

# Dynamic Meteorology: lecture 14

**19/1/2022 (Wednesday, 13:15-15:00)**

Sections **1.36 & 1.38** (part G), **1.42** (part I)

## **Vertical motion in statically stable atmosphere part 1**

Poleward flow is up; equatorward flow is down

“Rapids” in the jet stream (jet streaks) (1.36) (part G)

Accelerations in a jet streak related to ageostrophic motion with upward component

Ageostrophic motion related to changing temperature gradients (“frontogenesis”)

Mathematical definition of a front, frontogenesis and frontolysis (Q-vector) (1.42)

Quasi-geostrophic thermodynamic equation (1.38)

## **After this lecture: five presentations of project 5**

**Lisa, Tacey and Johanna** – NAO, temp and pressure Europe and USA

**Dennis, Bente and Jorge** – winter temps NL, MJO and ENSO

**Jade and Andrea** – winter temps in NL, zonal wind and pressure in Scandinavia

**Maarten, Carlo and Jonna** – Snowfall in Siberia, AO and winter temps in NL

**Judith, Sara, Tijmen and Reinaart** – Winter temps in Western Europe and Canada and polar jet stream

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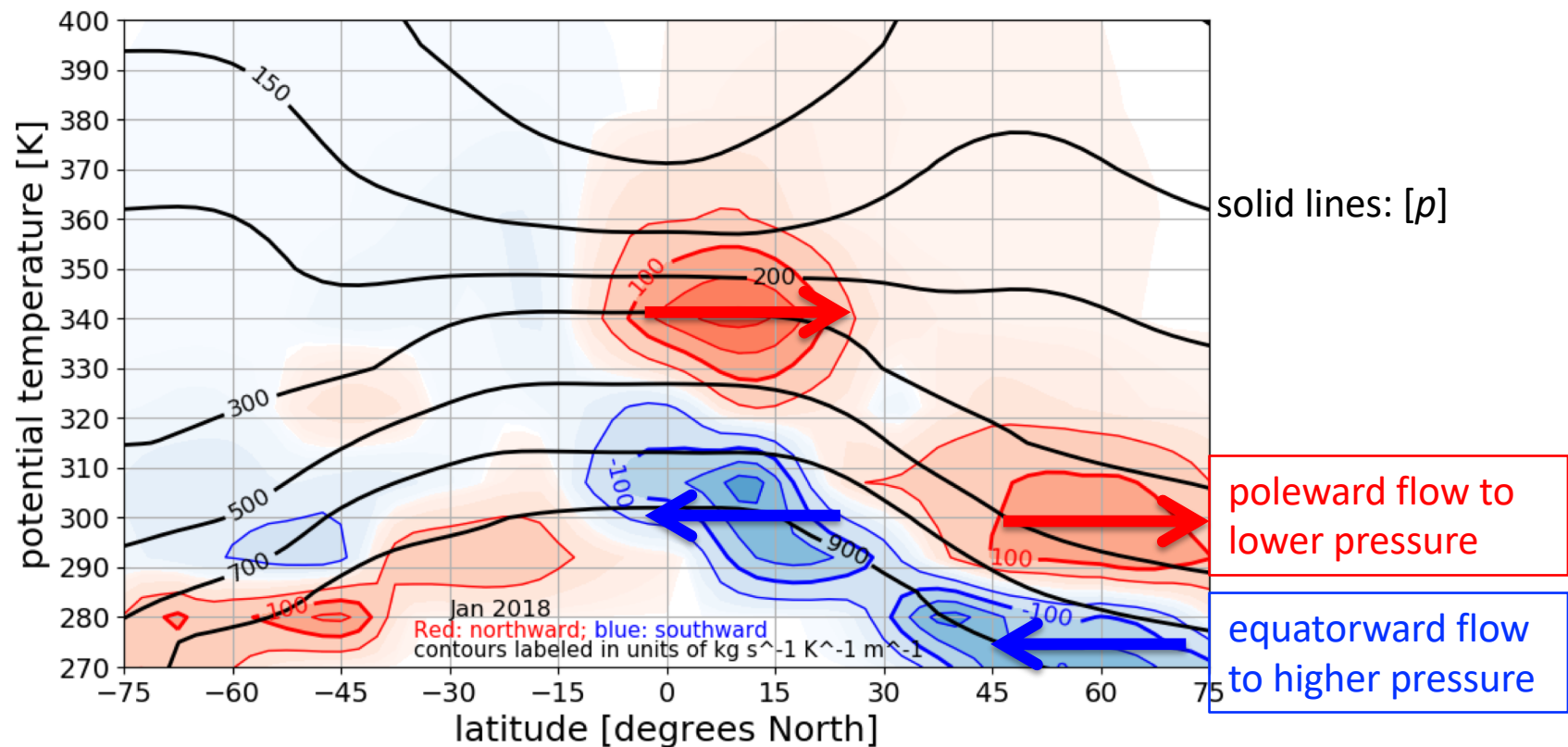
(<http://www.uu.nl/~nvdelden/dynmeteorology.htm>)

# Zonal/time mean meridional mass flux\*

$$* = \frac{1}{T} \int_0^T [\nu \sigma] dt \quad [\text{kg m}^{-1} \text{K}^{-1} \text{s}^{-1}]$$

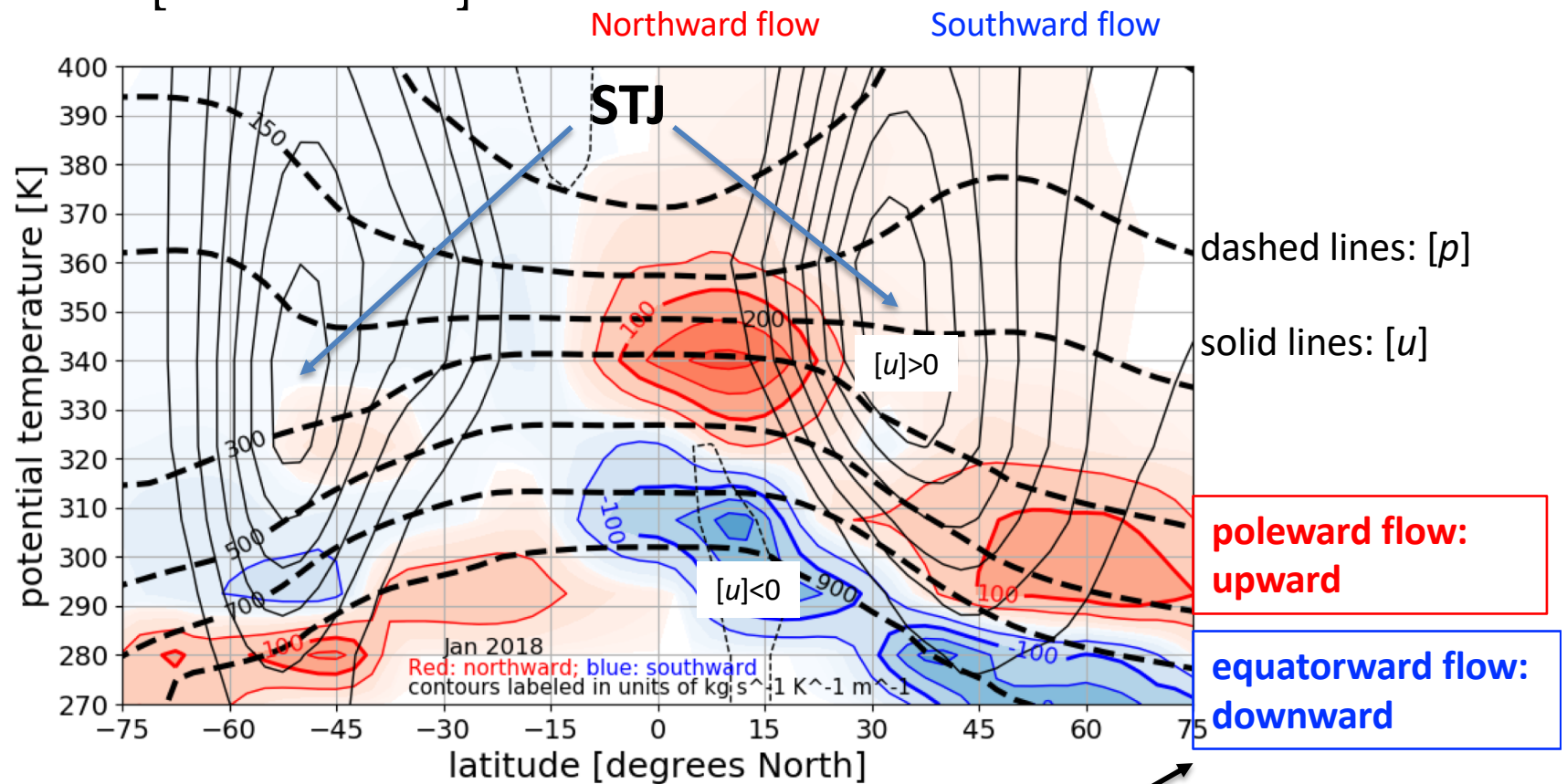
→  
Northward flow

←  
Southward flow



# Zonal/time mean meridional mass flux\*

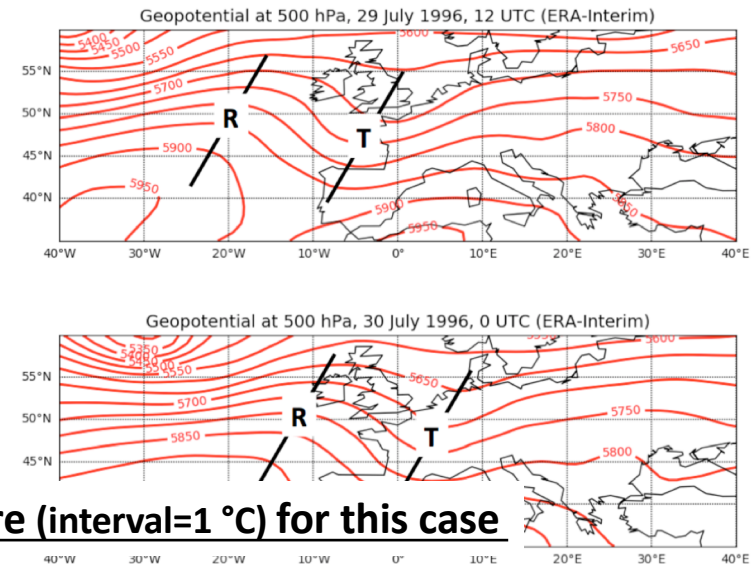
$$* = \frac{1}{T} \int_0^T [\nu \sigma] dt \quad \left[ \text{kg m}^{-1} \text{K}^{-1} \text{s}^{-1} \right]$$



Why and how does this work ?

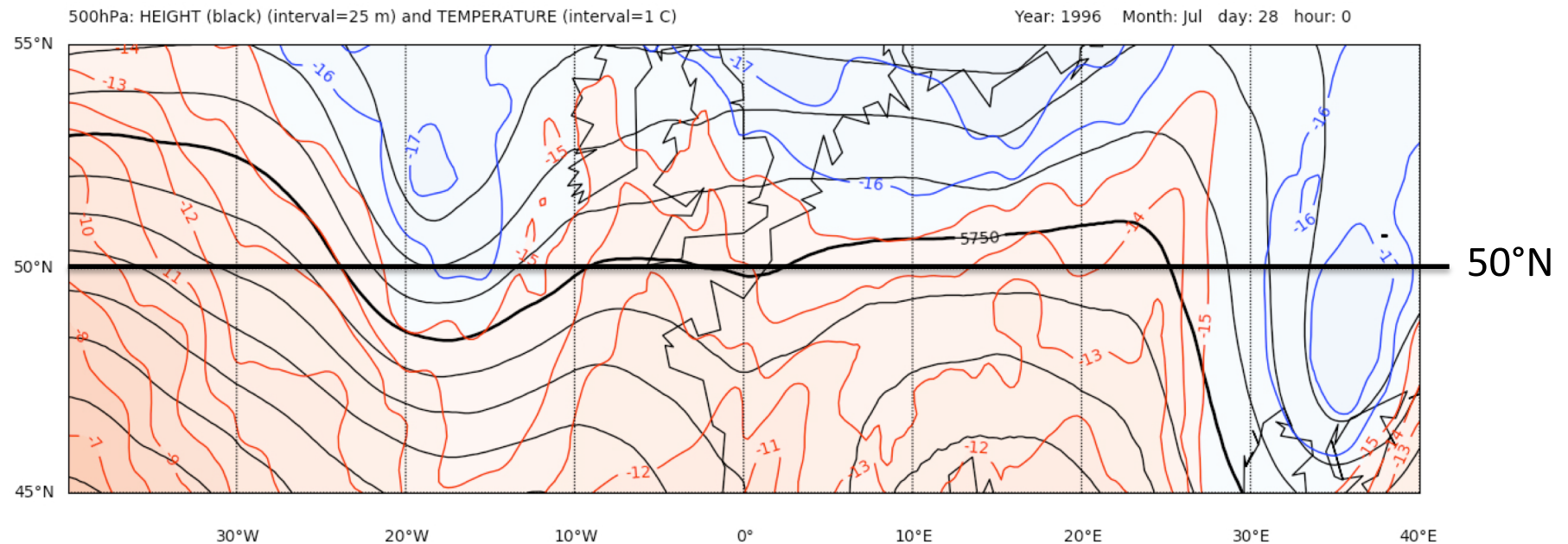
# Illustrate with ideal Rossby wave case of July 1996

## Lecture 11: Rossby waves in July 1996 Fig. 1.111



### Animation of the 500 hPa height (interval=25 m) and temperature (interval=1 °C) for this case

ERA-5





# Rossby wave trough connected to a positive PV-anomaly

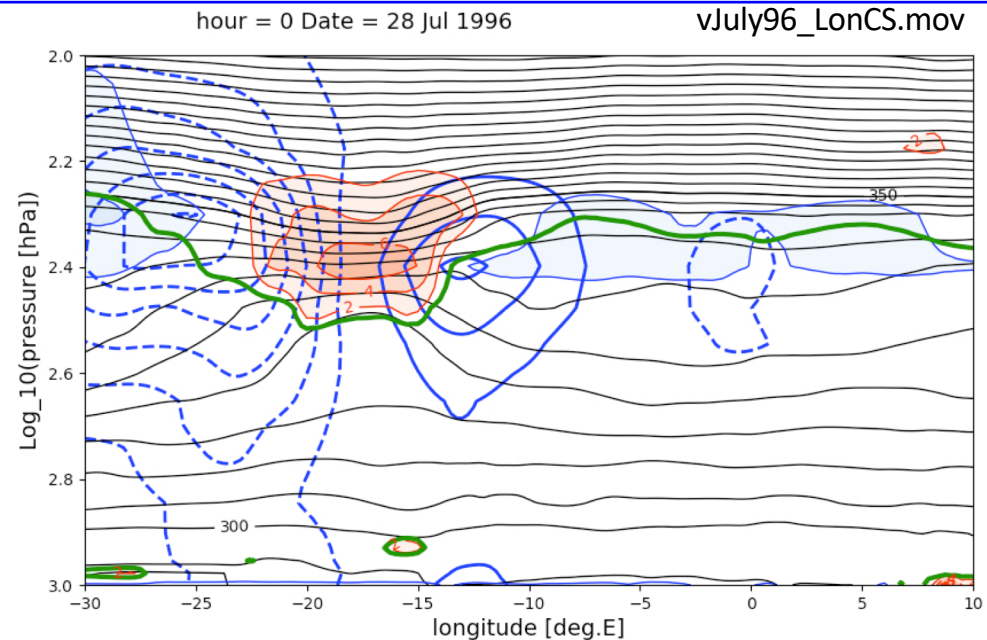
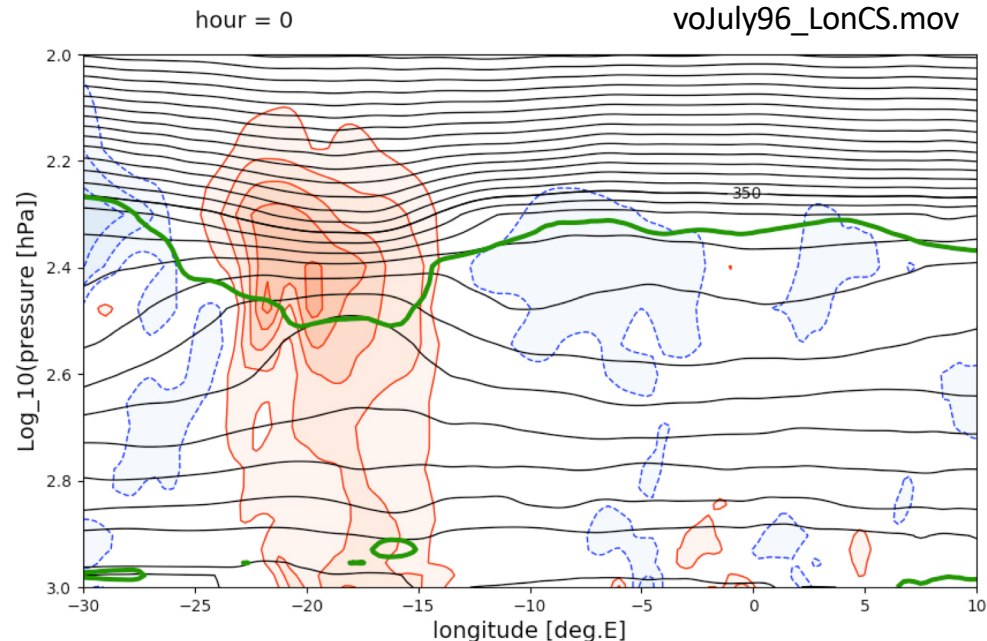
Rossby waves in July 1996 :  
cross section at 50°N.

**Animation 1** of PV-anomaly (red:positive) (interval=2 PVU), potential temperature (interval=5 K) and meridional velocity (solid blue: into the page or northward) (interval=5 m/s).

**Poleward (equatorward) motion on the east (west) side of the PV-anomaly, consistent with Kleinschmidt's principles.**

**Animation 2** of vorticity-anomaly (blue=negative; red=positive) and potential temperature

**PV-anomaly is restricted to the tropopause region while the vorticity anomaly reaches to the earth's surface, with Kleinschmidt's principles (example of "action at a distance").**



# Moving PV-anomaly “induces” vertical motion

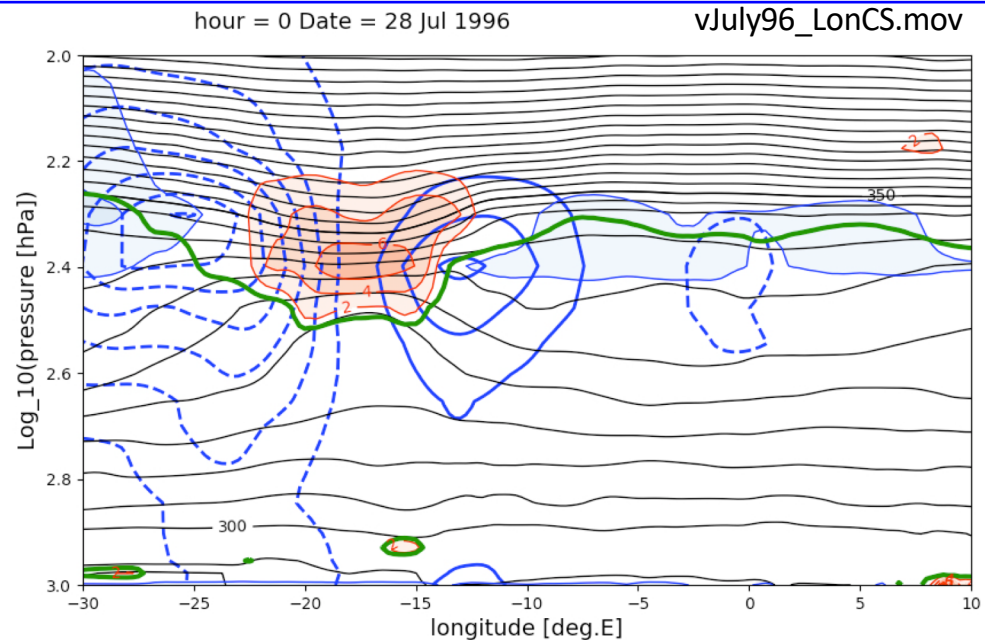
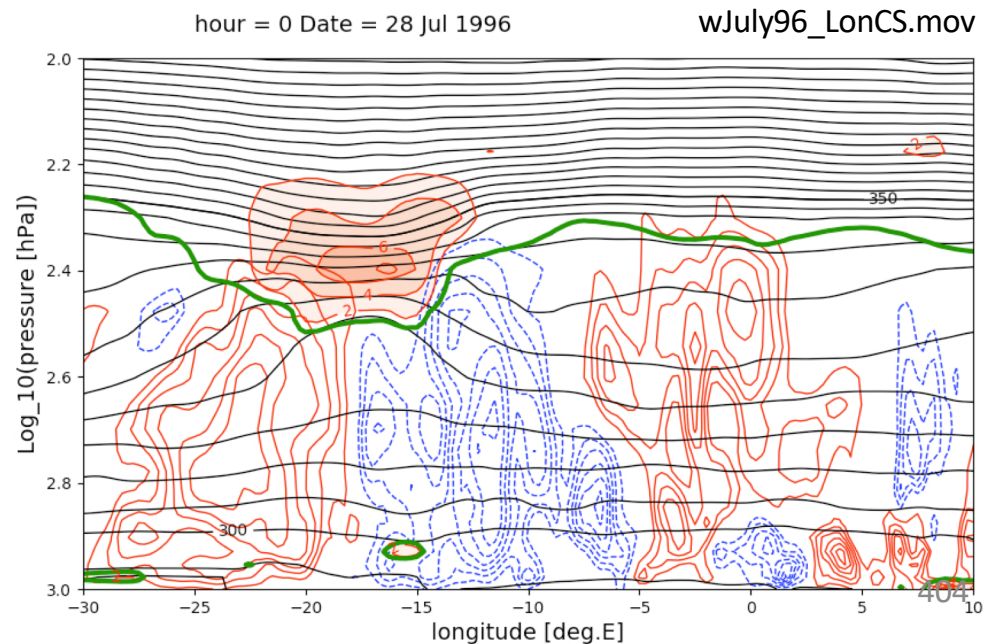
Rossby waves in July 1996 :  
cross section at 50°N.

**Animation 1** of PV-anomaly (red:positive) (interval=2 PVU), potential temperature (interval=5 K) and meridional velocity (solid blue: into the page or northward) (interval=5 m/s)

Upward bulging isentropes below positive PV-anomaly, with Kleinschmidt’s third principle.

**Animation 3** of PV-anomaly, potential temperature and vertical velocity (blue=up; red=down) (interval=0.02 Pa/s)

Poleward flow on the east side of the moving PV-anomaly is upward  
Equatorward flow on the west side of the moving PV-anomaly is downward



# Ageostrophic vertical motion is connected to changing temperature gradients (frontogenesis)

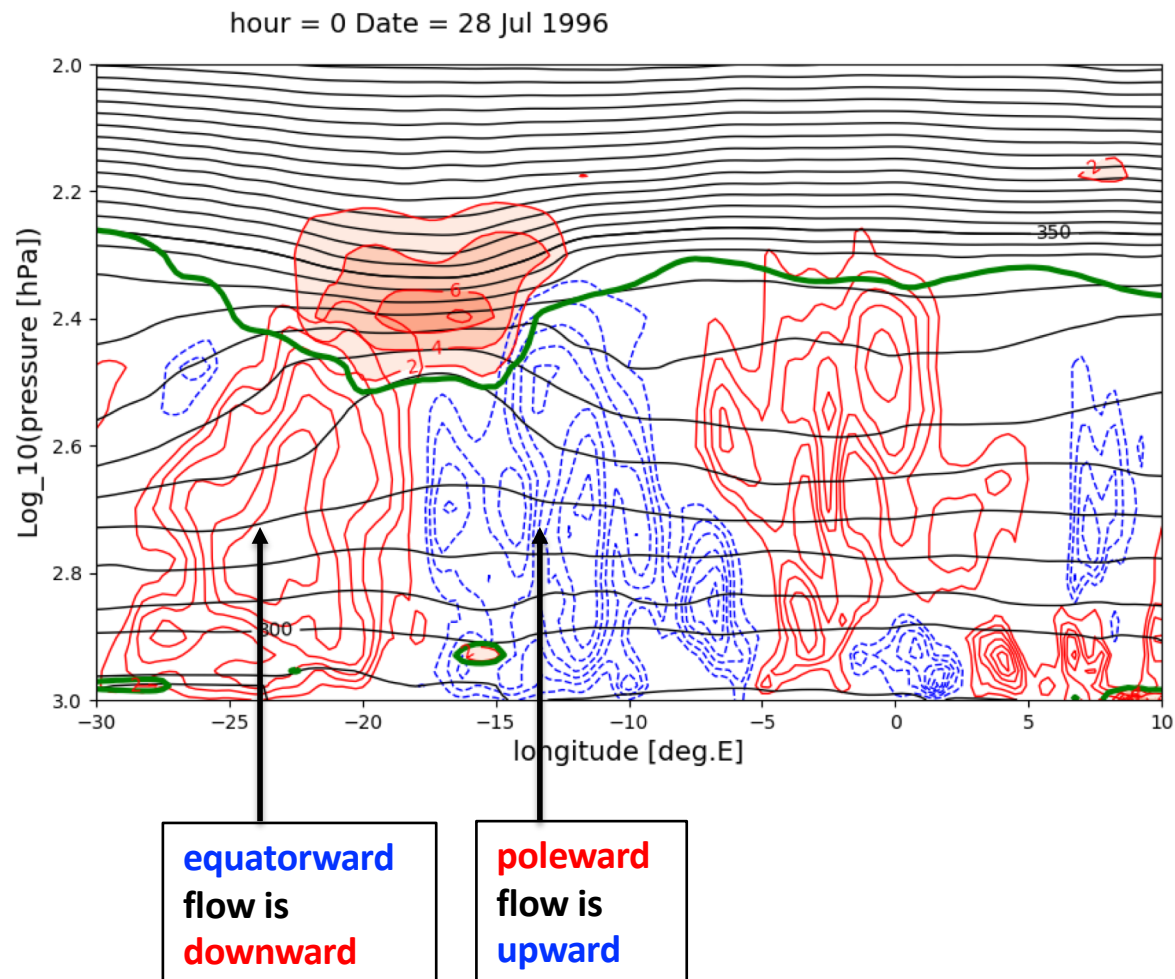
**Figure:** PV-anomaly, potential temperature and vertical velocity  
(blue=up; red=down)  
(interval=0.02 Pa/s)

Motion is **upward** (**downward**) on the **east** (**west**) side of the moving PV-anomaly

**Frontogenesis** (**frontolysis**) on the west side of the moving PV-anomaly.

**Frontogenesis:** increasing horizontal temperature gradient

**Frontolysis:** decreasing horizontal temperature gradient



# Ageostrophic motion

Equation of motion in pressure coordinates:

$$\frac{d\vec{v}}{dt} = -f\hat{k} \times \vec{v} - \vec{\nabla}\Phi$$

geostrophic wind:

$$f\hat{k} \times \vec{v}_g = -\vec{\nabla}\Phi$$

$$\vec{v} = \vec{v}_g + \vec{v}_a$$

Ageostrophic wind

# Ageostrophic motion is related to acceleration/deceleration

$$\frac{d\vec{v}}{dt} = -f\hat{k} \times \vec{v} - \vec{\nabla}\Phi \longrightarrow$$

$$\frac{d\vec{v}}{dt} = -f\hat{k} \times \vec{v}_a$$

geostrophic wind:

$$f\hat{k} \times \vec{v}_g = -\vec{\nabla}\Phi$$

$$\vec{v}_a = \frac{1}{f} \hat{k} \times \left( \frac{d\vec{v}}{dt} \right)$$

$$\vec{v} = \vec{v}_g + \vec{v}_a$$

Ageostrophic wind



Section 1.36:

# Accerations/decelerations in the jet stream: conceptual model of a “jetstreak” \*

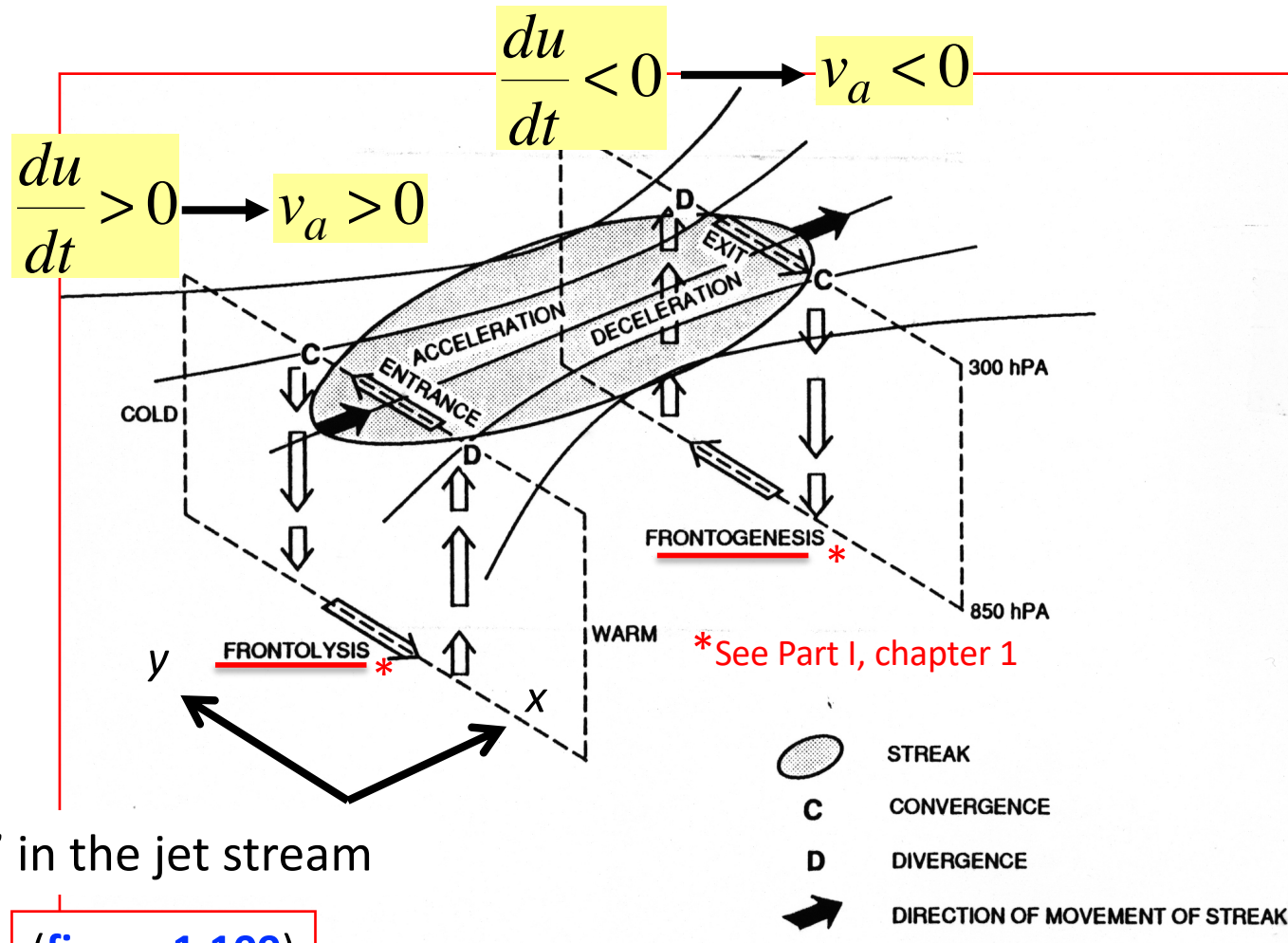
Previous slide:

$$\vec{v}_a = \frac{1}{f} \hat{k} \times \left( \frac{d\vec{v}}{dt} \right)$$

or

$$v_a = \frac{1}{f} \frac{du}{dt}$$

$$u_a = -\frac{1}{f} \frac{dv}{dt}$$



\* Jetstreak = “rapid” in the jet stream

(figure 1.109)

# Jetstreaks in the layer, 330-350 K on 17 Jan. 2007

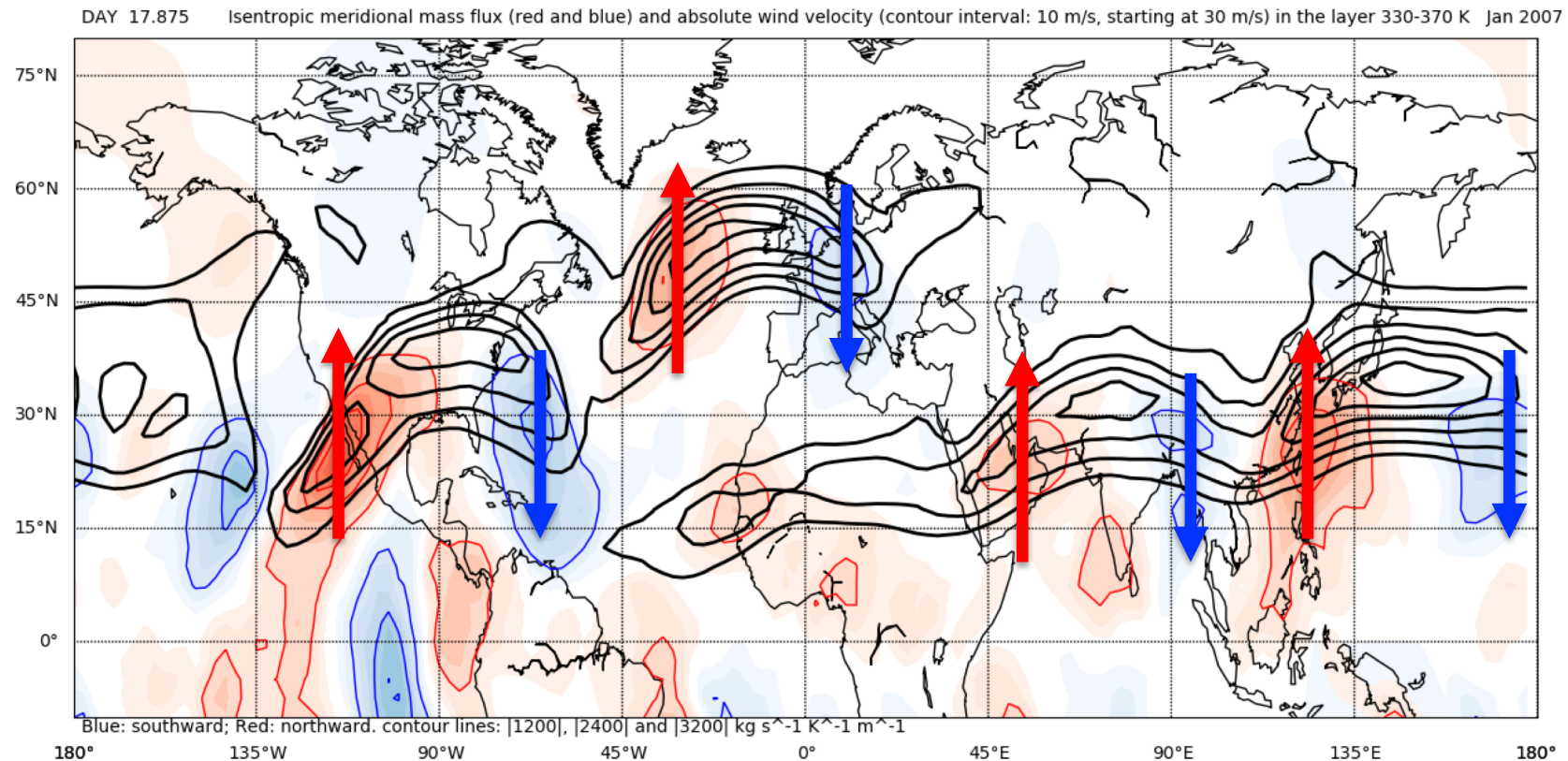
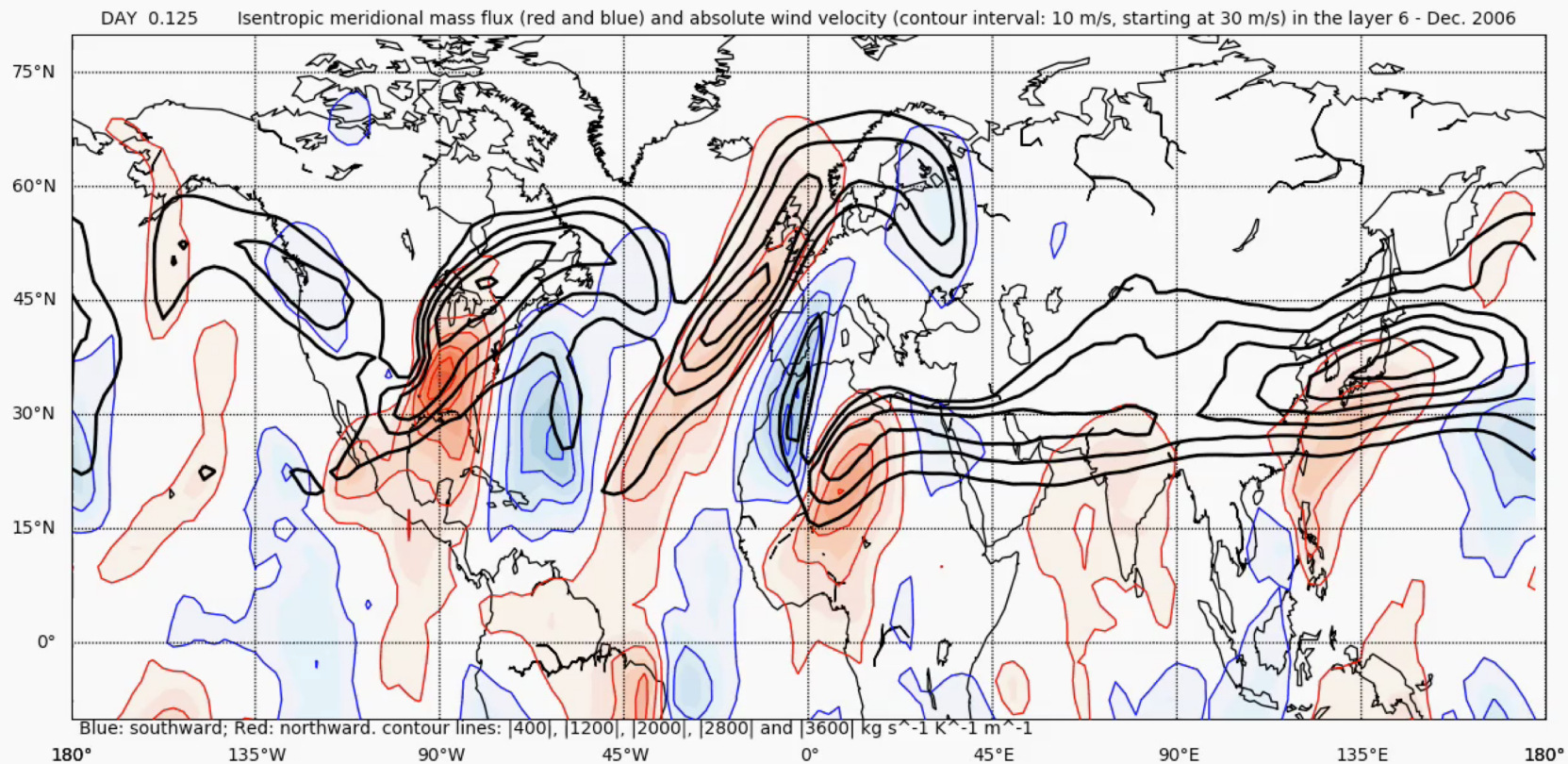


**FIGURE 1.110.** Meridional mass flux (red and blue contours/shading) and absolute isentropic wind velocity (black contours) in the layer between the isentropic surfaces corresponding to  $\theta=330 \text{ K}$  and  $\theta=370 \text{ K}$  in the time interval between 18 and 24 UTC, 17 Jan 2007. Red (northward mass flux) and blue (southward mass flux) contours are drawn at intervals of  $\pm 1200 \text{ kg s}^{-1} \text{ K}^{-1} \text{ m}^{-1}$ . Black contour interval is  $10 \text{ m s}^{-1}$ , starting at  $30 \text{ m s}^{-1}$ . Wind velocities exceeding  $70 \text{ m s}^{-1}$  are observed in jet streaks near Ireland and just east of Japan. These jet streaks move slowly eastward. A northward mass flux is observed in the jet entrance and a southward mass flux is observed in the jet exit, as predicted by the qualitative explanation (figure 1.109). The jet streaks are located in the ridges of a planetary wave pattern

# Jetstreaks are manifestations of Rossby waves

Animation of abs. wind velocity and MMF in December 2006 (330-350 K)



Absolute isentropic wind velocity (**black contours**) and meridional mass flux (MMF) (**red: northward mass flux, blue: southward mass flux**) in layer between 330 K and 350 K (350 K corresponds approx. to 200 hPa)



# A front separates air masses

An air mass is characterised by a homogeneous distribution of a materially conserved quantity.

In adiabatic circumstances, *materially* conserved quantities are

1. Potential temperature
2. Potential vorticity

isentropic tropopause is a front

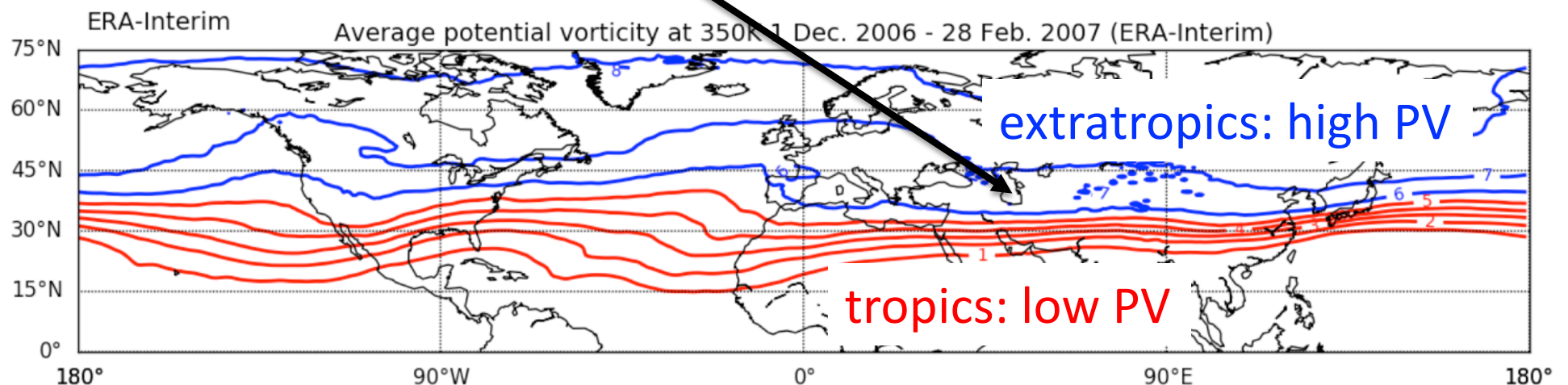
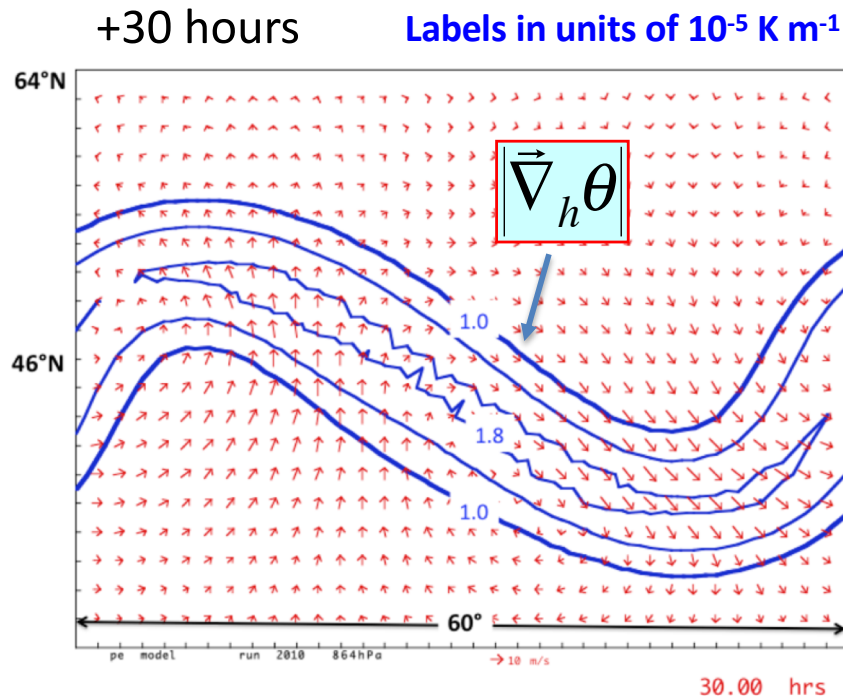


Figure 1.58

# The standard definition of the intensity of a front is terms of potential temperature



## Model simulation:

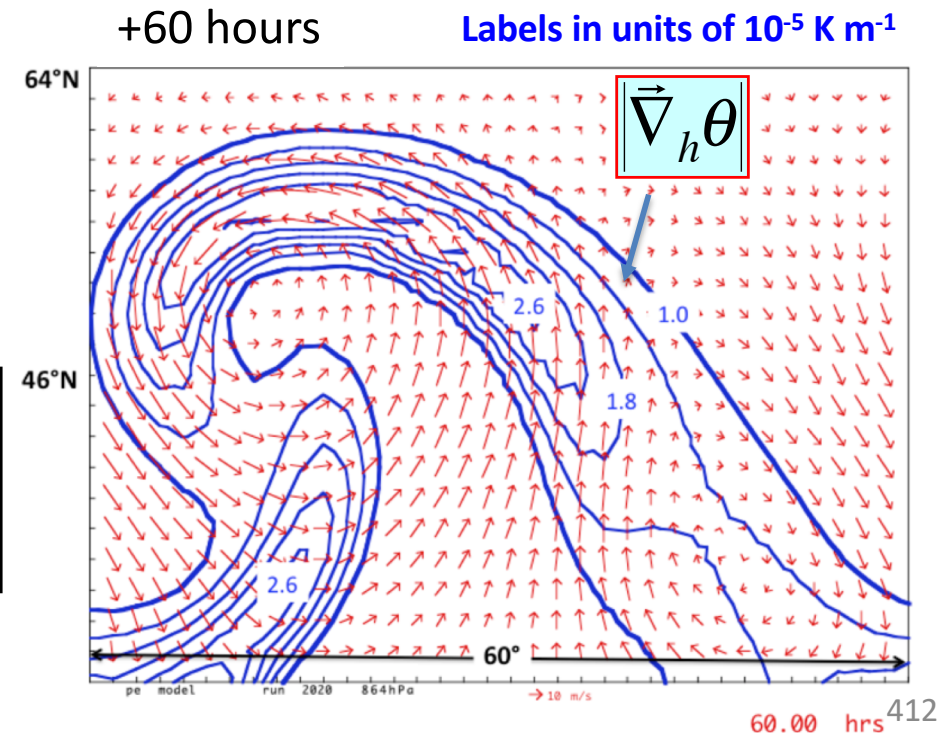
blue contours: *absolute value temperature gradient* on isobaric surface (at 864 hPa).

Red arrows: wind-vector.

Figure 1.129

**Frontogenesis:** a remarkable increase of the absolute temperature gradient within 30 hours !

$$\nabla_h \equiv \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, 0 \right)$$





# The "frontogenesis function"

The intensity of a front can be measured by the quantity  $(\vec{\nabla}_h \theta)^2$ .

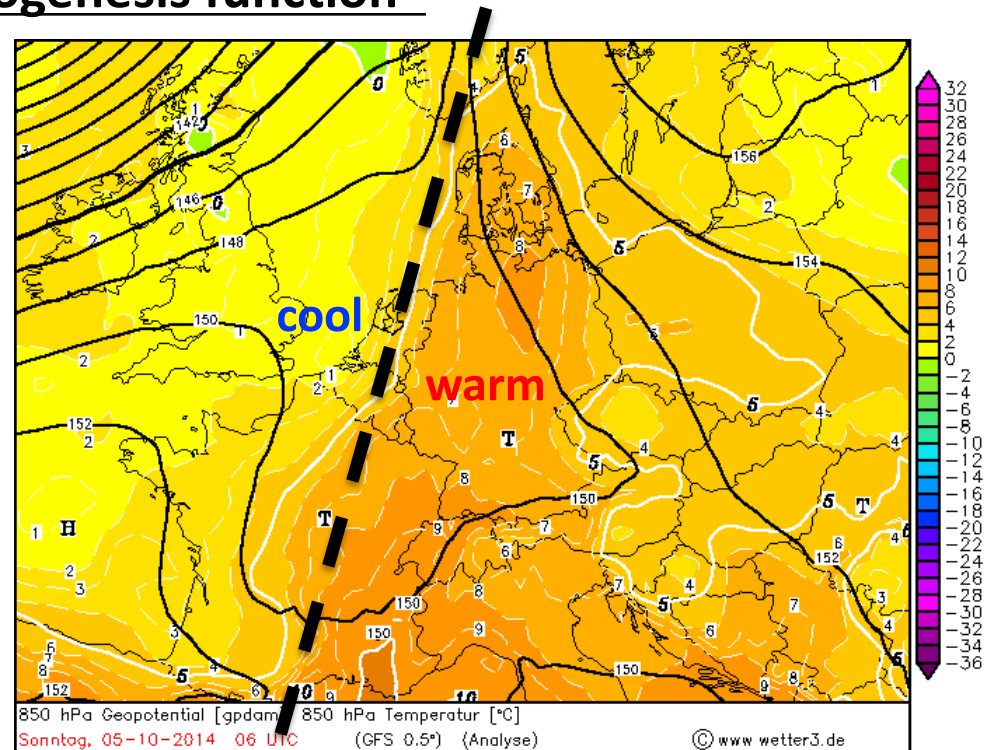
Intensity change is measured by

$$\frac{d(\vec{\nabla}_h \theta)^2}{dt} = 2 \frac{d\vec{\nabla}_h \theta}{dt} \cdot \vec{\nabla}_h \theta \equiv 2\vec{Q} \cdot \vec{\nabla}_h \theta$$

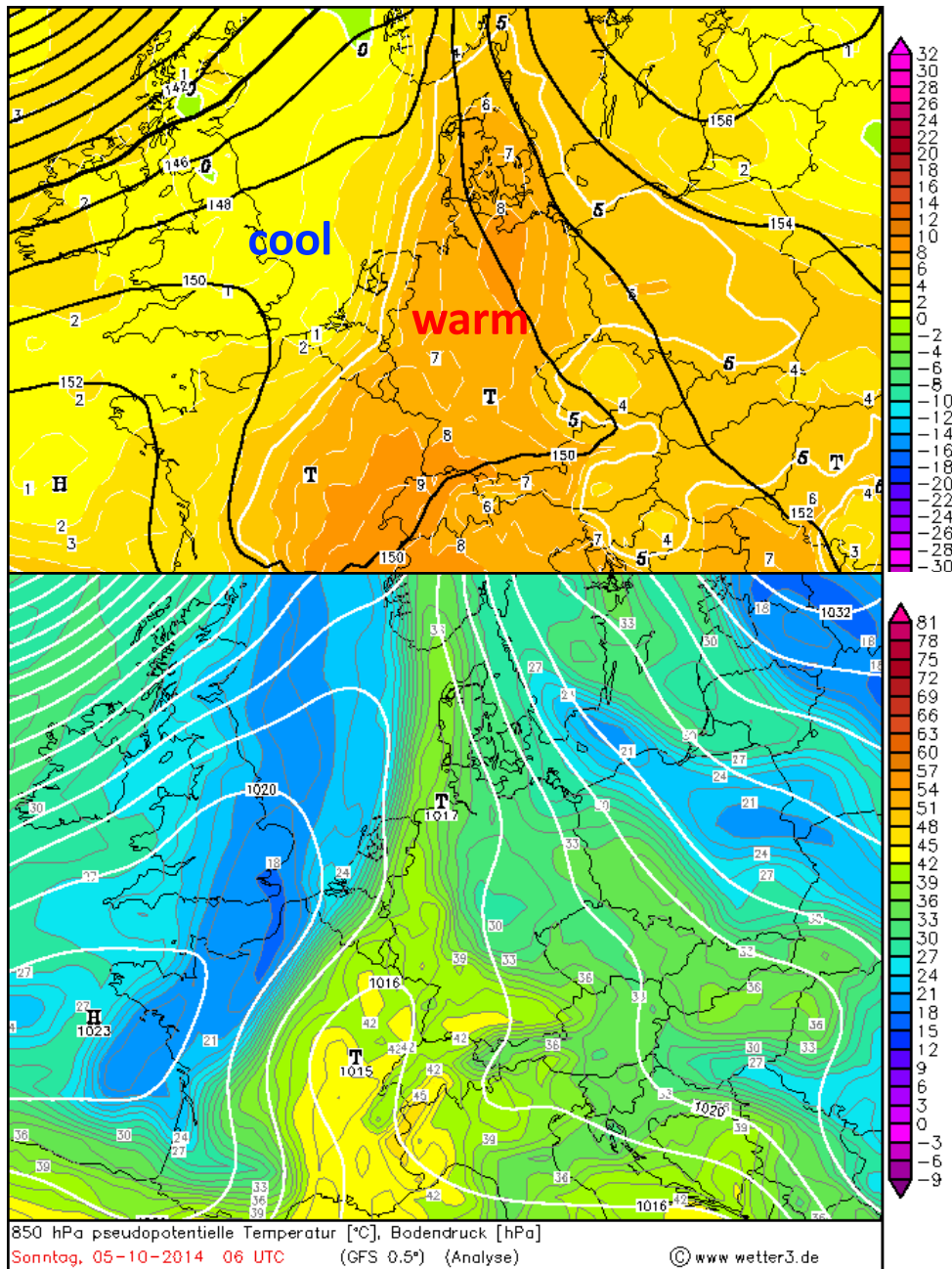
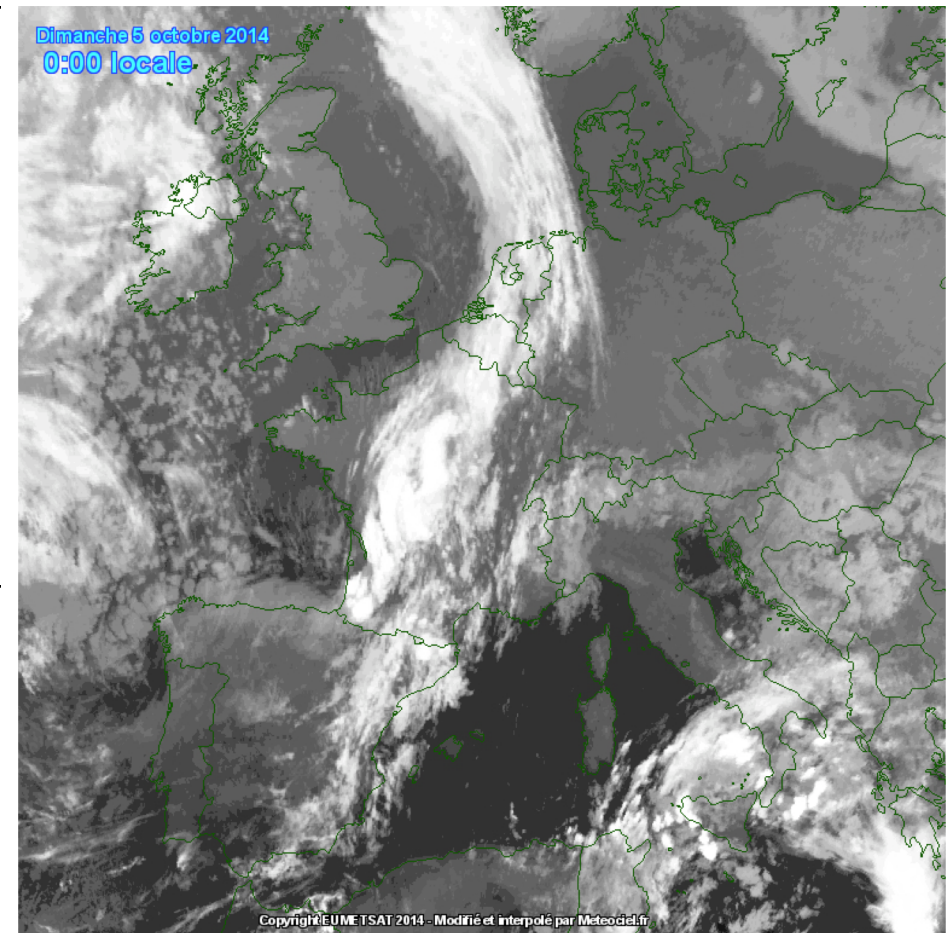
eq.1.341

↑  
"frontogenesis function"

Can we understand the formation of fronts with this definition of frontal intensity?



# Can we understand why fronts “produce” clouds ?



Can we understand vertical motion around fronts?

# Q-vector and frontogenesis

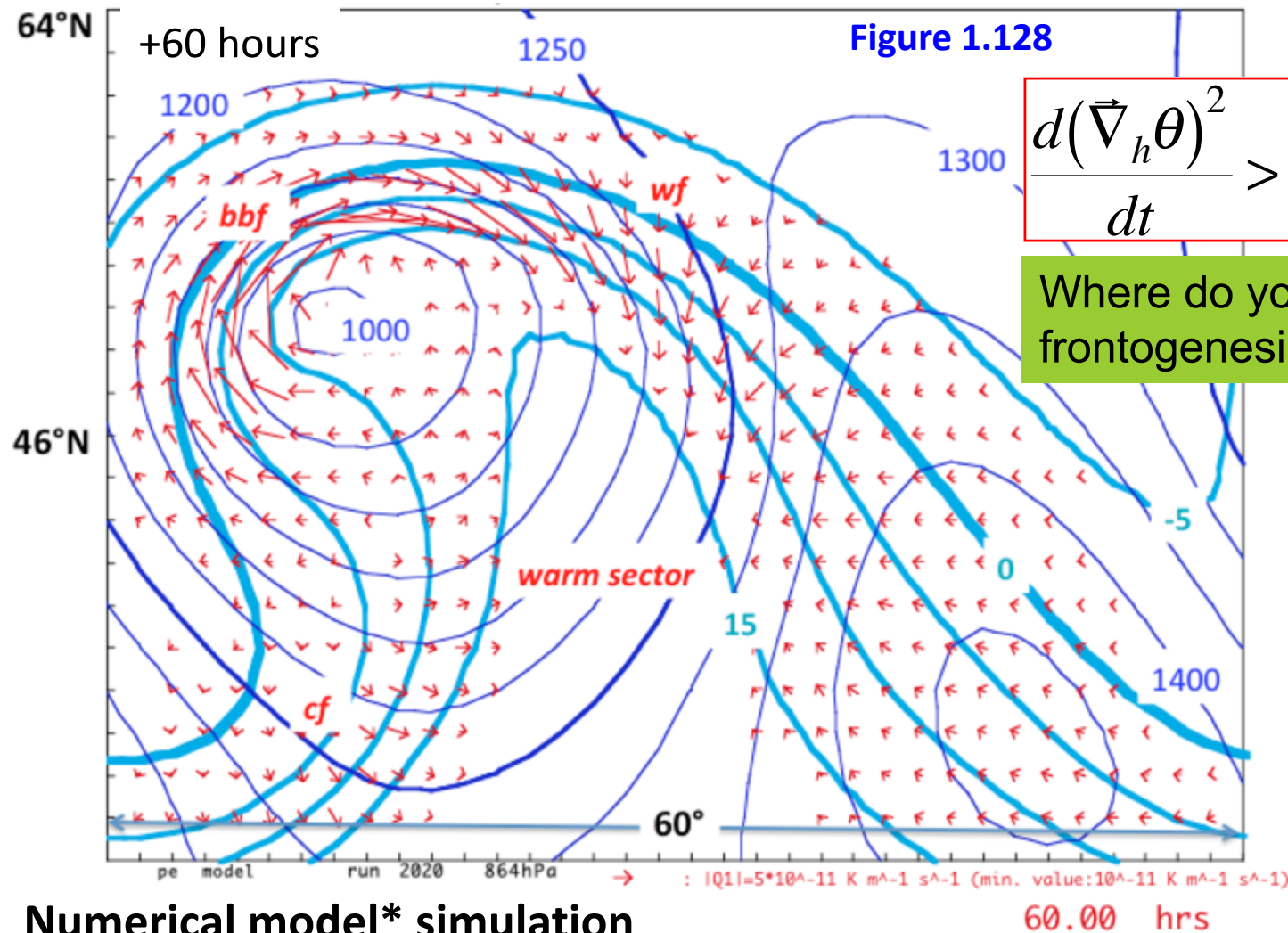
**“Q-vector”**:  $\vec{Q} \equiv (Q_1, Q_2) \equiv \frac{d\vec{\nabla}_h \theta}{dt}$

$$\frac{d(\vec{\nabla}_h \theta)^2}{dt} = 2 \frac{d\vec{\nabla}_h \theta}{dt} \cdot \vec{\nabla}_h \theta \equiv 2\vec{Q} \cdot \vec{\nabla}_h \theta \quad \text{eq.1.341}$$

If the **Q-vector** is **perpendicular** to isentrope, potential temperature gradient increases/decreases (frontogenesis/frontolysis)

If the **Q-vector** is **parallel** to isentrope, potential temperature gradient changes direction (isentrope rotates), but front does not intensify

# Frontogenesis in a baroclinic wave



$$\frac{d(\vec{\nabla}_h \theta)^2}{dt} > 0 \text{ if } 2\vec{Q} \cdot \vec{\nabla}_h \theta > 0$$

Where do you observe frontogenesis in this example?

## Numerical model\* simulation

Blue contours: geopotential height labeled in m (864 hPa)

cyan contours: isotherms on isobaric surface labeled in °C

Red arrows: Q-vector.

\*Model-description in chapter 10

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<https://webpace.science.uu.nl/~delde102/BaroclinicLifeCycle.htm>



# Equation governing $Q_1$

**x-component** of the frontogenesis function:

$$\frac{d}{dt} \left( \frac{\partial \theta}{\partial x} \right)^2 = 2 \frac{\partial \theta}{\partial x} \frac{d}{dt} \left( \frac{\partial \theta}{\partial x} \right) \equiv 2 \frac{\partial \theta}{\partial x} \underset{\uparrow}{Q_1}$$

x-component of Q-vector

If **adiabatic**:

$$\frac{\partial}{\partial x} \left( \frac{d\theta}{dt} \right) = \frac{\partial}{\partial x} \left( \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) = 0$$

$$\left( \frac{\partial}{\partial t} \frac{\partial \theta}{\partial x} + u \frac{\partial}{\partial x} \frac{\partial \theta}{\partial x} + v \frac{\partial}{\partial y} \frac{\partial \theta}{\partial x} + w \frac{\partial}{\partial z} \frac{\partial \theta}{\partial x} + \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} + \frac{\partial v}{\partial x} \frac{\partial \theta}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial \theta}{\partial z} \right) = 0$$

$$\frac{d}{dt} \frac{\partial \theta}{\partial x} \equiv Q_1 = - \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} - \frac{\partial v}{\partial x} \frac{\partial \theta}{\partial y} - \frac{\partial w}{\partial x} \frac{\partial \theta}{\partial z}$$

# Derivation of equation for $Q_2$

Result of previous slide:

$$\frac{d}{dt} \frac{\partial \theta}{\partial x} \equiv Q_1 = -\frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} - \frac{\partial v}{\partial x} \frac{\partial \theta}{\partial y} - \frac{\partial w}{\partial x} \frac{\partial \theta}{\partial z}$$

y-component of the frontogenesis function:

$$\frac{d}{dt} \left( \frac{\partial \theta}{\partial y} \right)^2 = 2 \frac{\partial \theta}{\partial y} \frac{d}{dt} \left( \frac{\partial \theta}{\partial y} \right) \equiv 2 \frac{\partial \theta}{\partial y} Q_2$$

↑  
y-component of Q-vector

Derive an expression for  $Q_2$   
assuming adiabatic conditions

# Interpretation of frontogenetic terms

From previous slides:

$$\frac{d}{dt} \frac{\partial \theta}{\partial x} \equiv Q_1 = -\frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} - \frac{\partial v}{\partial x} \frac{\partial \theta}{\partial y} - \frac{\partial w}{\partial x} \frac{\partial \theta}{\partial z} \quad \text{eq.1.343a}$$

$$\frac{d}{dt} \frac{\partial \theta}{\partial y} \equiv Q_2 = -\frac{\partial u}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial v}{\partial y} \frac{\partial \theta}{\partial y} - \frac{\partial w}{\partial y} \frac{\partial \theta}{\partial z} \quad \text{eq.1.343b}$$

Which two terms in the equations above expresses the frontogenetic processes illustrated in figures **a** and **b**, shown on the right?

**a. Confluence:**  
 $u \equiv -\alpha x; \quad v \equiv \alpha y$

**b. Tilting/shear:**  
 $u \equiv 0; \quad v \equiv \alpha x$

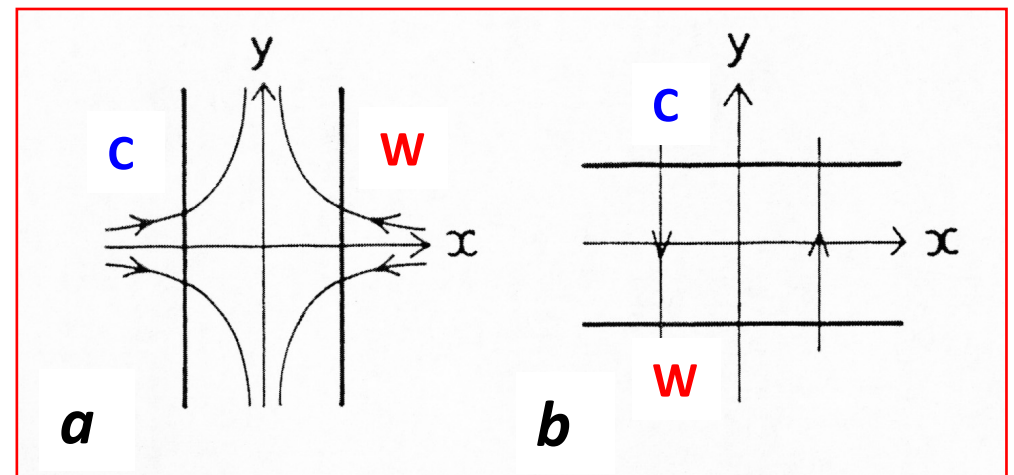
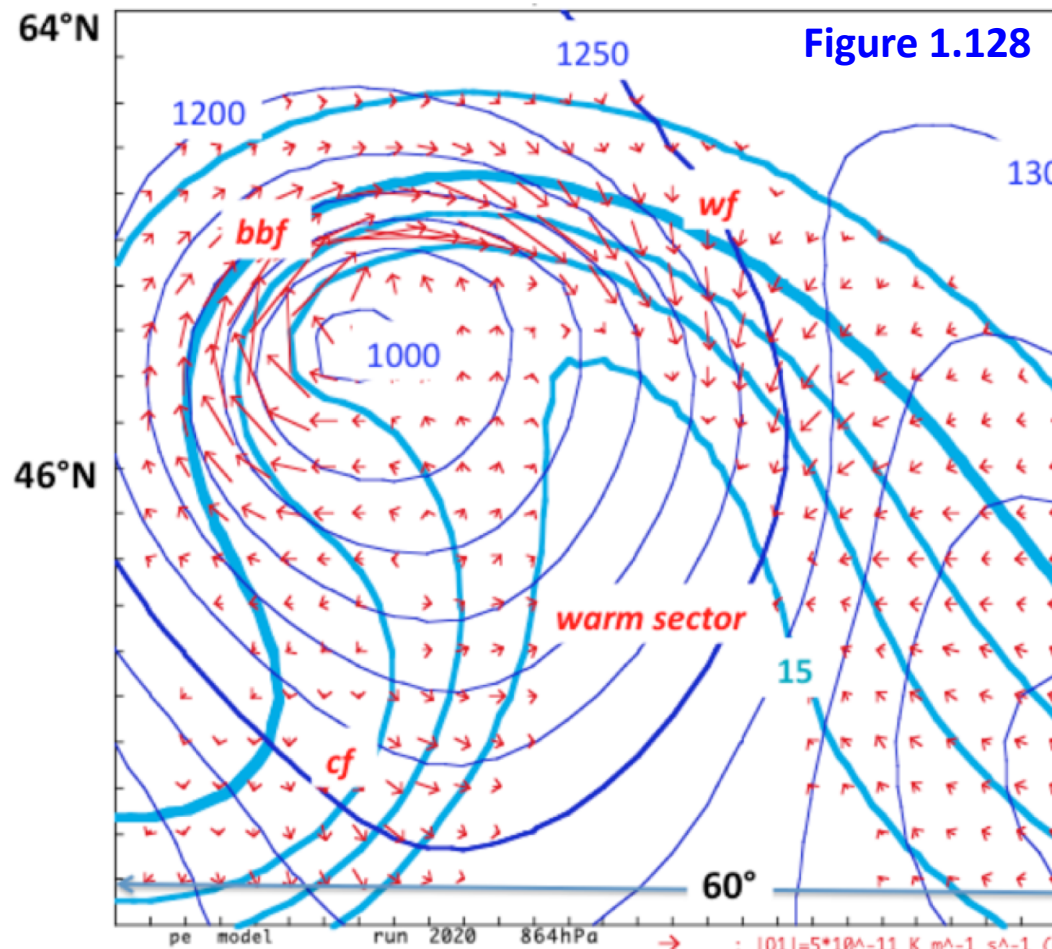


**Figure 1.127** **Arrows:** streamlines/x- or y-axis; **C:**Cold; **W:**Warm

# Dynamical role of frontogenesis



Absolute value of temperature gradient increases if Q-vector points up the existing temperature gradient, as can be observed in the cold front (*cf*) and in the warm front (*wf*) in the case shown here.

This process of frontogenesis brings the atmosphere out of thermal wind balance and this, in turn, requires an adjustment, which is manifest as a “cross-frontal vertical circulation”, which *redistributes momentum and heat in such a way that thermal wind balance is retained*.

**This adjustment process is described using the quasi-geostrophic equations**

60.00 hrs



# Dynamical consequences of frontogenesis

Frontogenesis disturbs thermal wind balance. The atmosphere responds by re-adjusting to thermal wind balance. This process is necessarily accompanied by vertical motion, as we shall see next.

Thermal wind balance:  
(in *pressure coordinates*)

$$\frac{\partial u_g}{\partial p} = \frac{R}{pf_0} \frac{\partial T}{\partial y}; \quad \frac{\partial v_g}{\partial p} = -\frac{R}{pf_0} \frac{\partial T}{\partial x}$$

eq.1.347

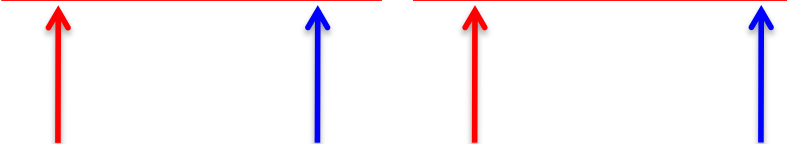
Derive these two equations

**Frontogenesis is associated with changing temperature gradients: it disturbs thermal balance**

Thermal wind balance:  
(in pressure coordinates)

$$\frac{\partial u_g}{\partial p} = \frac{R}{pf_0} \frac{\partial T}{\partial y}; \quad \frac{\partial v_g}{\partial p} = -\frac{R}{pf_0} \frac{\partial T}{\partial x}$$

eq.1.347



Changing temperature gradient must be associated with changing thermal wind

In order to evaluate the consequences of this process we need an equation governing the time evolution of temperature in a quasi-geostrophic atmosphere

To study the the process of frontogenesis and its dynamical consequences we need the ...

## Quasi-geostrophic thermodynamic equation

In pressure coordinates:

$$\frac{\partial T}{\partial t} + u \left( \frac{\partial T}{\partial x} \right)_p + v \left( \frac{\partial T}{\partial y} \right)_p - \left( \frac{\alpha}{c_p} - \frac{\partial T}{\partial p} \right) \omega = + \frac{J}{c_p} \quad (\text{eq. 1.195})$$

In short:

$$\frac{\partial T}{\partial t} + u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} - S_p \omega = + \frac{J}{c_p}$$

Static Stability parameter (see problem 1.36)  
( $S_p > 0$ )

Quasi-geostrophic approximation:

$$u \approx u_g; v \approx v_g$$

$$\frac{\partial T}{\partial t} + u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} - S_p \omega = + \frac{J}{c_p}$$

**Vertical motion,  $\omega$ , is retained !**

**Why?**

# Equations needed to study the dynamics of frontogenesis in a quasi-geostrophic atmosphere

Thermal wind balance:  $\frac{\partial u_g}{\partial p} = \frac{R}{pf_0} \frac{\partial T}{\partial y}$   $\frac{\partial v_g}{\partial p} = -\frac{R}{pf_0} \frac{\partial T}{\partial x}$  eq.1.347

and  $\frac{\partial T}{\partial t} + u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} - S_p \omega = + \frac{J}{c_p}$  (previous slide)

also:  $\frac{\partial u_g}{\partial t} + u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} = f_0 v_a + \beta y v_g$  (1.247a)

Based upon the above equations, the next lecture investigates the relation between vertical motion in a statically stable atmosphere and frontogenesis/frontolysis.

**Next:**

# Dynamic Meteorology: lecture 15

**21/1/2022 (Friday, 9:00-10:45)**

Sections **1.42-1.43** (part I)

## **Vertical motion in statically stable atmosphere part 2**

Ageostrophic motion is due to frontogenesis

Large-scale vertical motion: the omega-equation

Relation between Q-vector and large-scale vertical motion

**Case of problem 1.35** : vertical velocity according to the **omega equation**

## **After lecture 15: eight presentations of project 5**

**Abhi, Filippo, Maren and Martin** - NAO, EA, temp Florida, temp Middle East

**Jim, Koen and Derk** – Tropical cyclones and NAO/NAM

**Famke, Marijn and Hannah** – AA, jet stream shift and weakening of storm tracks

**Laura, Elia and Emilia** – cyclones, SST, wind and moisture

**Rens, Marjolein and Marius** - hurricanes, SST, jet and rising CO2 levels

**Jos and Ben** - Sea Ice, NAO, SOI, Winter temp NL

**Anja, Rainette, Joao and Mattia** – Sea ice in arctic and Antarctic, (ant) arctic oscillations and global mean temp

**Lorenzo, Fiona and Lotje** – Himalayan ice thickness, rising CO2, rising temp and precipitation

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(<http://www.uu.nl/~nvdelden/dynmeteorology.htm>)