

PROJECT 1 - PROBLEM 1.6 in the lecture notes (p. 53)
(in couples) (weight final grade 15%)

Model of a parcel moving vertically under influence of buoyancy

In this small research project you will construct your first (Lagrangian) numerical simulation model.

The following ten theoretical concepts or topics come up for discussion.

1. Equation for the vertical acceleration of an air parcel under influence of pressure and gravity
2. Hydrostatic balance
3. Boussinesq approximation
4. Buoyancy
5. Adiabatic process and (dry-adiabatic) lapse rate
6. Potential temperature and potential temperature conservation
7. Static stability / instability
8. Buoyancy oscillations and Brunt Väisälä frequency
9. Convective Available Potential Energy (CAPE)
10. Lagrangian model

(a) Please study **sections 1.1-1.5** and **Box 1.5**.

(b) Construct a numerical model, which calculates the vertical position and vertical velocity of a dry air-parcel, which is initially at rest just above the earth's surface. Initially this air-parcel is warmer than its environment. The potential temperature, θ , of the air-parcel is given by

$$\theta(z,t) = \theta_0(z) + \theta'(z,t)$$

where, $\theta_0(z)$, which is the potential temperature of the environment of this air-parcel, is a function of height, z , only

The equations governing the vertical position above the earth's surface, z , the vertical velocity, w , and the potential temperature, θ , are

$$\frac{dz}{dt} = w, \quad \frac{dw}{dt} = g \frac{\theta'}{\theta_0}, \quad \frac{d\theta}{dt} = 0.$$

Note that potential temperature is conserved.

Define an air-parcel with a potential temperature, $\theta = \theta^*$. Assume that this air-parcel is at the surface of the earth, and that $\theta^* > \theta_0$ at this level. Write a program that computes the time evolution of the height and vertical velocity of this air-parcel, as it moves through the environment while it neither mixes with this environment, nor disturbs the environment. The potential temperature of the environment of the air-parcel is computed from

$$\theta_0(z) = \theta_0(0) + \Gamma z$$

In other words, in this example θ_0 is a simple analytical function of height. For simplicity, Γ is assumed constant (e.g. 5 K km^{-1}). If the air-parcel is 1°C warmer than its environment initially, it will accelerate upwards. The program computes the height of the air-parcel at equally spaced points in time. Assume that $\theta_0(0) = 300 \text{ K}$

On the basis of analytical theory in this section *and in Box 1.5* (on CAPE) (so please study **Box 1.5!**), the following questions should be answered (approximately). **What maximum upward**

velocity will it attain? At which height will it reach this velocity? What maximum height will it attain? Eventually the air-parcel will move up and down with a constant frequency. Compute this frequency. This information will be a guide to your choice of the time step.

Numerical approximation of differential equations is not an exact science. Finding the best numerical scheme is a matter of “educated trial and error”. This means that you should check the consistency of the analytical theory with your numerical results. The most obvious and, at first sight, best numerical scheme does not necessarily give the best results (tip: the *semi-implicit Euler method* (https://en.wikipedia.org/wiki/Semi-implicit_Euler_method) is a simple and accurate numerical method). Use Python with numpy and matplotlib (see “A Hands-On Introduction to Using Python in the Atmospheric and Oceanic Sciences” by Johnny Lin (<http://www.johnny-lin.com/pyintro/>)). Describe the numerical method that you used to obtain the numerical results.

(c) Repeat all the steps of part (b) with the following more realistic lapse rate of the environmental potential temperature distribution.

$\Gamma = -0.04$ K/m if $z \leq 50$ m (super - adiabatic surface layer);

$\Gamma = 0$ if $50 \text{ m} < z < 2 \text{ km}$ (well - mixed boundary layer, BL);

$\Gamma = 40$ K/km if $2 \text{ km} \leq z \leq 2.25 \text{ km}$ (temperature inversion at the top of the BL);

$\Gamma = 5$ K/km if $z > 2.25 \text{ km}$ (free atmosphere)