Book: Dynamical Oceanography
Author: Dijkstra, H. A.; email: H.A.Dijkstra@uu.nl
Chapter: 6, Exercise: 6.2: Sverdrup flow and Stommel boundary layer
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Given is the same dimensionless wind-stress field as in exercise 6.1, over an ocean basin with $x \in [0, 1]$ and $y \in [0, \pi]$.

a. Determine the dimensionless Stommel boundary layer solution for this wind-stress field.

The Sverdrup flow stream function ψ^0 is:

$$\psi^0(x, y) = 0.6(1 - x)\cos[6(y - \frac{\pi}{6})]$$

and hence the Stommel boundary layer solution is given by equation (6.61):

$$\hat{\psi}(\lambda, y) = \left(1 - e^{-\lambda}\right)\psi^0(0, y)$$

b. Determine now the dimensional pressure p_*, the sea surface elevation h_* and the meridional transport $\Phi_*(y_*)$ defined by

$$\Phi_*(y_*) = \int_0^L v_*(x_*, y_*)\ dx_*$$

where L is the basin length.

Using equation (6.2) the dimensional streamfunction in the interior is given by

$$\psi_*^0(x_*, y_*) = UL\psi^0(x, y) = 0.6\frac{\tau_0}{\rho D\beta_0 L}(L - x_*)\cos[6(\frac{y_*}{L} - \frac{\pi}{6})]$$

and the pressure $p_* = \rho f_0 \psi_*$. The sea surface height follows from the normal stress balance at the surface, explained at page 97:

$$h_* = \frac{Uf_0L}{g}p = \frac{f_0}{g}\psi_*^0$$

and hence h_* and p_* have the same pattern. Obviously, $\Phi_* = 0$ because of mass conservation as it is the total transport through a zonal section.

c. Sketch (or plot) these fields for typical values of the parameters as in Table 6.1. Is the flow in the Stommel boundary layer in geostrophic balance?

- For all fields the pattern is the same as in exercise 6.1b. A typical value of ψ_*^0 is given by $LU = \tau_0/\rho D\beta_0 \approx 1.56 \times 10^4$ m^{2}s^{-1}. Typical values of p_* are h_* then $p_* \approx 1.56 \times 10^3$ Pa and $h_* \approx 0.16$ m.
- Yes, the Stommel boundary layer solution is in geostrophic balance.