Book: Dynamical Oceanography
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Chapter: 3, Exercise: 3.2: Properties of the Coriolis acceleration
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a. Show, in the same way as in section 3.1.2, that the rotation of the $(\mathbf{e}_1, \mathbf{e}_3)$ *plane induces an acceleration* $-2\Omega w \cos\theta$ *in the* $\mathbf{e}_1$ *direction and an acceleration* $2\Omega u \cos\theta$ *in the* $\mathbf{e}_3$ *direction.*

The vertical plane spanned by e_1 and e_3 rotates clockwise with angular velocity $\Omega\cos\theta$. Consider the movement of fluid parcels in the plane (e_1, e_3). Within a time Δt the coordinate system (e_1, e_3) rotates over an angle $\Delta t\Omega\cos\theta$. A parcel which moves at $t = 0$ uniformly along e_3 with velocity w arrives after Δt in point A, with $|OA| = w\Delta t$. With respect to the rotating coordinate system, the parcel will have an displacement (in the negative e_1 direction): $-w(\Delta t)^2\Omega\cos\theta$. The acceleration is uniform in the negative direction of e_1 and when denoted by a_1^c, the displacement can be written as $\frac{1}{2}a_1^c(\Delta t)^2$. From the last two relations it follows that $a_1^c = -2\Omega w\cos\theta$ in the e_1.

In the same way, an expression can be derived for the component of the apparent acceleration in the direction of e_3. A fluid parcel which moves uniformly in the e_1 direction with velocity u is displaced with respect to the rotation coordinate system , in Δt, in the positive e_3 direction. In the same way as before, it follows that the displacement is $u(\triangle t)^2\Omega\cos\theta$ and $a_3^c = 2\Omega u\cos\theta$ in the e_3 direction.

In section 3.1.2 we derived the expression for the Coriolis acceleration $\vec{a}^c$.

b. Show that $\vec{a}^c \perp \vec{\Omega}$ *and* $\vec{a}^c \perp \vec{v}$.

As $\vec{a}^c = -2\vec{\Omega}\wedge\vec{v}$, the acceleration vector is orthogonal to $\vec{\Omega}$ and $\vec{v}$. This can also be shown by using the scalar product of vectors. For example, given that $\vec{a_c} = (2\Omega(v\sin\theta - w\cos\theta), -2\Omega u\sin\theta, 2\Omega u\cos\theta)$ and $\vec{v} = (u, v, w)$ it results that $\vec{a_c}\cdot\vec{v} = 2\Omega u(v\sin\theta - w\cos\theta) - 2v\Omega u\sin\theta + w2\Omega u\cos\theta = 0$. Therefore, $\vec{a_c}\perp\vec{v}$.