**Book: Dynamical Oceanography**
**Author: Dijkstra, H. A.; email: H.A.Dijkstra@uu.nl**
**Chapter: 8, Exercise: 8.3: Coastal current**
**Version: 2**

*Consider a steady, geostrophic and parallel flow on the western side of a coast at $x = 1$. The density field perpendicular to the coast is given by $\rho = \rho(x)$ with $\partial\rho/\partial x < 0$; this can occur for example due to the freshwater outflow of a river at the coast. The water is well mixed vertically and the height of the sea surface is given by $z = h(x)$ while the flat bottom is located at $z = -1$.*

*All quantities describing the flow are independent of the coordinate $y$. Assume that inertia and friction can be neglected, that $|h| \ll 1$ and that*

$$\bar{v} \equiv \int_{-1}^{h} v\, dz = 0$$

*a. Derive the equation determining the slope of the sea surface in terms in the density field.*

Ignoring atmopsheric pressure, the hydrostatic balance can be integrated to give

$$p(x, z) = \rho(h - z)$$

From the geostrophic balance it follows that

$$v = -p_x \to v(x, z) = \rho'(h - z) + \rho h'$$

where the primes indicate differentiation to $x$. An equation for $h$ is obtained by mass conservation as

$$\int_{-1}^{h} v(x, z)\ dz = 0 \to \rho'(\frac{h^2}{2} + h + \frac{1}{2}) + \rho h'(h + 1) = 0$$

which is a differential equation in $h$ in terms of a given density distribution. When $|h| \ll 1$ it follows that

$$h' \approx -\frac{\rho'}{2\rho}$$

*b. Determine the velocity field of the flow and calculate the depth at which $v = 0$; this is the so-called 'level of no motion'.*

Once $h$ has been determined, the velocity distribution follows from (using the result under a.)

$$v(x, z) = \rho'(h - z) - \frac{\rho'}{2}$$

The level of no motion is the $z$-value where $v = 0$ and it is given by

$$h - z = -\frac{\rho h'}{\rho'} \to z = h - 1/2$$

*c. Make a sketch of the sea surface height and the velocity field. Provide a physical interpretation of the result.*

The imposed zonal density gradient induces vertical shear in the meridional velocity through thermal wind balance. If the vertically integrated velocity vanishes, the velocity must be nonzero at the top of the water column. Therefore, there must be a gradient in sea surface height to induce this velocity at the top of the water column.