Book: Dynamical Oceanography
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Chapter: 6, Exercise: 6.1: Sverdrup flow
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Given is the dimensionless wind-stress field

$$\vec{\tau} = \begin{pmatrix} \tau^x \\ \tau^y \end{pmatrix} = 0.1 \begin{pmatrix} \sin 6(y - \frac{\pi}{6}) \\ 0 \end{pmatrix}$$

over an ocean basin with $x \in [0, 1]$ *and* $y \in [0, \pi]$. *The Sverdrup flow is assumed to satisfy the kinematic condition at the eastern boundary.*

a. Determine the Sverdrup streamfunction $\psi^0(x, y)$.

The Sverdrup balance is

$$v^0 = \frac{-\partial \tau^x}{\partial y} + \frac{\partial \tau^y}{\partial x} = -0.6 \cos[6(y - \frac{\pi}{6})]$$

The geostrophic streamfunction ψ^0 is

$$\psi^0 = \int -0.6 \cos[6(y - \frac{\pi}{6})]dx = -0.6x \cos[6(y - \frac{\pi}{6})] + g(y)$$

where the unknown function g(y) is determined from the condition $\psi^0 = 0$ for $x = 1$. Hence,

$$g(y) = 0.6 \cos[6(y - \frac{\pi}{6})]$$

So, the stream function ψ^0 is:

$$\psi^0(x, y) = 0.6(1 - x) \cos[6(y - \frac{\pi}{6})]$$

b. Draw (or plot) the streamline pattern for $\frac{\pi}{12} < y < \frac{5\pi}{12}$ *and interpret the result in terms of a vorticity balance.*

See figure 6.4a for a plot of the streamline pattern. When using $\frac{\pi}{12} < y < \frac{5\pi}{12}$ and the given wind-stress, your plot should have the same pattern, but a different amplitude (0.6 instead of 1.0). The vorticity balance of the interior is the Sverdrup balance: vorticity input by the wind is balanced by norh-south motions on the sphere.