PROBLEM 7.1 In how far is the zonal mean zonal wind in balance and what causes departures from balance?
Equation 7.1 has more than one equilibrium solution. In other words, the balanced zonal mean zonal wind, \( \langle u \rangle \), can take on different values for the same meridional gradient of the Montgomery streamfunction. Investigate the relation between the meridional gradient of the Montgomery streamfunction and the zonal mean zonal wind in the Northern Hemisphere, in one month in winter and in one month in summer, using the ERA-Interim re-analysis on isentropic levels. Compare your results to the solution of eq. 7.1. Do this for the following three isentropic surfaces, corresponding to \( \theta = 350 \) K, \( \theta = 475 \) K and \( \theta = 600 \) K. Departures of \( \langle u \rangle \) from the balanced wind are caused by eddy meridional fluxes of potential vorticity substance (section 1.25), which give rise to meridional accelerations. Explore the relation between the zonal mean meridional wind, \( \langle v \rangle \), and the zonal mean eddy meridional flux of potential vorticity substance, \( \langle v^* \zeta^* \rangle \). Compare your results to the left panel of figure 7.19.

PROBLEM 7.3. The surf zone: zonal mean zonal wind and mass distributions that result from meridional mixing of potential vorticity

\[ \text{Figure 7.25.} \] Schematic illustration of the effect of eddy mixing on the distribution of potential vorticity. The initial monotonically increasing distribution is shown by the dashed line, and the step-like distribution after eddy-mixing by the solid line (Hoskins and James, 2014, figure 18.6).

The PV-distribution in the layer between \( \theta = 370 \) K and \( \theta = 550 \) is characterised by a reversed isentropic gradient of \( Z^* \) (figure 7.7). This is the consequence of meridional “mixing” of potential vorticity by eddies under the constraint of material conservation of potential vorticity. According to Dritschell and McIntyre (2008) (see also Hoskins and James (2014, chapter 18), inhomogeneous meridional mixing of potential vorticity leads to the emergence of step-like structures in the zonal mean potential vorticity distribution in the middle to high latitudes, as is indicated schematically in figure 7.25. Potential vorticity mixing, which occurs in a restricted band of latitudes, called “the surf zone”, makes the zonal mean potential
vorticity uniform within this surf zone and creates merional PV-gradients at the edge of the surf zone. It is interesting to note that the reference potential vorticity, $Z_{\text{ref}}$ (section 7.3), depends on latitude. According to the solution of the PV-inversion equation (7.12), $Z_{\text{ref}}$ is associated with a zonal mean zonal wind, $|u|=0$. Mixing of $Z_{\text{ref}}$ in a range of latitudes between the latitudes, $\phi=\phi_S$ and $\phi=\phi_N$, will create PV-steps at these latitudes. The isentropic tropopause at $=350$ K, for instance, is an approximate PV-step, which is established partly by inhomogeneous lateral PV-mixing. If the atmosphere adjusts to balance, this will lead to non-zero zonal winds at and in the vicinity of these latitudes. Perform a detailed investigation of the influence of eddy PV-mixing on the zonal mean state of the atmosphere. In which way will the zonal wind change when $Z_{\text{ref}}$ is mixed, and how will this affect the pressure distribution in isentropic coordinates? For this problem you will need to use the PV-inversion code.

PROBLEM 8.1. Latitude dependence of the radiative equilibrium temperature at equinox

Using the theory in Boxes 2.1 and 2.4, determine the approximate dependence on $y$ (latitude) at different isobaric levels in the atmosphere (e.g. at 500 hPa and at 300 hPa and ) of the radiative equilibrium temperature at equinox in an atmosphere with one well mixed greenhouse gas. In which way might this change if the effect of water vapour as a greenhouse gas is taken into account, and what consequences might this have for the Hadley circulation, if we believe the Held-Hou model?

PROBLEM 8.2 Extension of the Held-Hou model to the case of heating centred off the equator

Lindzen and Hou (1988) (see also James (1994) in the list of reference at the end of this chapter) extended the Held-Hou model to the more realistic case involving heating centred off the equator (such as during solstice). What is the consequence for the curve of $\theta_M$ (eq. 8.5) if the maximum of $\theta_E$ is located at latitude of 10N° (see figure 8.4)?

PROBLEM 10.1. Analysis of a simulation of consecutive baroclinic life-cycles on a large domain

Investigate the output of a simulation of several consecutive life cycles of unstable baroclinic waves on a large domain. Relate the appearance of growing baroclinic waves to frontogenesis, represented by the Q-vector, and the associated pattern of vertical motion. Is this pattern of vertical motion in agreement with the predictions made by the solution of the omega equation (eq. 9.31)? Describe the variations in the intensity of the Ferrel circulation and correlate these variations with the zonal mean eddy meridional heat flux. The zonal mean meridional heat flux in pressure coordinates is usually calculated by evaluating

$$[v\theta] = [v][\theta] + [v^* \theta^*]$$

(see section 11.3 for the definition of square bracket and the asterisk). The first term on the r.h.s. of this equation represents the heat flux due to the Ferrel circulation, while the second term represents the eddy heat flux. Can you identify a relation between the first and second term from the output of the simulation? Model data and Python-software will be provided.
**PROBLEM 10.2. Analysis of a simulation of a “storm track” by “downstream development”**

In reality, the atmosphere is baroclinically unstable in restricted areas, such as on the west side of the Atlantic and Pacific oceans. Wave-like disturbances start growing in these areas, but the most intense cyclones develop further downstream in weakly baroclinic regions. This phenomenon, which is known as “downstream development”, leads to the formation of a storm track consisting of unstable baroclinic waves in different stages of their development. In this exercise the output of a simulation of such a storm track is provided. Read the paper by Chang (1993) and analyse the dynamics of the simulated storm track in relation to the content of this paper. Model data and Python-software will be provided.

**PROBLEM 10.3. The isentropic tropopause**

The isentropic tropopause at approximately 30°N, in the layer between $\theta=330$ and $\theta=370$ K is characterised by an intense latitudinal isentropic potential vorticity gradient, which resembles a so-called “PV-step” (chapter 7) (figure 10.26). Despite frequent disruptions by waves and eddies, the *time mean* isentropic tropopause remains remarkably sharp and zonally symmetric. How is this possible? Is this due to the zonal symmetry of the isentropic mass and potential vorticity substance flux in the upper branch of the Hadley circulation? Is this due to the zonal symmetry of the downwelling cross-isentropic branch of the Hadley circulation? In other words, how zonally symmetric are the upper and downward branches of the Hadley circulation?

![Figure 10.26](image)

**Figure 10.26.** Time-average distribution of potential vorticity ($Z$) between 1 December 2009 and 28 February 2010 on the 350 K isentropic surface in the “Middleworld” on the Northern hemisphere, as a function of latitude ($°N$) and longitude ($°E$). Red contours correspond to 1, 2, 3, 4 and 5 PVU (tropical air). Blue contours correspond to 6, 7, 8, 9, 10 PVU and higher (extra-tropical air).

The zonal mean isentropic meridional mass flux can be separated into a mean component and an eddy-component as follows:

$$ [v\sigma] = [v][\sigma] + [v^*\sigma^*]. $$

Similarly, the zonal mean cross-isentropic meridional mass flux can be separated into a mean component and an eddy-component as follows:

$$ [v\sigma] = [v]\left[\frac{d\theta}{dt}\right] + [v^*\left(\frac{d\theta}{dt}\right)^*] $$
Investigate the above questions by comparing the mean isentropic mass flux and eddy isentropic mass flux at $\theta=340$ K and by comparing the mean cross-isentropic mass flux and eddy cross-isentropic mass flux at $\theta=340$ K (figure 10.25). Data will be provided.

**PROBLEM 11.1. Meridional circulation forced by eddy flux easward momentum**

(a) Devise an analytical expression, similar to eq. 11.49, that best approximates the distribution of the eddy flux of eastward momentum in figure 11.9 and solve the Kuo-Eliassen equation with only this forcing effect included: i.e.

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{f_0^2 \rho}{S_p R} \frac{\partial^2 \psi}{\partial p^2} = \frac{f_0^2 \rho}{S_p R} \frac{\partial^2 [v^* u^*]}{\partial \rho \partial y}.$$  \hspace{1cm} (11.57)

Will the eddy flux of eastward momentum also force an indirect meridional circulation and thus reenforce the indirect circulation that is forced by eddy heat flux?

(b) Can you add the individual solutions of eqs. 11.45 and 11.57 in order to obtain solution of the full equation:

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{f_0^2 \rho}{S_p R} \frac{\partial^2 \psi}{\partial p^2} = \frac{f_0^2 \rho}{S_p R} \frac{\partial^2 [v^* u^*]}{\partial \rho \partial y} - \frac{1}{S_p} \frac{\partial^2 [v^* T^*]}{\partial y^2}.$$  \hspace{1cm} (11.58)

**PROBLEM 11.4. Potential vorticity distribution in the winter polar vortex**

Using the ERA-Interim reanalysis data (http://apps.ecmwf.int/datasets/data/interim-full-moda/levtype=pt/), of monthly mean potential vorticity on $\theta=600$ K, construct a covariance matrix of the zonal mean potential vorticity for this isentrope, similar to the covariance matrix due to Li and Wang (J. Li and J.X.L. Wang, 2003: A modified zonal index and its physical sense. Geophys.Res.Lett., 30, 1632) (http://ljp.gcess.cn/dct/page/65607). Define an index, which is a measure of a coherent extra-tropical oscillation in zonal mean potential vorticity on this isentrope, similar to the oscillation in sea level pressure that was identified by Li and Wang, leading to their definition of the sea-level NAM-index. What physical processes does this index reflect? Investigate the relation between the monthly NAM-index of Li and Wang and your index for the northern hemisphere winters of 2006-2007 and 2009-2010.

**PROBLEM 11.5. The Sudden Stratospheric Warmings of the winters of 1984-1985 and 2008-2009**

Based on the ERA-Interim reanalysis (http://apps.ecmwf.int/datasets/data/interim-full-daily/levtype=pl/), compute both components of the EP-flux vector (every 6 hours) at 100 hPa at 60°N (a measure of the intensity of wave propagation into the stratosphere and towards the pole) and investigate its relation with the major stratospheric warmings of January 1985 and January 2009. The SSW is reflected in the average temperature and in the average potential vorticity over the polar cap (north of 60°N) at 10 hPa, and also in the zonal mean zonal wind at 60°N.
PROBLEM 11.6. Northern Annular Mode during the winters of 2006-2007 and 2009-2010
Investigate the relation between both components of the EP-flux vector at 100 hPa at 60°N, during the winters of 2006-2007 and 2009-2010 and the daily mean NAM-index of Li and Wang (problem 11.4). Make a graph of both quantities. Discuss the characteristics of these two northern hemisphere winters. Interpret your result.

PROBLEM 11.7. Cold tropical tropopause
Temperatures at the tropical tropopause at about 100 hPa, which are approximately the lowest measured anywhere below the stratopause (figure 12.1), exhibit a yearly cycle with lowest values observed in January or February and highest values observed in July or August. Demonstrate this by constructing a Hovemoller plot of the zonal mean, monthly mean temperature at 100 hPa for the years 1979-2015, based on the ERA-Interim reanalysis (http://apps.ecmwf.int/datasets/data/interim-full-daily/levtype=pl/). Explain the yearly cycle and investigate trends in the data.

PROBLEM 12.1. Time lag between Solar forcing and thermal response
Using the theory of section 2.4, estimate the radiative (thermal) response time of the atmosphere in the case discussed here, i.e. for an atmosphere, containing one well-mixed greenhouse gas, with a total optical path equal to 1.8 (eq.12.12). Why are we allowed to neglect the daily cycle of insolation in computing the radiatively determined state. Do you think that the response time of the system “Atmosphere+Earth’s surface” is much longer than the response time of the atmosphere if $C=10^7$ J K$^{-1}$ m$^{-2}$?

PROBLEM 12.3. Comparison of the radiative equilibrium isentropic density to the radiatively determined isentropic density
Plot the radiative equilibrium isentropic density (eq. 12.14) as a function of potential temperature at the equator and at 60°N, on 20 March, 20 June, 20 September and 20 December (using eqs. 12.11 and 12.12 and the definition of potential temperature), for the parameter values indicated in the caption of figure 12.12, and compare the outcome with the radiatively determined isentropic density, shown in figure 12.12. Use the formula’s given in Box 2.1 to compute $Q$. Discuss the dynamical consequences of the differences.

PROBLEM 12.4. Accuracy of a statement in a well-known scientific journal
The first sentence of a relatively recent (2013) paper in the Quarterly Journal of the Royal Meteorological Society states, “Low latitudes experience a net gain of radiative energy, while high latitudes experience a net deficit; as a result of this imbalance, the oceans and the atmosphere transport heat poleward”. Discuss the accuracy of this statement.

PROBLEM 12.5. Testing the model parametrisations concerning the water cycle with reanalysis data
Investigate the realism for a particular year of the model parametrisations concerning the water cycle (section 12.7). Restrict the analysis to the zonal mean. You need to retrieve the following fields from the ERA-Interim website (http://apps.ecmwf.int/datasets/): net
radiation at the Earth’s surface, sensible heat flux from the Earth’s surface to the atmosphere, evaporation at the Earth’s surface, precipitation, precipitable water vapour and the temperature and dewpoint at 2 m (from which you can retrieve the relative humidity) as a function of latitude (section 1.10). The principal model assumptions are that (1) the relative humidity at the ground is 75%, (2) total precipitation is instantaneously in balance with total evaporation, (3) the Bowen ratio is 0.25 at all latitudes, (4) local precipitation is 80% of the locally evaporated water, except in the ITCZ, (5) total precipitable water vapour is equal to the density of water vapour near the surface times the scale height (eq. 12.30).

The following 5 questions constitute “what-if’’ questions.

**PROBLEM 12.6 Influence of Earth’s rotation rate**
How would the Hadley circulation and the zonal mean zonal jets be affected if we would double the value of the Coriolis parameter?

**PROBLEM 12.7 Influence of carbon dioxide (CO₂)**
How would the Hadley circulation and the zonal mean zonal jets be affected if we would double the carbon dioxide concentration? What will happen to the annual mean global mean temperature at the surface? What will happen to the annual mean global mean temperature at the 10 hPa?

**PROBLEM 12.8 Influence of precipitation in the ITCZ**
How would the zonal mean state of the atmosphere be affected if we would decrease the fraction, $f_{locprec}$, of the evaporation that is converted to local precipitation from 0.8 to 0.6?

**PROBLEM 12.9 Influence of thermal inertia**
How does the value of the thermal inertia coefficient of the earth’s surface ($C$) affect the (seasonal cycle of the) subtropical jet and the zonal wind in the stratosphere? In other words, what will happen to the modelled subtropical jets and stratospheric zonal winds if we repeat RUN 6 (table 12.2) with $C=5\times10^7$ J K⁻¹ m⁻² instead of with $C=10^7$ J K⁻¹ m⁻²?

**PROBLEM 12.10 Influence of ozone**
What will happen to the tropopause and the jets if we neglect the effect of absorption of Solar radiation by ozone in the stratosphere? Would there still be zonal mean zonal wind reversals?

**PROBLEM 12.12 Stratospheric wind reversals**
Investigate the time of the Spring and Autumn zonal mean zonal wind reversal at a latitude of $±45°$, at $θ=600$ K for at least 5 years, using daily ERA-Interim reanalysis data. Compute the area weighted amplitude of the daily mean zonal mean PV-anomaly (eq. 12.5) north of 65°N, as a function of time (see e.g. figure 12.4). Is there a connection between the wind reversal and the reversal of the sign of the PV-anomaly? Identify the differences between the southern and northern hemisphere and explain these differences. Compare the time of zonal wind reversal at $θ=600$ K in the reanalysis with the simulated time of zonal wind reversal in RUN 4 (figure 12.29).
PROBLEM 12.13. Held and Hou’s theory of the Hadley circulation
In the light of insights gained in the present chapter, on the interaction between radiation and
dynamics, and the role of the water cycle and the Earth’s surface, evaluate or criticise (i.e.
give your opinion on) the theory due to Held and Hou of the Hadley circulation, which is
described in section 8.2.

PROBLEM 12.14. Tape-recorder effect
The seasonal cycle of tropical tropopause temperature (problem 11.7) yields a
corresponding seasonal cycle in saturation mixing ratios of air at this level. Since air travels
upwards through this level into the stratosphere (the upward branch of the Brewer Dobson
circulation), alternating dry and moist layers are formed in the lower stratosphere which
move slowly upwards. This effect is called the tape-recorder effect. Detect this tape-
recorder effect in ERA-Interim data (http://apps.ecmwf.int/datasets/data/interim-full-
daily/levtype=pl/ ) and deduce the upward vertical velocity from this data.

PROBLEM 12.15. Controversies in climate science
Climate scientists, like other scientists, do not always agree. Especially when the water cycle
is involved, a climate problem becomes very difficult. Identify a topic in climate science,
which has been, or is, controversial and write an concise essay (no more than 1500 words)
on this topic, describing the phenomenon, the history of theories of this phenomenon, the
controversy in the scientific literature and your own position. Examples of topics that are
and/or have been controversial are the following.

1. The influence of clouds on climate. An interesting example is the discussion about the
“infrared iris effect” with Dick Lindzen (Harvard, USA) and Dennis Hartmann (Seattle,
USA) as opponents (section 2.19).

2. The theory of the growth of tropical cyclones. Water plays a very important role in the
growth of tropical cyclones. Opponents of a theory called “Conditional Instability of the
Second Kind” (CISK) have qualified this theory, which is due to Jules Charney and Arnt
Eliassen (the founders of quasi-geostrophic theory!), as a “a lengthy dead-end road in
atmospheric science”. The alternative theory is known by the acronym, “WISHE”, which
stands for “Wind Induced Surface Heat Exchange”. Michael Montgomery (Monterey,
USA) and Roger Smith (Munich, Germany) oppose Kerry Emanuel (MIT, Boston, USA) on
this issue.

3. Do annular modes exist? Jim Wallace and David Thompson (Seattle, USA)) think that
the “Annular Mode”, as a zonally symmetric mode of extra-tropical variability, really
exists, while the “North Atlantic Oscillation” (NAO) is a local manifestation of this
annular mode. Maarten Ambaum (Reading, UK) criticizes this viewpoint.

4. Can we predict the climate with numerical models? Freeman Dyson (Princeton) says
“no”…“Climatologists do not understand climate. Their models are full of fudge factors”. Many
of these “fudge factors” have to do with either the modelling (parametrisation) of the
water cycle (clouds, evaporation and precipitation) or with the modelling of potential
vorticity-flux by planetary eddies and waves. Many authors of the IPCC-report tend to agree
with Dyson, but nevertheless think that this does not mean that using models to understand
climate is a hopeless endeavor. Investigate whether this controversy has a lot to do with the **definition of “climate”**.

5. **How sensitive is our climate to increase of carbon dioxide concentrations?** The debate about “Climate Sensitivity”, which is defined as the change of the global mean, annual mean surface equilibrium temperature, resulting from a doubling of CO$_2$-concentration, has been alive since Svante Arrhenius proposed that this number is 5°Ç in 1896. Richard Lindzen’s estimate of climate sensitivity, based on the observed warming of the past 50 years, is 1°C. Write an essay on the debate on Earth’s Climate Sensitivity and the usefulness of Arrhenius’ definition.

6. **Because governments have a monopoly on the financial support of scientific research, scientific research is not or hardly driven by intellectual curiosity.** Is this true and, if true, what consequences does this have for the advancement of scientific knowledge?

**PROBLEM 12.16. Realism of the semi-grey radiative transfer scheme**
The highly simplified description of radiative transfer in the two-dimensional General Circulation Model, apparently, does not prevent the model from producing a very realistic seasonal cycle of zonal mean zonal wind, the zonal mean temperature and the tropopause. List the most important aspects of the physics of radiative transfer that have been neglected and discuss the reasons why these aspects are not detrimental to the simulation of the zonal mean General Circulation.

**PROBLEM 12.17. What questions remain?**
The two-dimensional primitive equation model of the General Circulation of Earth’s atmosphere, which is developed in this chapter, gives a reasonably realistic portrayal of how the zonal mean wind and associated temperature distribution is established. Yet, some important questions on the General Circulation remain to be addressed. Give a concise overview of the questions that have been answered, including the answers (in no more 1000 words), and give a list of questions that remain?