Book: Dynamical Oceanography
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Chapter: 5, Exercise: 5.4: Ekman layer at an eastern boundary
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Ekman layers can also occur near continental boundaries. Consider a flow near an east coast $x = x_E$ *driven by wind stress* $\tau^y = \tau_0$, $\tau^x = 0$, *where* τ_0 *is constant. Bottom friction can be neglected.*

a. Write down the zonal momentum equations in this case (integrated over depth) and determine the Ekman pumping at the surface.

Neglecting inertia and meridional friction, the vertically integrated momentum balances are given by

$$-f\bar{v} = -\frac{1}{\rho}\bar{p}_x + A_H \bar{u}_{xx}$$

$$f\bar{u} = -\frac{1}{\rho}\bar{p}_y + A_H \bar{v}_{xx} + \frac{\tau_0}{\rho D}$$

where the bar indicates vertical average. The Ekman transports are given by $M^x_{E*} = \tau_0/(\rho f)$ and $M^y_{E*} = 0$.

b. Give a physical explanation of the upwelling velocity in the northern hemisphere, in case $\tau_0 < 0$.

A southward meridional wind stress ($\tau_0 < 0$) induces an Ekman transport away from the coast and because of continity there is upwelling near the coast.

c. Introduce a coordinate $\lambda = (x_E - x)/\delta$ *and derive that the dominant zonal momentum balance is given by*

$$u + (\frac{A_H}{f})^2 u_{xxxx} \approx 0$$

In the boundary layer, the frictional terms become dominant and hence balance the Coriolis acceleration. Hence, we have the approximate balances

$$-f\bar{v} \approx A_H \bar{u}_{xx}$$

$$f\bar{u} \approx A_H \bar{v}_{xx}$$

and by differentiating the first equation twice to x and using the second, we find

$$\bar{u} + (\frac{A_H}{f})^2 \bar{u}_{xxxx} \approx 0$$

d. Determine the lateral Ekman boundary layer scale δ.

By putting in the scales coordinate λ mentioned under c., we find a balance in the equation above only when

$$(\frac{A_H}{f})^2 \frac{1}{\delta^4} \approx 1 \rightarrow \delta = \sqrt{\frac{A_H}{f}}$$