Book: Dynamical Oceanography
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For Poincaré waves, the phase speed is given by (7.15). The group velocity is defined as:

$$\mathbf{C}_g = \begin{pmatrix} \frac{\partial \sigma}{\partial k} \\ \frac{\partial \sigma}{\partial l} \end{pmatrix}$$

a. Determine the group velocity for Poincaré waves.

We determine the group velocity from (7.14) as follows

$$C_{gx} = \frac{\partial \sigma}{\partial k} = \pm \frac{C_0^2 k}{\sqrt{f^2 + C_0^2(k^2 + l^2)}}$$

$$C_{gy} = \frac{\partial \sigma}{\partial l} = \pm \frac{C_0^2 l}{\sqrt{f^2 + C_0^2(k^2 + l^2)}}$$

b. Are these waves dispersive?

The group velocity is not equal to the phase velocity and hence Poincaré waves are dispersive. This is most easily seen from the following relation:

$$\mathbf{C}_g = |\mathbf{k}| \begin{pmatrix} \frac{\partial C}{\partial k} \\ \frac{\partial C}{\partial l} \end{pmatrix} + C \frac{\mathbf{k}}{|\mathbf{k}|},$$

where C is the phase speed. Note that when $f = 0$ the dispersion relation reduces to that of normal gravity waves, $\sigma = \pm C_0 |\mathbf{k}|$, and these waves are nondispersive. So, in this case rotation introduces dispersive effects in the waves.