Book: Dynamical Oceanography
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Chapter: 4, Exercise: 4.2: Tornado
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A tornado consists of a thin vortex tube. Assume that the vorticity is constant over the cross section of the tube.

a. Show that locally the vorticity decreases when the thickness of the vortex tube increases.

For the vortex tube bounded by two closed contours C_1 and C_2 enclosing surfaces with areas A_1 and A_2 and average vorticities ω_1 and ω_2, respectively, the Helmholtz theorem gives

$$\omega_1 A_1 = \omega_2 A_2 \rightarrow \omega_2 = \omega_1 \frac{A_1}{A_2}$$

When the thickness of the vortex tube at C_1 is larger than that at C_2 ($A_1 > A_2$), then $\omega_1 < \omega_2$.

b. About 10 m from the center of a tornado one measures wind speeds of 200 km/hr. Determine the pressure variations when the tornado passes by.

This is a classical application of Bernoulli's law (formally only valid under particular conditions), which states that the function

$$H(r) = \frac{1}{2}v^2 + \frac{p}{\rho} + \xi$$

where v is the velocity, p the pressure, ρ the density, r the distance from the center of the tornado and ξ the gravitational potential, is constant along flow trajectories.
When a flow particle moves horizontally from the centre of the tornado (where the velocities are small) once it passes, Bernoulli's law gives

$$H(0) = \frac{p(0)}{\rho} + \xi = H(r_0) = \frac{1}{2}v^2(r_0) + \frac{p(r_0)}{\rho} + \xi$$

and hence with $r_0 = 10$ m and $v(r_0) = 55.6$ ms^{-1}, we find

$$p(r_0) - p(0) = -\frac{\rho}{2}v^2(r_0) = -1543 \text{ Nm}^{-2}.$$

Assuming a roof of a house with an area of 90 m^2 and a thickness of about 0.1 m and a density of 10^3 kgm^{-3}, its weight is 10^4 kg. The upward force due to the tornado is $90 \times 1543 = 138870$ N and hence such a tornado can easily lift the roof.