

07/12/2022 (Wednesday, 13:15)

Dynamical Meteorology: lecture 7

Sections 1.18-1.21

Stability of geostrophic balance,
Inertial oscillations & inertial instability
The thermal wind
Meridional exchange of warm and cold air masses
“Optimum” slope of trajectories in the meridional plane & conversion of
potential energy into kinetic energy
Conceptual model of a middle latitude cyclone (section 1.21)

Tutorial after this lecture (7)

Project 3

Problem 1.17

Make a weather forecast for next Sunday 11 December

Deadline for project 3 is end of Friday 9 December 2022

(a.j.vandelden@uu.nl)

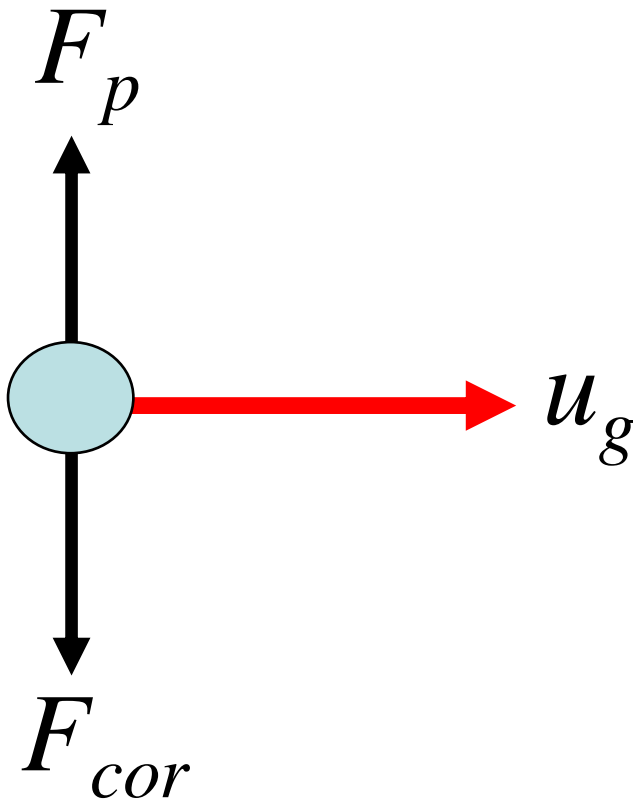
(<http://www.uu.nl/~nvdelden/dynmeteorology.htm>)

geostrophic zonal flow

Assume: no pressure gradient in x-direction

Assume a “geostrophic” **zonal flow**, u_g :

$$fu_g \equiv -\theta \frac{\partial \Pi}{\partial y}$$



geostrophic zonal flow

(section 1.18)

Assume: no pressure gradient in x-direction

Assume a “geostrophic” **zonal flow**, u_g :

$$fu_g \equiv -\theta \frac{\partial \Pi}{\partial y}$$

Equations of motion:

x-component: $\frac{du}{dt} = fv = f \frac{dy}{dt} \rightarrow du = fdy$

y-component: $\frac{dv}{dt} = -\theta \frac{\partial \Pi}{\partial y} - fu = f(u_g - u)$

geostrophic zonal flow

(section 1.18)

Assume: no pressure gradient in x-direction

Assume a “geostrophic” **zonal flow**, u_g :

$$fu_g \equiv -\theta \frac{\partial \Pi}{\partial y}$$

Equations of motion:

x-component: $d(u - fy) \equiv dM = 0$

Absolute momentum per unit mass:

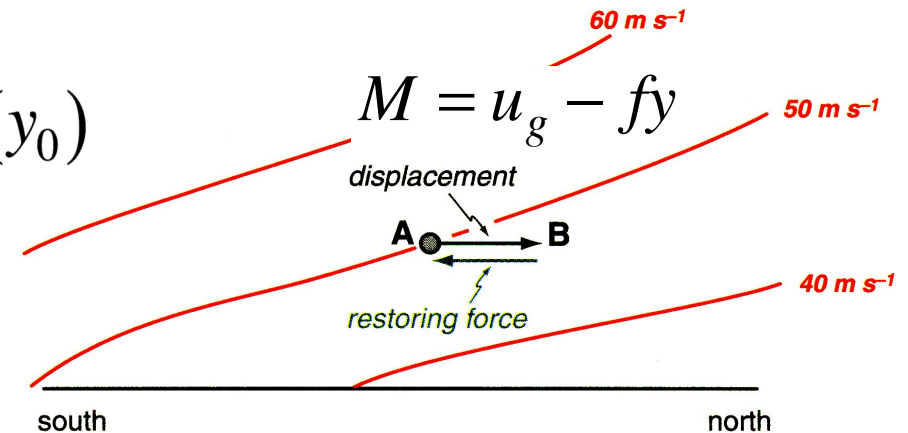
$$M \equiv u - fy$$

y-component: $\frac{dv}{dt} = f(u_g - u)$

Stability analysis: parcel method

Air parcel at $y = y_0$ moves to $y = y_0 + \delta y$

Since $d(u - fy) = 0 : u(y_0 + \delta y) - f\delta y = u(y_0)$



Isopleths of M in a meridional/vertical cross section of the atmosphere in geostrophic balance in the middle latitudes.

Note: in this figure u_g depends on y and z .

Figure: Markowski and Richardson, 2010

Stability analysis: parcel method

Air parcel at $y = y_0$ moves to $y = y_0 + \delta y$

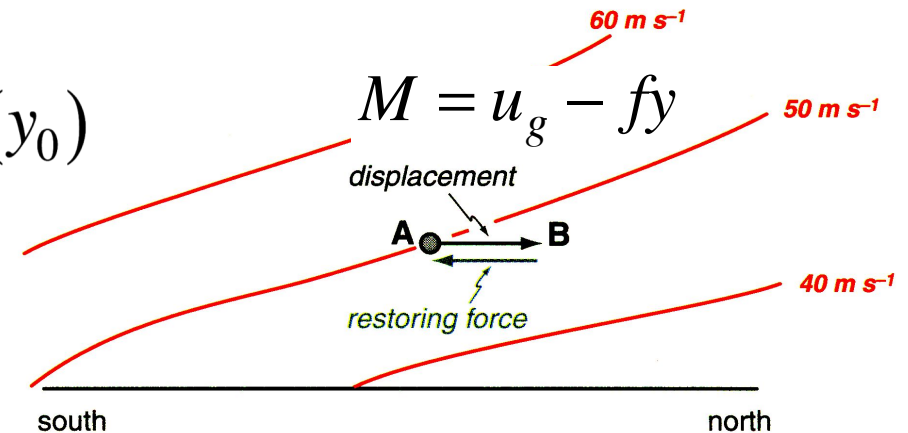
Since $d(u - fy) = 0 : u(y_0 + \delta y) - f\delta y = u(y_0)$

Geostrophic wind environment:

$$u_g(y_0 + \delta y) \approx u_g(y_0) + \frac{\partial u_g}{\partial y}(y_0)\delta y$$

Because $\frac{dv}{dt} = f(u_g - u)$

$$\frac{dv(y_0 + \delta y)}{dt} = f(u_g(y_0 + \delta y) - u(y_0 + \delta y)) = f\left[u_g(y_0) + \frac{\partial u_g}{\partial y}\delta y - u_g(y_0) - f\delta y\right]$$



Isopleths of M in a meridional/vertical cross section of the atmosphere in geostrophic balance in the middle latitudes.

Note: in this figure u_g depends on y and z .

Figure: Markowski and Richardson, 2010

Stability of geostrophic balance

Previous slide:

$$\frac{dv(y_0 + \delta y)}{dt} = f(u_g(y_0 + \delta y) - u(y_0 + \delta y)) = f\left[u_g(y_0) + \frac{\partial u_g}{\partial y}\delta y - u_g(y_0) - f\delta y\right]$$

or

$$\frac{d^2\delta y}{dt^2} = f\left[\frac{\partial u_g}{\partial y} - f\right]\delta y$$

Equation governing meridional acceleration: inertial stability

Previous slide:

$$\frac{d^2 \delta y}{dt^2} = -f \left[f - \frac{\partial u_g}{\partial y} \right] \delta y$$

or

$$\frac{d^2 \delta y}{dt^2} = -F^2 \delta y$$

with

$$F \equiv \sqrt{f \left[f - \frac{\partial u_g}{\partial y} \right]}$$

The solution: $\delta y = \text{Re}[A \exp(-iFt)]$

If $F^2 < 0$ then **inertial instability**;

If $F^2 > 0$ then **inertial stability (inertial oscillation)** ←

Inertial frequency, $|F|$ is about 10^{-4} s^{-1}

(positive in middle latitudes of the Northern Hemisphere)

Inertial frequency is a function of vorticity!

$$F \equiv \sqrt{f \left[f - \frac{\partial u_g}{\partial y} \right]}$$

Inertial frequency, $|F|$ is about 10^{-4} s^{-1} (in middle latitudes)

Absolute vorticity:

(defined precisely on the next slide)

$$\zeta_{\text{abs}} \equiv f - \frac{\partial u_g}{\partial y}$$

planetary vorticity

relative vorticity

Inertial instability if vorticity < 0 (*anticyclonic* flow)

Absolute value of anti-cyclonic vorticity never larger than $|f|$!

Relative vorticity

Relative vorticity vector is defined as the curl of the relative velocity vector:

$$\vec{\omega} \equiv \vec{\nabla} \times \vec{v}$$

Relative to the Earth's surface

In (x,y,z) coordinate system:

$$\vec{v} \equiv (u, v, w) \quad \vec{\nabla} \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Therefore:

$$\vec{\omega} \equiv \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

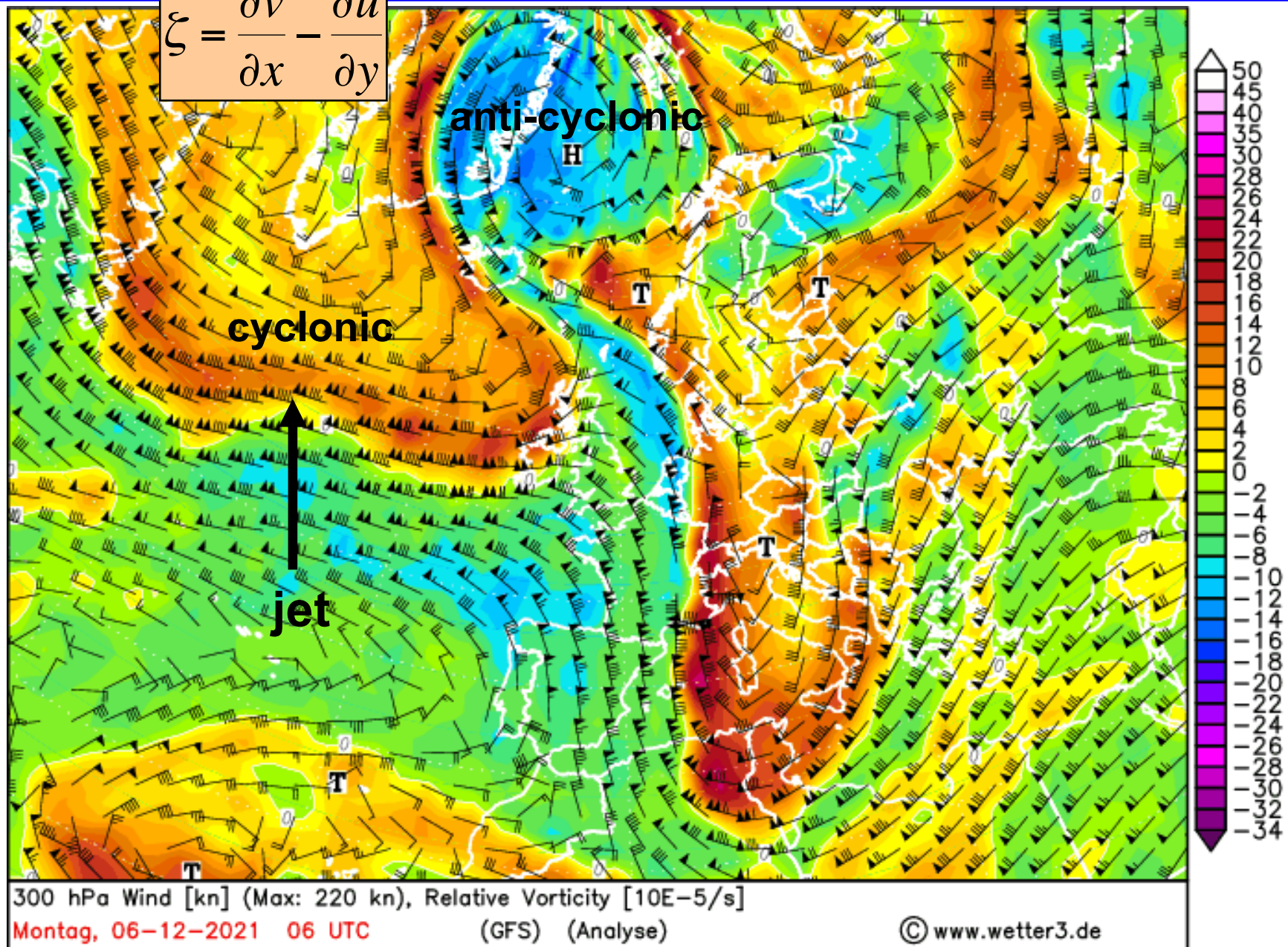
The vertical component of the relative vorticity is

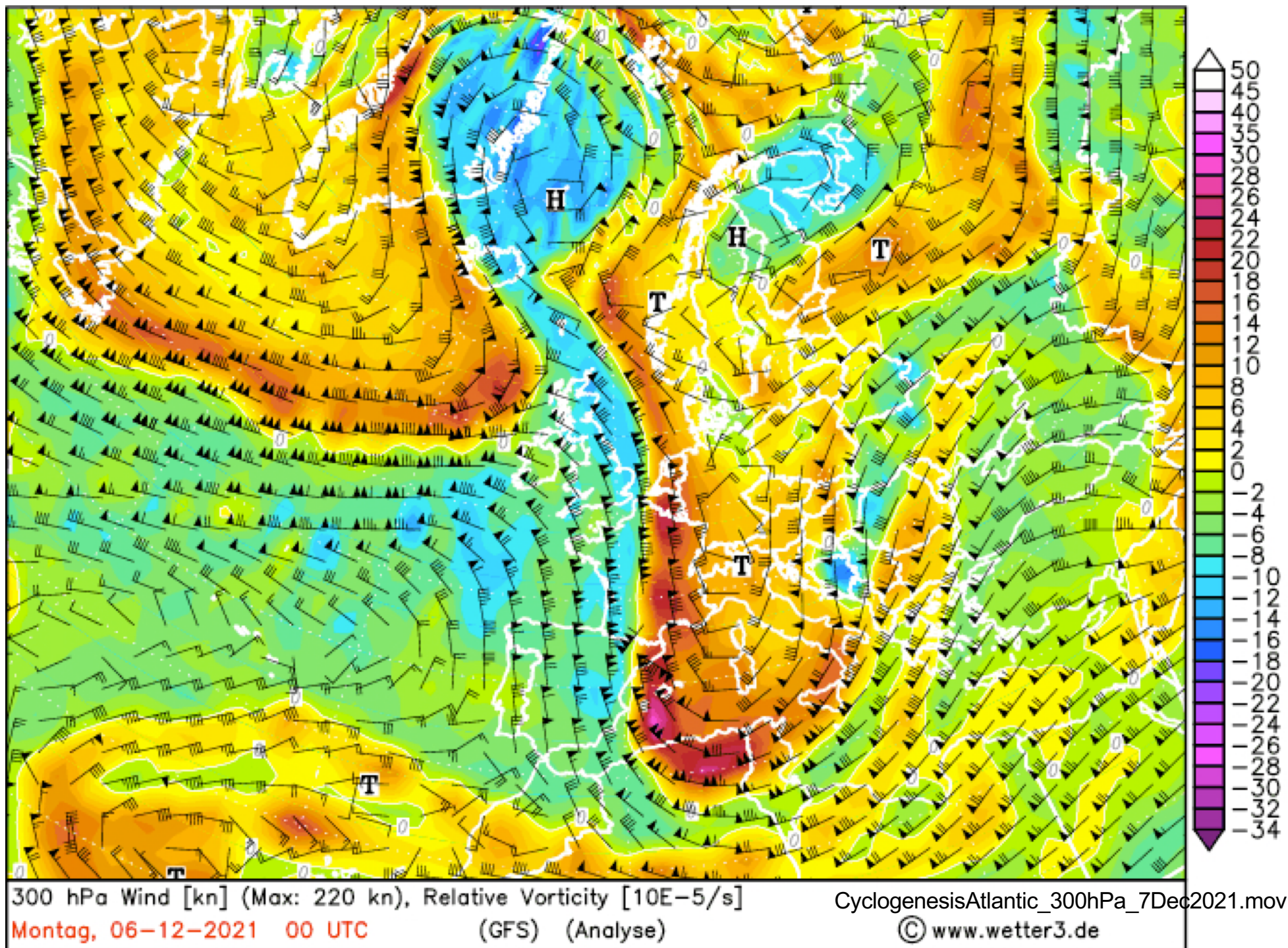
$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Problem: write expressions for the other two (horizontal) components of the relative vorticity

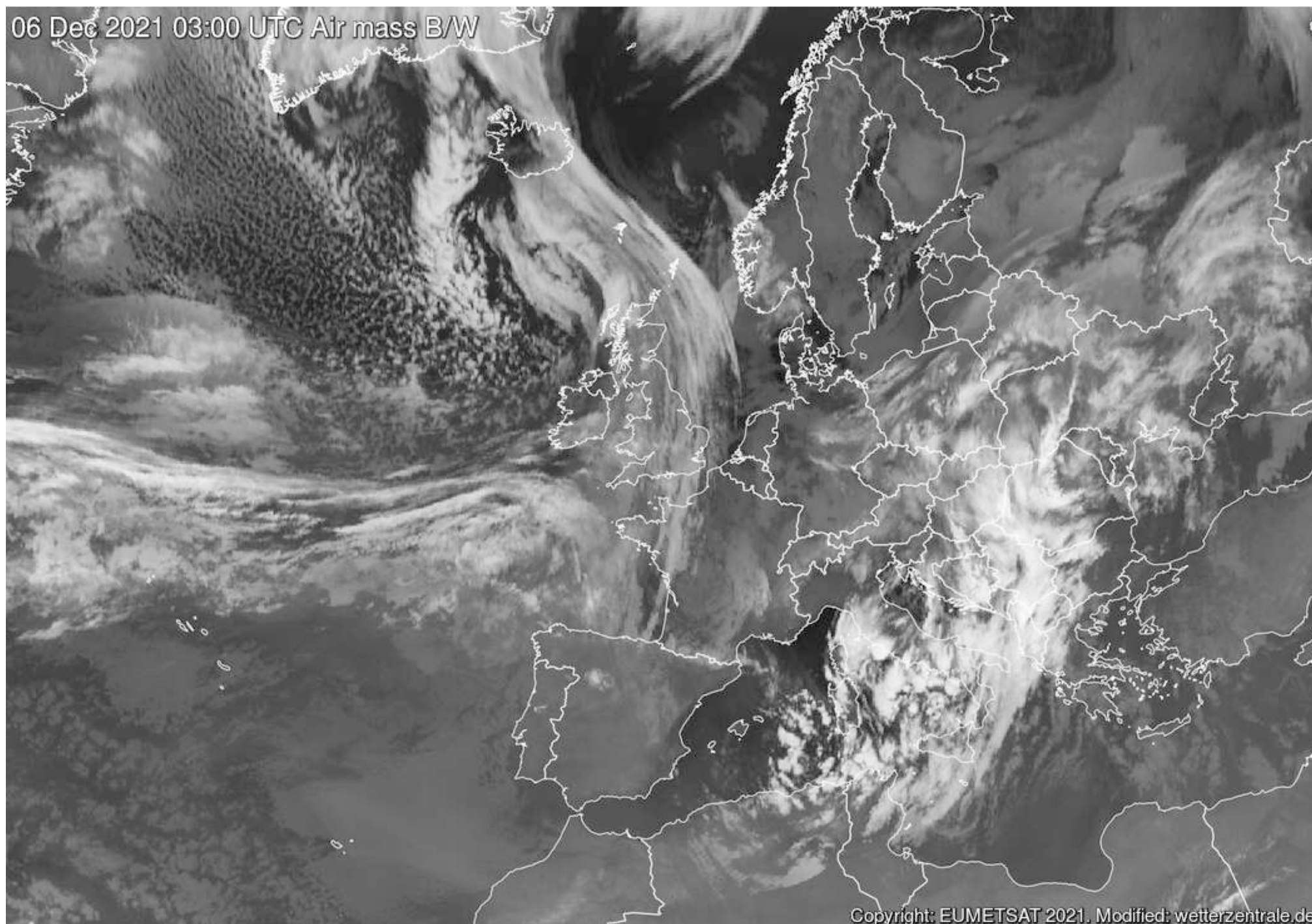
Relative vorticity around a jet at 300 hPa

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$





06 Dec 2021 03:00 UTC Air mass B/W



Relative vorticity around a jet

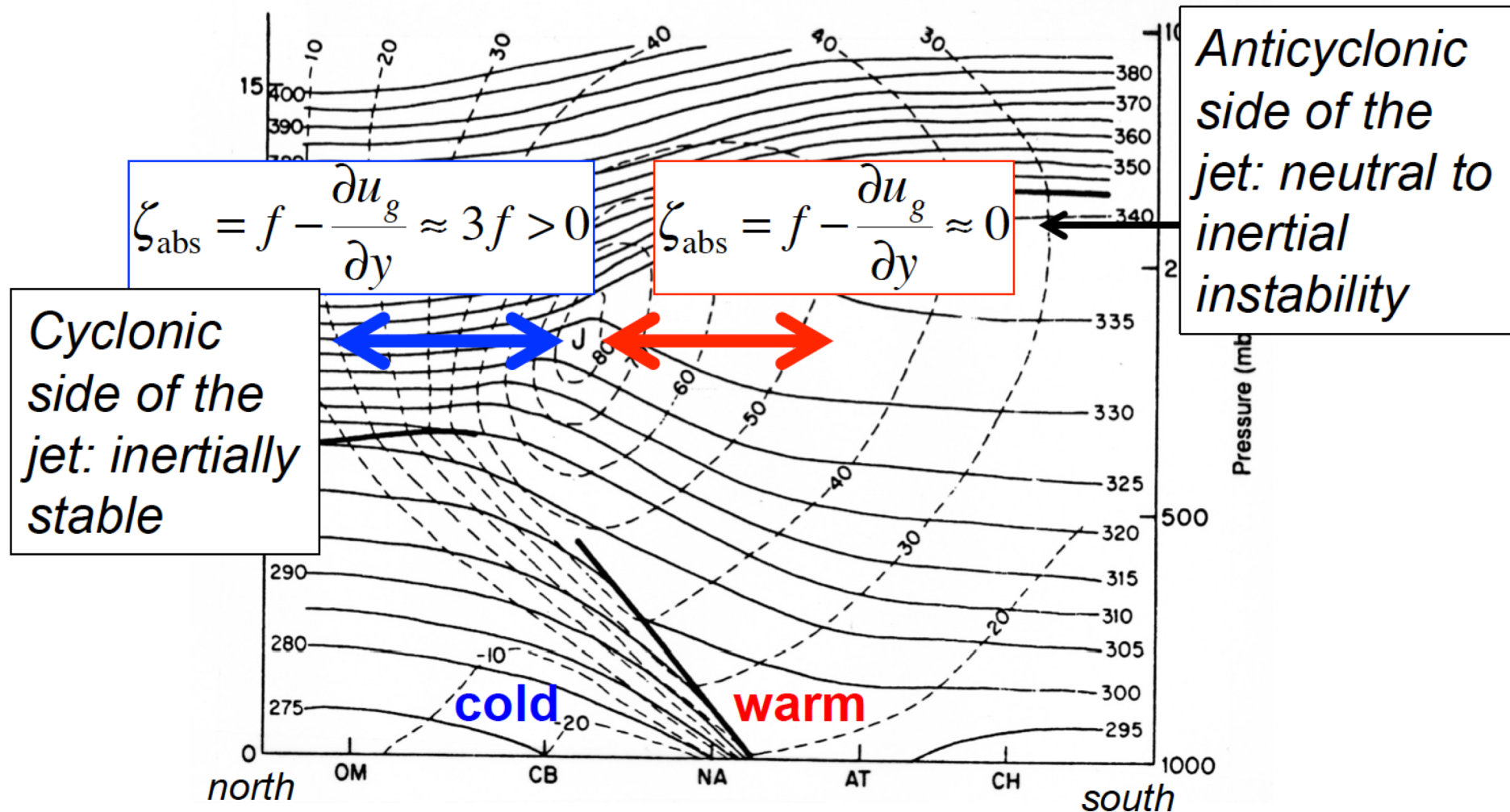


FIGURE 1.39: Isentropes (thin solid lines, labeled in Kelvin) and isotachs (isopleths of the zonal velocity) (dashed lines, m s^{-1}) in a vertical section through a cold front. The y-coordinate is positive towards the left. Heavy lines mark the tropopause and frontal boundaries. The section extends approximately 1200 km in the horizontal direction (Palmen, E. and C.W. Newton, 1969). .

Geostrophic and hydrostatic balance => thermal wind balance

If zonal flow is in **geostrophic balance**: $\theta \frac{\partial \Pi}{\partial y} + f u_g = 0$

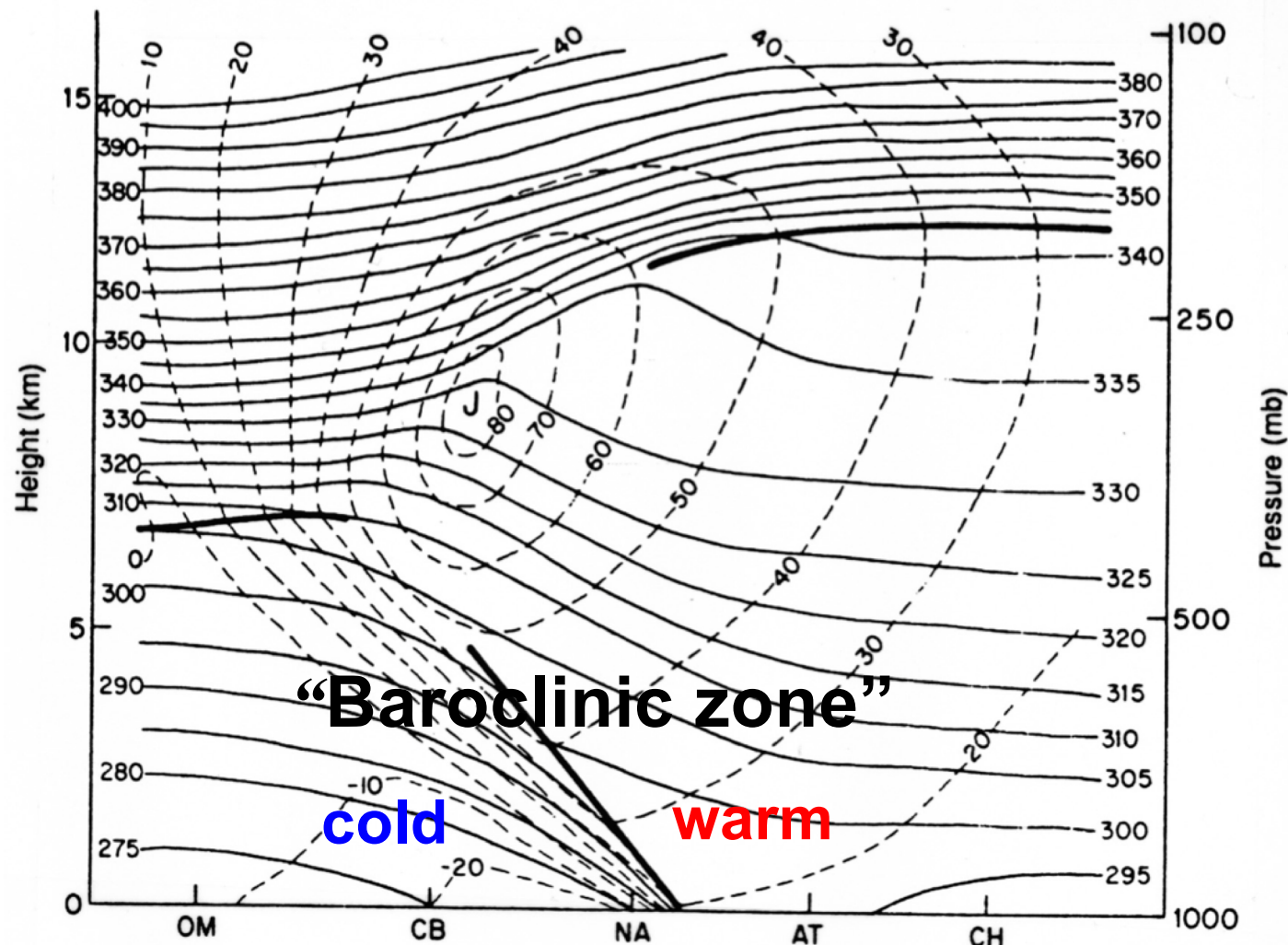
and in **hydrostatic balance**: $\theta \frac{\partial \Pi}{\partial z} + g = 0$

Check this using
scale analysis !

Thermal wind balance : $\frac{\partial u_g}{\partial z} \approx - \frac{g}{f \theta} \frac{\partial \theta}{\partial y}$ (section 1.19)

Vertical gradient of the geostrophic wind is related to a horizontal gradient in potential temperature.

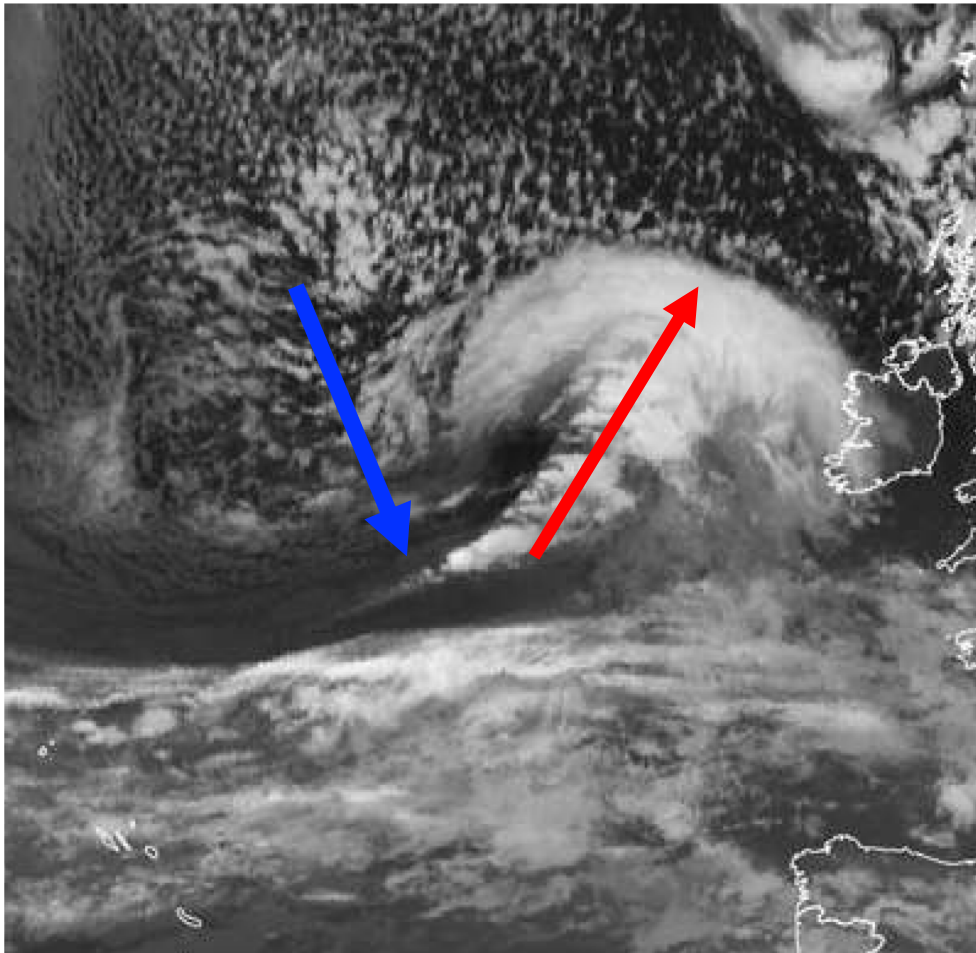
Jet: manifestation of the thermal wind



*Vertical shear of the zonal wind is related to the horizontal temperature gradient in a “**baroclinic zone**”*

FIGURE 1.39: Isentropes (thin solid lines, labeled in Kelvin) and isotachs (isopleths of the zonal velocity) (dashed lines, m s^{-1}) in a vertical section through a cold front. The y-coordinate is positive towards the left. Heavy lines mark the tropopause and frontal boundaries. The section extends approximately 1200 km in the horizontal direction (Palmen, E. and C.W. Newton, 1969).

Exchange of cold and warm air masses in a baroclinic zone may be self-sustaining



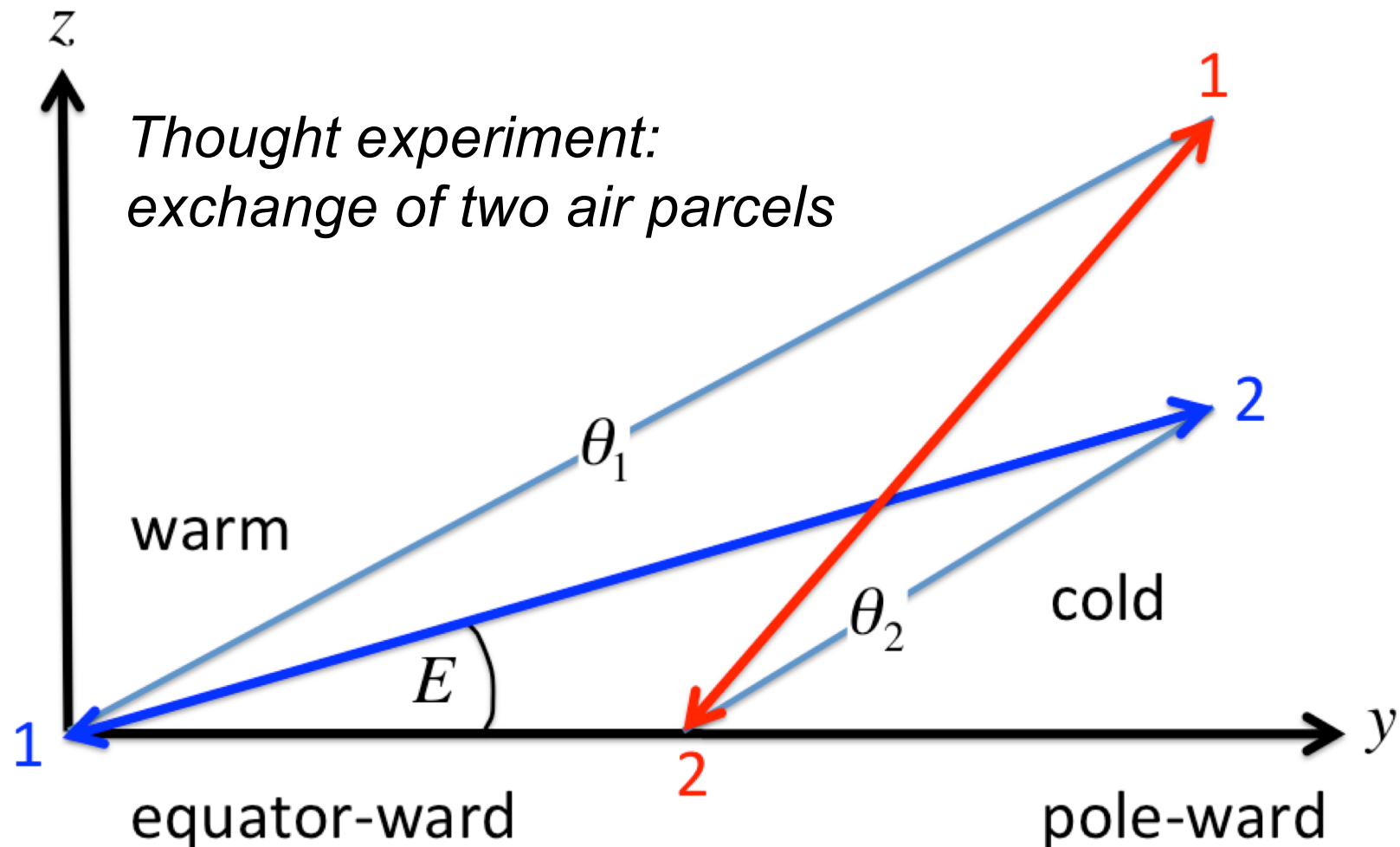
Decending cold air

Rising warm air

6 December 2021, 20 UTC

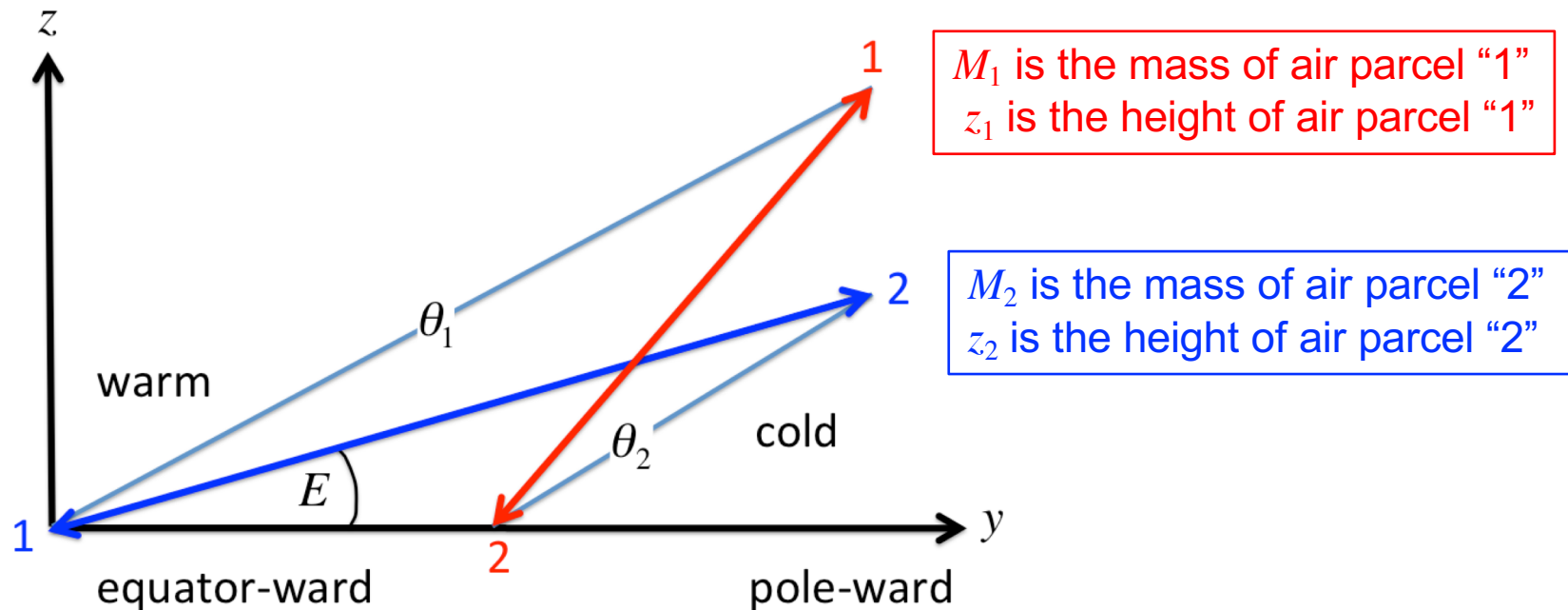
Energy conversions in a baroclinic zone

Let us study the exchange of potential and kinetic energy in meridional exchange of air parcels with different potential temperatures.



Energy conversions in a baroclinic zone

Exchange of two air parcels

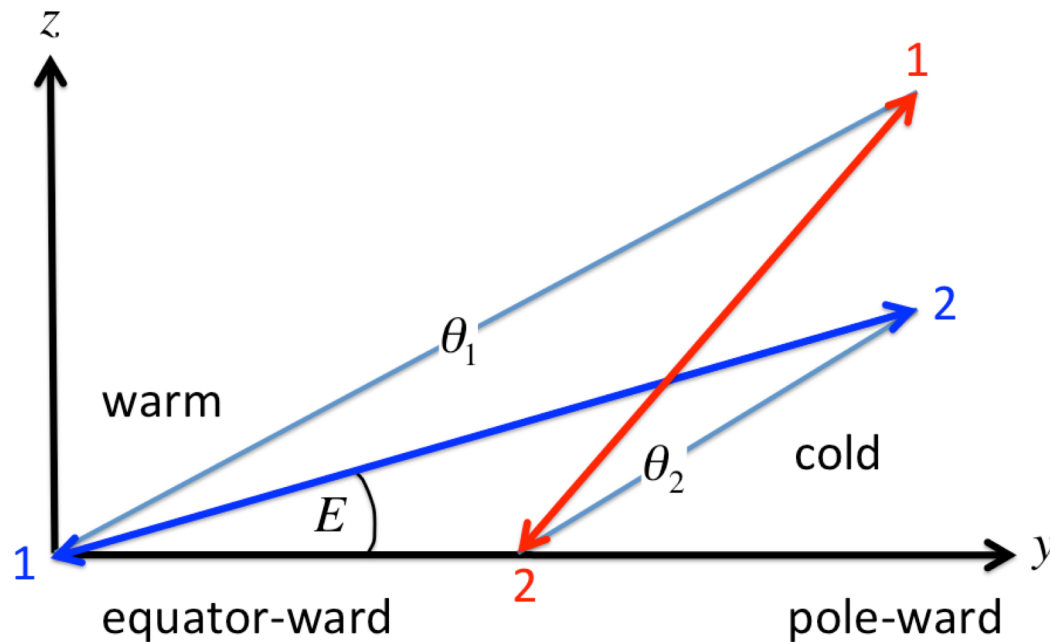


Change in potential energy:

$$\Delta PE = (M_1 g z_2 + M_2 g z_1) - (M_1 g z_1 + M_2 g z_2) = M_1 g (z_1 - z_2) \left(\frac{M_2}{M_1} - 1 \right)$$

Energy conversions in a baroclinic zone

Exchange of two air parcels



With:

$$p_1 V_1^\gamma = p_2 V_2^\gamma \quad \textbf{(Problem 1.2)}$$

$$\theta = \frac{C}{\rho} p^{1/\gamma} \quad \textbf{Show this}$$

$$C = \text{constant} \quad \gamma \equiv \frac{c_p}{c_v}$$

Ratio of masses is:

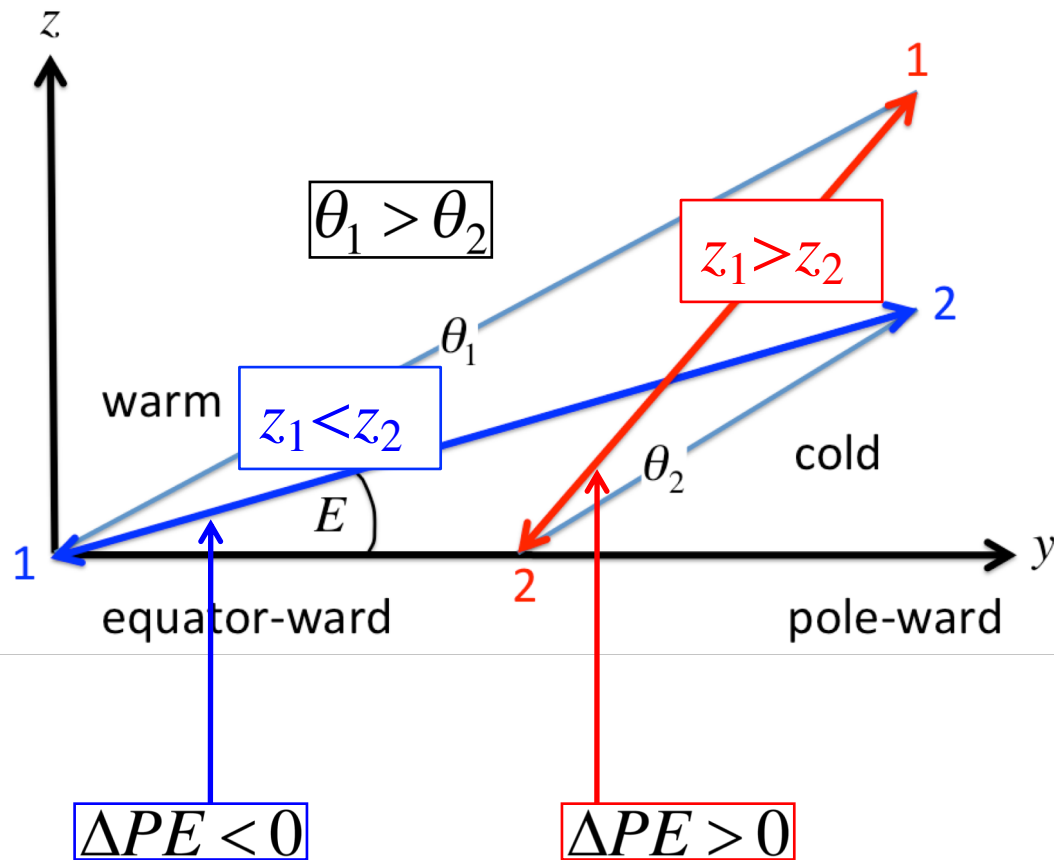
$$\frac{M_2}{M_1} = \frac{\rho_2 V_2}{\rho_1 V_1} = \frac{\rho_2 p_1^{1/\gamma}}{\rho_1 p_2^{1/\gamma}} = \frac{\theta_1}{\theta_2}$$

Change in potential energy:

$$\Delta PE = M_1 g (z_1 - z_2) \left(\frac{\theta_1 - \theta_2}{\theta_2} \right)$$

Energy conversions in a baroclinic zone

Exchange of two air parcels



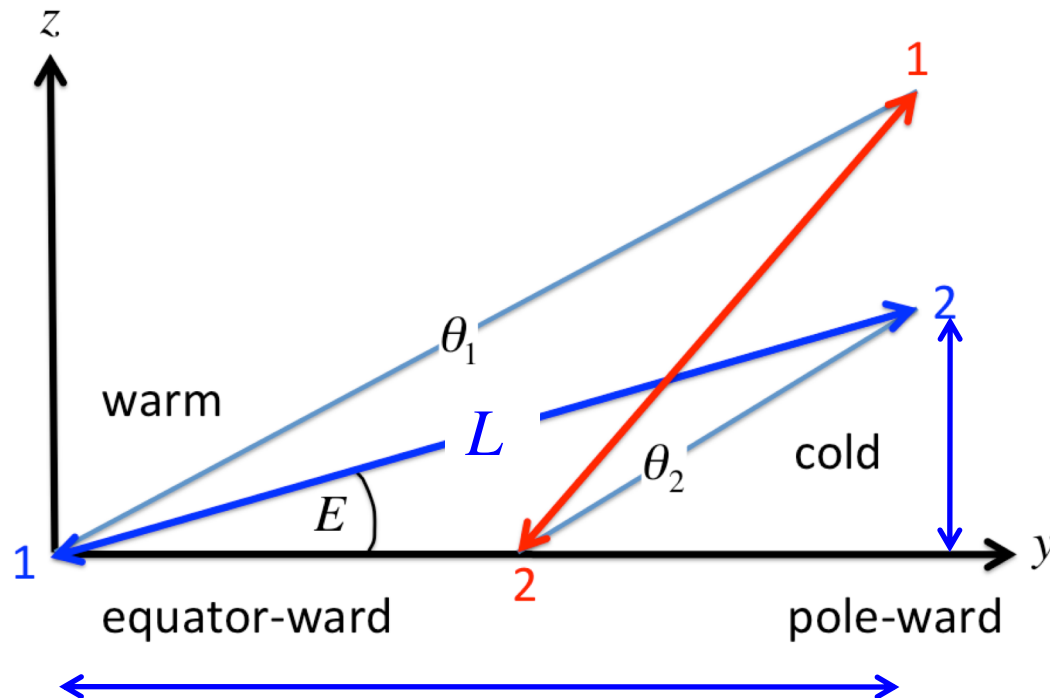
Change in potential energy:

$$\Delta PE = M_1 g (z_1 - z_2) \left(\frac{\theta_1 - \theta_2}{\theta_2} \right)$$

Increase of KE

Decrease of KE (kinetic energy)

Slope, E , of the baroclinic trajectory



Change in potential energy:

$$\Delta PE = M_1 g (z_1 - z_2) \left(\frac{\theta_1 - \theta_2}{\theta_2} \right)$$

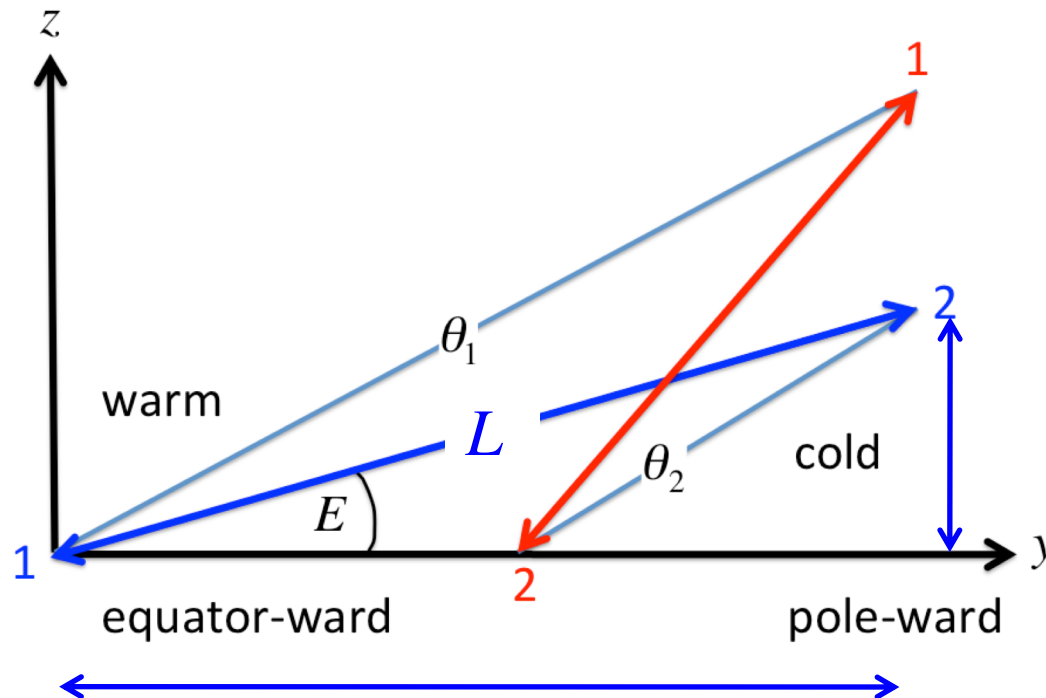
Exchange along blue line:

$$\cos E = \frac{(y_2 - y_1)}{L}$$

Potential temperature difference in terms of slope of trajectory:

$$\theta_2 - \theta_1 = (y_2 - y_1) \frac{\partial \theta}{\partial y} + (z_2 - z_1) \frac{\partial \theta}{\partial z} = \left(\frac{\partial \theta}{\partial y} \cos E + \frac{\partial \theta}{\partial z} \sin E \right) L$$

Slope, E , of the baroclinic trajectory



Change in potential energy:

$$\Delta PE = M_1 g (z_1 - z_2) \left(\frac{\theta_1 - \theta_2}{\theta_2} \right)$$

$$\theta_2 - \theta_1 = \left(\frac{\partial \theta}{\partial y} \cos E + \frac{\partial \theta}{\partial z} \sin E \right) L$$

$$(z_2 - z_1) = L \sin E$$

With this:

$$\Delta PE = \frac{M_1 g L^2}{\theta_2} \sin E \left(\frac{\partial \theta}{\partial y} \cos E + \frac{\partial \theta}{\partial z} \sin E \right)$$

Slope, E , of the baroclinic trajectory

Repeat from previous slide:

$$\Delta PE = \frac{M_1 g L^2}{\theta_2} \sin E \left(\frac{\partial \theta}{\partial y} \cos E + \frac{\partial \theta}{\partial z} \sin E \right)$$

Kinetic energy is maximum when ΔPE is a minimum

Minimizing this expression with respect to E yields:

(do this yourself)

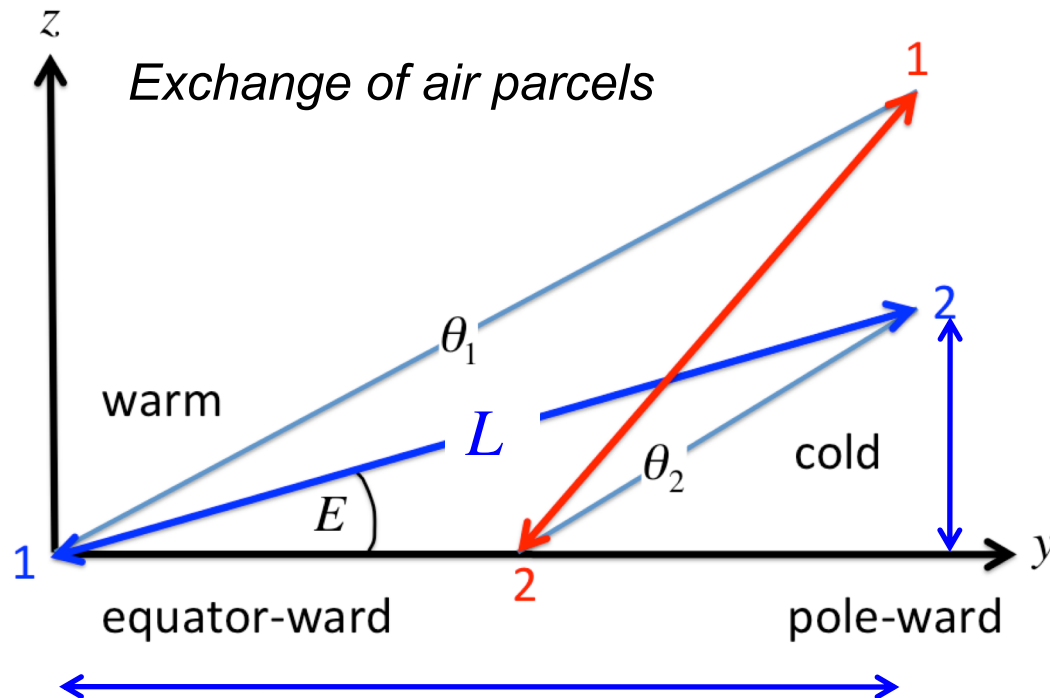
$$\tan 2E = - \frac{\partial \theta / \partial y}{\partial \theta / \partial z}$$

Use:

$$\begin{aligned} 2 \sin x \cos x &= \sin 2x \\ (\cos x)^2 - (\sin x)^2 &= 1 - 2(\sin x)^2 = \cos 2x \end{aligned}$$

The “preferred” slope of parcel trajectories is half the slope of the isentropic surfaces in the environment

Slope, E , of the baroclinic trajectory



Therefore, if

$$\tan 2E = - \frac{\partial \theta / \partial y}{\partial \theta / \partial z}$$

i.e. if the slope of the parcel trajectories is half the slope of the isentropes in the environment, potential energy is reduced most strongly, leading to kinetic energy production.

Problem 1.17. Assume that all the potential energy is converted into kinetic energy. Derive an approximate expression for the final velocity of each air parcel after the exchange. What is this velocity (approximately) if $L=200$ km? Assume that $\partial \theta / \partial y \approx -5 \times 10^{-5} \text{ K m}^{-1}$ and $\partial \theta / \partial z \approx 5 \times 10^{-3} \text{ K m}^{-1}$

Baroclinic instability in three dimensions

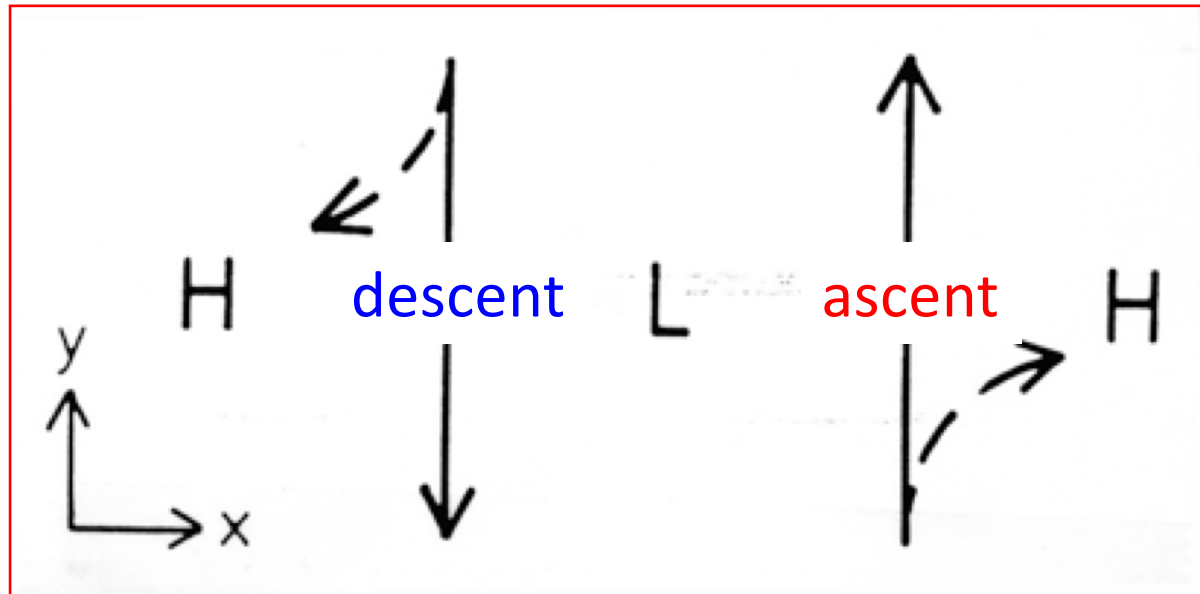


FIGURE 1.42. The motion of air, as seen in the plane of the trajectories. The dashed vectors are those that would occur in the absence of pressure variation indicated by L (low pressure) and H (high pressure), and the continuous vectors are the actual balanced motions.

Question is: what mechanism forces these meridional motions?
This question will be answered in the second part of the course.

Baroclinic instability in three dimensions

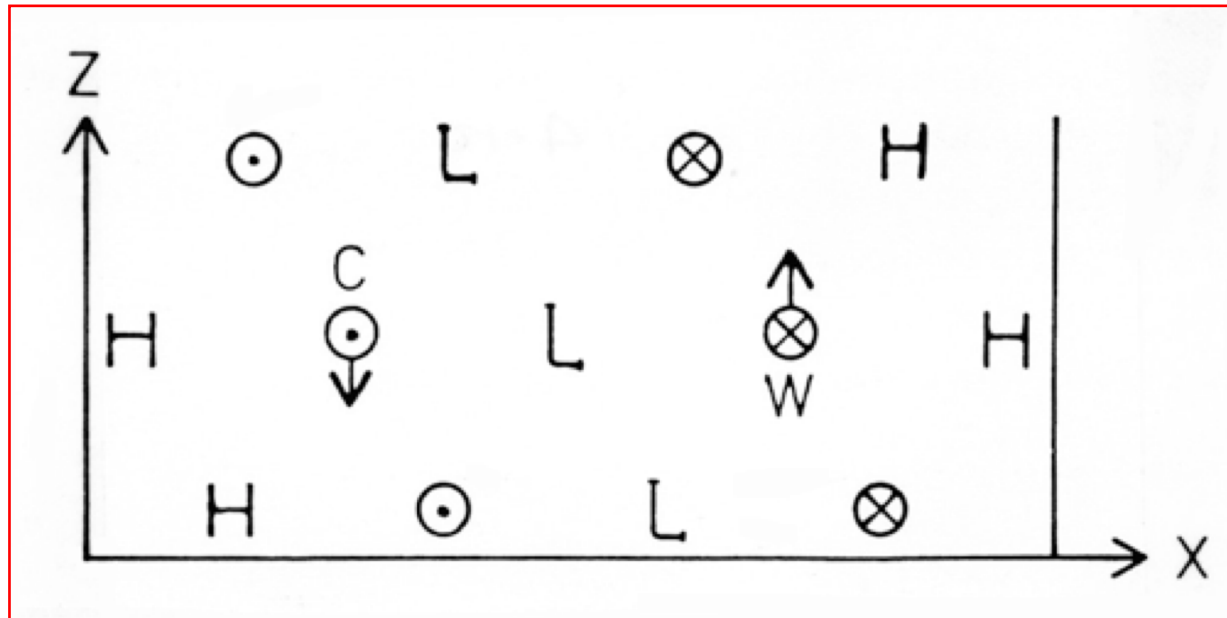


FIGURE 1.43. A longitude-height section showing **poleward**, **equatorward** vertical motion of **warm** (W) and **cold** (C) air, and the longitudinal variation in pressure (H=high pressure; L=low pressure) for a growing “baroclinic disturbance”.

Wave tilts westward with height!
This is a very important property of baroclinic waves

Conceptual model - unstable baroclinic wave

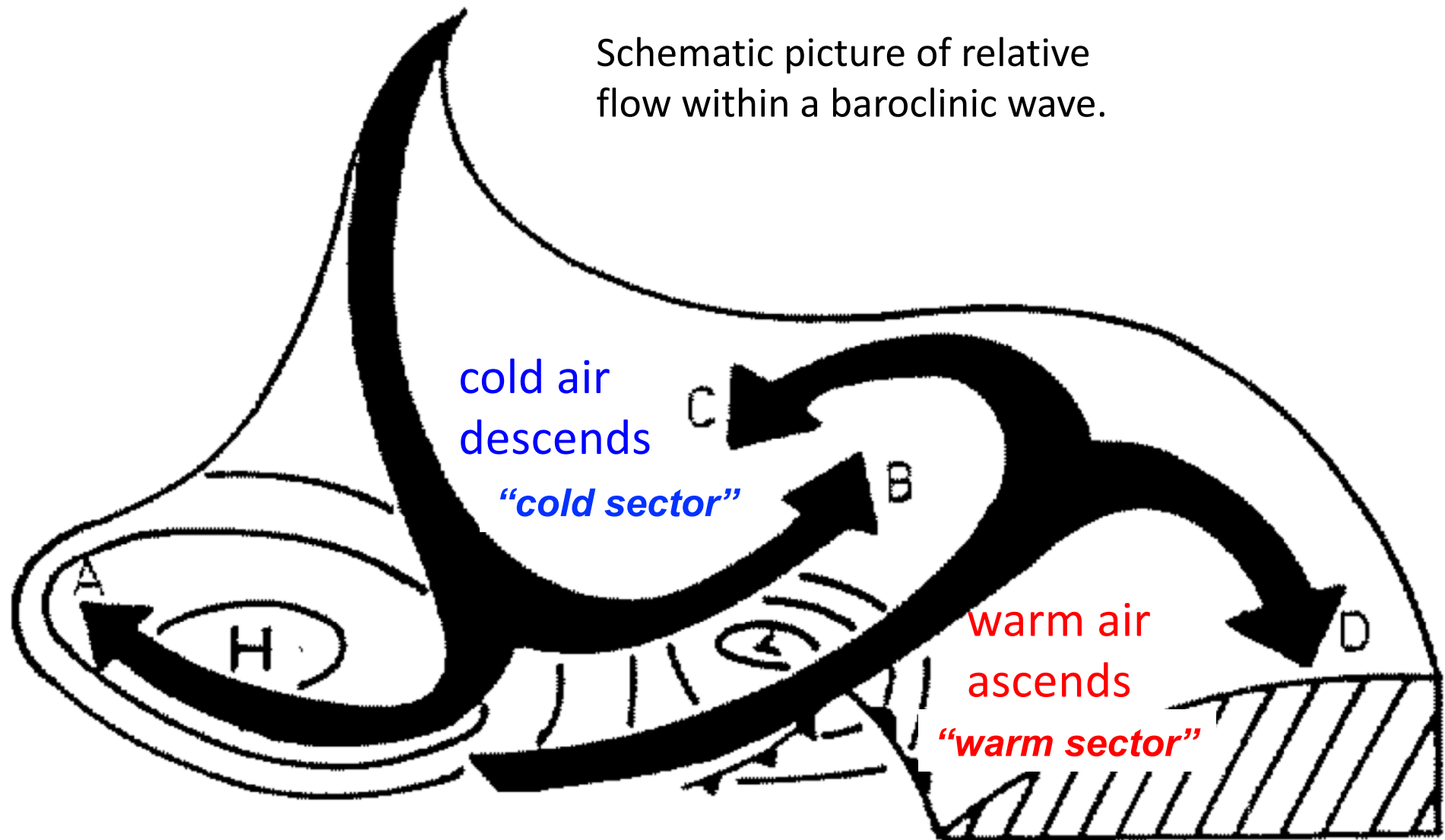
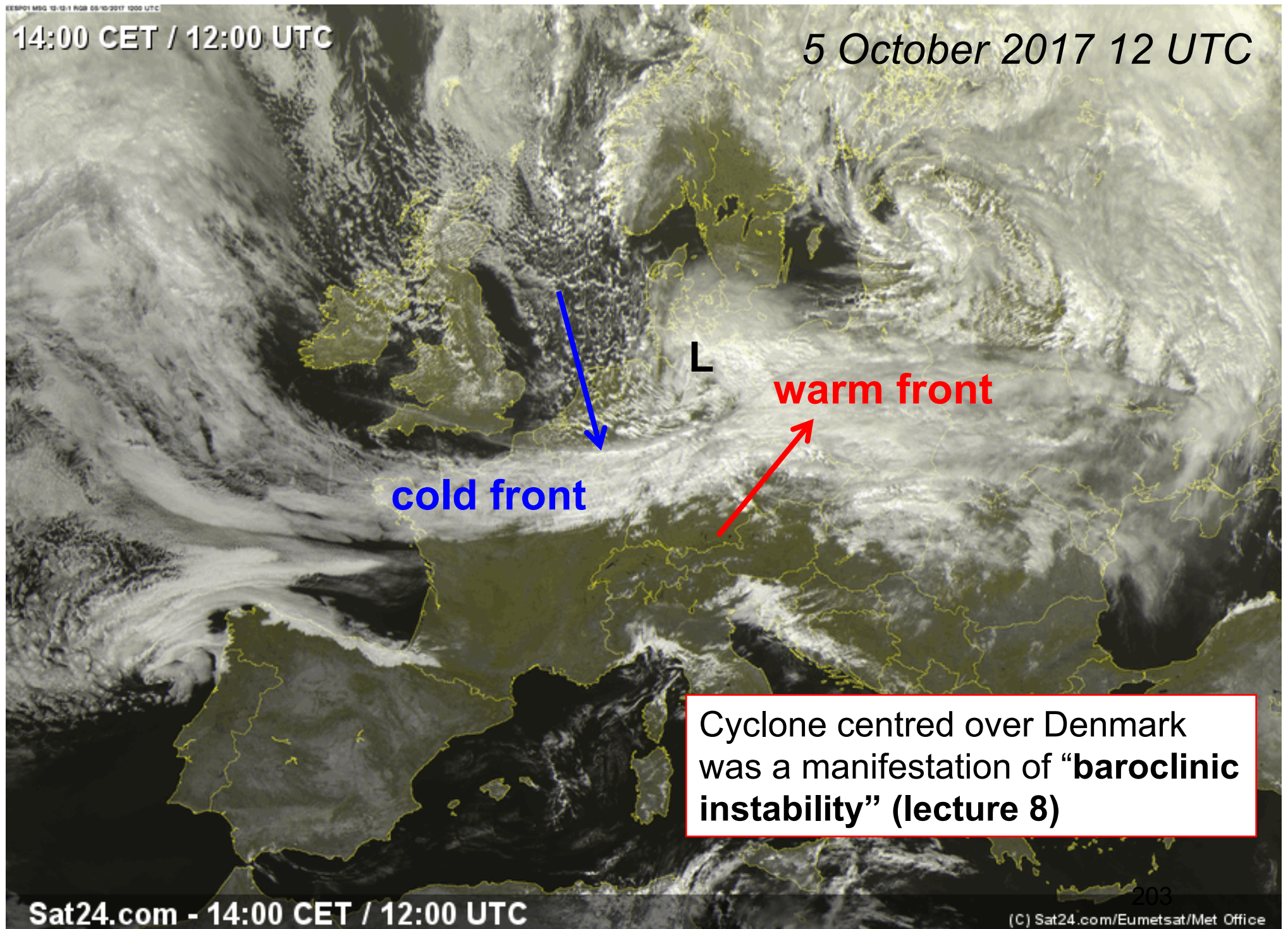


Figure 1.44

14:00 CET / 12:00 UTC

5 October 2017 12 UTC



Cyclone centred over Denmark was a manifestation of “**baroclinic instability**” (lecture 8)

Typical sky in a region of slantwise upward motion due to baroclinic instability

Typical sky in the “warm sector”



Jan Sluijters 1881-1957: *“Landschap in tegenlicht”*

**Typical sky in a region of slantwise downward motion
due to baroclinic instability over a relative warm ocean**

Typical sky in the “cold sector”



Hendrik Willem Mesdag 1831-1915, *“Vissereschepen in de branding”*

We will return to “baroclinic instability” several times in this course and look at it from different viewpoints.

On understanding a complicated problem

Anyone who wants to analyze the properties of matter in a real problem might want to start by writing down the fundamental equations and then try to solve them mathematically. Although there are people who try to use such an approach, these people are the failures in this field; the real successes come to those who start from a physical point of view, people who have a rough idea where they are going and then begin by making the right kind of approximations, knowing what is big and what is small in a given complicated situation. These problems are so complicated that even an elementary understanding, although inaccurate and incomplete, is worth while having, and so **the subject will be one that we shall go over again and again, each time with more accuracy...**

Richard Feynman (1963) in The Feynman Lectures on Physics, chapter 39. vol. 1.

Next:

Dynamical Meteorology: lecture 8

09/12/2022 (Friday, 09:00-10:45)

Sections 1.21 and 1.22

Circulation and vorticity

Large-scale vortices in the atmosphere (polar vortex and tropical cyclone)

Potential vorticity and isentropic density

The tropopause: lapse rate, dynamical, cold point, isentropic

Underworld, Middleworld and Overworld

Tutorial after this lecture (7) (Wednesday 7 December 2022):

Make a weather forecast for next Sunday 11 December 2022

Project 3

Problem 1.17

Deadline for project 3 is Friday 9 December 2022

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(<http://www.uu.nl/~nvdelden/dynmeteorology.htm>)