

Dynamic Meteorology: lecture 4

Moisture in the atmosphere and its effect (parts B and C of lecture notes)

Dry adiabatic *lapse rate*

Clausius Clapeyron equation

Water vapour is an ideal gas

Relative humidity

Dew point temperature

Mixing ratio in terms of partial pressures (Dalton's law)

Dew point lapse rate and the Lifted condensation level

Saturated ascent, latent heat release and conditional instability

Sections 1.13(B) (ozone)
is not part of DYME

Tutorial after this lecture:

problem 1.6 (project 1)

problem 1.8 (p. 78)

problem 1.11 (p. 89)



Lago di Bolsena (Italy): view of growing puffy (cumulus) clouds over the mountains to the west of the lake on 25 July 2010, 17:23 LT.

Dry-Adiabatic Lapse Rate

The hydrostatically “neutral state”:

$$\frac{d\theta_0}{dz} = 0$$

is equivalent to

$$\frac{dT_0}{dz} = -\frac{g}{c_p}$$

(page 22-23, eq. 1.20)

From which basic equations is this result derived?

Dry-Adiabatic Lapse Rate

Measuring the vertical profile of (potential) temperature by radiosonde

De Bilt, 26 Sep. 2013, 12 UTC

Launching a radiosonde at KNMI
on 30 November 2012, 12 UTC

PRES hPa	HGHT m	TEMP C	D
1017.0	4	16.0	
1014.0	28	14.6	
1000.0	142	13.2	
936.0	693	8.3	
925.0	791	7.4	
864.0	1348	2.6	
850.0	1481	6.4	-
849.0	1491	6.4	-
822.0	1754	4.9	-1
803.0	1945	3.8	-1
786.0	2119	5.0	-2
781.0	2171	5.4	-3
700.0	3057	0.8	-4
659.0	3537	-2.5	-4
647.0	3683	-2.3	-4
624.0	3969	-3.3	-3



Temperature and potential temperature profiles in the lower Atmosphere

De Bilt, 26 Sep. 2013, 12 UTC

potential temperature, θ_0

PRES hPa	HGHT m	TEMP C	DWPT C	RELH %	MIXR g/kg	DRCT deg	SKNT knot	THTA K	THTE K	THTV K
1017.0	4	16.0	8.0	59	6.66	20	8	287.8	306.7	288.9
1014.0	28	14.6	5.6	55	5.65	22	9	286.6	302.8	287.6
1000.0	142	13.2	5.2	58	5.57	30	12	286.4	302.3	287.3
936.0	693	8.3	3.1	70	5.13	40	14	286.8	301.5	287.7
925.0	791	7.4	2.7	72	5.05	35	13	286.9	301.4	287.8
864.0	1348	2.6	1.5	92	4.96	19	9	287.5	301.8	288.4
850.0	1481	6.4	-7.6	36	2.55	15	8	292.8	300.6	293.3
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822.0	1754	4.9	-13.2	26	1.69	5	10	294.1	299.4	294.4
803.0	1945	3.8	-17.2	20	1.24	339	11	294.9	298.9	295.1
786.0	2119	5.0	-29.1	6	0.44	315	12	298.0	299.5	298.1
781.0	2171	5.4	-32.6	4	0.32	315	13	298.9	300.1	299.0
700.0	3057	0.8	-44.2	2	0.11	320	21	303.3	303.8	303.4
659.0	3537	-2.5	-44.5	2	0.11	314	25	304.9	305.3	304.9
647.0	3683	-2.3	-44.3	2	0.12	312	26	306.7	307.2	306.8
624.0	3969	-3.3	-39.3	4	0.20	309	28	308.8	309.5	308.8

Will an air parcel at the surface, with a **temperature of 16.0°C** and a **dewpoint temperature of 8.0°C**, accelerate upwards spontaneously?

Temperature and potential temperature profiles in the lower Atmosphere

De Bilt, 26 Sep. 2013, 12 UTC

potential temperature, θ_0

PRES hPa	HGHT m	TEMP C	DWPT C	RELH %	MIXR g/kg	DRCT deg	SKNT knot	THTA K	THTE K	THTV K
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Up to what approximate height will it accelerate upwards?

Temperature and potential temperature profiles in the lower Atmosphere

De Bilt, 26 Sep. 2013, 12 UTC

potential temperature, θ_0

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Up to a height between 1348 m and 1481 m. Why?

Temperature and potential temperature profiles in the lower Atmosphere

De Bilt, 26 Sep. 2013, 12 UTC

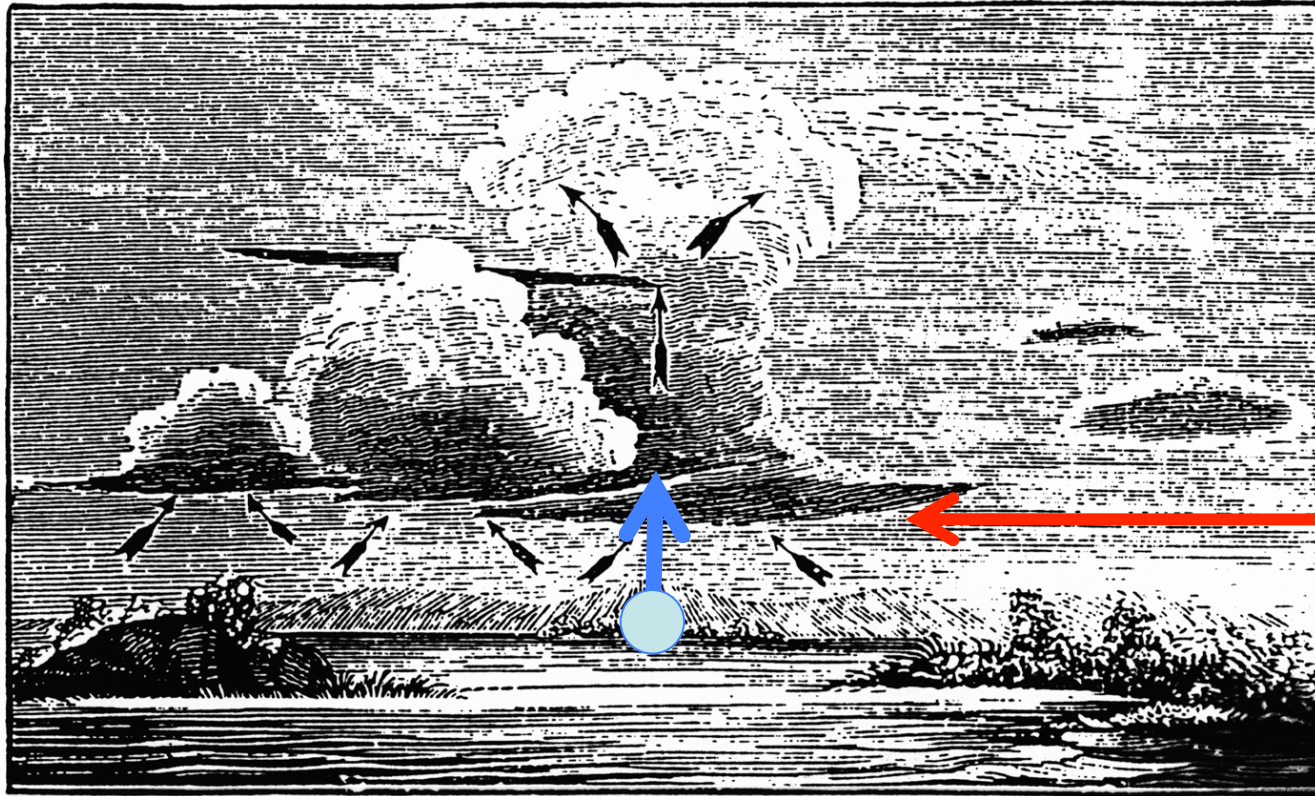
potential temperature, θ_0

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Will it reach the “Lifted Condensation Level” (LCL), i.e. will clouds form?

Moist (cumulus) convection

(Espy, 1841)



LCL

An air parcel will cool as it ascends. At some level it will become saturated. This level is the **Lifted Condensation Level (LCL)**. This is the level of the cloud base.

Determining the Lifted condensation level (LCL)

We will learn that the LCL can be determined with information of the **dewpoint temperature**, T_d , and the **temperature**, T , at the **surface**...

T_d is the temperature, which an air parcel would have if it were cooled to **saturation** at constant pressure



What determines saturation?

Clausius-Clapeyron equation

An equation of state, giving the water vapour equilibrium pressure, p_e



in equilibrium (subscript, e) with the liquid phase

$$\frac{\partial \ln p_e}{\partial T} = \frac{L_v}{R_v T^2} \quad (\text{eq. 1.50})$$

Notation in meteorology: $e_s \equiv p_e$



Subscript, s, stands for “saturation”

very large number!



L_v : latent heat of evaporation/condensation ($=2.5 \times 10^6 \text{ J kg}^{-1}$)

R_v : specific gas constant for water vapour ($=461.5 \text{ J K}^{-1} \text{ kg}^{-1}$)

Water vapour as an ideal gas

Clausius-Clapeyron equation:

$$\frac{\partial \ln e_s}{\partial T} = \frac{L_v}{R_v T^2}$$

Water vapour is an ideal gas :

$$e = \rho_v R_v T$$

$$\rho_v = \frac{e}{R_v T}$$

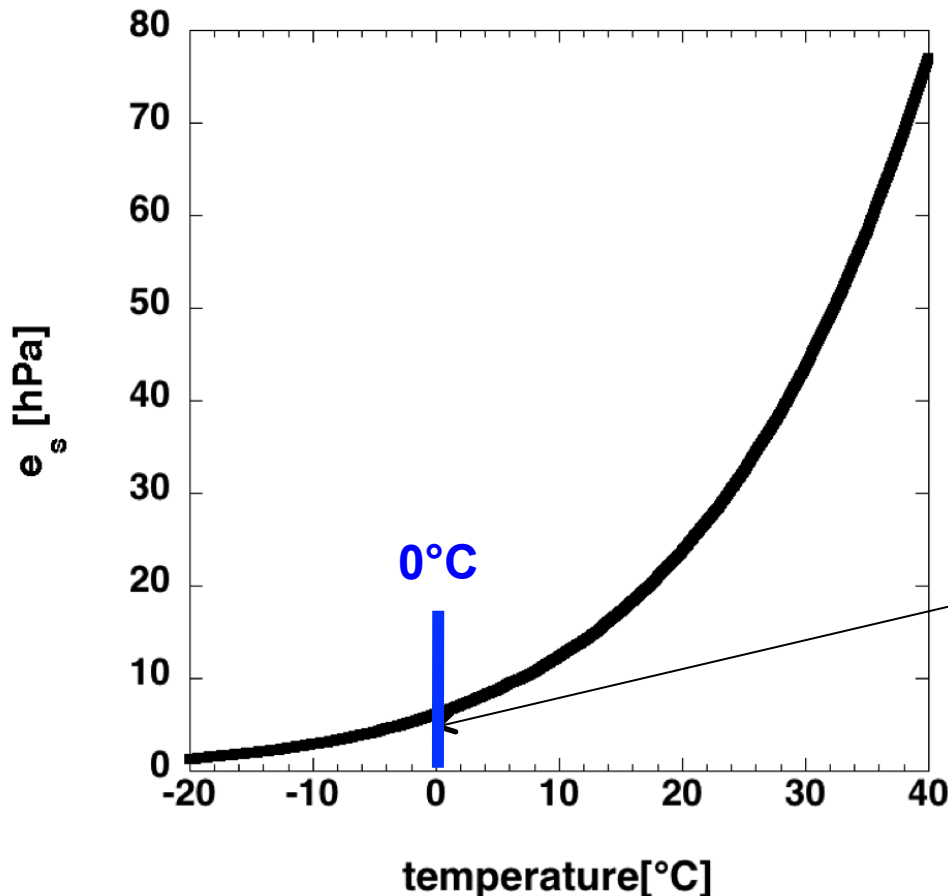
e is the partial pressure exerted by water vapour in the air

ρ_v : water vapour density in the air

Integral of Clausius Clapeyron equation

Clausius-Clapeyron equation:

$$\frac{\partial \ln e_s}{\partial T} = \frac{L_v}{R_v T^2}$$



Integrate:

$$\ln e_s - \ln e_{s,0} = -\frac{L_v}{R_v} \left(\frac{1}{T} - \frac{1}{T_0} \right)$$

By definition, $e_{s,0}$ is the equilibrium or “saturated” vapour pressure at $T_0=0^\circ\text{C}$ (6.1 hPa).

Relative Humidity and dew point temperature

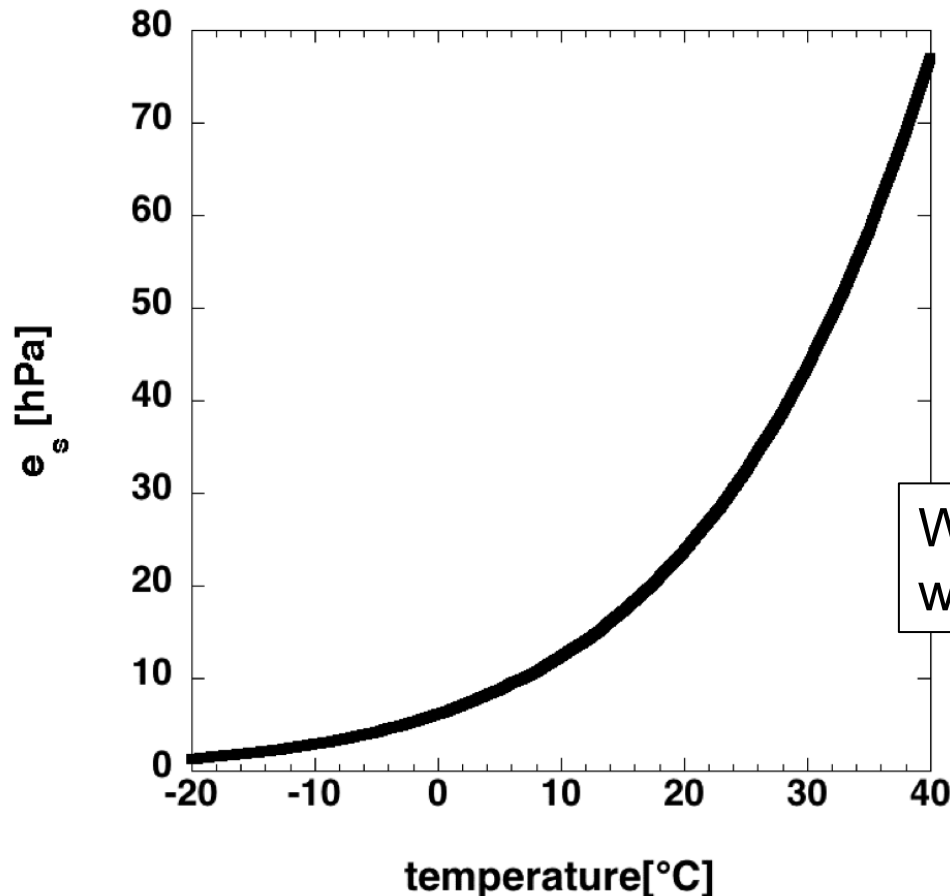
Relative humidity is defined as the ratio,

$$RH \equiv \frac{e}{e_s(T)} = \frac{e_s(T_d)}{e_s(T)}$$

T_d = dew point temperature

$$e \equiv e_s(T_d)$$

Warm air can “hold” more water vapour than cold air



Relative humidity in a Convective Layer

$$T_s \approx 20^\circ\text{C}; RH_s \approx 45\%$$

In a simulation with a cloud model (labels: %)

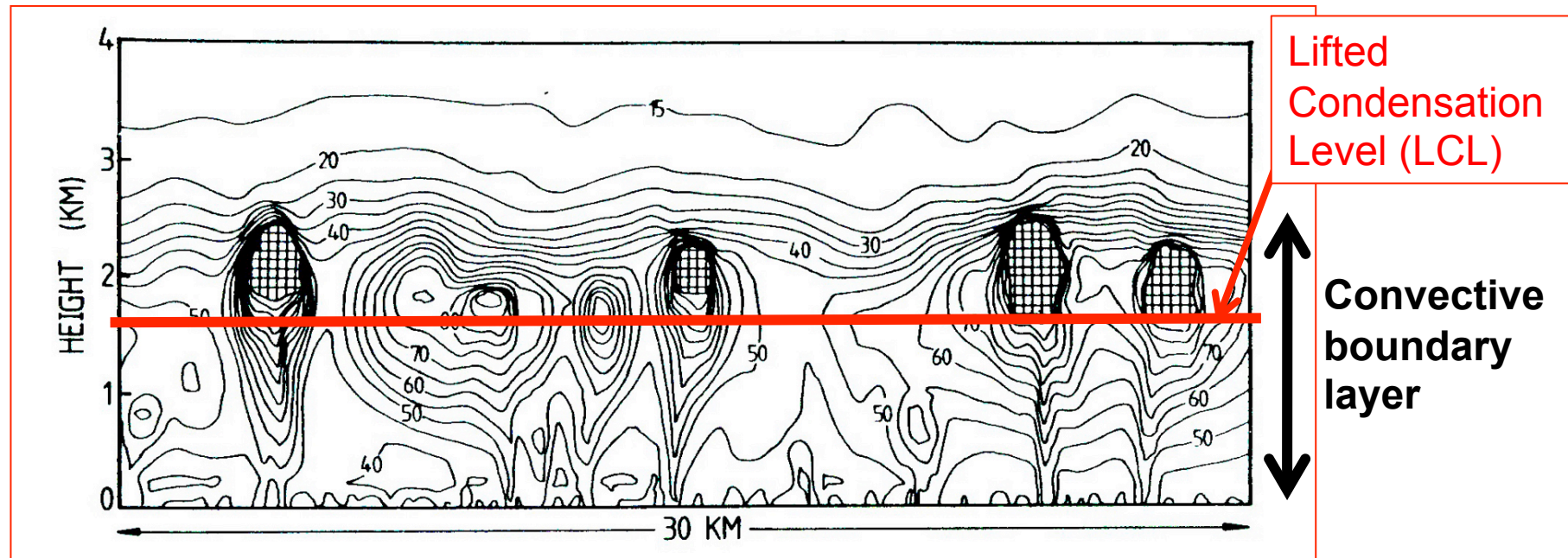


FIGURE 1.28. Simulation of the **relative humidity** in the **atmospheric boundary** layer at midday in June in The Netherlands. The hatched regions correspond to clouds (regions where the relative humidity is above 100%). The cloud cover in this case is about 20. **Relative humidity is reasonably constant at the surface, but not in the cloud layer!! Why?** In problem 1.11 on page 89 of the lecture notes you are asked to determine the lifted condensation level with information of the rel. humidity and the temperature at the ground.

Mixing Ratio: definition

Water vapour mixing ratio (definition): $r_v \equiv \frac{\rho_v}{\rho_d}$ (subscript, d , stands for “dry”)

Dry air and water vapour as ideal gases:

Dry air: $p_d = \rho_d R_d T$

Water vapour: $e = \rho_v R_v T$

Relation to partial pressures: $r_v \equiv \frac{\rho_v}{\rho_d} = \frac{e}{R_v} \frac{R_d}{p_d} \equiv \frac{\varepsilon e}{p_d} \approx \frac{\varepsilon e}{p}$ with $\varepsilon \equiv \frac{R_d}{R_v}$

Dalton's law: total pressure is sum of partial pressures, i.e. $p = p_d + p_e$

$$p_d \gg p_e$$

$$L_v \equiv L_c \equiv L$$

Finding the Lifted Condensation Level

Substitute $e_s=e$ and $T=T_d$ in the **Clausius Clapeyron** equation:

$$\frac{de}{dT_d} = \frac{Le}{R_v T_d^2} \longrightarrow \frac{de}{dT_d} \approx \frac{L r_v p}{R_v \epsilon T_d^2} \quad (1)$$

Previous slide:

$$e \approx \frac{r_v p}{\epsilon}$$

$$\frac{de}{dT_d} \approx \frac{r_v}{\epsilon} \frac{dp}{dT_d} \quad (2)$$

r_v is constant (see problem 1.8, p. 78) !



Lago di Bolsena (Italy): view of growing puffy (cumulus) clouds over the mountains to the west of the lake on 25 July 2010, 17:23 LT.

Finding the Lifted Condensation Level

Substitute $e_s=e$ and $T=T_d$ in the **Clausius Clapeyron equation**:

$$\boxed{\frac{de}{dT_d} = \frac{Le}{R_v T_d^2}} \longrightarrow \boxed{\frac{de}{dT_d} \approx \frac{L r_v p}{R_v \epsilon T_d^2}} \quad (1)$$

Previous slide: $e \approx \frac{r_v p}{\epsilon}$

r_v is constant (see problem 1.8, p. 78) !

$$\boxed{\frac{de}{dT_d} \approx \frac{r_v}{\epsilon} \frac{dp}{dT_d}} \quad (2)$$

Eqs. (1) and (2) $\rightarrow$ eq. for “**dew point lapse rate**”:

$$\boxed{\frac{dT_d}{dp} = \frac{R_v T_d^2}{Lp}}$$

With the **hydrostatic equation**:

$$\boxed{\frac{dT_d}{dz} = -\frac{\rho g R_v T_d^2}{Lp} \approx -\frac{g R_v T_d^2}{L R_d T} \approx -\frac{g T_d}{L \epsilon}}$$

Finding the *lifted condensation level*

Dew point lapse rate:

$$\Gamma_{dew} \equiv -\frac{dT_d}{dz}$$

Previous slide:

$$\frac{dT_d}{dz} \approx -\frac{gT_d}{L\varepsilon} \approx -\frac{10 \times 285}{2.5 \times 10^6 \times 0.62} \approx -0.0018 \text{ K m}^{-1}$$

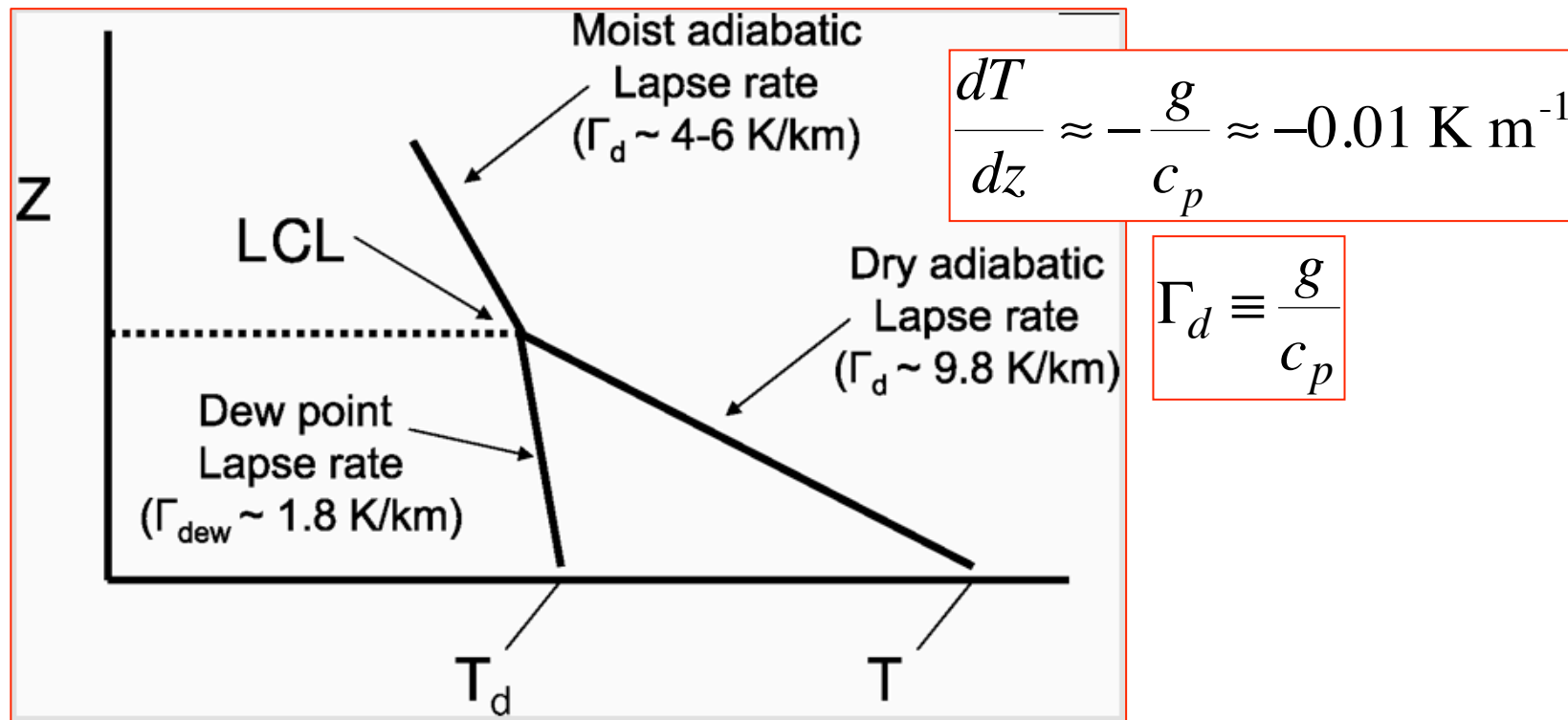


Figure 1.27

Finding the *lifted condensation level*

Dew point lapse rate

$$\Gamma_{dew} \approx \frac{gT_d}{L\epsilon} \approx 1.8 \text{ K km}^{-1}$$

$$\Gamma_{dry} \approx \frac{g}{c_p} \approx 10 \text{ K km}^{-1}$$

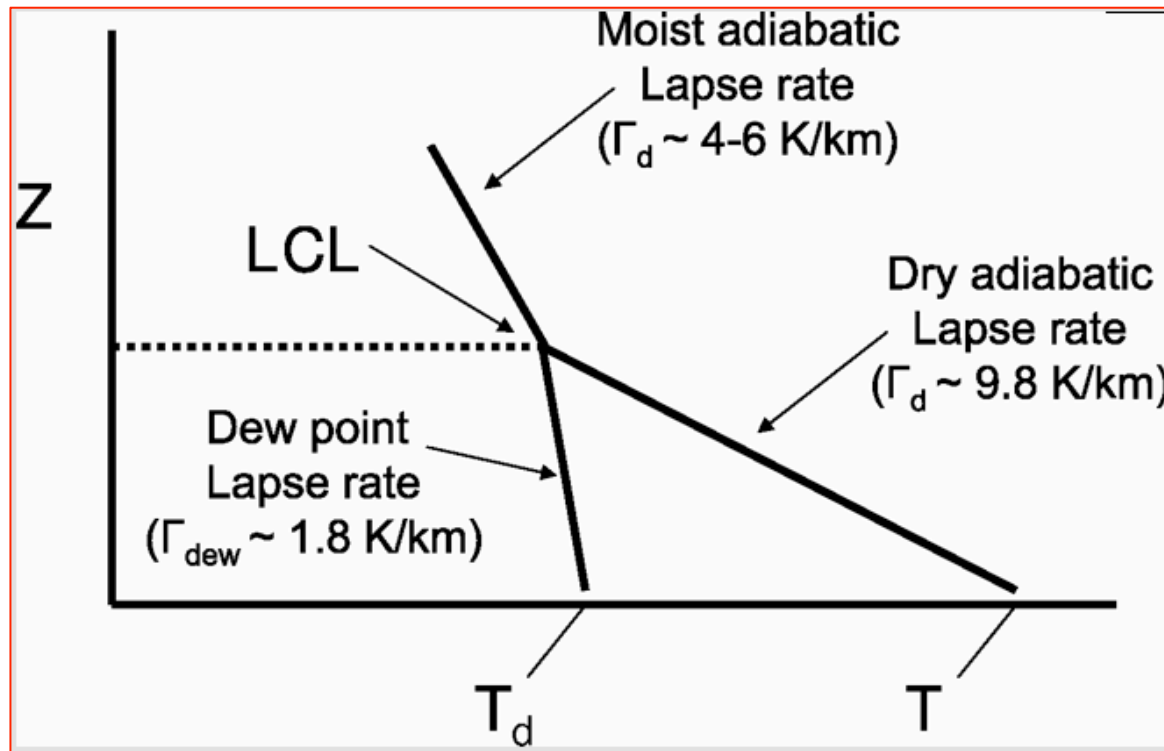


Figure 1.27

Extra problem

When is

$$\Gamma_{dew} > \Gamma_d?$$

Investigate the consequences for cloud formation, precipitation and global water cycle if this is indeed the case.

Radiosonde measurements

De Bilt, 26 Sep. 2013, 12 UTC

θ

PRES hPa	HGHT m	TEMP C	DWPT C	RELH %	MIXR g/kg	DRCT deg	SKNT knot	THTA K	THTE K	THTV K
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624.0	3969	-3.3	-39.3	4	0.20	309	28	308.8	309.5	308.8

Determine the lifted condensation level (LCL). Will clouds form?

Radiosonde measurements

De Bilt, 26 Sep. 2013, 12 UTC

θ

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647.0	3683	-2.3	-44.3	2	0.0	4	0	303.8	303.4	303.4

The lifting condensation level is found by solving:

$$T_s - \Gamma_d z_{LCL} = T_{ds} - \Gamma_{dew} z_{LCL} \longrightarrow 16.0 - \left(\frac{g}{c_p} z_{LCL} \right) = 8.0 - \left(\frac{1.8}{10^3} z_{LCL} \right)$$

$$\longrightarrow z_{LCL} = 1 \text{ km}$$

Yes: clouds will form!

What happens above the LCL?

If the air parcel continues its ascent after reaching the LCL, condensation of water vapour will occur, which will be accompanied by release of latent heat.....

The **rate of heating** due to condensation is mJ (m is mass of air parcel):
(Joules per second)

$$mJ = -L \frac{dm_v}{dt} \xrightarrow{\text{?}} J \approx -L \frac{dr_s}{dt}$$

r_s is saturation mixing ratio

L ($=2.5 \times 10^6 \text{ J kg}^{-1}$) is the latent heat of condensation

m_v is the mass of water vapour in the air parcel

J is the heat transfer to the air parcel per unit mass, per unit time

Latent heat release in updraught

The rate of heating due to condensation is mJ (m is mass of air parcel):

$$mJ = -L \frac{dm_v}{dt} \longrightarrow J = -L \frac{dr_s}{dt}$$

Change in r_s following the motion is primarily due to *ascent* (stationary state) and Latent heat release only in the *updraught* !

Ascent: $\frac{dr_s}{dt} \cong w \frac{dr_s}{dz} \text{ for } w > 0; \longrightarrow J = -Lw \frac{dr_s}{dz}$

Descent: $\frac{dr_s}{dt} \cong 0 \text{ for } w \leq 0.$

Saturated ascent and unsaturated decent

Assume that $\theta = \theta_0(z) + \theta'$, with $\theta' \ll \theta_0$. Then:

$$\frac{d\theta}{dt} = \frac{d\theta'}{dt} + w \frac{d\theta_0}{dz} = \frac{J}{\Pi}$$

$$J=0 \text{ if } w < 0: \quad N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz} \rightarrow \frac{d\theta'}{dt} = \frac{-\theta_0}{g} N^2 w \text{ if } w \leq 0;$$

$J > 0$ if $w > 0$ and air is saturated:

$$\frac{d\theta'}{dt} \approx \frac{-\theta_0}{g} N_m^2 w \text{ if } w > 0,$$

“Moist” Brunt Väisälä frequency

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$J > 0$ if $w > 0$ and air is saturated:

$$\frac{d\theta'}{dt} \approx \frac{-\theta_0}{g} N_m^2 w \text{ if } w > 0,$$

$$J = -L \frac{dr_s}{dz} w \text{ if } w > 0 \rightarrow N_m^2 \equiv N^2 + \frac{gL}{\theta_0 \Pi_0} \frac{dr_s}{dz},$$

N_m is “moist” Brunt Väisälä frequency

Conditional instability

Previous slide:

$$\frac{d\theta}{dt} \approx \frac{d\theta'}{dt} + w \frac{d\theta_0}{dz} = \frac{J}{\Pi}$$

$$\left\{ \begin{array}{l} \frac{d\theta'}{dt} = \frac{-\theta_0}{g} N^2 w \text{ if } w \leq 0; \\ \frac{d\theta'}{dt} \approx \frac{-\theta_0}{g} N_m^2 w \text{ if } w > 0, \end{array} \right.$$

$$N_m^2 \equiv N^2 + \frac{gL}{\theta_0 \Pi_0} \frac{dr_s}{dz}$$

Frequently we observe that: $N_m^2 < 0$ and $N^2 > 0$.

In these circumstances the atmosphere is statically unstable only with respect to saturated upward motion.

This is called conditional instability.

Next lecture: Wednesday 30 November 2022, 13:15-15:00 (room 001 BBG)

Dynamic Meteorology: lecture 5

Moisture in the atmosphere and its effect (2) (parts B and C of lecture notes)

Equivalent potential temperature

Potential instability

Tephigram

The influence of earth's rotation (parts D of lecture notes)

Material derivative in a rotating coordinate system on the sphere

The influence the Coriolis force on the sea-breeze circulation

A “second mode” of convection

Inertial frequency

Tutorial after this lecture (4) (Friday 25 November 2022) :

problem 1.6 (project 1)

problem 1.8 (p. 78)

problem 1.11 (p. 89)