Book: Dynamical Oceanography
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Chapter: 12, Exercise: 12.2: Equatorial upwelling
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In section 12.2.2 the Ekman layer equations (12.7) were given in case of linear friction with damping coefficient a_s. Assume that $\tau_*^y = 0$.

a. Determine the horizontal Ekman velocities u_{E*} and v_{E*} in terms of the wind stress τ_*^x.

The linear system of equations (12.7) becomes

$$a_s u_{E*} - \beta_0 y_* v_{E*} = \frac{\tau_*^x}{\rho H_E}$$

$$a_s v_{E*} + \beta_0 y_* u_{E*} = 0$$

which is easily solved for u_{E*} and v_{E*} as

$$u_{E*} = \frac{a_s \frac{\tau_*^x}{\rho H_E}}{a_s^2 + \beta_0^2 y_*^2}$$

$$v_{E*} = \frac{-\beta_0 y_* \frac{\tau_*^x}{\rho H_E}}{a_s^2 + \beta_0^2 y_*^2}$$

b. Explain why $v_{E*} = 0$ at the equator.

From the expression for v_{E*} it is seen that $v_{E*} = 0$ at $y_* = 0$. On the equator, the Coriolis acceleration vanishes and hence v_{E*} is totally determined by the local meridional wind-stress which was assumed to be zero.

c. Determine the upwelling velocity w_{E*} at the equator for a constant zonal wind stress $\tau_*^x = -\tau_0$.

From (12.8), the vertical Ekman velocity is determined as

$$w_{E*} = H_E(\frac{\partial u_{E*}}{\partial x_*} + \frac{\partial v_{E*}}{\partial y_*}).$$

For $\tau_*^x = -\tau_0$ constant, it is seen from a. that u_{E*} does not depend on x_* and hence

$$w_{E*}(x_*, 0) = H_E \frac{\partial v_{E*}}{\partial y_*}|_{y_*=0} = \frac{\beta_0 \tau_0}{\rho a_s^2}.$$

d. With a wind stress amplitude $\tau_0 = 0.1$ Pa and a damping coefficient $a_s = 5.0 \times 10^{-6}$ s^{-1}, determine w_{E*} in m/day.

Putting in the numbers, we determine

$$w_{E*}(x_*, 0) = \frac{1.6 \times 10^{-11} \times 0.1}{10^3 \times 25 \times 10^{-12}} \sim 5.5 \text{ m/day}.$$