Wednesday 16 November 2022

Dynamical Meteorology: lecture 1

Preface and section 1.2-1.5 of the lecture notes

Introduction: phenomenolgy of atmospheric dynamics
Variables: pressure, temperature, density and velocity
Basic equations derived from budgets of mass, energy and momentum

Tutorial this afternoon (15:15-17:00)**:**

Study part A of the lecture notes (especially sections 1.2-1.5)

Problems 1.2 and 1.3 (see page 52 of the lecture notes)

Lectures 2 and 3 also serve as an introduction to Project 1

Theoretical concepts

PROJECT 1 - PROBLEM 1.6 in the lecture notes (p.53)
(in couples) (10%)

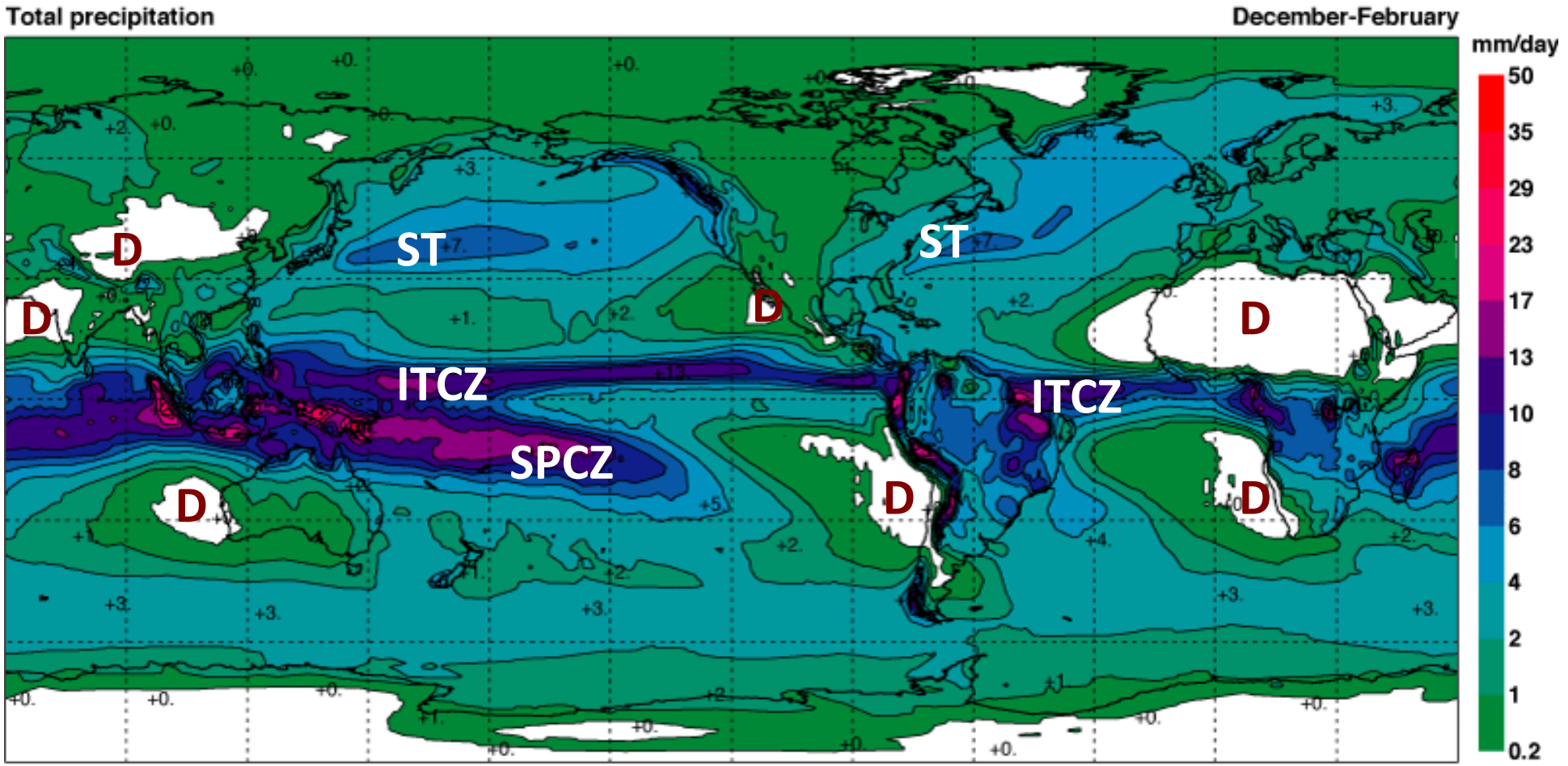
Model of a parcel moving vertically under influence of buoyancy

In this small research project you will construct your first (Lagrangian) numerical simulation model.

The following ten theoretical concepts or topics come up for discussion.

1. Equation for the vertical acceleration of an air parcel under influence of pressure and gravity
2. Hydrostatic balance
3. Boussinesq approximation
4. Buoyancy
5. Adiabatic process and (dry-adiabatic) lapse rate
6. Potential temperature and potential temperature conservation
7. Static stability / instability
8. Buoyancy oscillations and Brunt Väisälä frequency
9. Convective Available Potential Energy (CAPE)
10. Lagrangian model

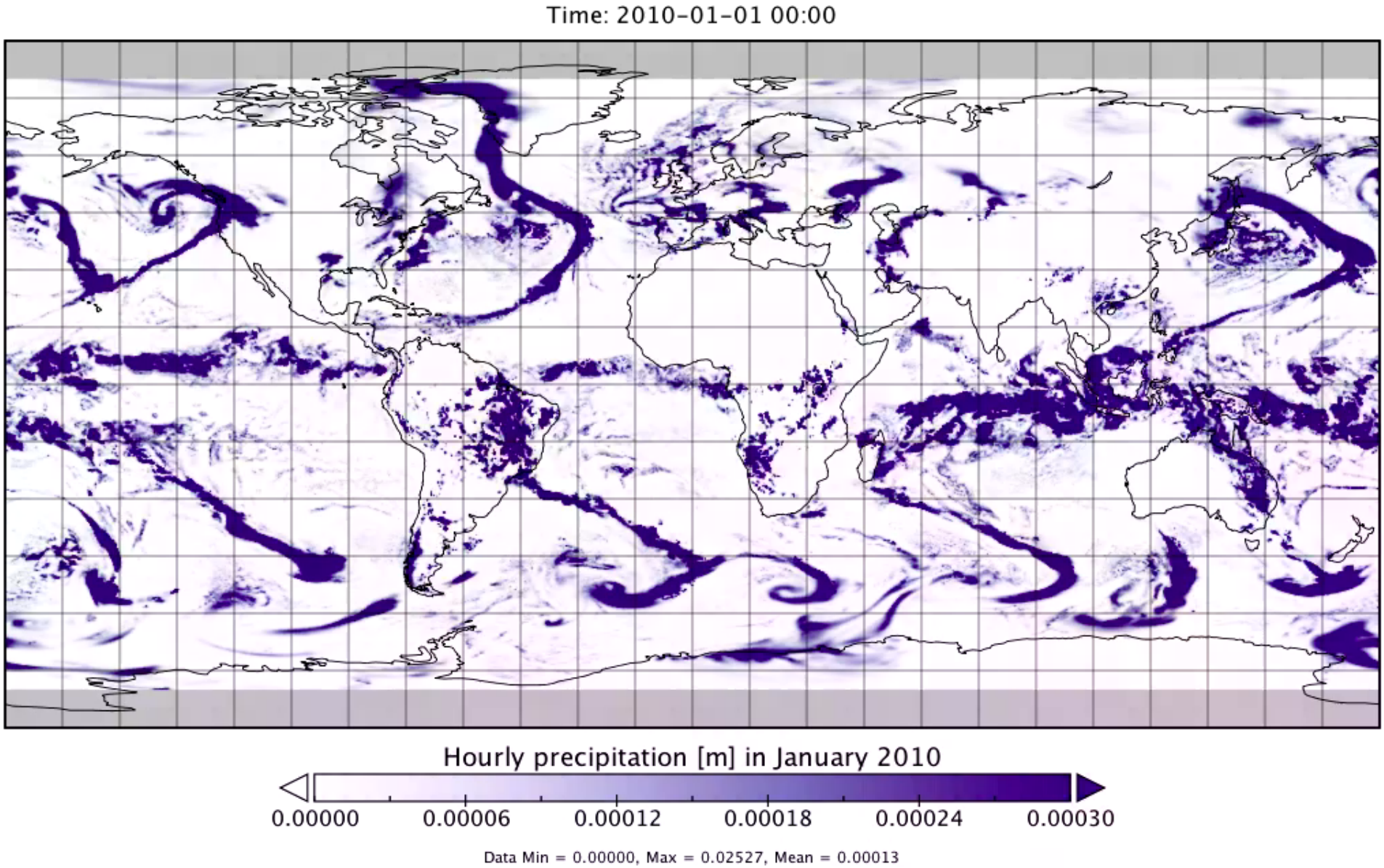
Total Precipitation & Climate Zones



ITCZ=Intertropical convergence zone, SPCZ=South Pacific convergence zone, ST=Storm track, D=Desert

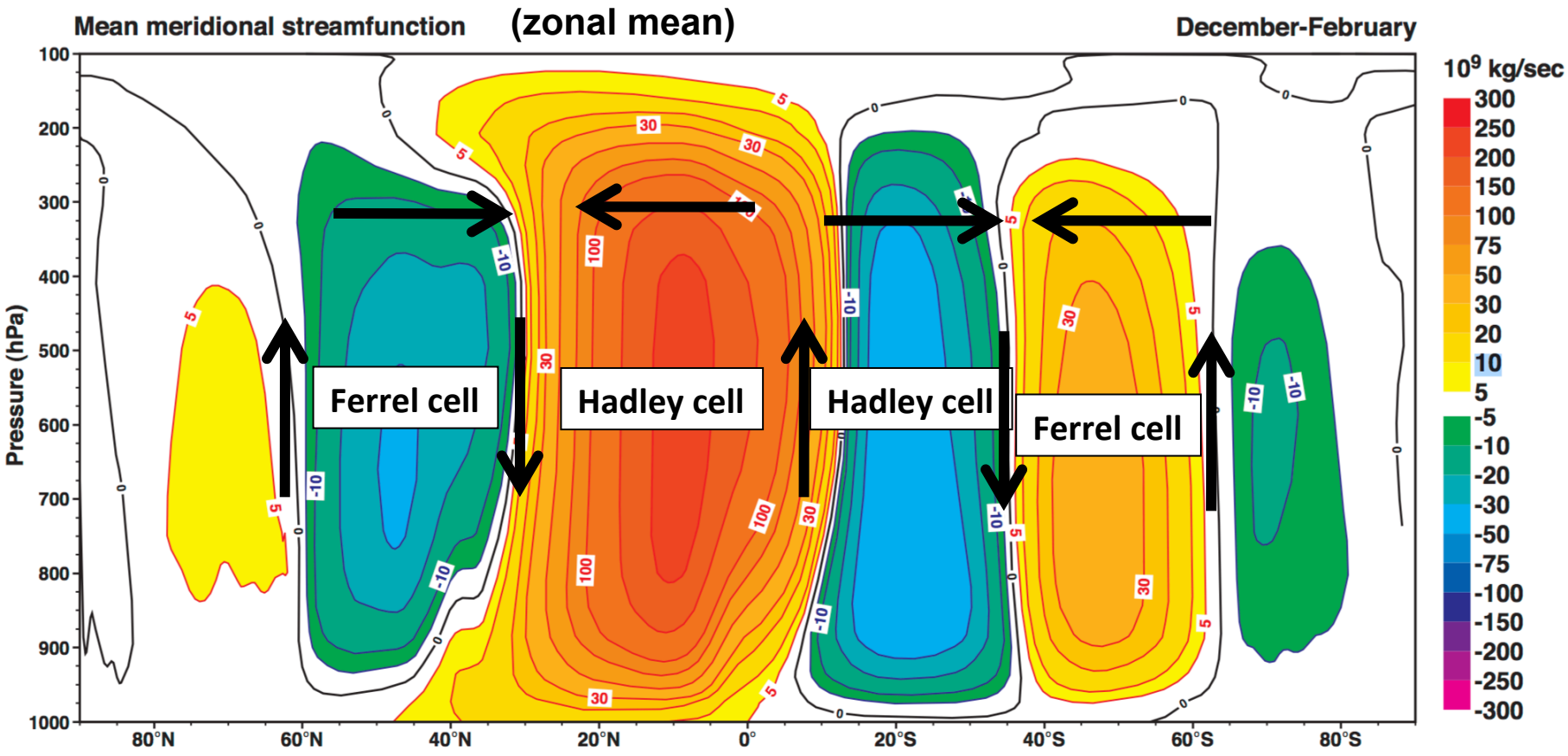
ERA-40

Re-analysis of hourly precipitation (ERA-5)



ERA5: https://rmets.onlinelibrary.wiley.com/doi/full/10.1002/qj.3803

Response is different from expectations



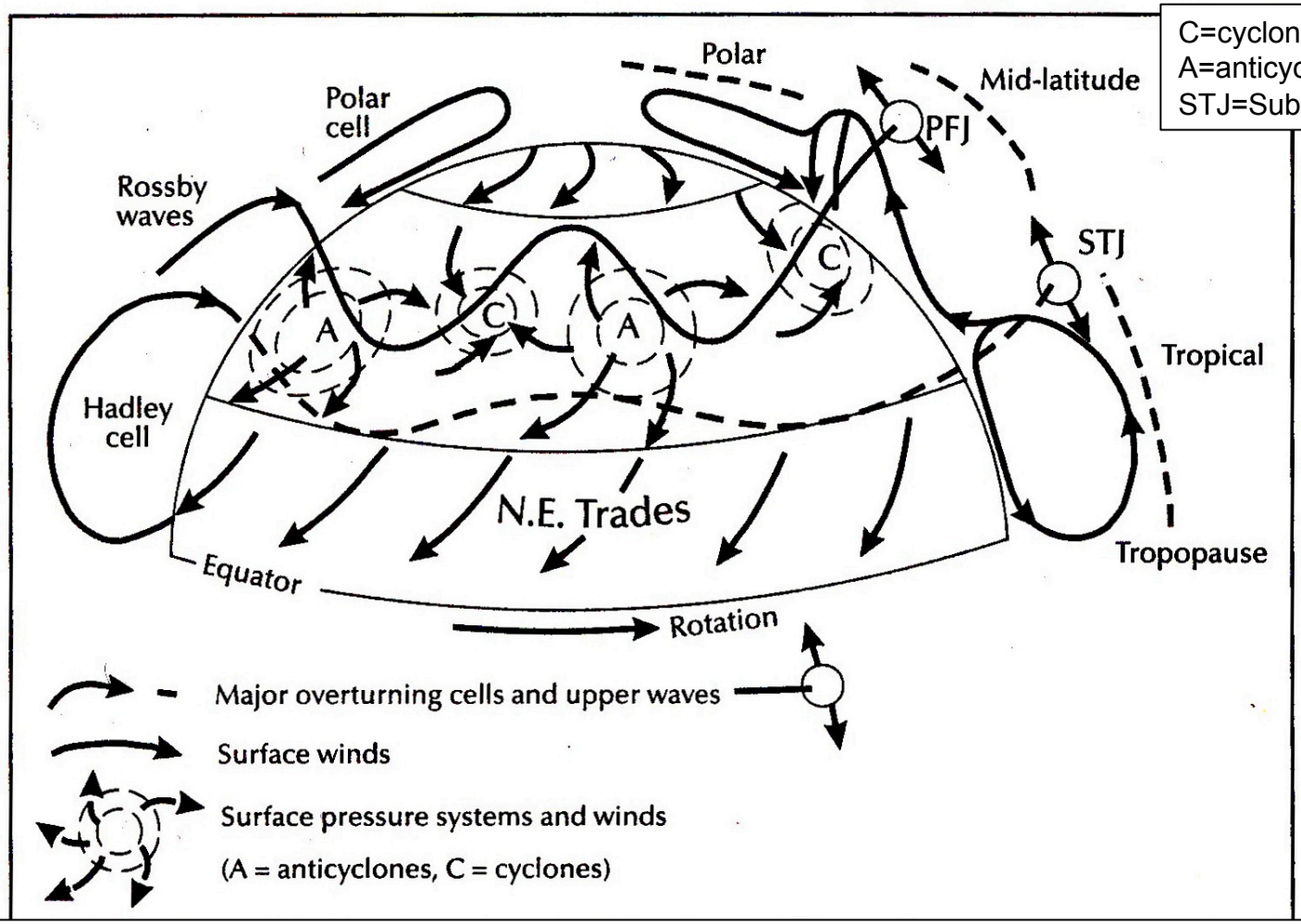
Hadley cell is the dynamical response to heating in the ITCZ

Ferrel cell is response to what?

ERA-40

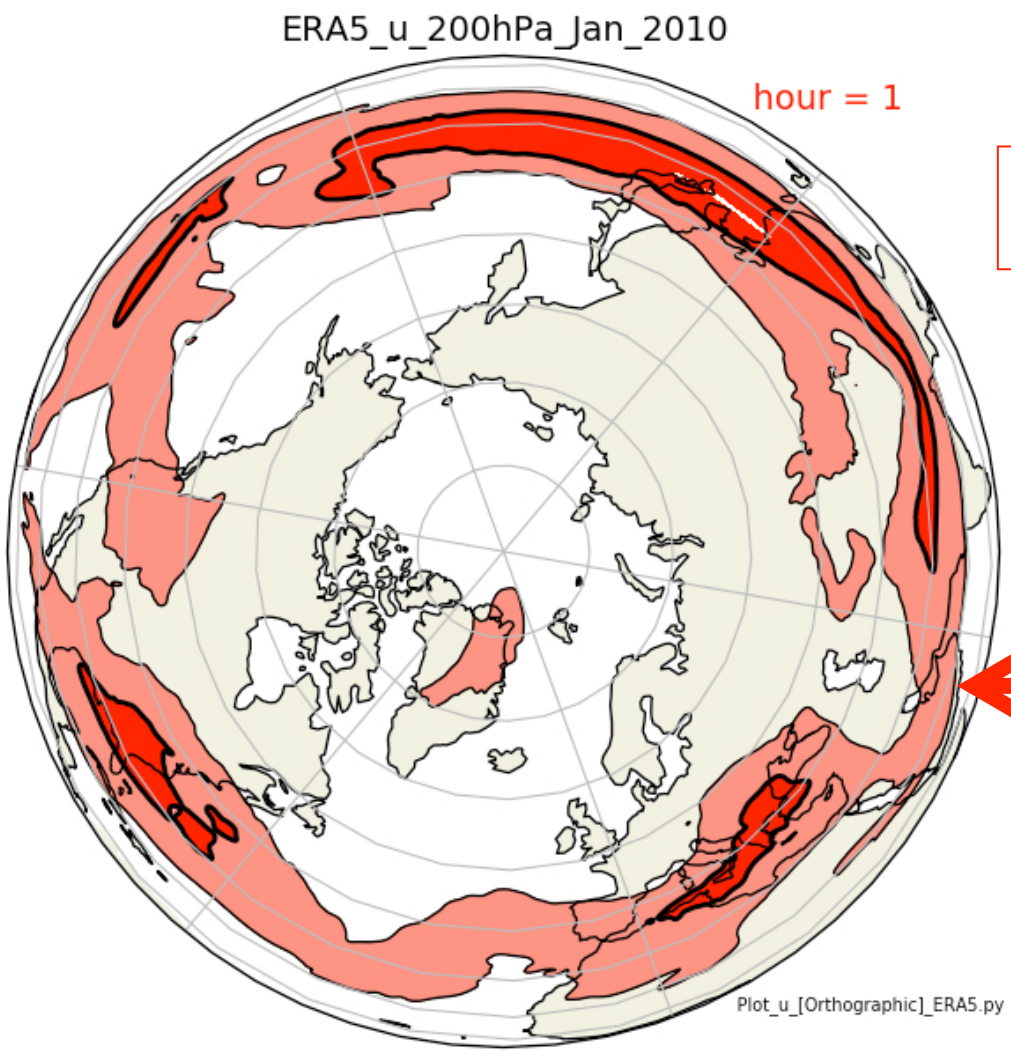
General Circulation

C=cyclone
A=anticyclone
STJ=Sub-Tropical Jet



Conceptual/schematic picture of the “General Circulation” in the northern hemisphere (from **Climatology, third edition**, by J.Hidore, J.Oliver, M.Snow & R.Snow, 2010, Prentice Hall).

Zonal wind velocity (u) in the upper troposphere



ERA5 re-analysis,January 2010)

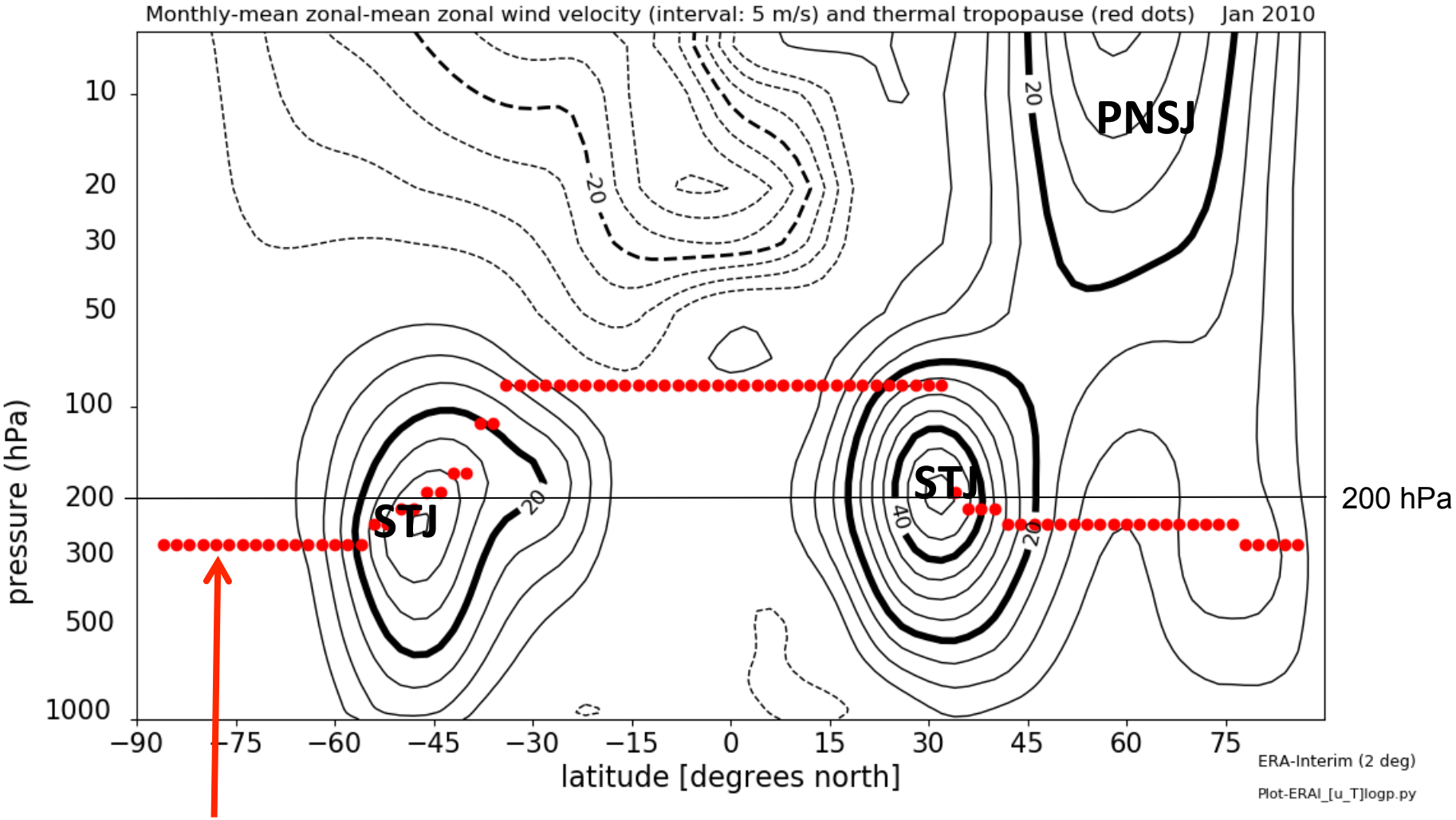
Salmon: $30<u<60$ m s^{-1}
Red: $u>60$ m s^{-1}

← Sub-Tropical Jet (STJ)

ERA5: https://rmets.onlinelibrary.wiley.com/doi/full/10.1002/qj.3803

Zonal mean zonal wind, *u*

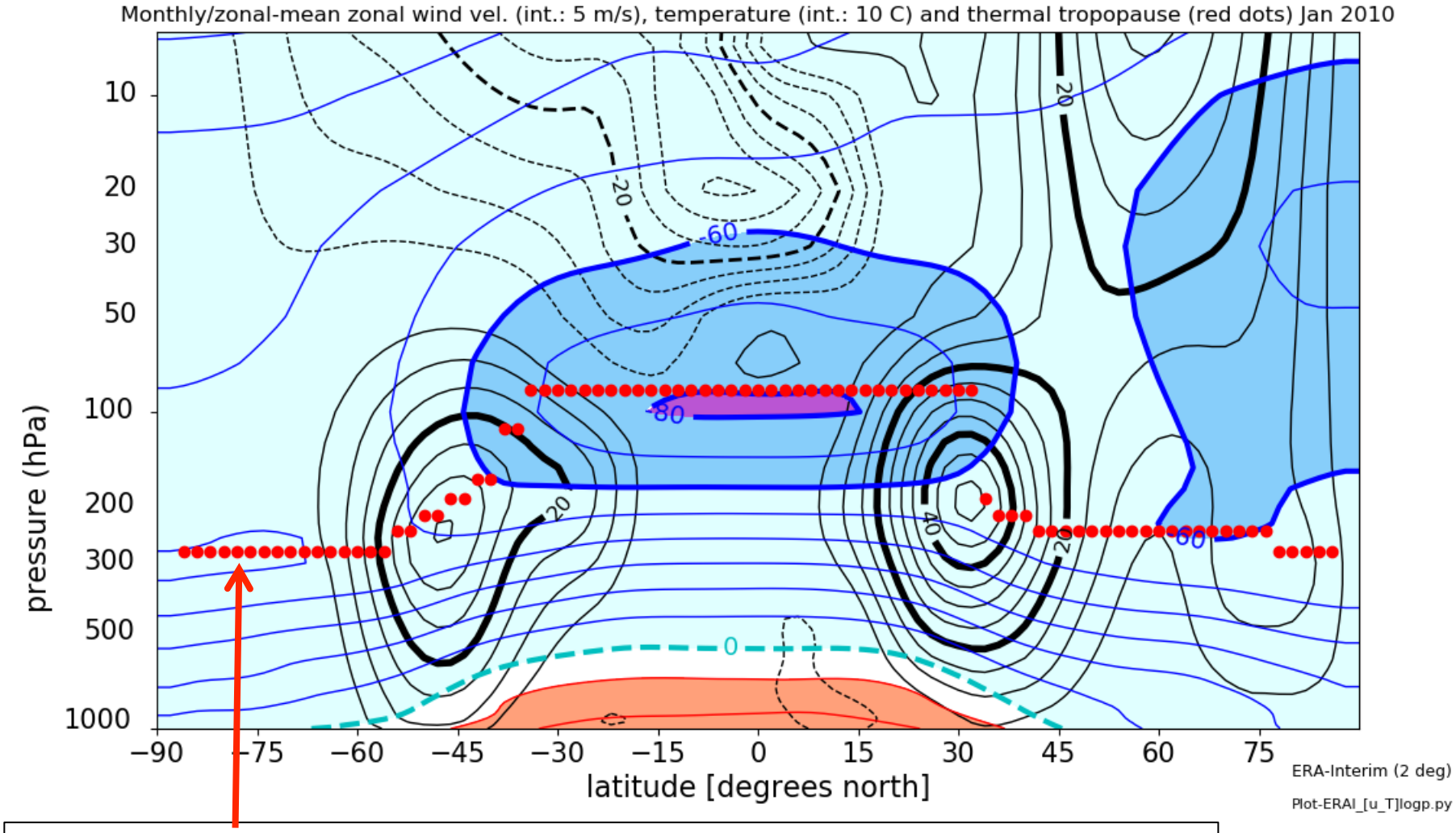
Subtropical jet (STJ), polar night stratospheric jet (PNSJ)



thermal tropopause (see pages 23-26, lecture notes)

Add zonal mean temperature, *T*

(contour interval = 10 °C)



Thermal tropopause is the level which vertical temperature gradient changes suddenly

(see pages 23-26, lecture notes)

What concepts do we need to describe these phenomena?

Object of study: an "**air parcel**"

Are you familiar with the thermodynamic concepts of **temperature** and **pressure** of a fluid or gas, …

and with the **conservation laws** or **budget equations** governing the fluid (air) parcel in a rotating frame of reference?

What concepts do we need to describe these phenomena?

Object of study: an "**air parcel**"

Are you familiar with the thermodynamic concepts of **temperature** and **pressure** of a fluid or gas, ...

What is pressure ?

and with the **conservation laws** or **budget equations** governing the fluid (air) parcel in a rotating frame of reference?

What is conserved ?

What concepts do we need to describe these phenomena?

Object of study: an "**air parcel**"

Are you familiar with the thermodynamic concepts of **temperature** and **pressure** of a fluid or gas, …

and with the **conservation laws** or **budget equations** governing the fluid (air) parcel in a rotating frame of reference?

What is pressure ?

Write your answer to this question on a piece of paper and hand it in

What concepts do we need to describe these phenomena?

Object of study: an "air parcel"

Density, ρ
Speed, $\vec{v}$
Pressure, p
Temperature, T

(**four unknowns**)

We need four equations to solve the "problem"

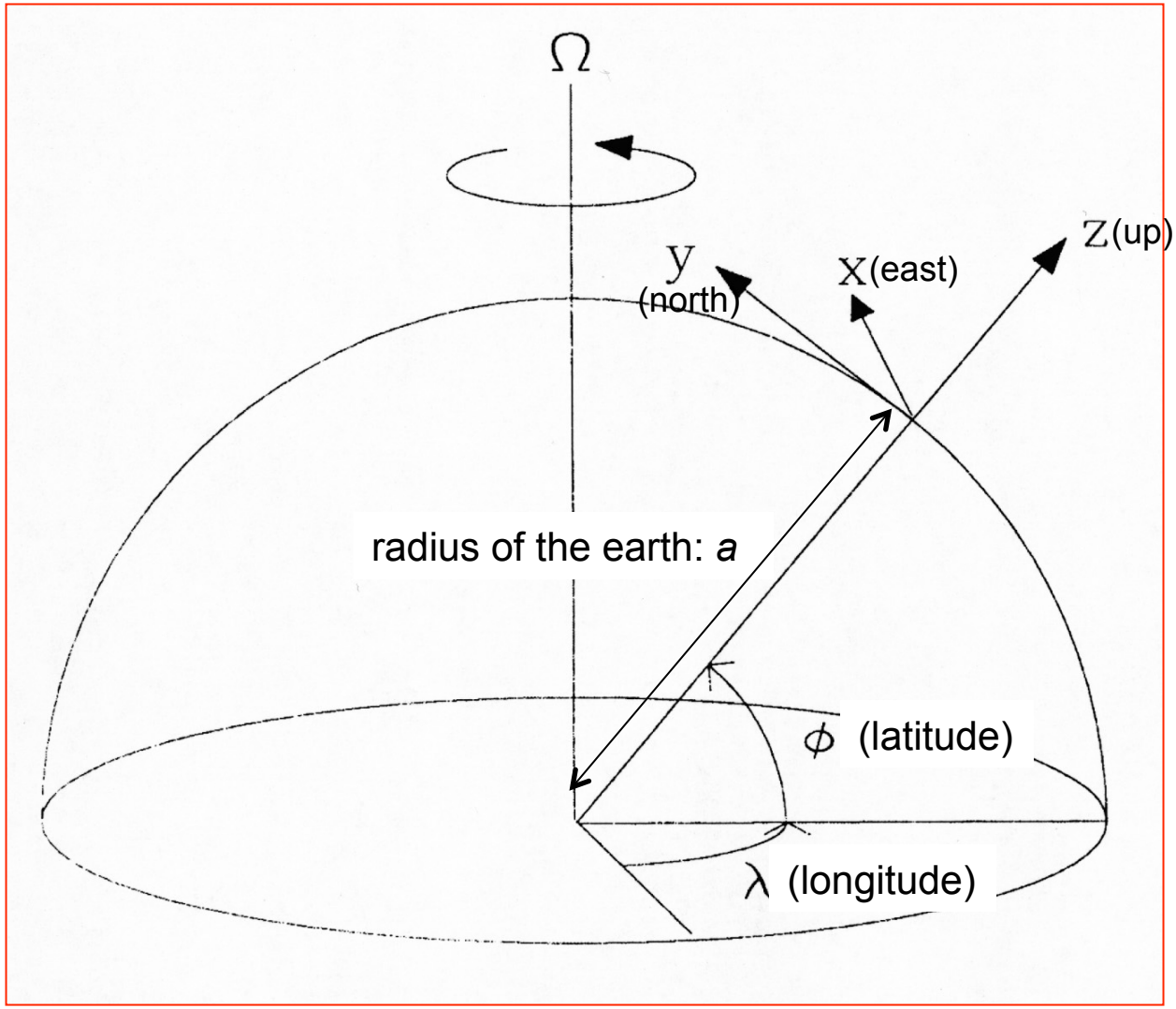
We need four equations to solve the "problem": what equations?

Equations describing the **budgets** of **momentum, mass and energy**

And the **equation of state**

This yields a closed system of four equations with four unknowns

The coordinate system



*see geophysical fluid dynamics and sections 1.2, 1.3 and 1.4 of lecture notes

The equations ("laws") of Dynamical Meteorology

Unknowns are velocity, density, temperature and pressure: $\vec{v}, \rho, T, p$

momentum

$$\frac{d\vec{v}}{dt} = -\alpha\vec{\nabla}p - g\hat{k} - 2\vec{\Omega}\times\vec{v} + \overrightarrow{Fr} \qquad \alpha \equiv \frac{1}{\rho}$$

Pressure gradient (1.3) | Gravity (1.3) | Coriolis (1.7) | Friction (1.8)

(explain symbols on the blackboard !!)

*see geophysical fluid dynamics and sections 1.2, 1.3 and 1.4 of lecture notes

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Pressure gradient (1.3) | Gravity (1.3) | Coriolis (1.7) | Friction (1.8)

mass

$$\frac{d\rho}{dt} = -\rho\vec{\nabla}\cdot\vec{v}$$

eq. 1.6, lecture notes

(explain symbols on the blackboard !!)

*see geophysical fluid dynamics and sections 1.2, 1.3 and 1.4 of lecture notes

The equations ("laws") of Dynamical Meteorology

Unknowns are velocity, density, temperature and pressure: $\vec{v}, \rho, T, p$

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Pressure gradient (1.3) | Gravity (1.3) | Coriolis (1.7) | Friction (1.8)

mass

$$\frac{d\rho}{dt} = -\rho\vec{\nabla}\cdot\vec{v}$$

eq. 1.6, lecture notes

energy

$$mc_v dT + pdV = 0$$

eq. 1.16, lecture notes

state

$$p = R\rho T$$

eq. 1.10b, lecture notes

(symbols explained on the blackboard)

These equations form a closed set !!

But what is missing?

These physical laws were known in 1904

The problem of weather prediction, considered from the viewpoints of mechanics and physics

VILHELM BJERKNES

If it is true, as any scientist believes[E1]*, that subsequent states of the atmosphere develop from preceding ones according to physical laws, one will agree that the necessary and sufficient conditions for a rational solution of the problem of meteorological prediction are the following:*

1. *One has to know with sufficient accuracy the state of the atmosphere at a given time.*
2. *One has to know with sufficient accuracy the laws according to which one state of the atmosphere develops from another.*

BJERKNES, V., 1904: Das Problem der Wettervorhersage, betrachtet vom Standpunkte der Mechanik und der Physik. – Meteorol. Z. **21**, 1–7.

We know the “laws”,

but we cannot calculate exactly how one state of the atmosphere develops into another

Why ?

Give three reasons

Why is there a limit to predictability?

1. Turbulence
2. Boundary conditions (initial value) not known exactly
3. Non-linearity

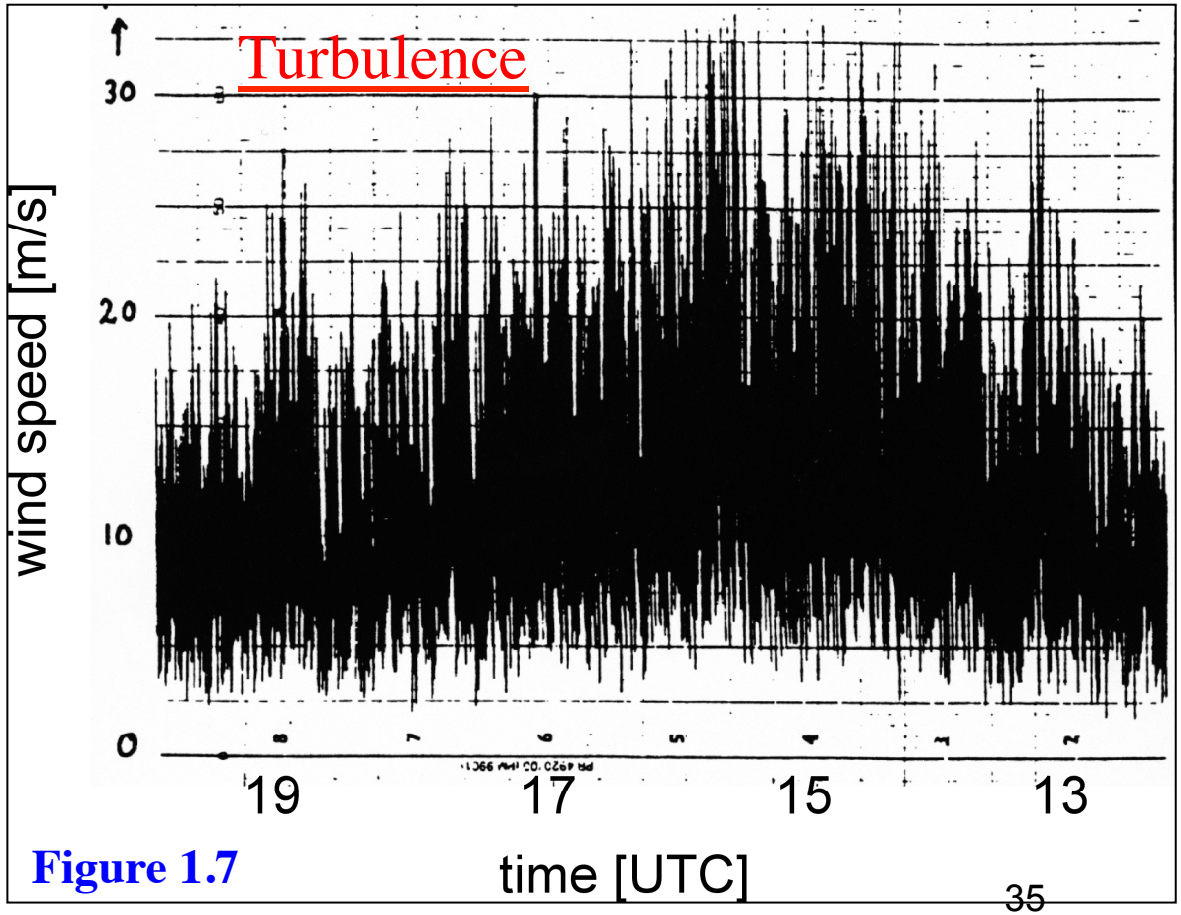


Figure 1.7

Windspeed in m s^{-1} on January 25, 1990 between 1300 and 1900 UTC (time runs from right to left) at Amersfoort (the Netherlands) during one of the most severe wind storms in the Netherlands in the twentieth century.

Richard Feynman (1965 Physics nobel-prize winner) about Meteorology in 1960

Meteorology as a branch of physics

Feynman Lectures in Physics:

“The theory of meteorology has never been satisfactorily worked out by the physicist. "Well," you say, "there is nothing but air, and **we know the equations of motions of air**". Yes, we do. "**So, if we know the condition of air today, why can't we figure out the condition of the air tomorrow?**".

First, we do not really know what the condition is today, because the air is swirling and twisting everywhere. It turns out to be very sensitive, and even unstable. If you have ever seen water run smoothly over a dam, and turn into a large number of blobs and drops as it falls, you will understand what I mean by unstable. You know the condition of the water before it goes over the spillway; it is perfectly smooth; but the moment it begins to fall, where do the drops begin? What determines how big the lumps are going to be? That is not known, because the water is unstable. Even a smooth moving mass of air, in going over a mountain turns into complex whirlpools and eddies. In many fields we find this situation of turbulent flow that we cannot analyze today. **Quickly we leave the subject of weather…**”

We will certainly *not* leave the subject of weather !

Momentum: $$\frac{d\vec{v}}{dt} = -\alpha\vec{\nabla}p - g\hat{k} - 2\vec{\Omega}\times\vec{v} + \overrightarrow{Fr}$$

Mass: $$\frac{d\rho}{dt} = -\rho\vec{\nabla}\cdot\vec{v}$$

Energy: $$mc_v dT + pdV = 0$$

State: $$p = R\rho T$$

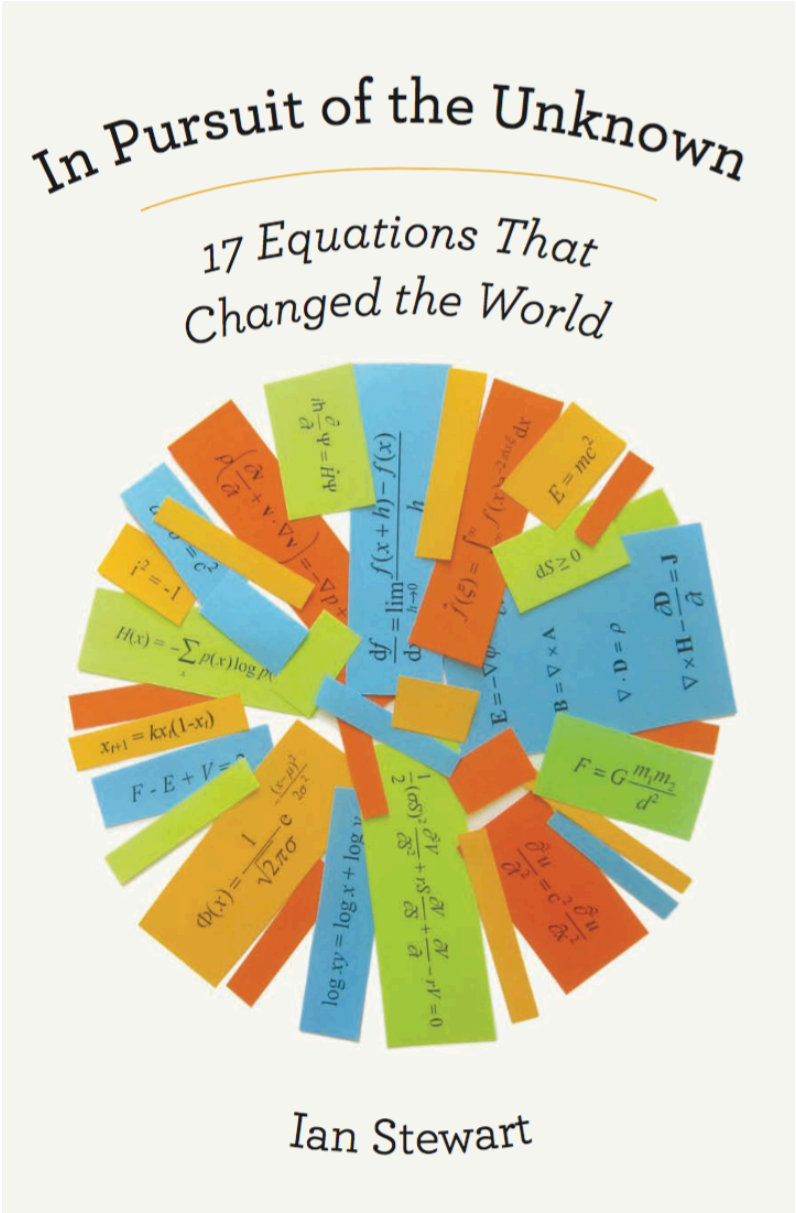
$\}^*$ $$c_p dT - \frac{1}{\rho}dp = 0$$

The analytical solution of these equations in general is **impossible** analytically, but since about 1950 we are using **computers** to obtain **numerical solutions**. Many **complex phenomena that we see in the atmosphere, such cyclones and fronts, are reproduced by numerical solutions of these equations** !!

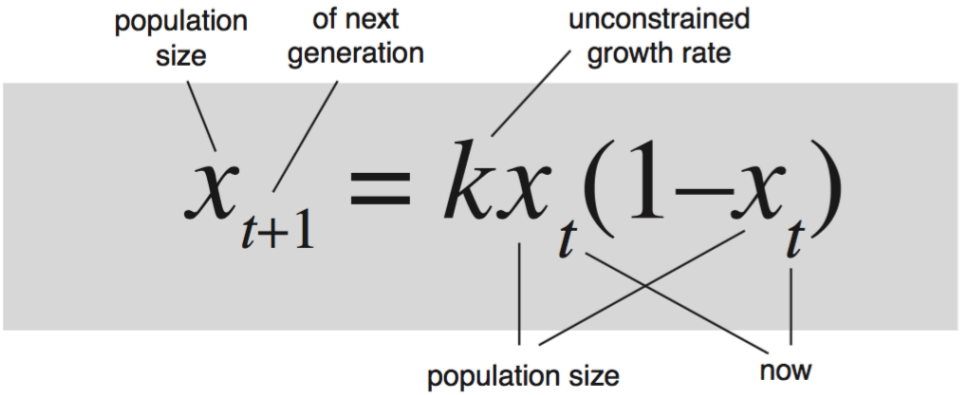
*also use: $R = c_p - c_v$

Implications of non-linearity illustrated (problem 0.1, p. 12 of the lecture notes)

Governing equations are non-linear



16 The imbalance of nature
Chaos Theory



Why is that important?

It is one of the simplest equations that can generate deterministic chaos – apparently random behaviour with no random cause.

What did it lead to?

The realisation that simple nonlinear equations can create very complex dynamics, and that apparent randomness may conceal hidden order. Popularly known as chaos theory, this discovery has innumerable applications throughout the sciences including the motion of the planets in the Solar System, weather forecasting, population dynamics in ecology, variable stars, earthquake modelling, and efficient trajectories for space probes.

Problem 0.1 (lecture notes)

Governing equations are non-linear

16 The imbalance of nature
Chaos Theory

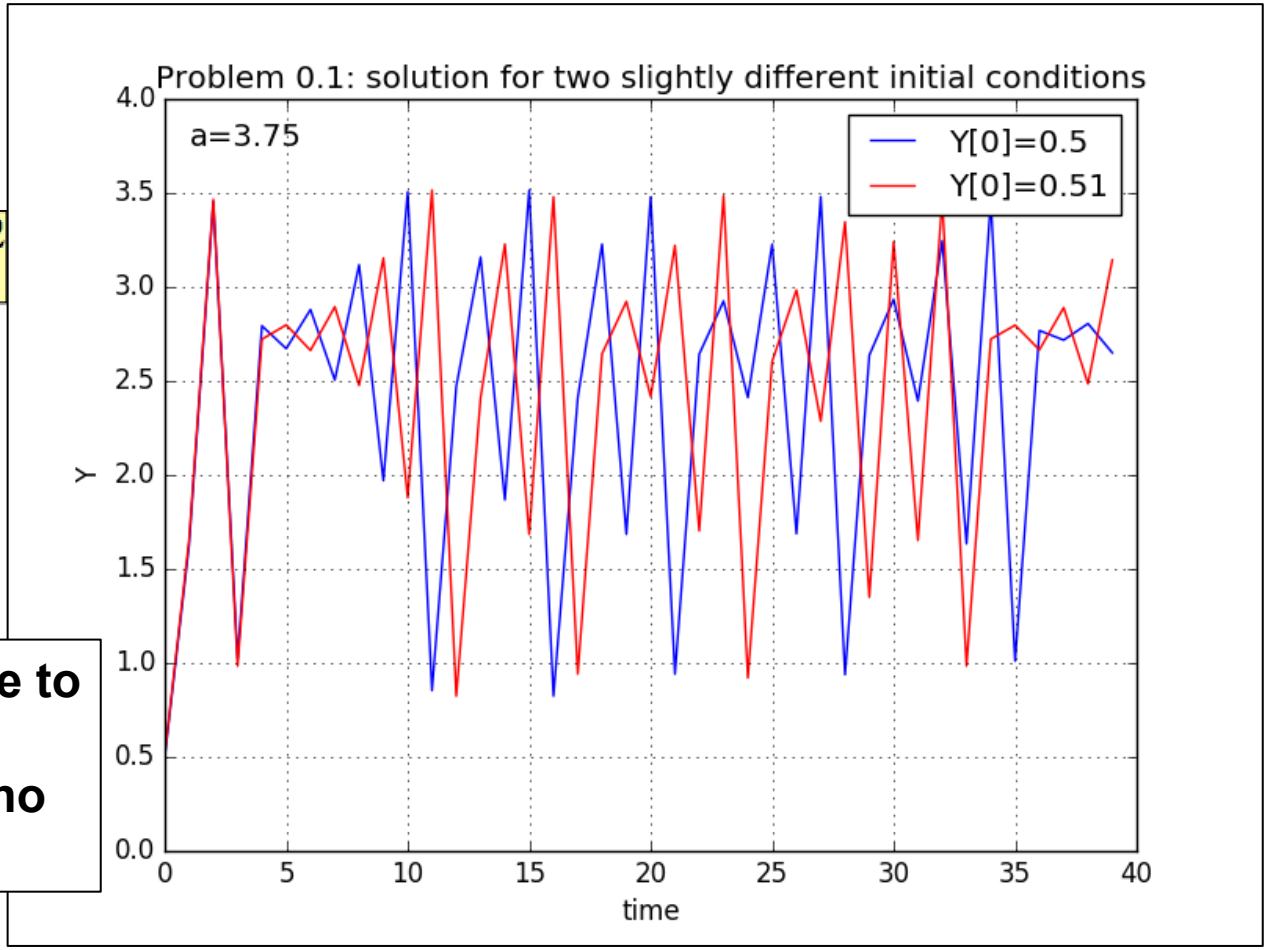
Here we iterate the following equation:

$$x[t+1] = ax[t] - (x[t])^2$$

Initial conditions:

x_0=0.5 x_0=0.51

Solution is very sensitive to initial conditions: random behaviour with no random cause.



Python script on next slide

Learn by example

Solution problem 0.1 in Python

See the Python-script, **Problem0.1AD.py**, on Blackboard (Assignments)

Problem0.1AD.py

Problem0.1AD.py 〉 No Selection

```
#Problem 0.1 AtmosphericDynamics

import numpy as np
import matplotlib.pyplot as plt

a = 3.75

len_t = 40
n_len = 2
#'d':double precision; 'l':long integer
Y = np.zeros((len_t, n_len))
t = np.zeros((len_t))

for n in range(n_len):
 i = 0
 t[i] = 0
 Y[i,0] = 0.5 # initial condition 1
 Y[i,1] = 0.51 # initial condition 2
 for i in range(len_t):
  if i>0:
   Y[i,n] = (a * Y[i-1,n]) - (Y[i-1,n] ** 2)
   t[i] = t[i-1]+1

plt.plot(t, Y[:,0],label='Y[0]='+str(Y[0,0]), color='blue')
plt.plot(t, Y[:,1],label='Y[0]='+str(Y[0,1]), color='red')
plt.axis([0,len(t),0,4.0])
plt.xlabel('time')
plt.ylabel('Y')
plt.title('Problem 0.1: solution for two slightly different initial conditions')
plt.legend()
plt.text(1, 3.75, 'a=3.75', fontsize=14, color='black')
plt.grid(True)
plt.savefig("test.png")
plt.show()
```

← import modules, **numpy** and **matplotlib**

← define arrays and fill with zero's

for-loop

Plot result of iteration

Next lecture (Friday 18 Nov. 2022, 09:00-10:45, **KBG 208** !):

Dynamic Meteorology: lecture 2

Sections 1.3-1.5 (boxes 1.1-1.4)

Potential temperature
Vertical acceleration of an air parcel (Boussinesq approximation)
Buoyancy (-oscillations) and static instability,
Brunt-Vaisala frequency

Tutorial this afternoon (Wednesday 16 November, 15:15-17:00)**:**

Study part A of the lecture notes (especially sections 1.2-1.5)

Problems 1.2 and 1.3 (see page 52 of the lecture notes)

Hint: equations needed for problem 1.2 are given on slide 37 (this lecture), or on pages 21 and 22 of the lecture notes.