Book: Dynamical Oceanography
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Chapter: 1, Exercise: 1.2: Potential temperature
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Consider in the table below some in-situ measurements of the temperature and salinity at a certain location in the Atlantic.

Depth(m)	*Temperature (°C)*	*Salinity (ppt)*
0	*18.909*	*32.574*
1000	*2.697*	*34.410*
2000	*1.868*	*34.600*
3000	*1.528*	*34.661*
4000	*1.456*	*34.679*
5000	*1.503*	*34.686*
5460	*1.547*	*34.688*

a. Investigate whether the water column is statically stable when a linear equation of state is assumed (with $\alpha_T = 10^{-4} K^{-1}$ *and* $\alpha_S = 7.6 \times 10^{-4}$*).*

Density is given by the following expression:

$$\rho_* = \rho_\infty(1 - \alpha_T(T - T_\infty) + \alpha_S(S - S_\infty))$$

with $\alpha_T = 10^{-4}$ and $\alpha_S = 7.6 \cdot 10^{-4}$. In this equation, ρ_∞, T_∞ and S_∞ are the reference density, a reference salinity and a reference temperature for which we take values at the surface.

Depth (m)	Temperature (°C)	Salinity (ppt)	$\frac{\rho_*}{\rho_\infty}$
0	18.909	32.574	1
1000	2.697	34.410	1.003017
2000	1.868	34.600	1.003244
3000	1.528	34.661	1.003324
4000	1.456	34.679	1.003345
5000	1.503	34.686	1.003346
5460	1.547	34.688	1.003343

When a linear equation of state is used, the water column does not appear to be statically stable between 5000 and 5460 m as the density decreases downward.

b. Why is the pressure dependence in the equation of state here important?

The pressure dependence in the equation of state is important for the temperature as a higher pressure leads to a higher temperature due to adiabatic compression.

c. Determine the potential temperature as a function of depth for these measurements.

The potential temperature is the in-situ temperature corrected for the adiabatic temperature gradient Γ and a good approximation is given by: $\vartheta = T + \Gamma z$. Γ is given for different depths as following:

$$0 - 1000 \text{ m} \rightarrow \text{very small (negligible)}$$
$$1000 - 5000 \text{ m} \rightarrow \Gamma = 0.14(^0C/km)$$
$$5000 - 9000 \text{ m} \rightarrow \Gamma = 0.19(^0C/km)$$

The potential temperature, salinity and relative density as a function of depth are then given by

Depth (m)	Temperature (°C)	Pot. Temperature (°C)	Salinity (ppt)	$\frac{\rho_*}{\rho_\infty}$
0	18.909	18.909	32.574	1
1000	2.697	2.697	43.410	1.003017
2000	1.868	1.728	34.600	1.003258
3000	1.528	1.248	34.661	1.003352
4000	1.456	1.036	34.679	1.003387
5000	1.503	0.943	34.686	1.003402
5460	1.547	0.900	34.688	1.003408

Note that the effect of pressure on salinity is very small and is neglected here.

d. Is the water column statically stable?

The water column is statically stable as the density now monotonically increases with depth.

A water parcel with an in-situ temperature of $T = 10°C$ and a salinity of $S = 35$ ppt moves adiabatically and without salinity changes from a depth of 2 km to a depth of 5 km.

e. What is the in-situ temperature and the potential temperature of the parcel at the final depth?

At 2 km, the potential temperature is about equal to the in-situ temperasture. Over this depth range from 2 - 5 km, we use the value of $\Gamma = 0.14(^0C/km)$. Because of adiabatic compression, the value of the in situ temperature increases to $T = 10 + 0.14 \times 3 = 10.52$°C. By contruction, the potential temperature remains constant, i.e., $\theta = 10$°C.