Book: Dynamical Oceanography
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Chapter: 2, Exercise: 2.2: Heat flux
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Consider a layer of seawater (heat capacity $C_p = 4.2 \times 10^3$ $Jkg^{-1}K^{-1}$*, density* $\rho = 1.027 \times 10^3$ kgm^{-3}*) with a thickness of 1 m and an area of* 10^4 m^2*.*

a. Determine how much energy is needed to raise the temperature of the water by 1 $^\circ C$*.*

The mass m of the water layer is $m = \rho V = 1.027 \times (10^3 \times 1 \times 10^4) = 1.027 \times 10^7$ kg. The energy E needed to raise the temperature by ΔT degrees is given by

$$E = C_p m \Delta T$$

With $\Delta T = 1^\circ$C, we find $E = 4.31 \times 10^{10}$ J = 43.1 GJ.

Assume that a constant heat flux $Q = 400$ Wm^{-2} *is applied over the area, for example due to a warmer atmosphere.*

b. How long does it take before the temperature of the water layer has increased by 1 $^\circ C$*?*

The mean temperature in the layer of depth h will change according to the balance

$$\rho h C_p \frac{dT}{dt} = Q$$

From this equation we estimate

$$T(t) \approx T(0) + \frac{Q}{\rho h C_p} t$$

and find with $T(t_*) - T(0) = 1$ that $t_* = 1.08 \times 10^4$ s.

During a seasonal cycle, there is heat uptake in an ocean region whereas during the winter the opposite occurs.

c. Assume that the upper 100 m of ocean water is in contact with the atmosphere and that the seasonal temperature change in the water is about 10° *C. Calculate the amount of energy which is stored (released) by the water layer.*

For the layer, we now have $h = 100$ m and hence $V = 10^6$ m^3. For the seasonal energy change E we find $E = \rho V C_p \Delta T = 4.31 \times 10^{13}$ J.

d. Determine the same quantity for a land surface and a seasonal temperature change of about 20° *C.*

We must estimate a thickness of the layer over which temperature changes occur and a soil density ρ_s. Dry sand has a density of 1602 kgm^3 and a heat capacity of 830 J(kg K)$^{-1}$ (see http://www.simetric.co.uk/si_materials.htm). For a layer of 1m of sand, the energy required is $E = 1602 \times 1 \times 10^4 \times 830 \times 20 = 2.66 \times 10^{11}$ J.