Book: Dynamical Oceanography
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Chapter: 10, Exercise: 10.1: Barotropic instability
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As we have seen in section 10.2 a necessary, but not sufficient, condition for barotropic instability of zonal flows is the Kuo criterium. This criterium implies that there can be no growth of perturbations on a background flow $U = U(y)$ if $U'' - \beta$ does not change sign with the flow domain. Consider now the dimensionless zonal flow

$$U(y) = \bar{U}(3y - 2y^3)$$

on the domain $-1 \leq y \leq 1$ on a β plane, where $\bar{U}$ is a constant amplitude.

a. Show that this flow is stable when $|\bar{U}|$ is smaller than $\beta/12$.

The second derivative of $U(y)$ is $-12\bar{U}y$. Therefore, $U'' - \beta = -12\bar{U}y - \beta$. If $\bar{U} > 0$, this quantity does not change sign when $0 < \bar{U} < \beta/12$. If $\bar{U} < 0$, then there is no change of sign when $-\beta/12 < \bar{U} < 0$ and hence when $|\bar{U}|$ is smaller than $\beta/12$ the flow is stable.

b. Is this flow unstable on an f-plane?

On the f-plane the Kuo criterium becomes the Rayleigh criterium. As $U'' = -12\bar{U}y$ always changes sign, the flow may be barotropically unstable (but the criterium cannot be used to prove stability of the flow).