Book: Dynamical Oceanography
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Chapter: 1, Exercise: 1.1: Mixing
Version: 2

A water mass A, with a temperature $T_A = 5°C$ *and a salinity* $S_A = 35.5$ *ppt, is mixed with a water mass B, with a temperature* $T_B = 2°C$ *and a salinity* $S_B = 34.5$ *ppt. The resulting water has a temperature of 3°C and a salinity of 34.85 ppt.*

a. Calculate the volume ratio of the water masses A and B in the mixture.

Water mass A (with volume V_A and density ρ_A) is mixed with water mass B (with volume V_B and density ρ_B) to give a water mass with a volume V, a temperature T, a salinity S, and a density ρ. Conservation of total mass, internal energy and salt imply that

$$\rho_A V_A + \rho_B V_B = \rho V$$

$$\rho_A V_A C_p T_A + \rho_B V_B C_p T_B = \rho V C_p T$$

$$\rho_A V_A S_A + \rho_B V_B S_B = \rho V S$$

Here C_p is the heat capacity of water, which we assume to be independent of temperature so that it cancels out. Furthermore, $\frac{\rho_A}{\rho} \approx 1$ and $\frac{\rho_B}{\rho} \approx 1$ are reasonable approximations because absolute density variations in the ocean are small. Hence

$$V_A T_A + (V - V_A) T_B = VT$$

$$V_A S_A + (V - V_A) S_2 = VS$$

Dividing by V and putting $\alpha = V_A/V$ gives

$$\alpha T_A + (1 - \alpha) T_B = T \rightarrow \alpha = \frac{T - T_B}{T_A - T_B}$$

$$\alpha S_A + (1 - \alpha) S_B = S \rightarrow \alpha = \frac{S - S_B}{S_A - S_B}$$

With the values of the temperatures and salinities as above we find $\alpha = 1/3$ and $\alpha = 0.35$, respectively, and hence we can take, for example, an average value of $V_A/V = \alpha = 0.34$ and $V_B/V = 1 - \alpha = 0.76$.

b. How can one determine this ratio graphically using a T-S diagram?

By equating both expressions for α, we see that there is a linear relationship between T and S, as

$$\frac{T - T_B}{T_A - T_B} = \frac{S - S_B}{S_A - S_B} \rightarrow T = T_B + (T_A - T_B)\frac{S - S_B}{S_A - S_B}$$

For the values of T_A, T_B and S_A, S_B above, this line is plotted in Fig. 1. The distances a and b in the figure represent the ratio's of volumes as $a : b \approx 2 : 1$, i.e., $a \approx 2b$. This can easily be seen by realizing that the sine of the angle between the line and the horizontal axis can be expressed as

$$\frac{T - T_B}{b} = \frac{T_A - T}{a} \rightarrow \frac{a}{b} = \frac{T_A - T}{T - T_B} \approx 2$$

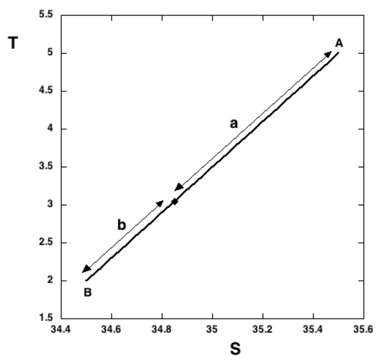

Figure 1: T-S diagram corresponding to the situation in exercise 1.1