

Dynamic Meteorology: lecture 5

Moisture in the atmosphere and its effect (continue from lecture 4)

Equivalent potential temperature (parts B and C of lecture notes)
Potential instability
Tephigram

The influence of earth's rotation (parts A and D of lecture notes)

Material derivative in a rotating coordinate system on the sphere
The influence of the Coriolis force on the Hadley circulation
A model of the sea breeze at the coast (problem 1.14)
Inertial oscillations; inertial frequency

introduction to project 3

Tutorial after this lecture (5):

Finish problem 1.6 (project 1); hand-in short report before end of today
Problems 1 and 2 in Box 1.8 (p. 88), problem 1.12 (p. 89)
Make a weather forecast for Sunday 4 December 2022

Conditional instability

$$\frac{d\theta}{dt} \approx \frac{d\theta'}{dt} + w \frac{d\theta_0}{dz} = \frac{J}{\Pi}, \quad \left\{ \begin{array}{l} \frac{d\theta'}{dt} = \frac{-\theta_0}{g} N^2 w \text{ if } w \leq 0; \text{ (unsaturated descent)} \\ \frac{d\theta'}{dt} \approx \frac{-\theta_0}{g} N_m^2 w \text{ if } w > 0, \text{ (saturated ascent)} \end{array} \right.$$

$$\downarrow$$

$$N_m^2 \equiv N^2 + \frac{gL}{\theta_0 \Pi_0} \frac{dr_s}{dz},$$

Frequently we observe that: $N_m^2 < 0$ and $N^2 > 0$.

In these circumstances the atmosphere is statically unstable only with respect to saturated upward motion.

This is called conditional instability.

Equivalent potential temperature

Previous slide:

$$N_m^2 \equiv N^2 + \frac{gL}{\theta_0 \Pi_0} \frac{dr_s}{dz} \quad N^2 \equiv \frac{g}{\theta_0} \frac{d\theta_0}{dz}$$

Define a **pseudo- or moist adiabatic process** in which a so-called **“equivalent potential temperature”**, θ_e , is constant, following saturated ascent.

By analogy simply define:

$$N_m^2 = \frac{g}{\theta_e} \frac{d\theta_e}{dz}$$

then

$$\theta_e \approx \theta \exp\left(\frac{Lr_s}{\theta \Pi}\right) \quad (\text{eq. 1.96a})$$

Equivalent potential temperature

For an saturated air parcel:

$$\theta_e \approx \theta \exp\left(\frac{Lr_s}{\theta \Pi}\right) = \theta \exp\left(\frac{Lr_s}{c_p T}\right) \quad (\text{previous slide})$$

For an unsaturated air parcel:

$$\theta_e \approx \theta \exp\left(\frac{Lr}{c_p T_{LCL}}\right)$$

actual mixing ratio

Temperature at the LCL

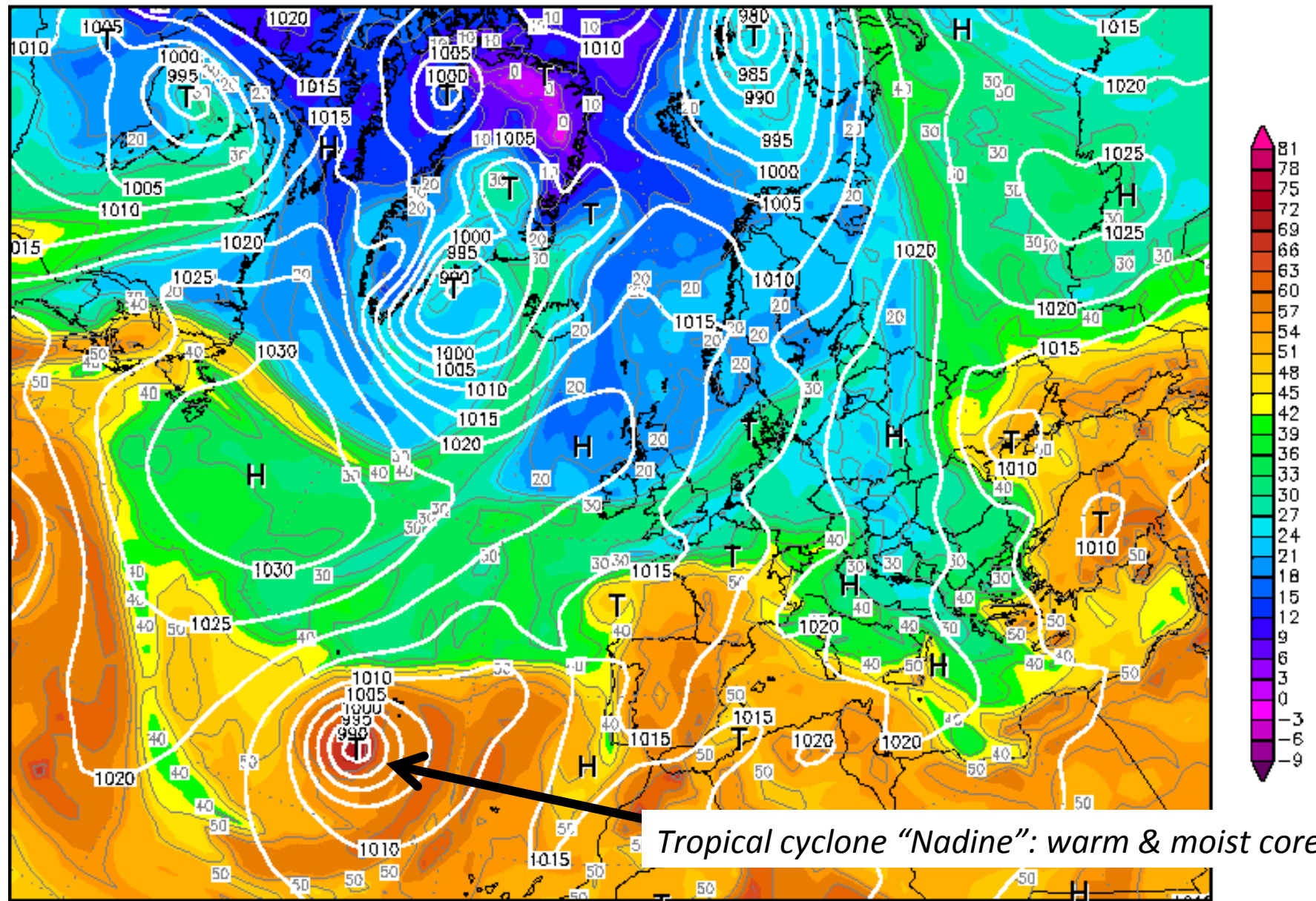
(LCL: lifting condensation level)

approximately conserved!

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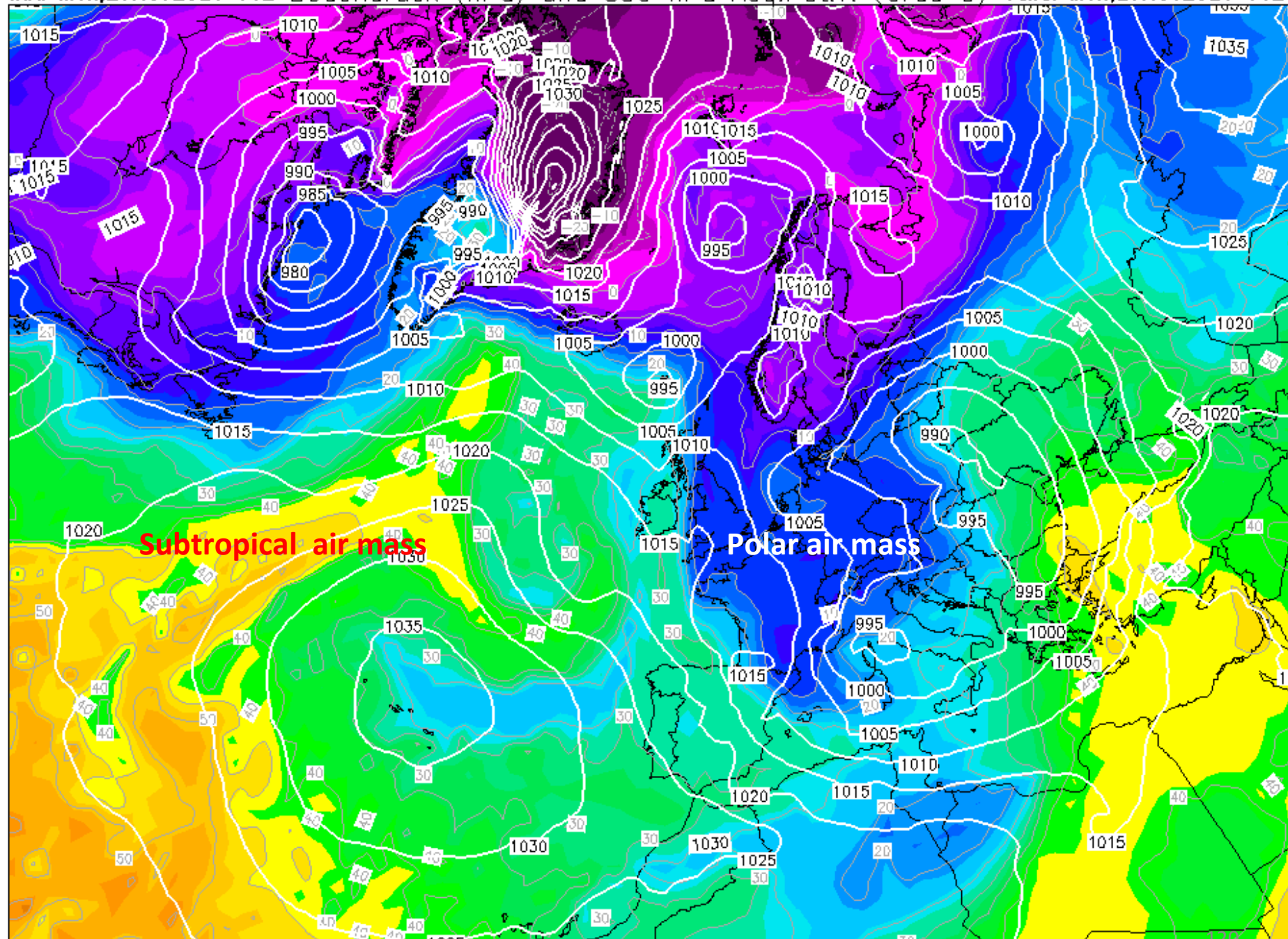
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Bodendruck (hPa) und 850 hPa Aeq.Pot.T. (Grad C)

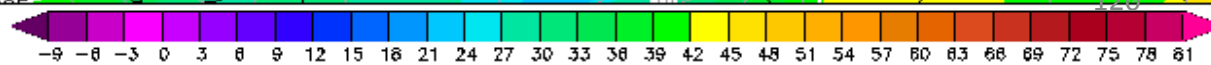


Daten: GFS-Modell des amerikanischen Wetterdienstes
(C) Wetterzentrale
www.wetterzentrale.de

Init: Mon,29NOV2021 06Z Bodendruck (hPa) und 850 hPa Aeq.Pot.T. (Grad C) Valid: Mon,29NOV2021 06Z



Data: GFS OPER 1.000°
WWW.WETTERZENTRALE.DE



Potential Instability: $\frac{\partial \theta_e}{\partial z} < 0$

De Bilt, 26 Sep. 2013, 12 UTC

θ θ_e

PRES hPa	HGHT m	TEMP C	DWPT C	RELH %	MIXR g/kg	DRCT deg	SKNT knot	THTA K	THTE K	THTV K
1017.0	4	16.0	8.0	59	6.66	20	8	287.8	306.7	288.9
1014.0	28	14.6	5.6	55	5.65	22	9	286.6	302.8	287.6
1000.0	142	13.2	5.2	58	5.57	30	12	286.4	302.3	287.6
936.0	693	8.3	3.1	70	5.13	40	14	286.8	301.5	287.6
925.0	791	7.4	2.7	72	5.05	35	13	286.9	301.4	287.6
864.0	1348	2.6	1.5	92	4.96	19	9	287.5	301.8	288.4
850.0	1481	6.4	-7.6	36	2.55	15	8	292.8	300.6	293.3
849.0	1491	6.4	-7.6	36	2.55	15	8	292.9	300.8	293.4
822.0	1754	4.9	-13.2	26	1.69	5	10	294.1	299.4	294.4
803.0	1945	3.8	-17.2	20	1.24	339	11	294.9	298.9	295.1
786.0	2119	5.0	-29.1	6	0.44	315	12	298.0	299.5	299.5
781.0	2171	5.4	-32.6	4	0.32	315	13	298.9	300.1	299.5
700.0	3057	0.8	-44.2	2	0.11	320	21	303.3	303.8	303.8
659.0	3537	-2.5	-44.5	2	0.11	314	25	304.9	305.3	304.9
647.0	3683	-2.3	-44.3	2	0.12	312	26	306.7	307.2	306.8
624.0	3969	-3.3	-39.3	4	0.20	309	28	308.8	309.5	308.8

$$\frac{\partial \theta}{\partial z} < 0$$

$$\frac{\partial \theta_e}{\partial z} < 0$$

Potential Instability up to 650 hPa (3600 m)

"Tephigram"

see Figure 1.29, p.86

<http://www.staff.science.uu.nl/~delde102/tephigram.pdf>

constant θ

constant pressure

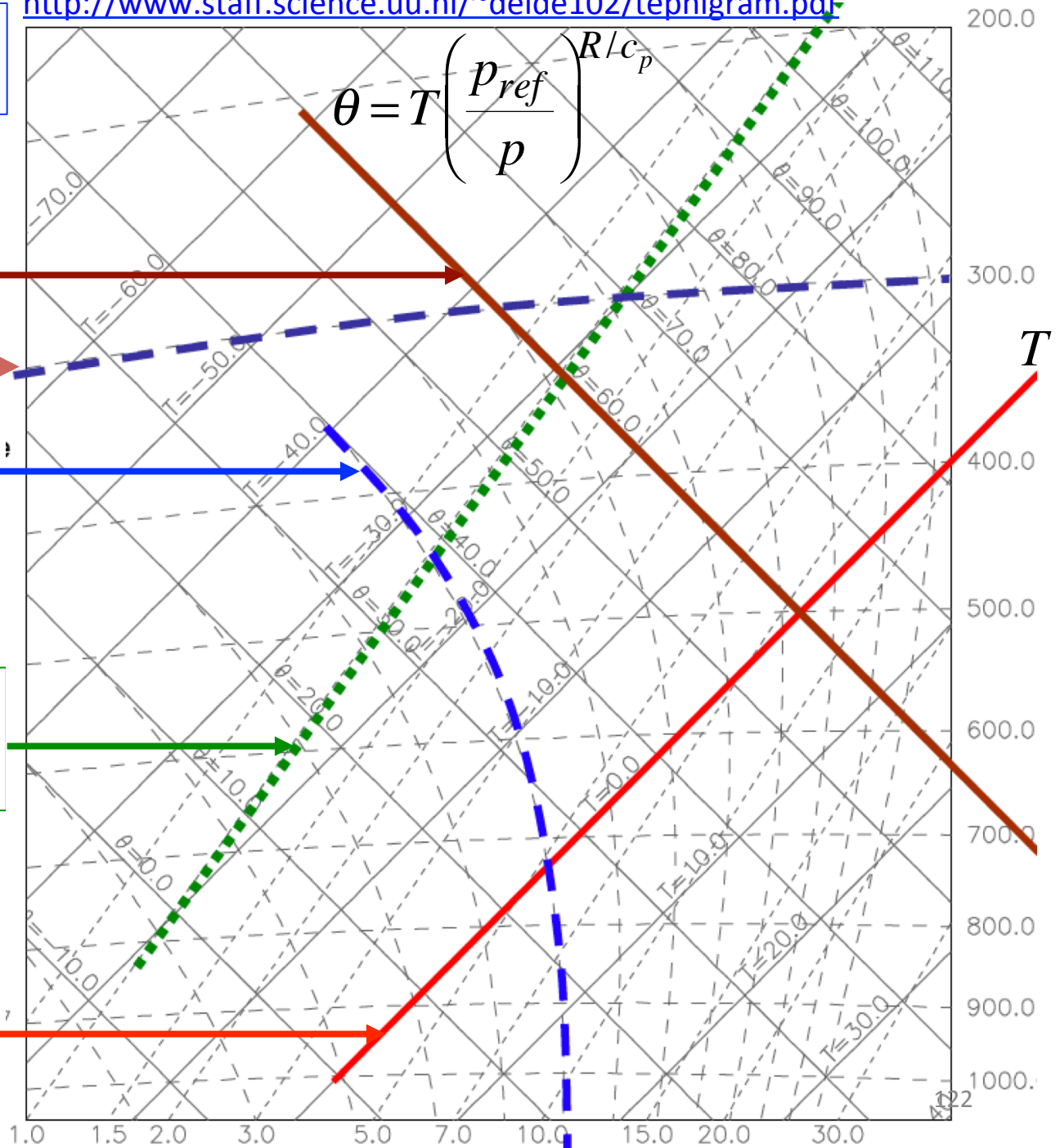
constant θ_e

$$\theta_e \approx \theta \exp\left(\frac{Lr_s}{\theta \Pi}\right)$$

constant saturation
mixing ratio

$$r_s = \frac{R_d e_s}{R_v p}$$

isotherm



The influence of earth's rotation

(parts A and D of lecture notes)

Governing equations

Material derivative in a rotating coordinate system on the sphere

Advection

The influence the Coriolis force as revealed by

(a) the Hadley circulation

(b) the sea-breeze circulation

Inertial oscillations and inertial frequency

* see **geophysical fluid dynamics** and **sections 1.2, 1.3, 1.4 and 1.7** of lecture notes

Back to the _____

The governing equations *

momentum

$$\frac{d\vec{v}}{dt} = -\alpha \vec{\nabla} p - g \hat{k} - 2\vec{\Omega} \times \vec{v} + \vec{F}r \quad \alpha \equiv \frac{1}{\rho}$$

Pressure gradient (1.3)

Gravity (1.3)

Coriolis (1.7)

Friction (1.8)

mass

$$\frac{d\rho}{dt} = -\rho \vec{\nabla} \cdot \vec{v}$$

eq. 1.6

(section)

energy

$$Jdt = c_v dT + p d\alpha$$

same as eq. 1.22 (p.23)

state

$$p = knT$$

eq. 1.10a

Unknowns:

$$\vec{v}, \rho, T, p$$

Material derivative of a scalar & advection

Scalar is a function of x, y, z and t :

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

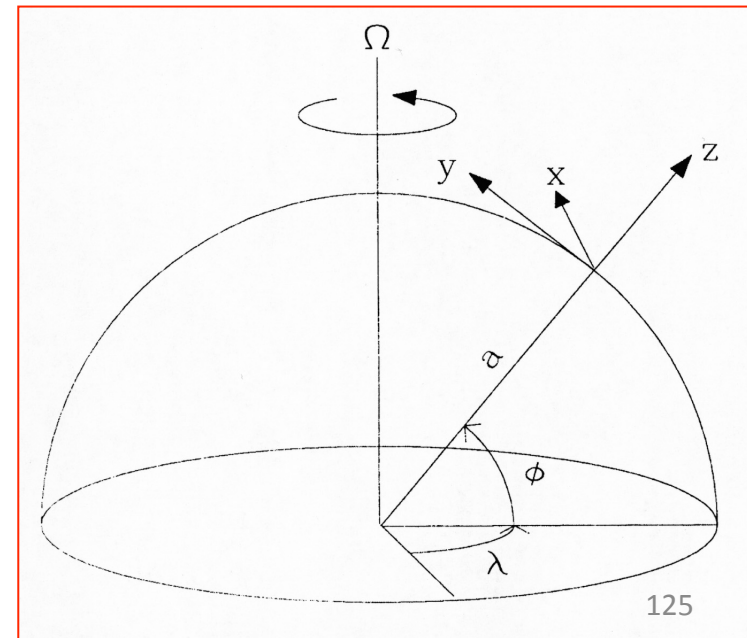
eq. 1.5

Material
derivative

Local
derivative

“Advection”

Non-linear!!!



12:35

$$\frac{d}{dt}(\dots) = 0$$

Conserved features:
liquid water in clouds

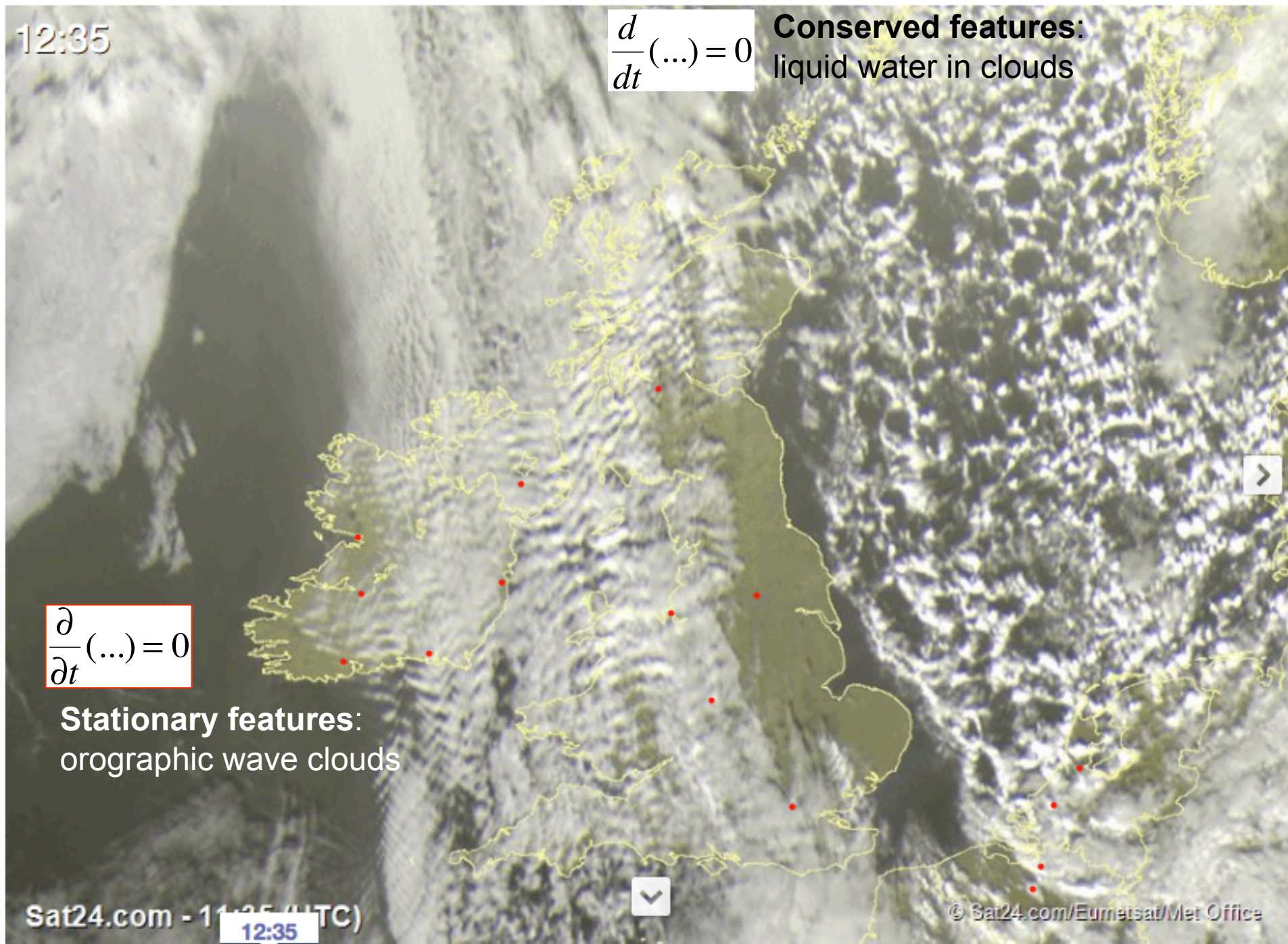
$$\frac{\partial}{\partial t}(\dots) = 0$$

Stationary features:
orographic wave clouds

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$$\frac{d}{dt}(\dots) = 0$$

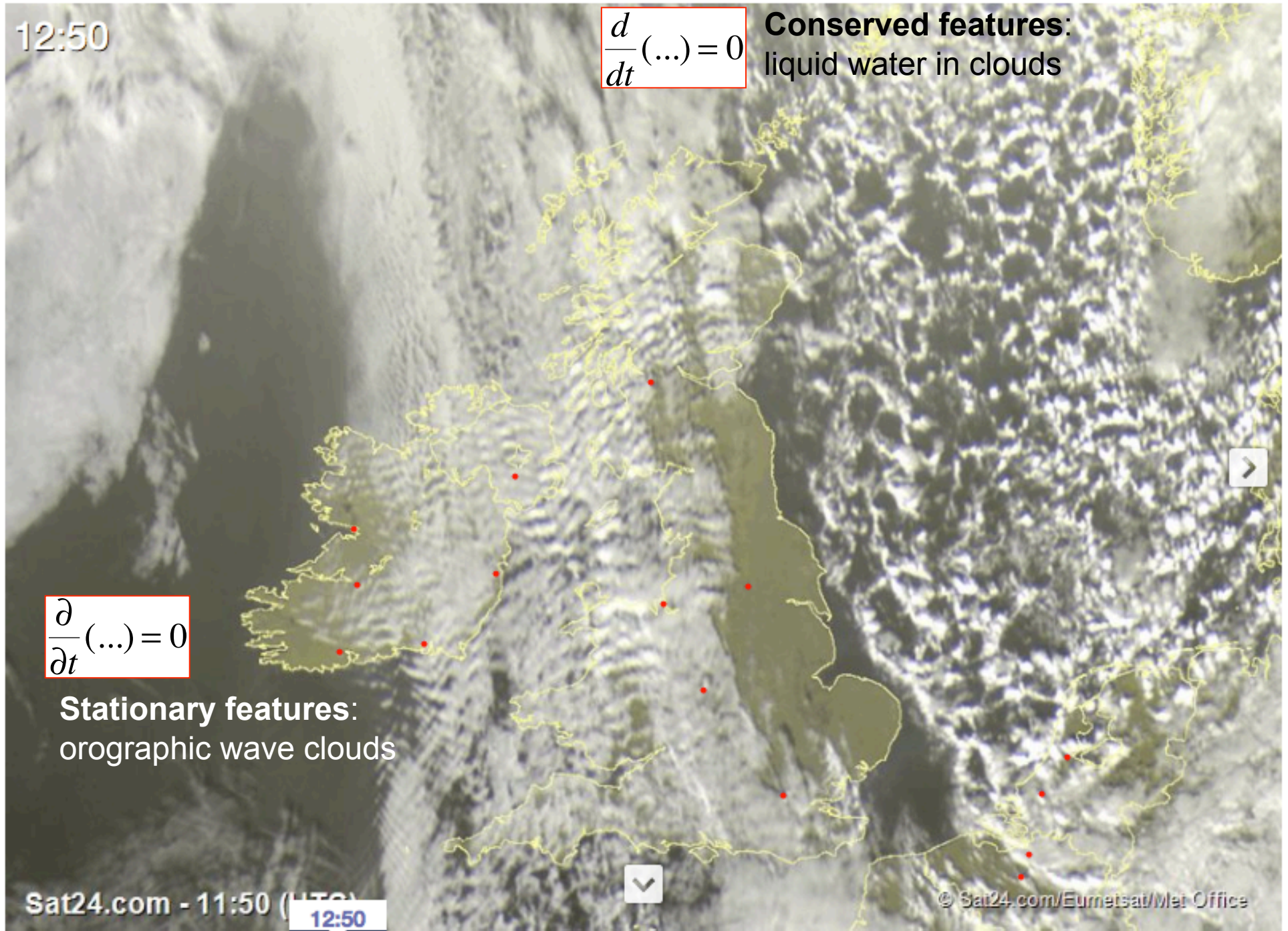
Conserved features:
liquid water in clouds

$$\frac{\partial}{\partial t}(\dots) = 0$$

Stationary features:
orographic wave clouds

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$$\frac{d}{dt}(\dots) = 0$$

Conserved features:
liquid water in clouds

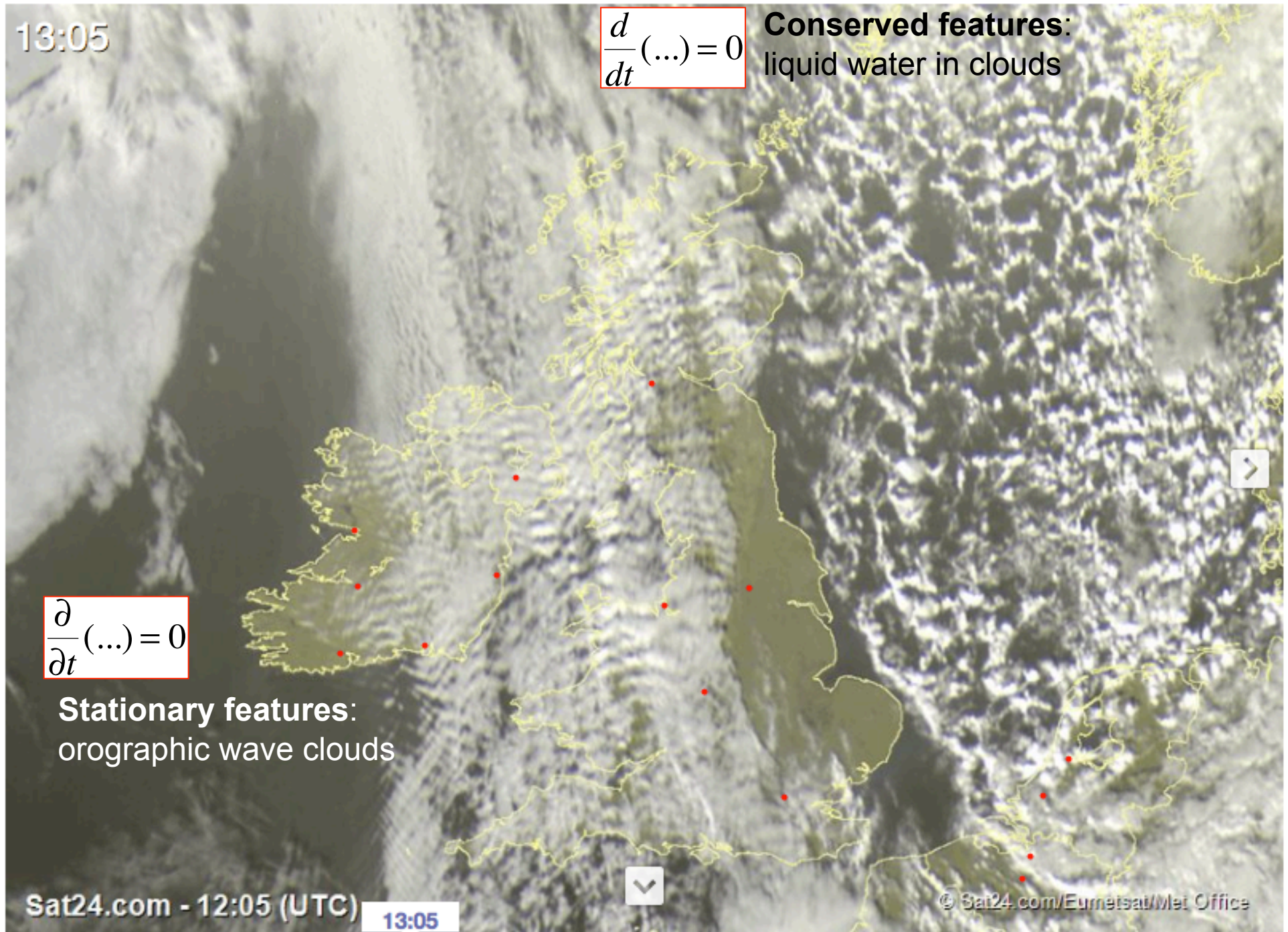
$$\frac{\partial}{\partial t}(\dots) = 0$$

Stationary features:
orographic wave clouds

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Material derivative *of a vector*

(for derivations see **geophysical fluid dynamics** and/or **Holton, chapters 1 and 2**)

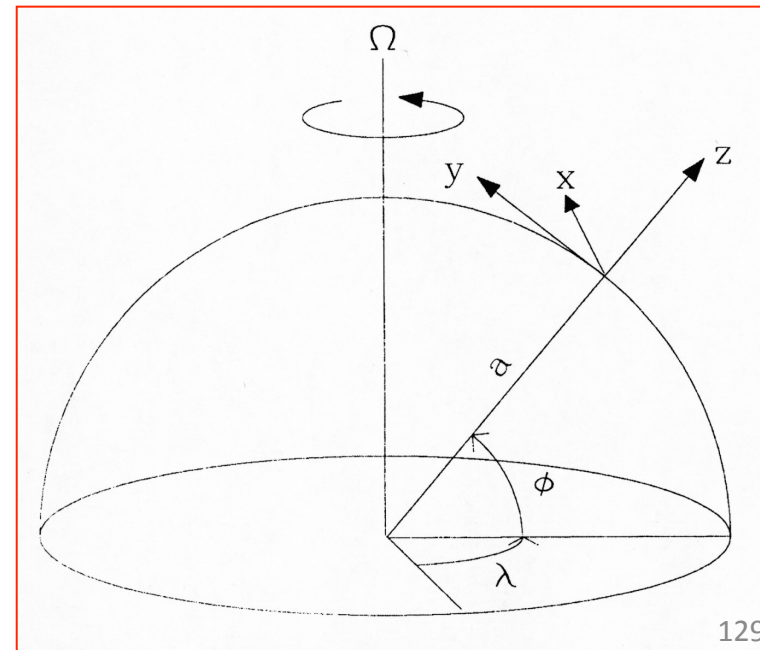
$$\frac{d\vec{v}}{dt} \equiv \left(\frac{du}{dt} - \frac{uv \tan \phi}{a} + \frac{uw}{a} \right) \hat{i} + \left(\frac{dv}{dt} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} \right) \hat{j} + \left(\frac{dw}{dt} - \frac{u^2 + v^2}{a} \right) \hat{k}$$

Additional terms due to curved coordinate system!!

eq. 1.40

These terms are frequently neglected in theoretical analysis (see **box 1.6**)

Coordinate system



“Coriolis effect” due to earth’s rotation

$$\frac{d\vec{v}}{dt} = -\alpha\vec{\nabla}p - g\hat{k} - 2\vec{\Omega} \times \vec{v} + \overline{Fr}$$

$$\vec{\Omega} \times \vec{v} = (w\Omega\cos\phi - v\Omega\sin\phi)\hat{i} + (u\Omega\sin\phi)\hat{j} - (u\Omega\cos\phi)\hat{k}$$

From “scale analysis”: *

$$2\vec{\Omega} \times \vec{v} \approx -(2\Omega v\sin\phi)\hat{i} + (2\Omega u\sin\phi)\hat{j} \equiv -fv\hat{i} + fu\hat{j}$$

f is the Coriolis Parameter: $f \equiv 2\Omega\sin\phi$

* $|w| \ll |u|$ and $|w| \ll |v|$ (see Holton, chapter 2 or geophysical fluid dynamics) 130

Equations in terms of potential temperature and Exner-function

$$\frac{d\theta}{dt} = \frac{J}{\Pi} \quad \leftarrow \text{eq. 1.23 or 1.33}$$

$$\frac{d\vec{v}}{dt} = -\theta \vec{\nabla} \Pi - g \hat{k} - 2\vec{\Omega} \times \vec{v} \quad \leftarrow \text{eq. 1.35+Coriolis}$$

$$\frac{d\Pi}{dt} = -\frac{R\Pi}{c_v} \nabla \cdot \vec{v} + \frac{RJ}{c_v \theta} \quad \leftarrow \text{eq. 1.37} \quad (\text{problem 1.7})$$

Three differential equations with three unknowns!

The influence of earth's rotation on atmospheric circulation patterns

The **Hadley circulation** and the **sea breeze** both constitute a magnificent illustration of the influence of rotation on the circulation !

What is the **Hadley circulation** ?

What is the **sea breeze** ?

First example of the influence of rotation:

Hadley circulation

George Hadley

Born 12 February 1685
London, England

Died 28 June 1768 (aged 83)

Nationality Great Britain

Occupation Lawyer, amateur meteorologist and physicist

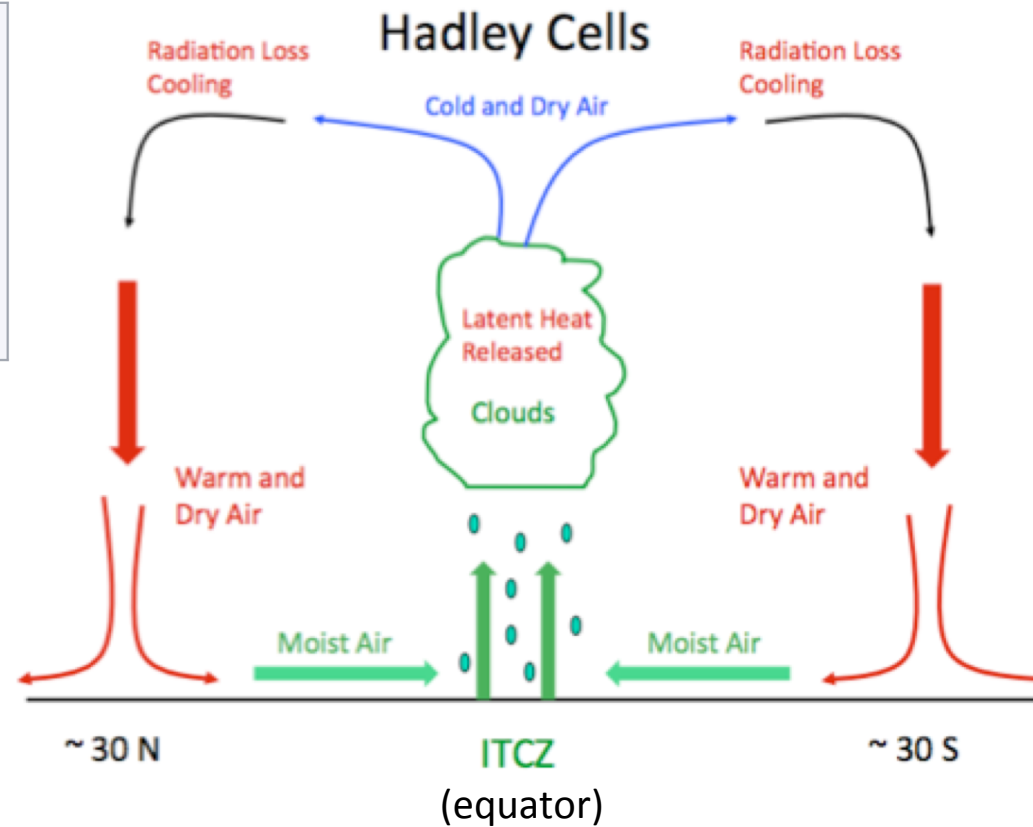
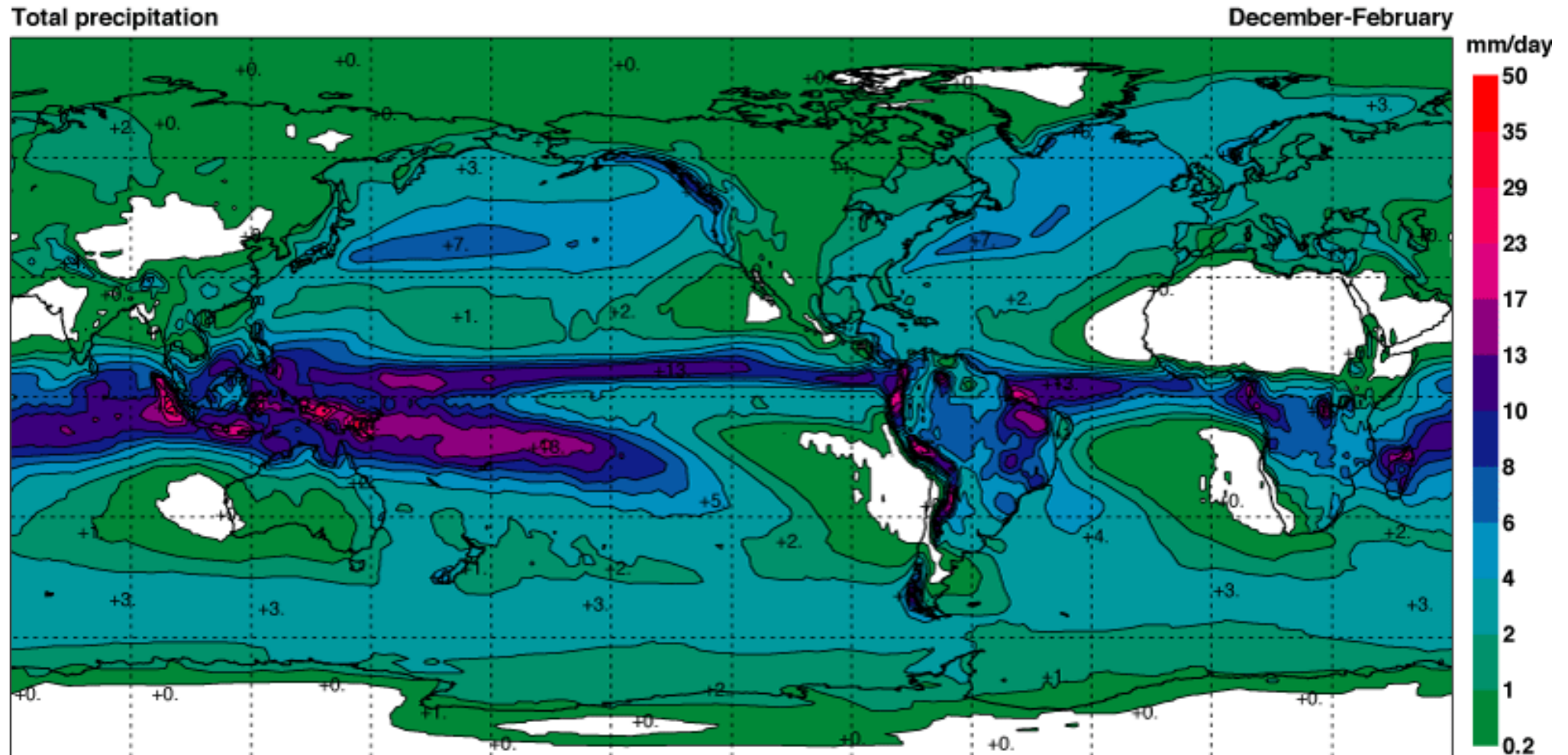


FIGURE 1.15. The time-average meridional circulation in the tropics, called the Hadley circulation, is driven principally by latent heat release in large convective clouds in the Inter-Tropical Convergence Zone (ITCZ). Water vapour is forced upwards over the ITCZ. There is compensating subsidence in the subtropics, which leads to a reduction of the relative humidity, cloudless skies and drought.

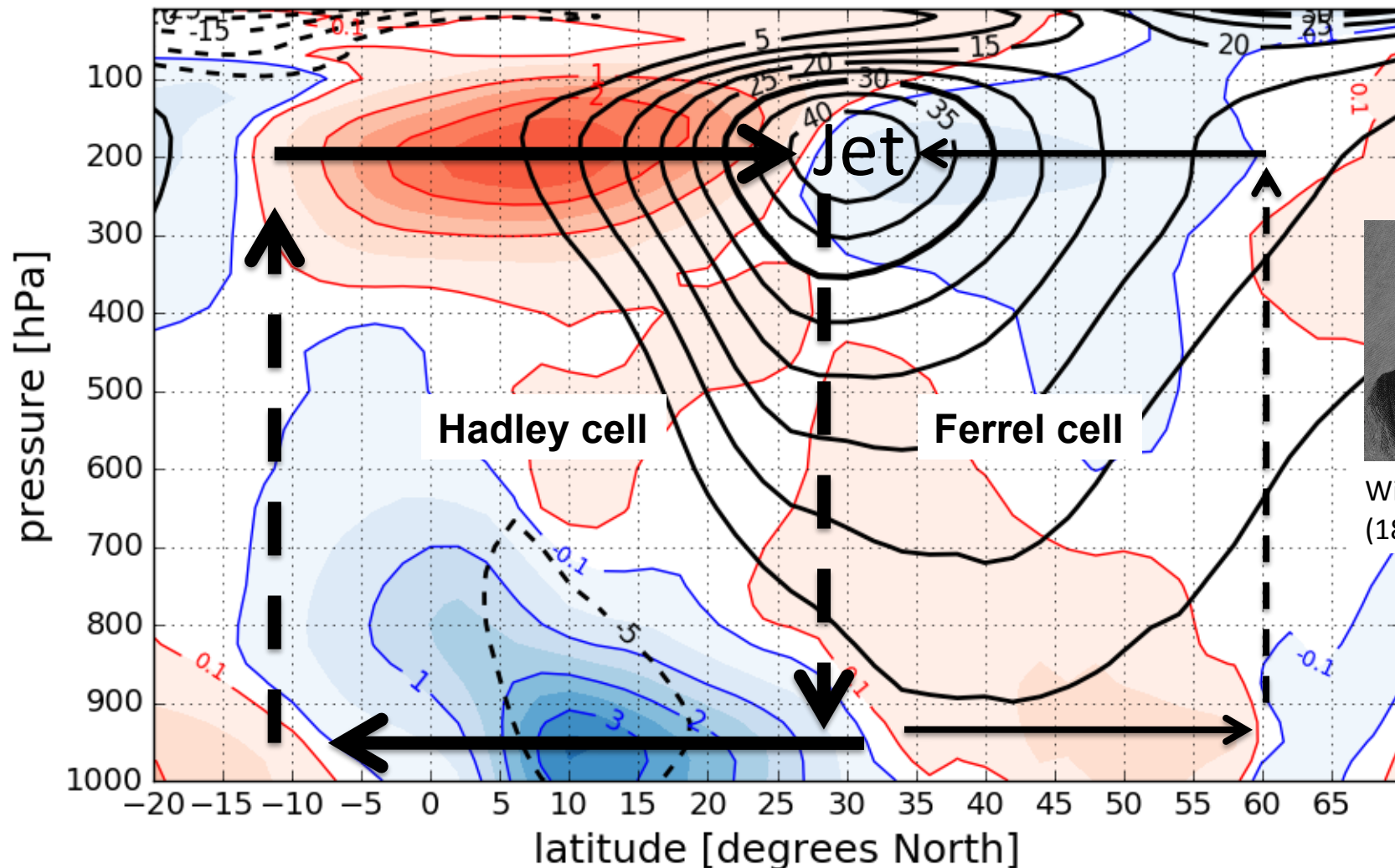
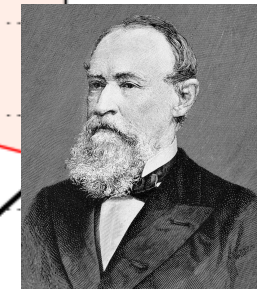
Precipitation at ITCZ & Hadley circulation



Precipitation maximum at the intertropical convergence zone (ITCZ) is a footprint of the so-called Hadley circulation (see next slide)

Winter Hadley cell and Ferrel cell

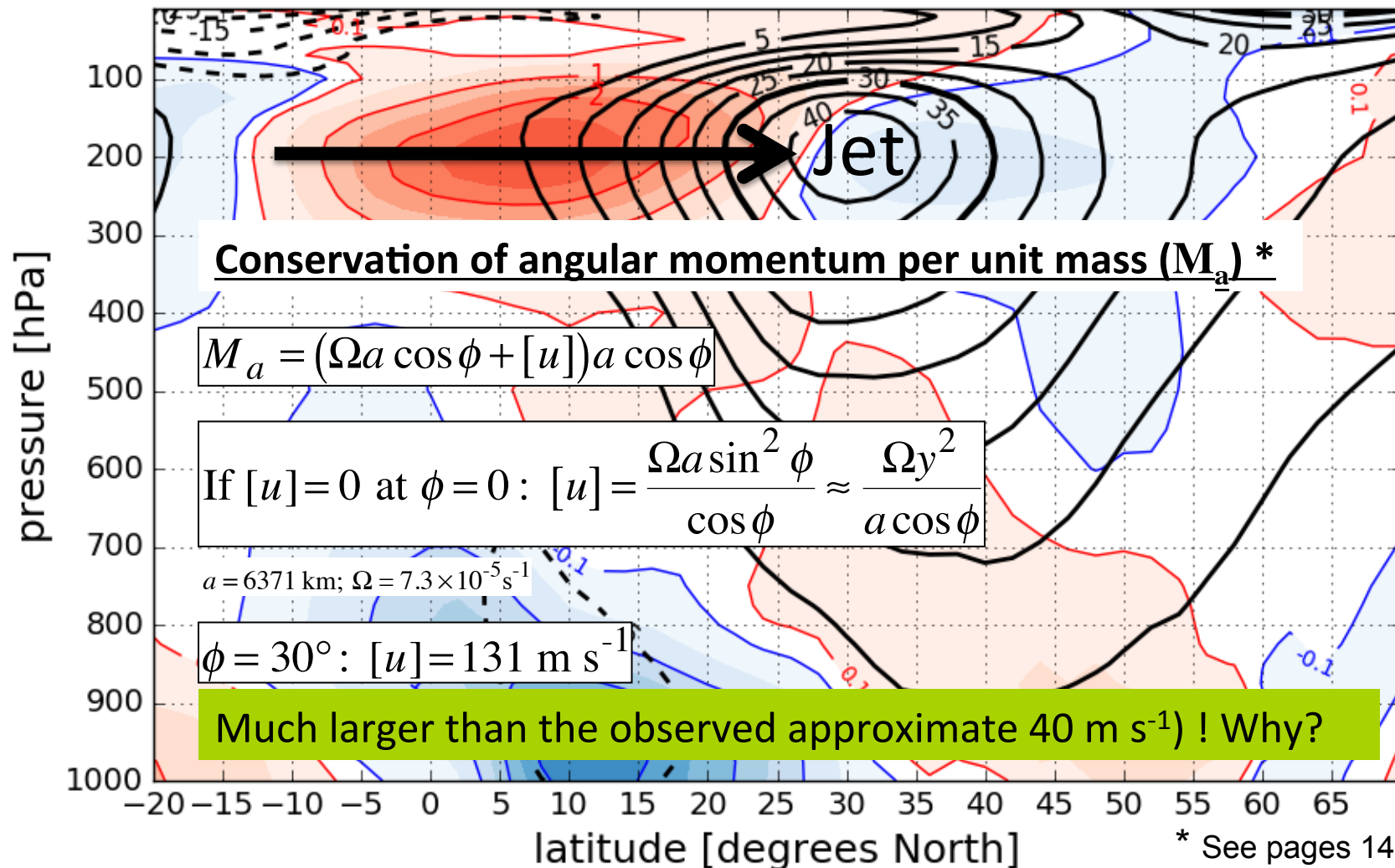
Ensemble-mean zonal-mean meridional velocity (red: toward north pole) (labeled in m s^{-1}) and zonal mean zonal velocity (black contours, labeled in m s^{-1}) in January (1979-2017).



William Ferrel
(1817-1891)

Winter Hadley cell

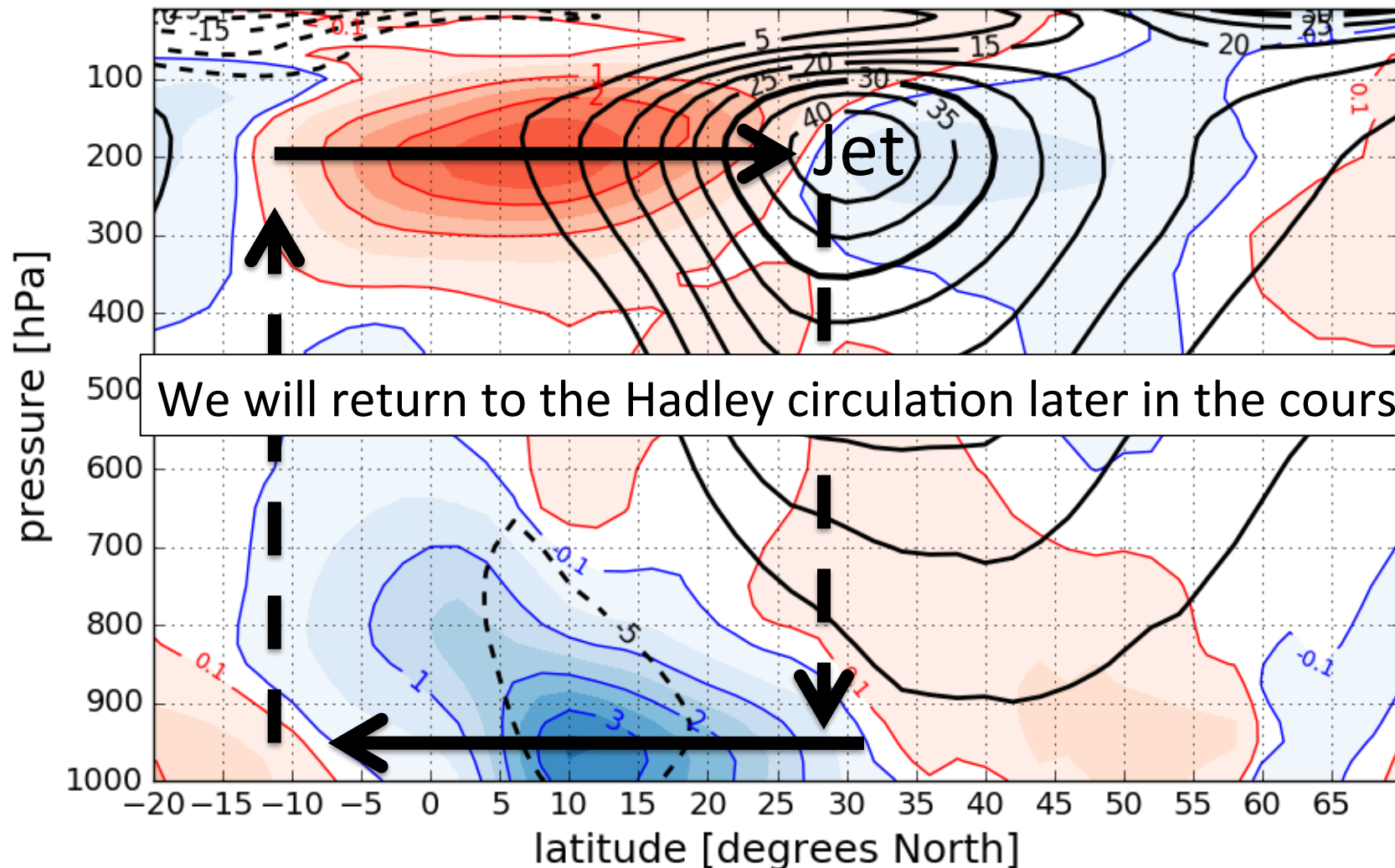
Ensemble-mean zonal-mean meridional velocity (red: toward north pole) (labeled in m s^{-1}) and zonal mean zonal velocity (black contours, labeled in m s^{-1}) in January (1979-2017).



* See pages 147-148

Winter Hadley cell

Ensemble-mean zonal-mean meridional velocity (red: toward north pole) (labeled in m s^{-1}) and zonal mean zonal velocity (black contours, labeled in m s^{-1}) in January (1979-2017).



Second example of the influence of rotation:

The sea breeze circulation

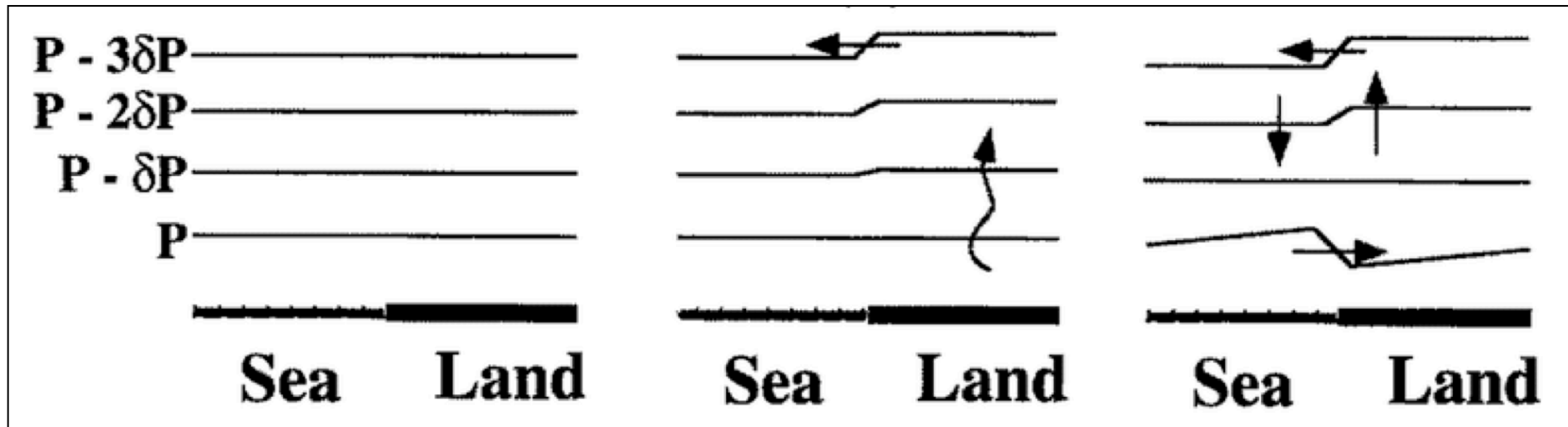


Figure 1.31

Three vertical cross-sections of the lowest 2 or 3 km of the atmosphere, schematically illustrating different stages of the development of the pressure gradients and the sea-breeze circulation.

The **left-hand diagram** shows the initial conditions.

The **middle diagram** shows the initiation of the circulation.

The **right-hand diagram** shows the final situation with fully-developed **sea breeze circulation**.

Sea breeze in the tropics

North-south component of the wind velocity (in m s^{-1}) above **Djakarta (Java)**, obtained from hourly mean values evaluated from balloon observations from May to November 1909 to 1915. Wind from the sea (sea breeze) is hatched. There is a pronounced return current with a maximum shortly before sunset. In the tropics the influence of rotation (Coriolis-effect) is negligible.

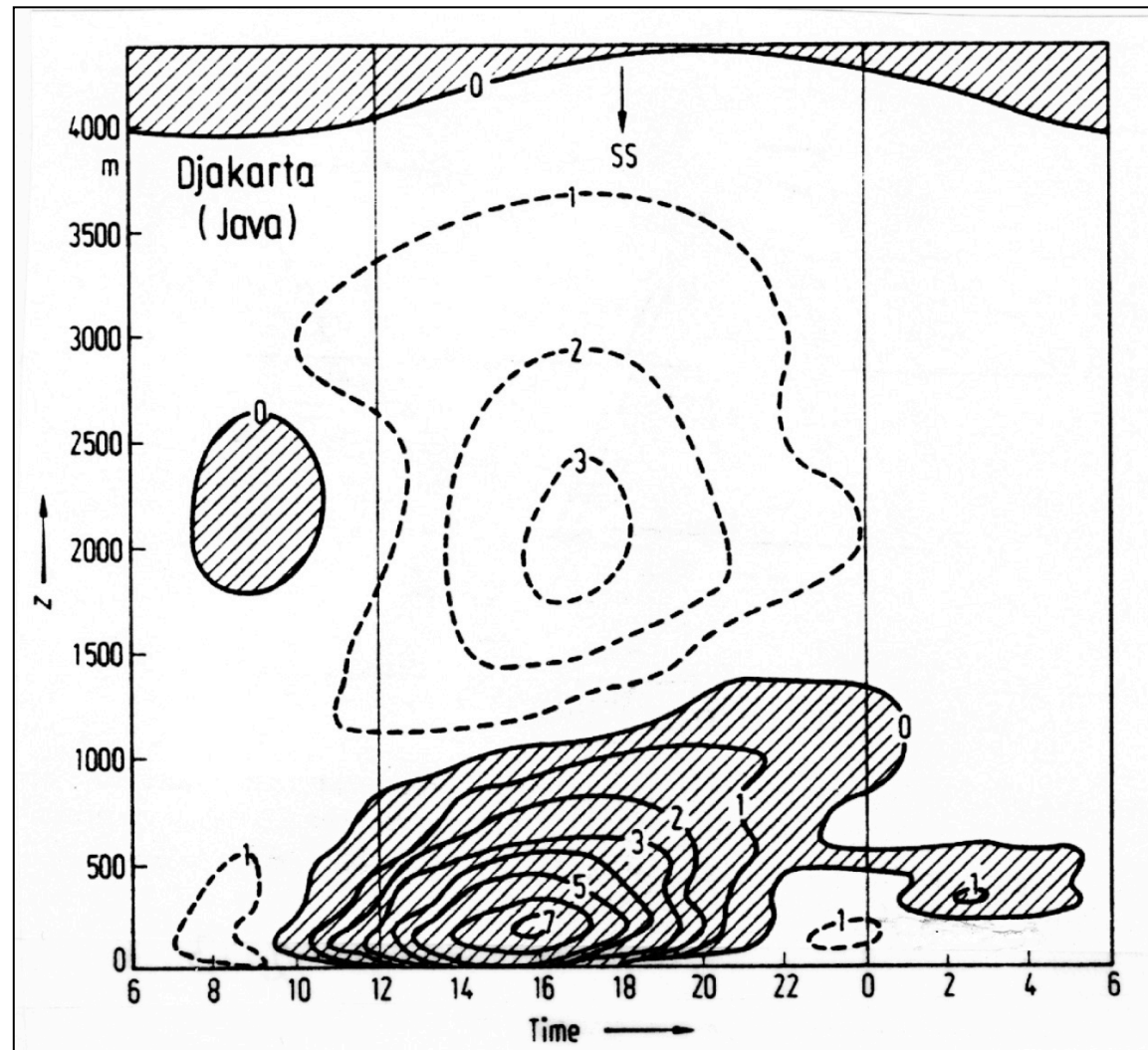
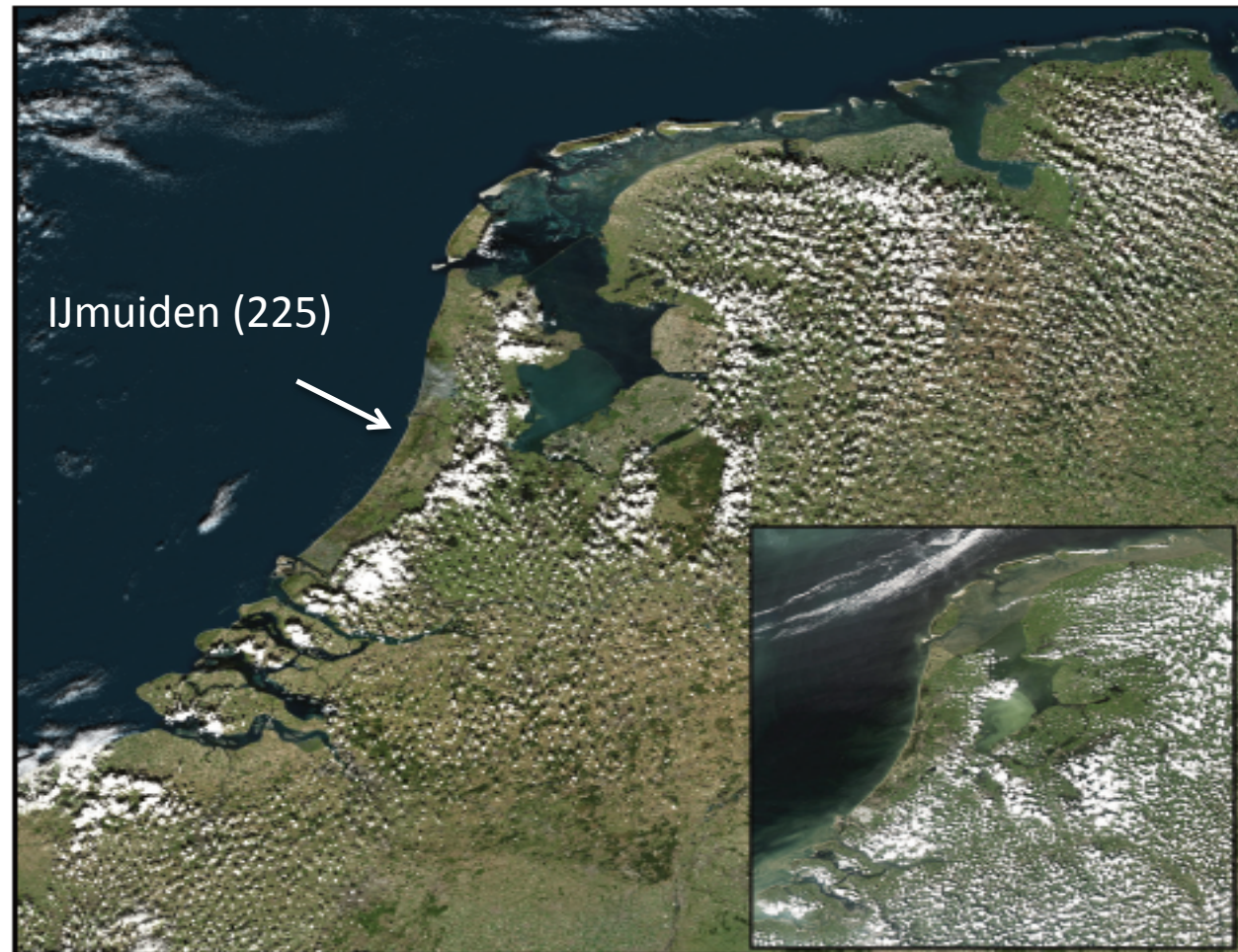


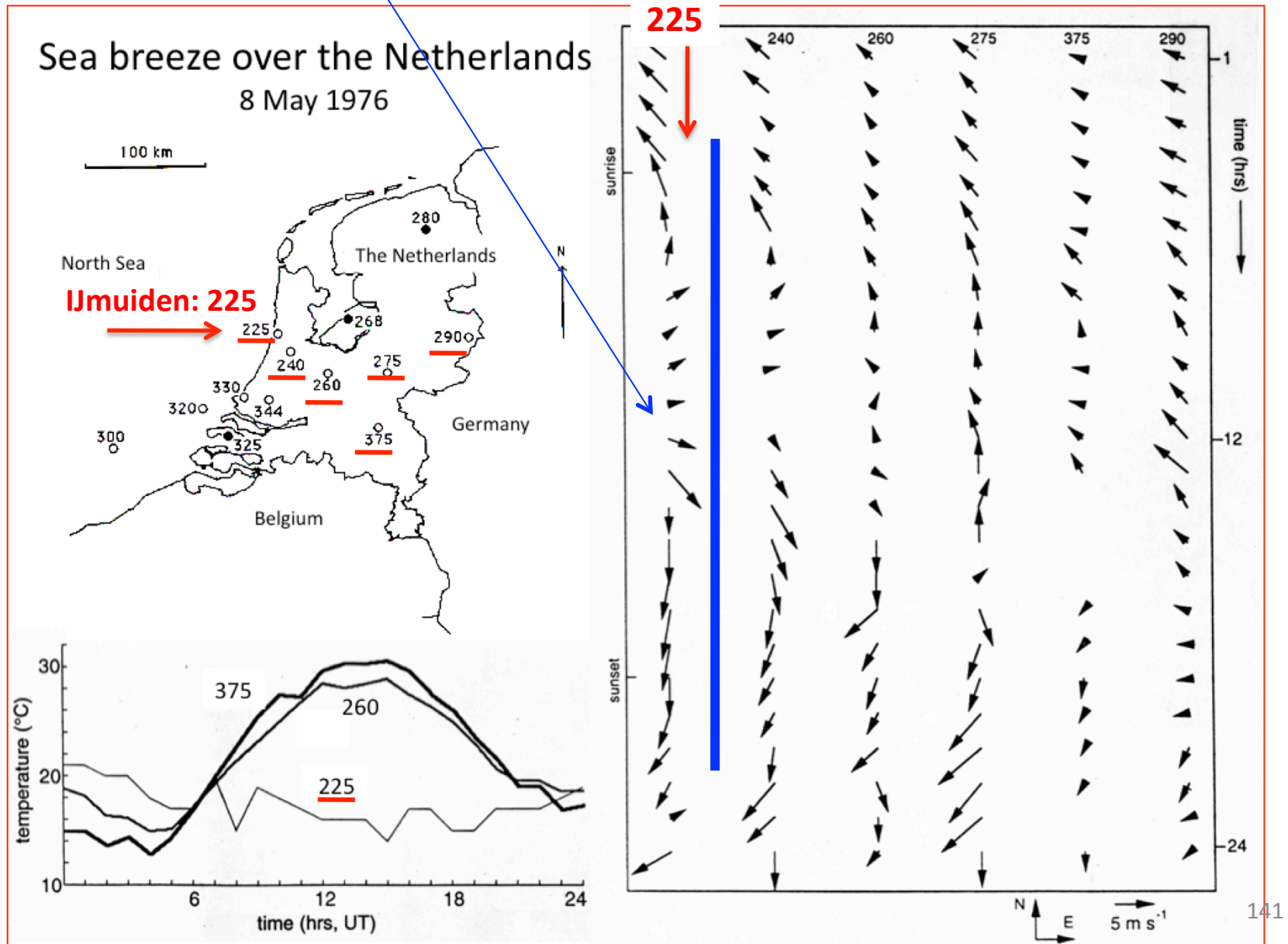
Figure 1.31

Evidence of sea breeze in the Netherlands

Computer model (DALES, Delft) simulation of cumulus clouds on 6 July 2004. In the right lower corner is the corresponding satellite image. Forced upward motion at the sea-breeze front leads more and larger clouds (Source PhD thesis by Jérôme Schalkwijk).



Clockwise turning of the sea breeze



NEXT lecture:

Project 3 (problem 1.14)

Simulation of the seabreeze and its clockwise turning at the coast during two days in May

Compare the model outcome with the observations at IJmuiden

Add the effect of friction to make the simulation more realistic



Next lecture: Friday 2 December 2022 (09:00-10:45), Room 001 BBG

Dynamic Meteorology: lecture 6

Sections 1.17-1.20

Introduction to project 3 (problem 1.14)

Theory of the seabreeze model

Box 1.9: Numerical approximation and model code/implementation

Box 1.10: model verification

Geostrophic balance

Stability of geostrophic balance,

Inertial oscillations & inertial instability

Tutorial after this lecture (5) (Wed. 30 November 2022):

Finish problem 1.6 (project 1); hand-in short report before end of today

Problem 1.12 (p. 89), problems 1 and 2 in Box 1.8 (p. 88),

Make a weather forecast for Sunday 4 December 2022