Book: Dynamical Oceanography
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Chapter: 5, Exercise: 5.1: Gulf Stream
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In this exercise, we consider the dominant balances in the horizontal momentum equations for the Gulf Stream. Characteristic length scales for the Gulf Stream are $L_x = 50$ km and $L_y = 500$ km. The characteristic velocity in east-west direction is about $U = 0.1$ m/s and that in north-south direction $V = 1.0$ m/s. Assume furthermore that the Gulf Stream is a steady current and take $A_H = 10^5 \ \ m^2 s^{-1}$ and $A_V = 0.1 \ m^2 s^{-1}$.

a. Show that in the meridional direction the Gulf Stream is not quite in geostrophic balance. Which other terms contribute to this momentum balance?

The meridional momentum balance in Cartesian coordinates is given by

$$\frac{Dv}{dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + A_H \nabla_H^2 v + A_V \frac{\partial^2 v}{\partial z^2}$$

Using $f = 10^{-4}$ s^{-1}, the ratio of horizontal frictional and Coriolis acceleration is

$$\frac{A_H V}{L_x^2 f U} \approx 4$$

and hence for this value of A_H the zonal flow is not in geostrophic balance. Of course, this is only a value which is used in low-resolution ocean-climate models. A realistic value of A_H would be much smaller!

b. Show that the zonal momentum equation is dominantly geostrophic.

The zonal momentum balance in Cartesian coordinates is given by

$$\frac{Du}{dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H \nabla_H^2 u + A_V \frac{\partial^2 u}{\partial z^2}$$

The ratio of horizontal frictional and Coriolis acceleration is

$$\frac{A_H U}{L_x^2 f V} \approx 0.04.$$

Advection and vertical friction give even smaller numbers and hence the meridional flow is in geostrophic balance.