

Dynamic Meteorology: lecture 3

Sections 1.3-1.5 and **Box 1.5**

Buoyancy-oscillations and hydrostatic instability

Brunt-Väisälä frequency

CAPE

Introduction to project 1 (problem 1.6) (Blackboard / Assignments)

Appendix: temperature profile determined by radiation

Tutorial after this lecture:

Working on project 1

Any other questions from part A

Making a weather forecast for Sunday 27/11

Analysis of the stability of hydrostatic balance employing the “parcel method”

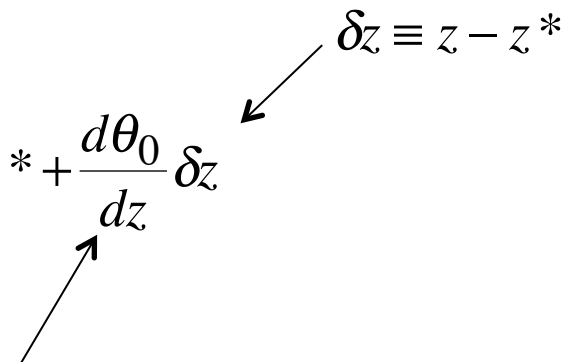
Lecture 2:

$$\frac{d^2 z}{dt^2} = g \frac{\theta'}{\theta_0}$$

An **air parcel** at $z=z^*$ has a potential temperature, $\theta = \theta^*$

Potential temperature of this air parcel is conserved!

Assume that in the **environment**: $\theta_0 = \theta^* + \frac{d\theta_0}{dz} \delta z$



$\frac{d\theta_0}{dz}$: “**lapse rate**” of the potential temperature

What happens when the vertical position of the air parcel is perturbed?

Analysis of the stability of hydrostatic balance employing the “parcel method”

Previous slide:

$$\frac{d^2 z}{dt^2} = g \frac{\theta'}{\theta_0}$$

An **air parcel** at $z=z^*$ has a potential temperature, $\theta = \theta^*$

Potential temperature of this air parcel is conserved!

Environment: $\theta_0 = \theta^* + \frac{d\theta_0}{dz} \delta z$

Buoyant force is proportional to

$$\frac{\theta'}{\theta_0} = \frac{\theta^* - \theta_0}{\theta_0} = \frac{\theta^* - \theta^* - \frac{d\theta_0}{dz} \delta z}{\theta_0} = -\frac{\frac{d\theta_0}{dz} \delta z}{\theta_0}$$

Therefore

$$\frac{d^2 \delta z}{dt^2} = -\frac{g}{\theta_0} \frac{d\theta_0}{dz} \delta z$$

Acceleration due to buoyancy

Previous slide:
$$\frac{d^2 \delta z}{dt^2} = -\frac{g}{\theta_0} \frac{d\theta_0}{dz} \delta z \equiv -N^2 \delta z \quad (1.29)$$

$$N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz}$$

N : Brunt-Väisälä-frequency

(1.31)

Acceleration due to buoyancy

$$\frac{d^2 \delta z}{dt^2} = -\frac{g}{\theta_0} \frac{d\theta_0}{dz} \delta z \equiv -N^2 \delta z \quad (1.29)$$

$$N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz}$$

N : Brunt-Väisälä-frequency

(1.31)

The solution to eq. 1.29:

$$\delta z = \exp(\pm i N t)$$

If $N^2 = \frac{g}{\theta_0} \frac{d\theta_0}{dz} < 0$

Exponential growth → **static instability**

If $N^2 = \frac{g}{\theta_0} \frac{d\theta_0}{dz} > 0$

oscillation → **static stability**

Stability of hydrostatic balance

depends on sign of vertical gradient of potential temperature

If	$\frac{d\theta_0}{dz} < 0$	exponential growth	→	static instability
If	$\frac{d\theta_0}{dz} > 0$	oscillation	→	static stability

Brunt-Väisälä frequency

$$N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz}$$

N is about $0.01\text{--}0.02 \text{ s}^{-1}$

Associated period is $\frac{2\pi}{N} \approx 300 - 600 \text{ s}$

Extra problem:

Demonstrate that the Brunt-Väisälä frequency is constant in an isothermal atmosphere.

What is the typical time-period of a buoyancy oscillation in the isothermal lower stratosphere?

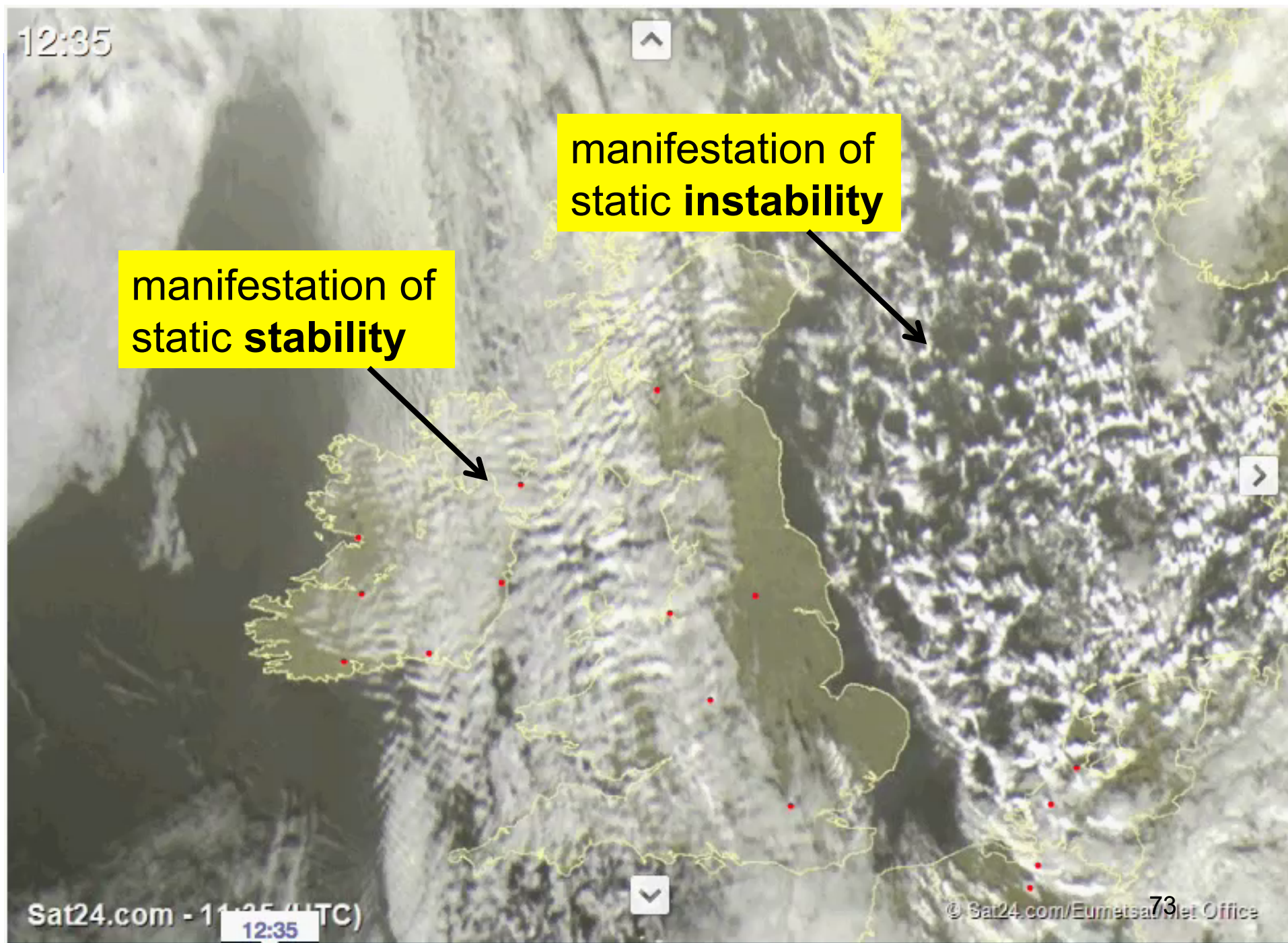
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manifestation of
static **instability**

manifestation of
static **stability**

Sat24.com - 11:25 (UTC)
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manifestation of static **instability**

Cumulus clouds

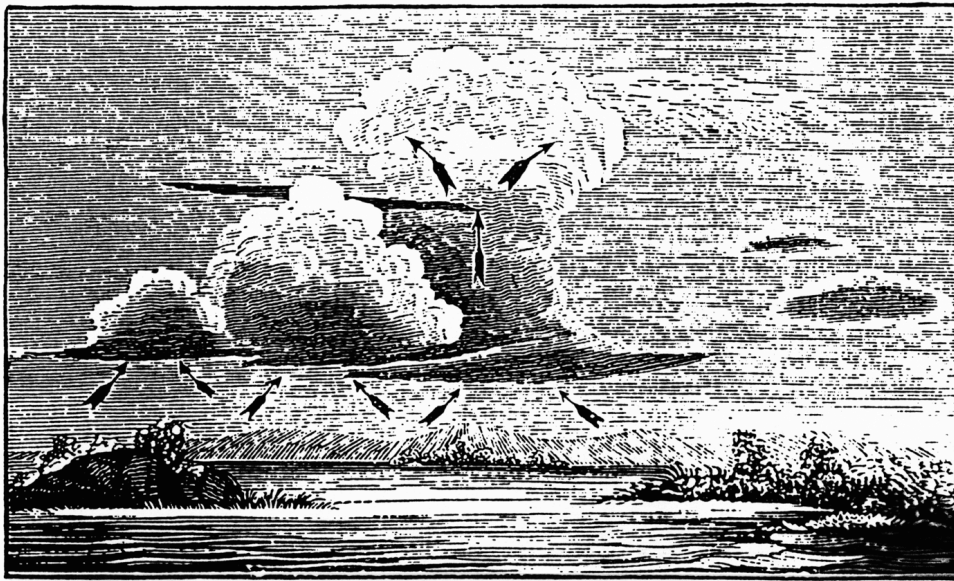


Figure 1.26: (Espy, 1841)



(Leslie Bonnema, 2011)

Convective Available Potential Energy (CAPE)

An air parcel, accelerating due to buoyancy, gains kinetic energy at the cost of Convective Available Potential Energy (CAPE). We calculate this CAPE as follows:

Vertical acceleration: $\frac{dw}{dt} = g \frac{\theta'}{\theta_0} \equiv Bg$ (lecture 2)

$B = \text{buoyancy}$

$B \equiv \frac{\theta'}{\theta_0}$

Assuming horizontal homogeneity:

$\left(\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0 \right)$

$\frac{dw}{dt} = \frac{\partial w}{\partial t} + w \frac{\partial w}{\partial z} = Bg.$

Assuming a stationary state this is:

$\left(\frac{\partial w}{\partial t} = 0 \right)$

or $w \frac{dw}{dz} = Bg$ $\longrightarrow$

Convective Available Potential Energy (CAPE)

Previous slide: $wdw = Bgdz.$

By integrating this equation, we deduce that an air parcel, which starts its ascent at a level z_1 with vertical velocity w_1 , will have a velocity w_2 at a height z_2 given by

$$w_2^2 = w_1^2 + 2 \times CAPE,$$

$$CAPE \equiv g \int_{z_1}^{z_2} B dz.$$

What is CAPE in the model-problem in **project 1 (problem 1.6)**? What is the associated maximum value of w ? Does your model reproduce this value?

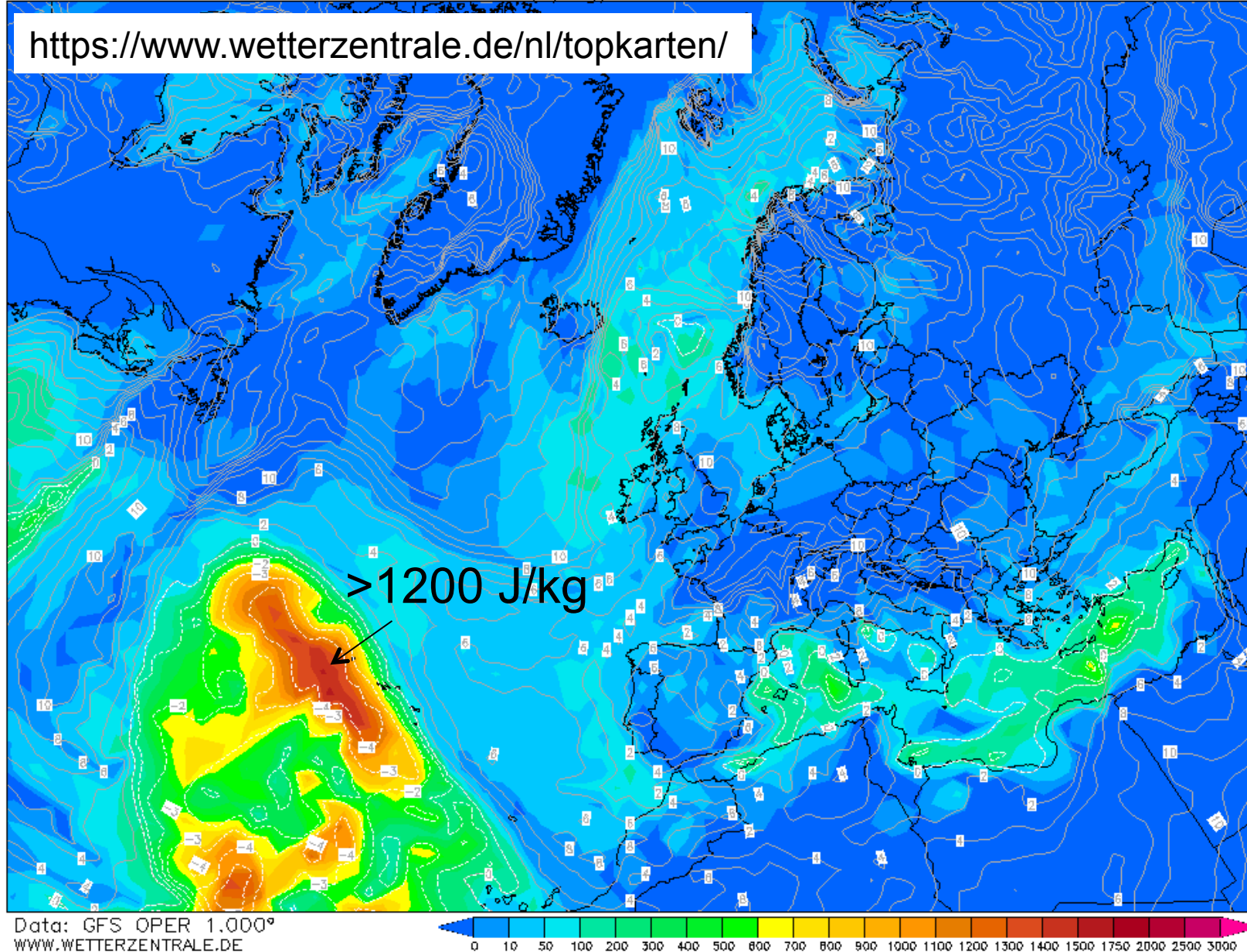
Predicted CAPE on 24 Nov. 2021

Init: Mon,22NOV2021 12Z

Surface CAPE (J/kg) und Lifted Index (°C)

Valid: Wed,24NOV2021 12Z

<https://www.wetterzentrale.de/nl/topkarten/>



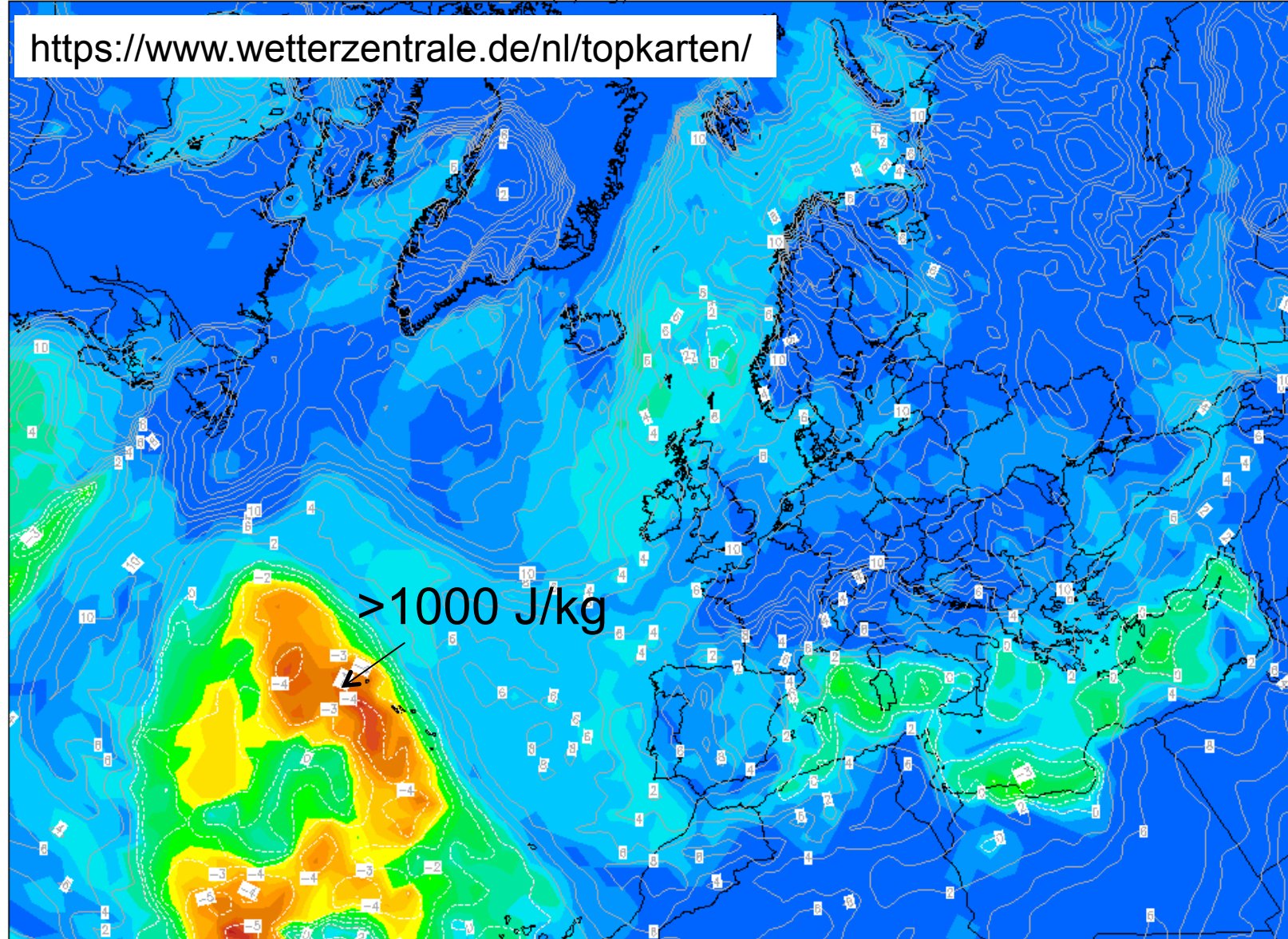
Predicted CAPE on 24 Nov. 2021

Init: Wed,24NOV2021 06Z

Surface CAPE (J/kg) und Lifted Index (°C)

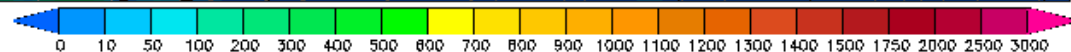
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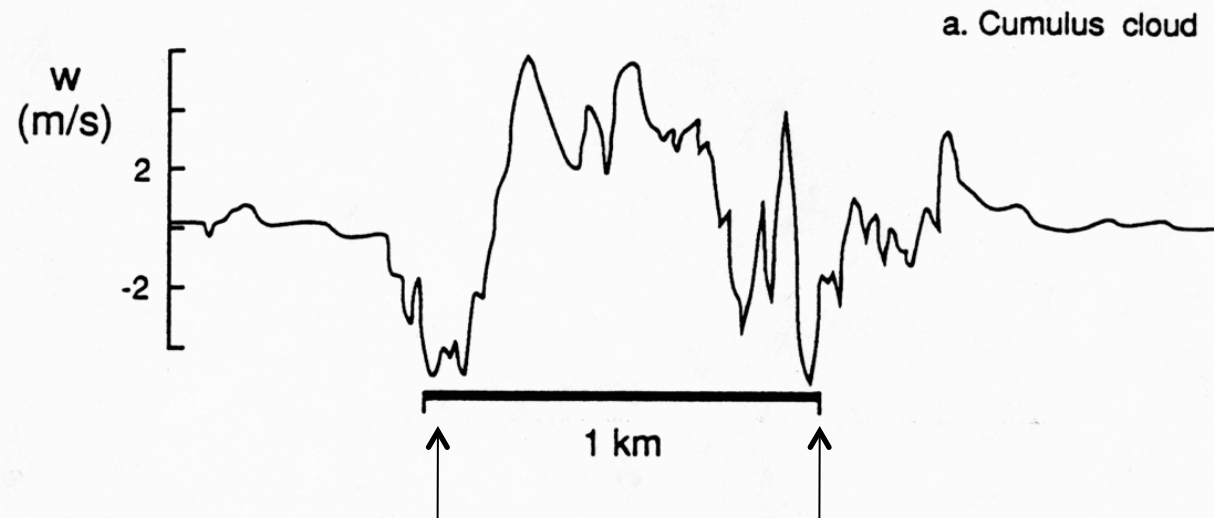


>1000 J/kg

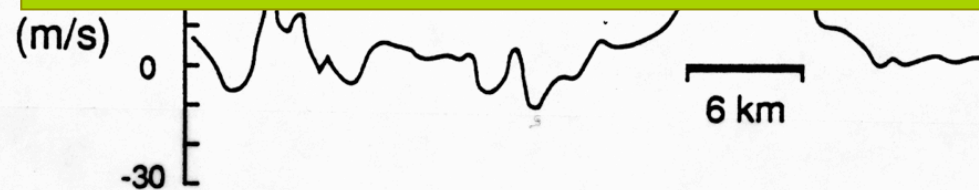
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Updraughts in cumulus clouds

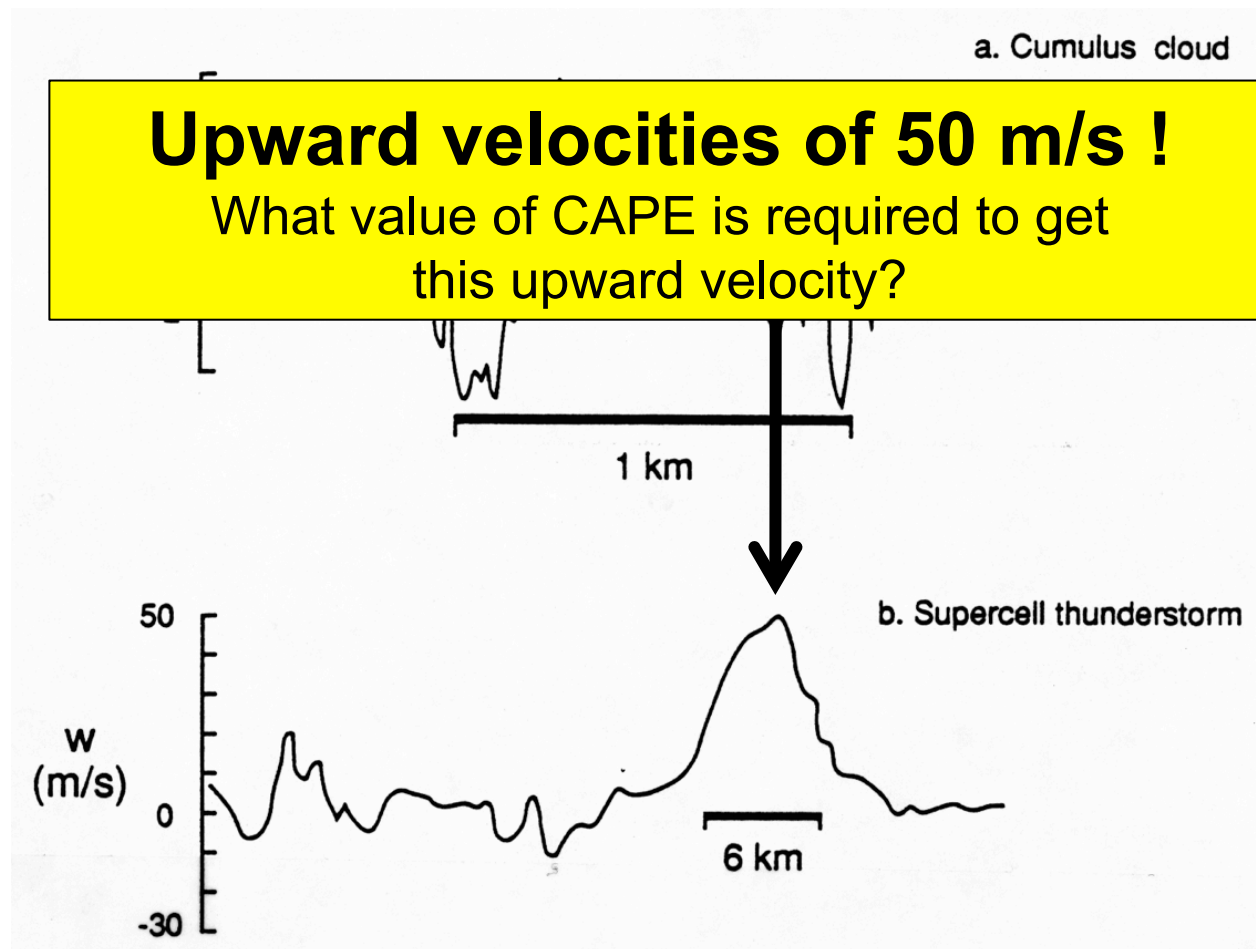


The relatively sharp downdraughts at the edge of the cumulus cloud are a typical feature of cumulus clouds. What effect is responsible for these downdraughts?



The vertical motion in **a fair weather cumulus cloud** 1.5 km deep, over a track about 250 m below cloud top

Updraughts in cumulus clouds



The vertical motion in (b) a **cumulo-nimbus cloud** more than 10 km deep, over a track 6 km above the ground

Introduction to Project 1

What theoretical concepts do you need to know?

PROJECT 1 - PROBLEM 1.6 in the lecture notes (p.53)
(in couples) (15%)

Model of a parcel moving vertically under influence of buoyancy

In this small research project you will construct your first (Lagrangian) numerical simulation model.

The following ten theoretical concepts or topics come up for discussion.

1. Equation for the vertical acceleration of an air parcel under influence of pressure and gravity
2. Hydrostatic balance
3. Boussinesq approximation
4. Buoyancy
5. Adiabatic process and (dry-adiabatic) lapse rate
6. Potential temperature and potential temperature conservation
7. Static stability / instability
8. Buoyancy oscillations and Brunt Väisälä frequency
9. Convective Available Potential Energy (CAPE)
10. Lagrangian model

Parcel model of a buoyancy oscillation in a stratified environment

Construct a numerical model (in e.g. Python; see next slide), which calculates the vertical position and vertical velocity of a dry air-parcel, which is initially at rest just above the earth's surface. Initially this air parcel is warmer than its environment. The governing equations are

$$\frac{dz}{dt} = w; \quad \frac{dw}{dt} = g \frac{\theta'}{\theta_0}; \quad \frac{d\theta}{dt} = 0 .$$

The potential temperature of the environment of the air parcel is

$$\theta_0 = \theta_0(0) + \Gamma z.$$

You start with a simple case in which the **lapse rate is constant**. In a second more realistic case, the **lapse rate in the environment of the air parcel is varies with height** (see the lecture notes, page 54)

Numerical method

See: https://en.wikipedia.org/wiki/Semi-implicit_Euler_method

Setting [edit]

The semi-implicit Euler method can be applied to a pair of equations of the form

$$\frac{dx}{dt} = f(t, v)$$

$$\frac{dv}{dt} = g(t, x),$$

In our case:

$$\frac{dz}{dt} = w, \quad \frac{dw}{dt} = g \frac{\theta'}{\theta_0}$$

where f and g are given functions. Here, x and v may be either scalars or vectors. The equations of motion in [Hamiltonian mechanics](#) take this form if the Hamiltonian is of the form

$$H = T(t, v) + V(t, x).$$

The differential equations are to be solved with the initial condition

$$x(t_0) = x_0, \quad v(t_0) = v_0.$$

The method [edit]

The semi-implicit Euler method produces an approximate [discrete](#) solution by iterating

$$v_{n+1} = v_n + g(t_n, x_n) \Delta t$$

$$x_{n+1} = x_n + f(t_n, v_{n+1}) \Delta t$$

where Δt is the time step and $t_n = t_0 + n\Delta t$ is the time after n steps.

The difference with the standard Euler method is that the semi-implicit Euler method uses v_{n+1} in the equation for x_{n+1} , while the Euler method uses v_n .

Appendix

Radiative equilibrium temperature

In an “ideal atmosphere”

(well mixed, one greenhouse gas, Solar radiation absorbed only by earth’s surface):

According to the solution of Schwarzschild’s equation for radiative equilibrium the temperature depends on pressure, p , as:

$$T = \left\{ \frac{Q}{2\sigma} \left(\frac{\sigma_a q_a p}{g} + 1 \right) \right\}^{1/4} \quad (\text{eq. 1.27a})$$

Absorbed Solar radiation

Stefan-Boltzmann constant

absorption cross-section

specific concentration of the absorber (CO_2)

See **boxes 1.1-1.4** in the lecture notes

(you do not need to study these boxes for the exam)

Appendix

Radiative equilibrium temperature

$$T = \left\{ \frac{Q}{2\sigma} (\delta + 1) \right\}^{1/4}$$

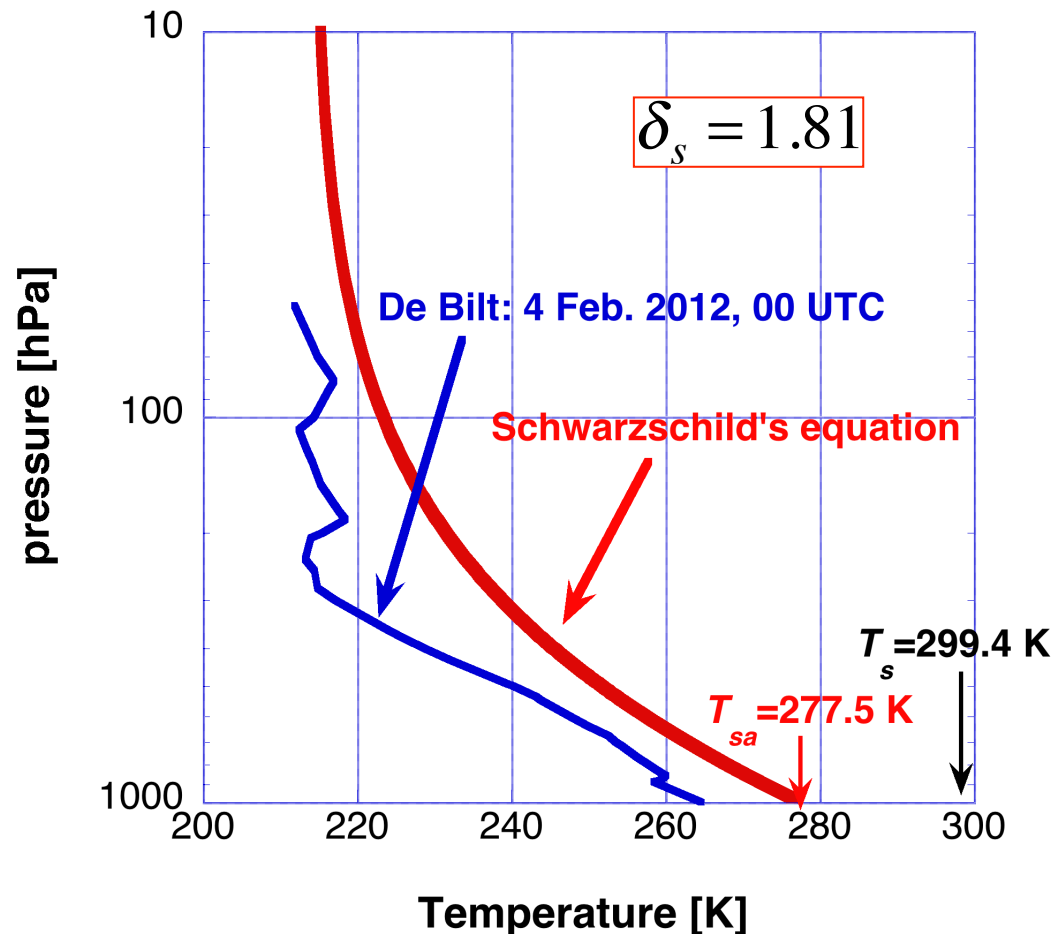
Optical path: $\delta = \sigma_a q_a p / g$

Optical path at surface:

$$\delta_s = \sigma_a q_a p_s / g$$



Surface pressure



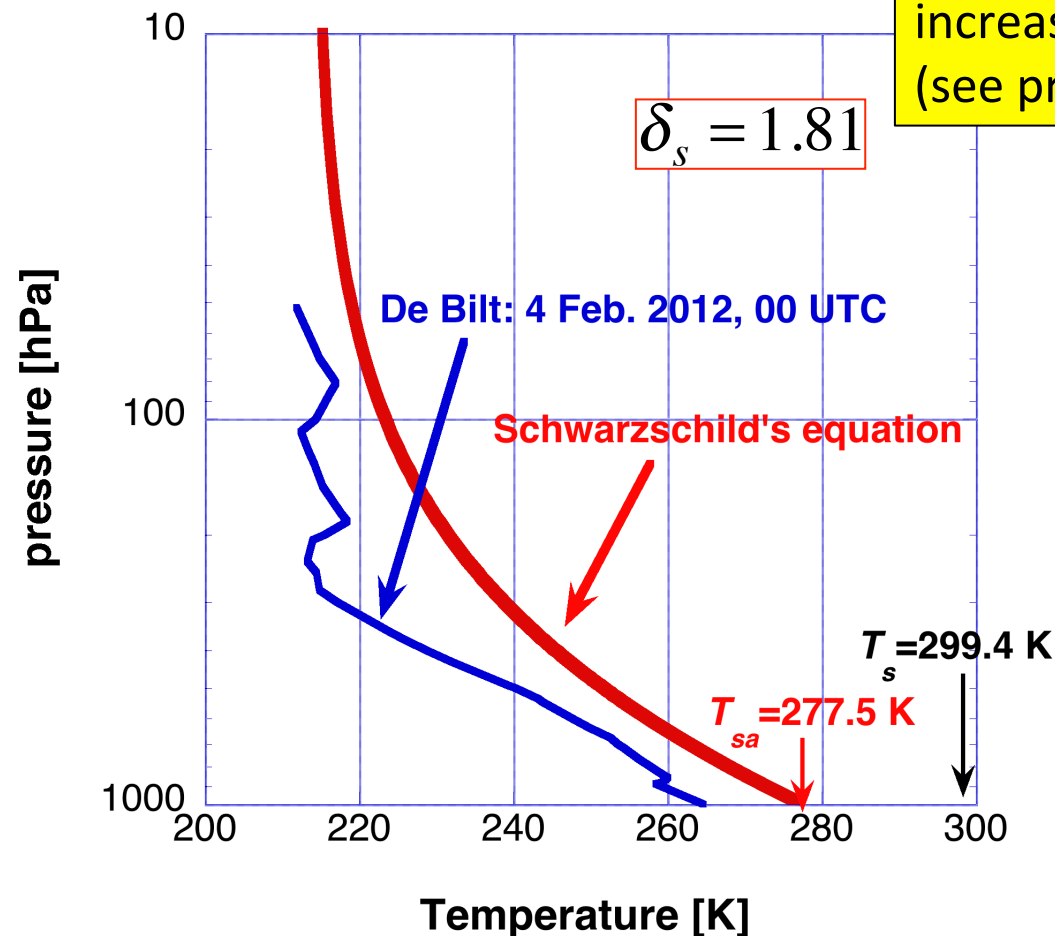
$$\begin{aligned} \sigma_a &= 0.3 \text{ m}^2 \text{ kg}^{-1}; \\ r_a &= 390 \text{ ppmv} \\ \alpha_p &= 0.3; \\ S_0 &= 1366 \text{ W m}^{-2}; \\ p_s &= 1000 \text{ hPa} \end{aligned}$$

Appendix

Radiative equilibrium temperature

$$T = \left\{ \frac{Q}{2\sigma} (\delta + 1) \right\}^{1/4}$$

It is easy to show that this solution corresponds to a statically stable stratification in which potential increases with height everywhere (see problem 1.5).

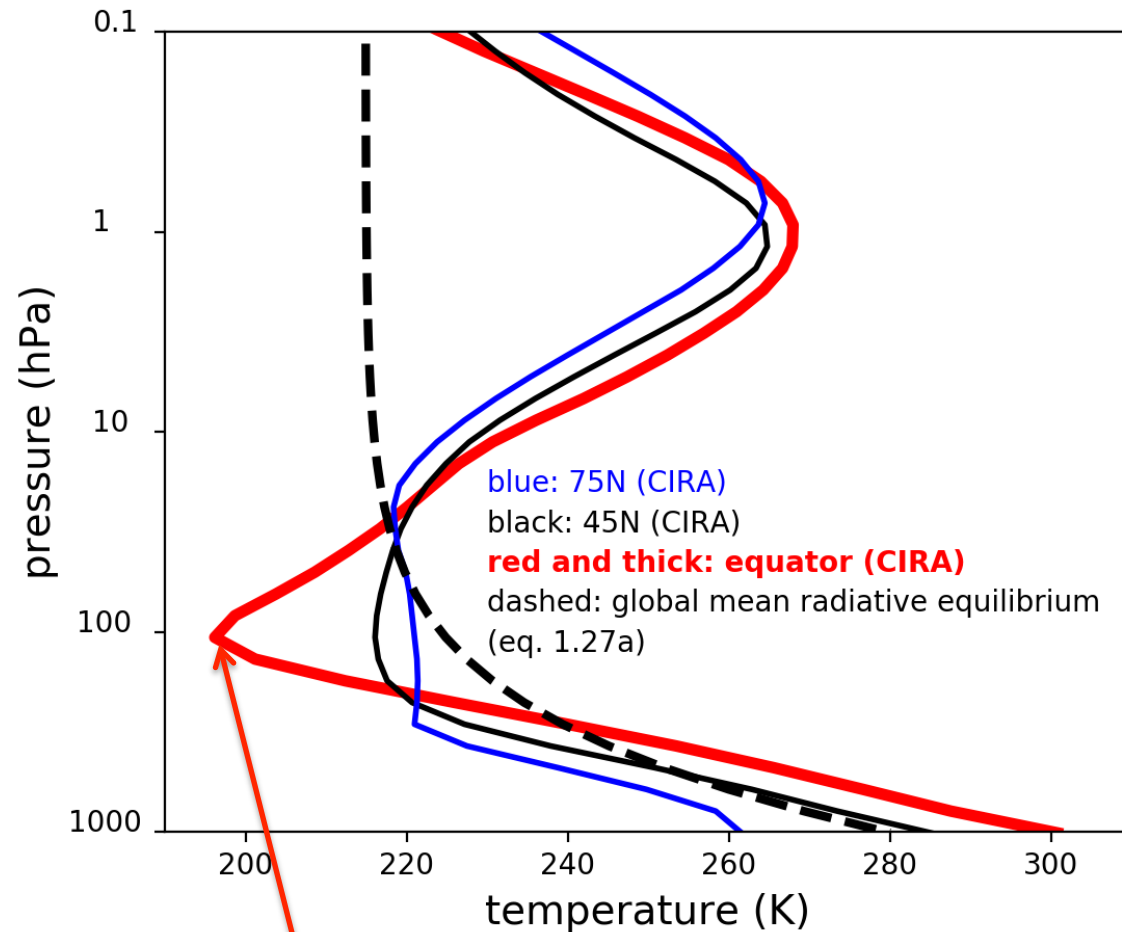


Appendix

Annual-mean temperature profiles

Annual-mean temperature as function of pressure according to the COSPAR International Reference Atmosphere (CIRA) at the **equator** (**solid thick red**), **45°N** (**black**), **75°N** (**blue**) and the **global mean, annual mean *theoretical radiative equilibrium* temperature** (**dashed line**), according to eq. 1.27a (also eq. 14 of **Box 1.4**), with $\sigma_m = 0.3 \text{ m}^2 \text{ kg}^{-1}$, $q_m = 400 \text{ ppm}$ *by volume* (see **section 1.9**) and $Q = 242 \text{ W m}^{-2}$ (the global, annual average absorbed solar radiation, assuming a planetary albedo of 0.3).

Figure 1.3 (p. 25)



cold point tropical tropopause

Dynamic Meteorology: lecture 4

Moisture in the atmosphere (parts B and C)

Dry adiabatic *lapse rate*

Clausius Clapeyron equation

Water vapour is an ideal gas

Relative humidity

Dew point temperature

Mixing ratio

Dew point lapse rate and the Lifted condensation level

Saturated ascent and latent heat release

Conditional instability

Tutorial after this lecture (Wed. 23 Nov. 2022)

Working on project 1 (in groups of 1-3 students)

Any other questions from part A

Make a weather forecast for Sunday 27/11