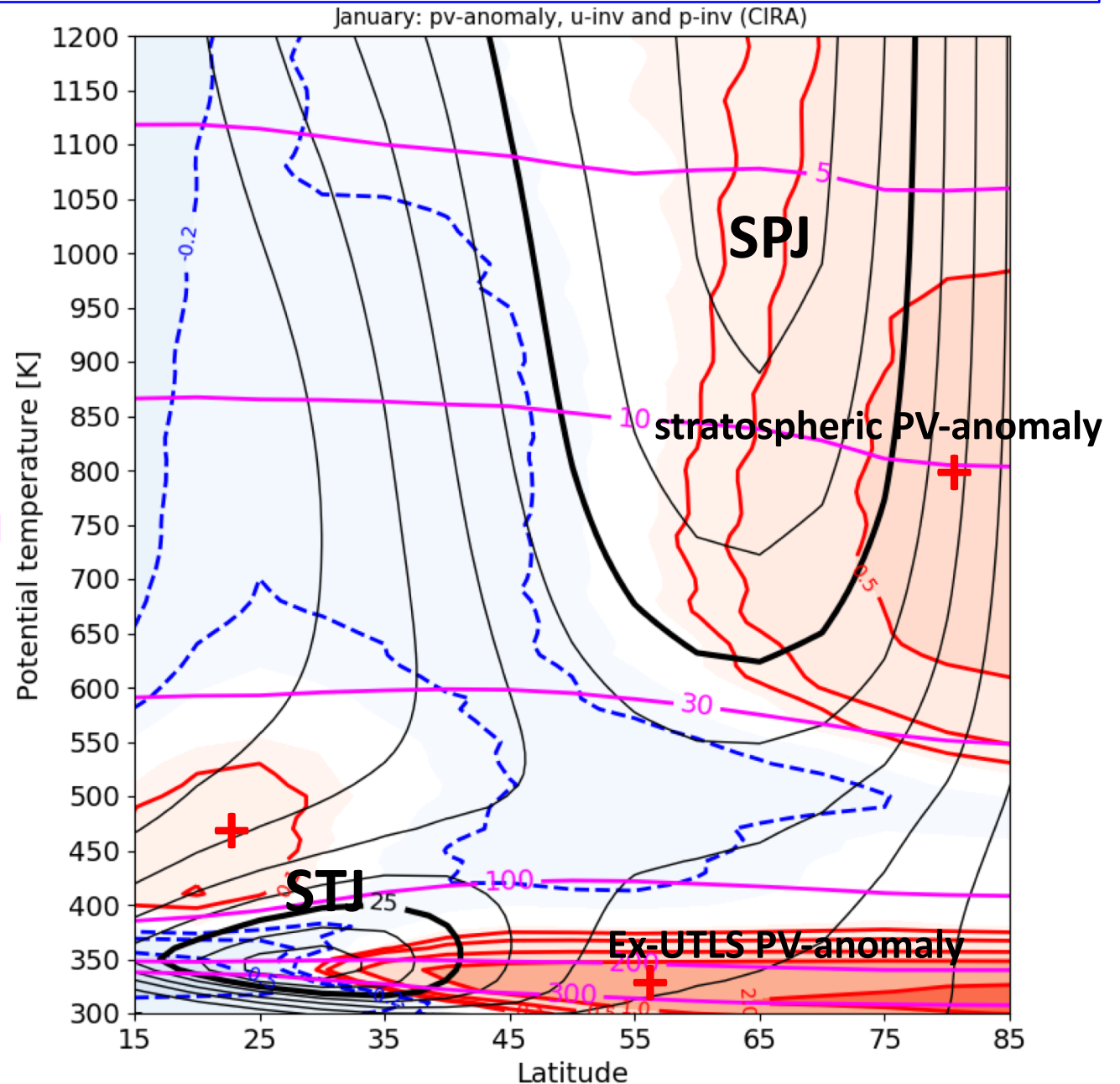
Black contours: $[u]_{\text{inverted}}$

$$Z^+ = \frac{Z'}{Z_{\text{ref}}}$$

Red & blue shading

$[p]_{\text{inverted}}$ magenta contours [hPa]

1. **Stratospheric polar jet (SPJ)** is induced by the stratospheric PV-anomaly
2. **Subtropical jet (STJ)** is induced by the Ex-UTLS PV-anomaly



Piecewise PV-inversion of UTLS-PV-anomaly

PV-anomaly, $\theta > 550$ K removed
Resulting PV-distribution inverted

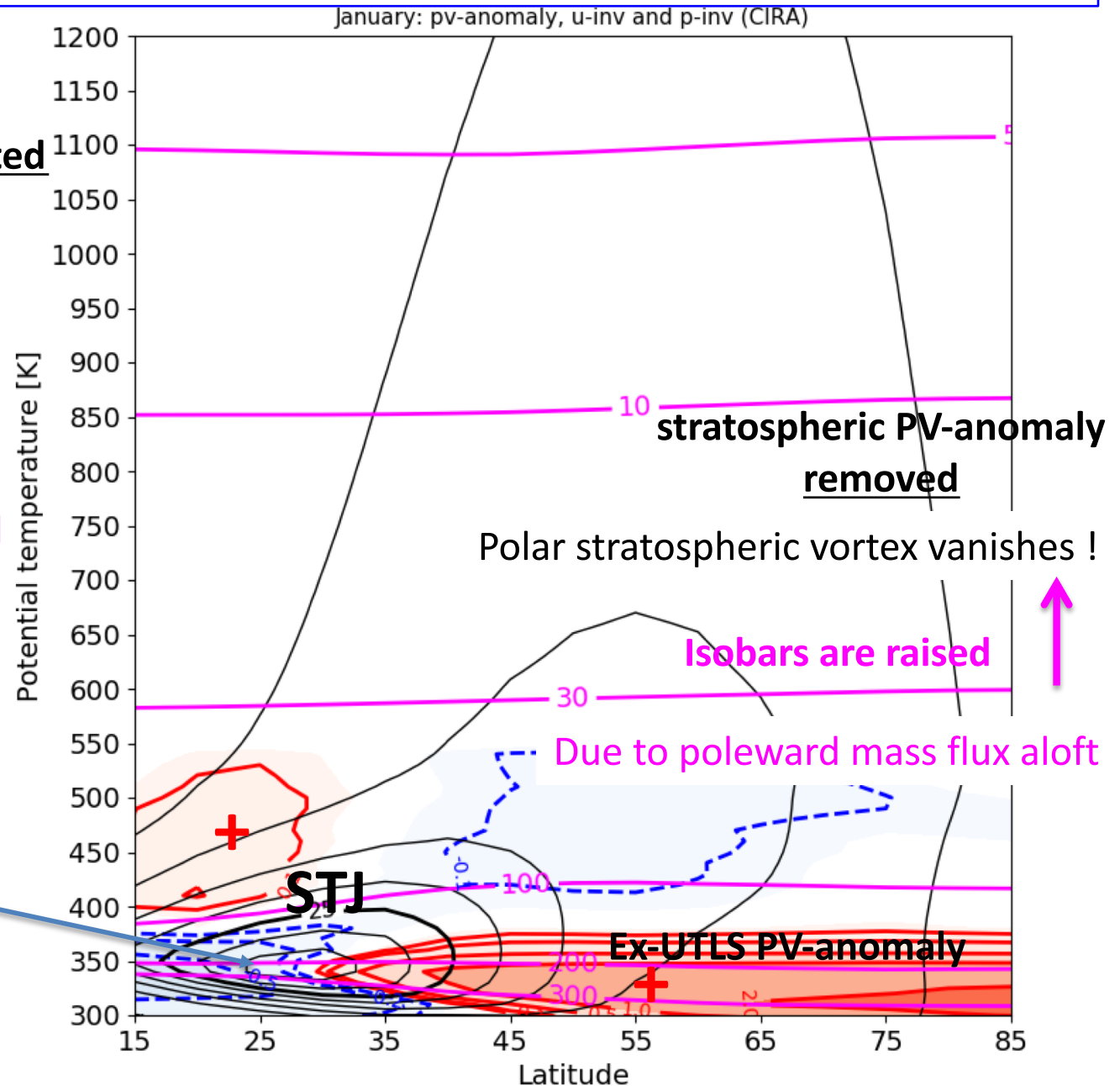
$[u]_{\text{inverted}}$

$$Z^+ = \frac{Z'}{Z_{\text{ref}}}$$

Red & blue shading

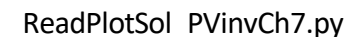
$[p]_{\text{inverted}}$ magenta contours [hPa]

Subtropical Jet hardly affected by the removal of the stratospheric PV-anomaly



UTLS = Upper Troposphere / Lower Stratosphere

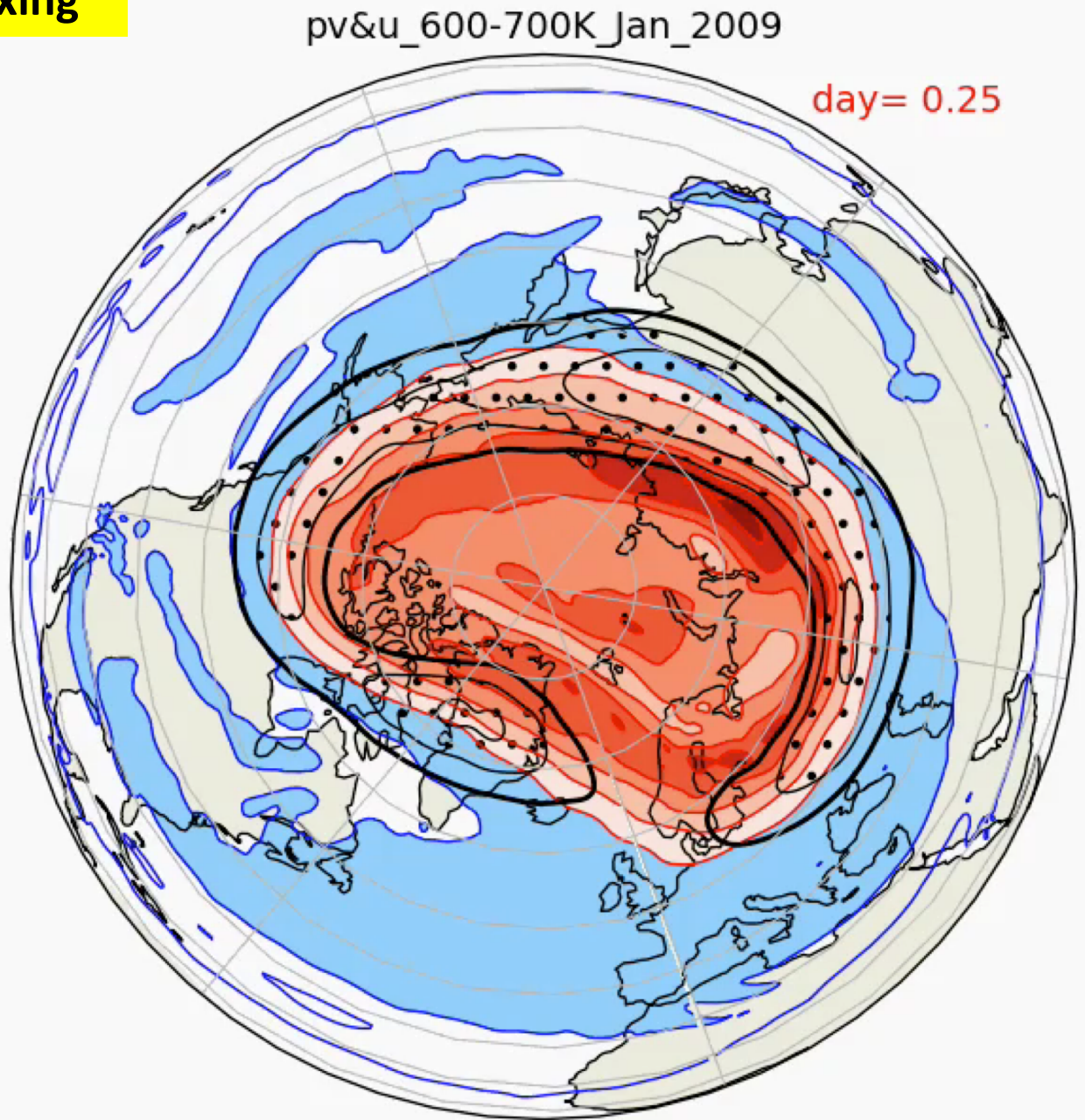
STJ is hardly affected , but westerly winds in middle latitudes increase



A clear-cut example of “PV-mixing”

Sudden Stratospheric Warming (SSW)

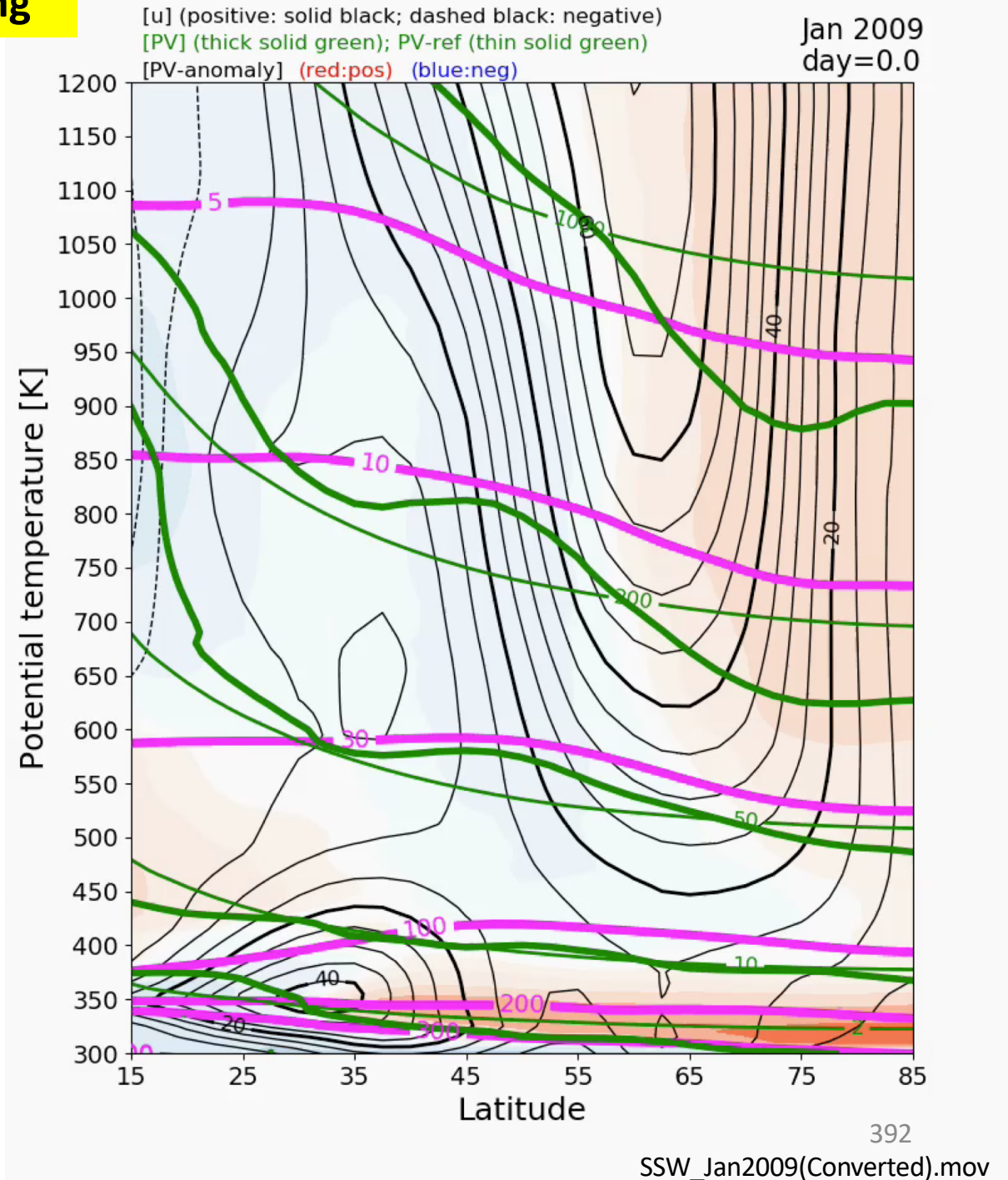
(section 1.29)



A clear-cut example of “PV-mixing”

Sudden Stratospheric Warming (SSW)

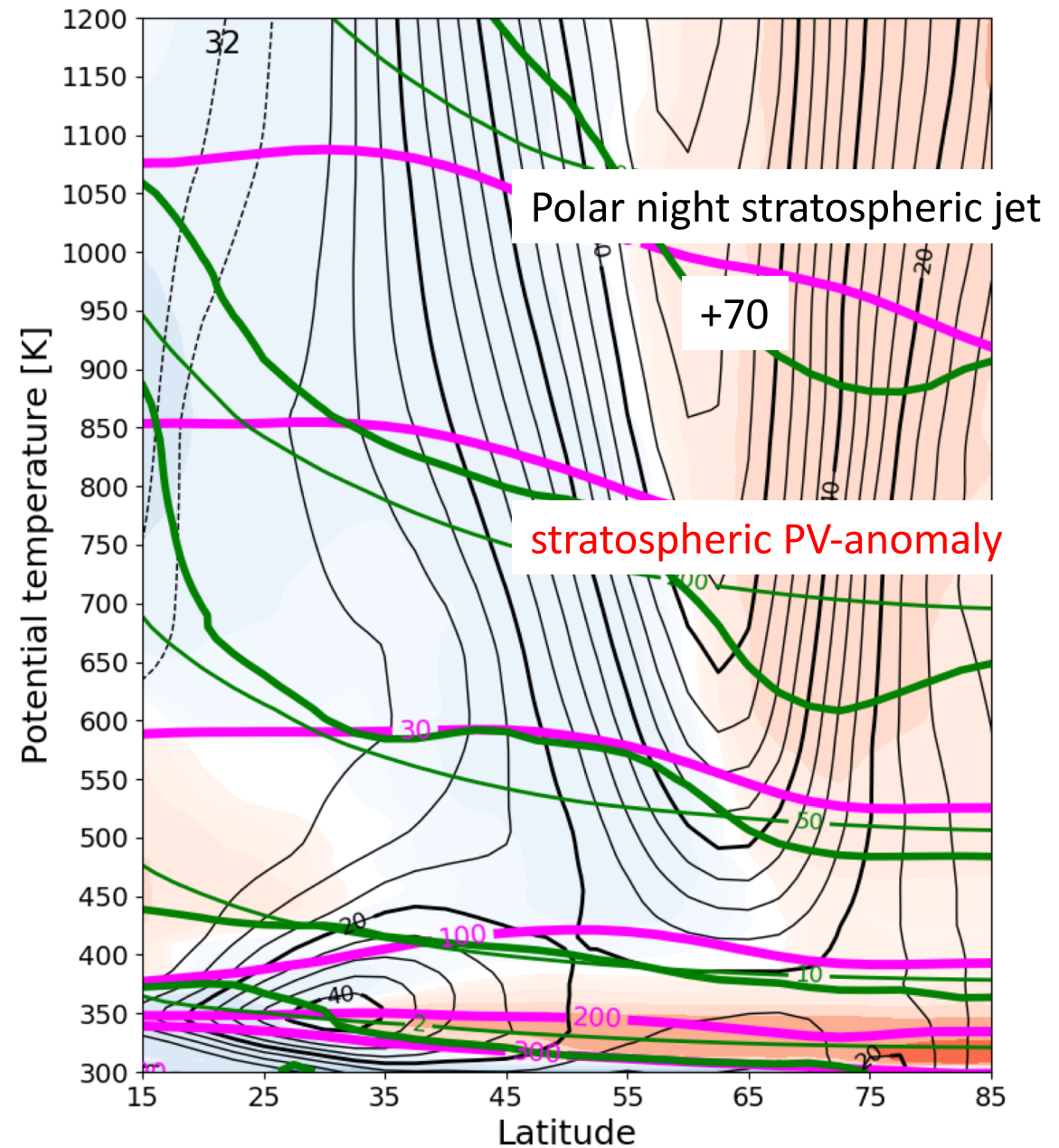
(section 1.29)



During an SSW the stratospheric PV-anomaly is destroyed by PV-mixing

Before the major Sudden Stratospheric Warming (SSW) of 24 January 2009

8 January, 2009, 00 UTC



During an SSW the stratospheric PV-anomaly is destroyed by PV-mixing

After the major **Sudden Stratospheric Warming (SSW)** of 24 January 2009

26 January, 2009, 00 UTC

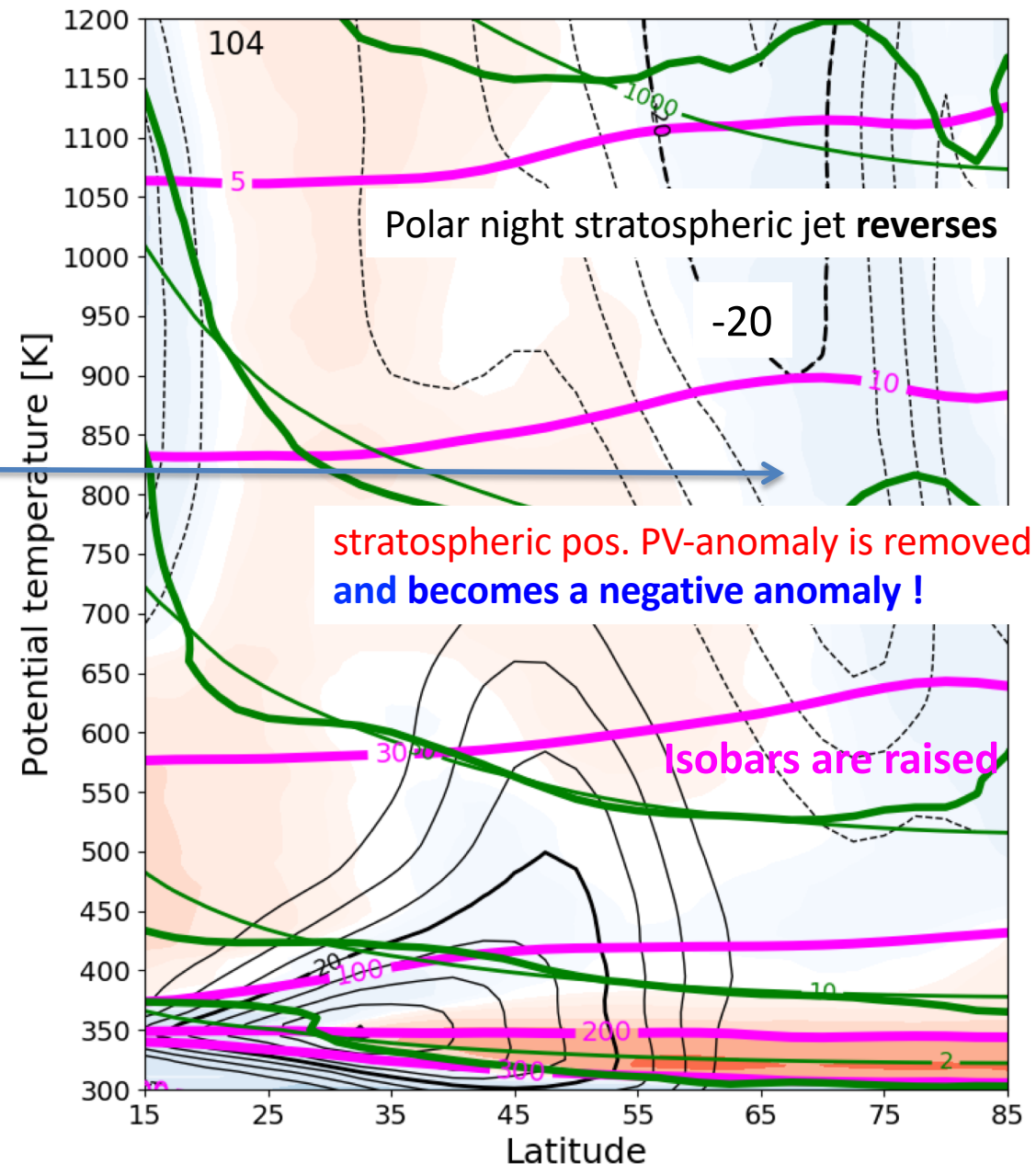
Positive stratospheric PV-anomaly is eliminated by isentropic mixing of PV by eddies.

The PV-anomaly is now a negative anomaly !

Polar night stratospheric vortex has vanished !

A zonal-mean zonal wind reversal has occurred !

Isobars are raised >> manifestation of the actual stratospheric warming

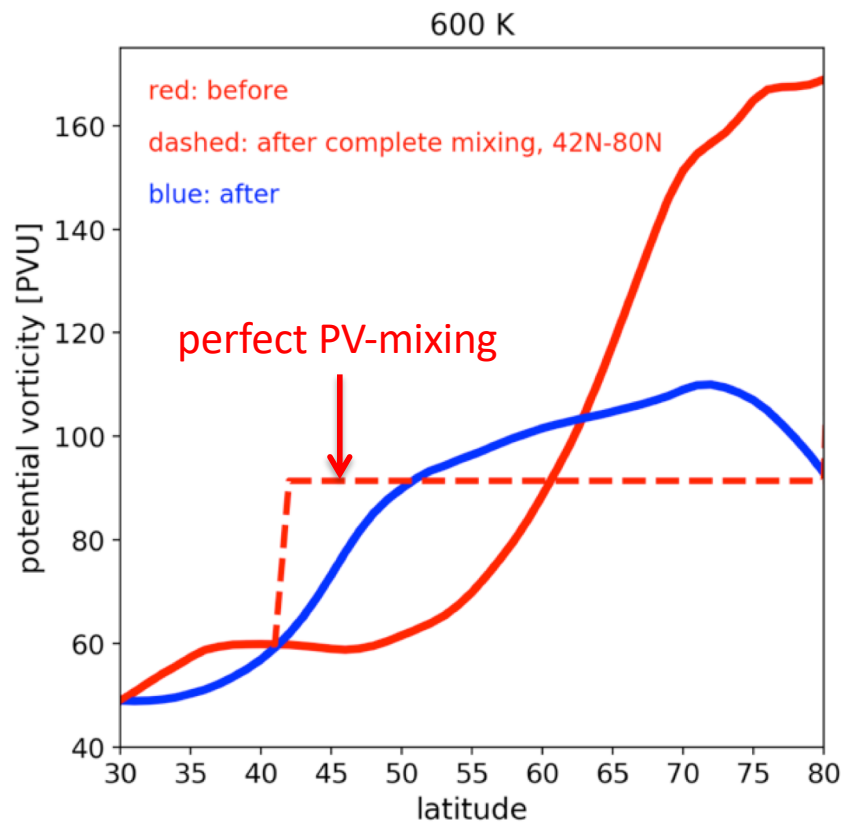


Sudden Stratospheric Warming

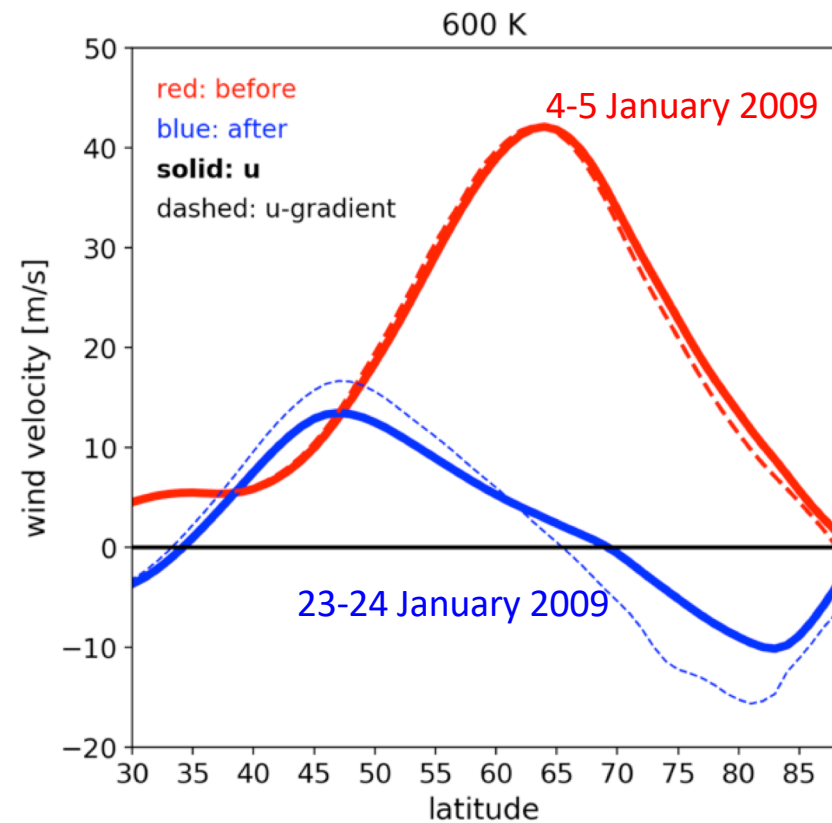
A Sudden Stratospheric Warming is an ideal example of fast meridional PV mixing in which high PV air is removed from the Polar Cap and replaced by low PV-air from lower latitudes.

Before: 4-5 January 2009

After: 23-24 January 2009



Zonal mean potential vorticity at $\theta=600$ K **before** (red solid curve) and **after** (blue solid curve) (fig. 1.124)



Zonal mean zonal wind at $\theta=600$ K **before** (red solid curve) and **after** (blue solid curve) (fig. 1.125)

All this is a beautiful illustration of

“Kleinschmidt’s Principles”

(section 1.33)

1b:

“Within an isolated air mass with abnormal potential vorticity, in thermal wind balance, the isentropic density deviates from the normal in the opposite sense as the potential vorticity”.

3:

“A cold air mass in thermal wind balance only remains cold as long as there are masses of high potential vorticity above it. When this condition is no longer fulfilled, the air sinks down and loses the character of a cold air mass”.

Next:

Dynamic Meteorology: lecture 14

19/1/2022 (Wednesday, 13:15-15:00)

Sections 1.36 (part G), 1.42-1.44 (part I)

Jet streaks and accelerations/decelerations in the jetstream (1.36) (part G)
Mathematical definition of a front, frontogenesis and frontolysis (Q-vector) (part I)
Quasi-geostrophic thermodynamic equation
Ageotropic motion due to frontogenesis
Omega-equation
Relation between Q-vector and vertical motion

Tutorial after this lecture (13) (online)

Project 5 (Principal Component Analysis)

Problems 1.29, 1.30, 1.33 (p. 224-227) and 1.37 (piecewise PV-inversion)
(...and problems 1.34 & 1.35)

Presentations of project 5 in January 2022

Each presentation: no more than 12 minutes to leave time for questions and discussion

First 6 presentations on **Wednesday 19 January 2022**, 15:15-17:00

Remaining presentations on **Friday 21 January 2021**, 11:00-12:45, 13:15-15:00

Simple structure of the presentation

1. Introduction (physics of the research question & hypothesis)
2. Data (time series)
3. Correlations between time series presented as a correlation matrix
4. Discussion of eigenvectors and corresponding eigenvalues.
5. Interpretation of the first and second principal components
6. Projection of original data on the first principal component
7. Conclusion: is the hypothesis verified?