Book: Dynamical Oceanography
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Chapter: 8, Exercise: 8.5: Rossby waves in the Pacific
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As we have seen in section 8.4, for a baroclinic Rossby wave with certain $\chi = \chi_n$ and with wavevector $\mathbf{k} = (k, l)$, the dimensionless streamfunction ψ is given by

$$\psi(x, y, t) = \Psi_0 \exp\left[i(kx + ly - \sigma t)\right]$$

with Ψ_0 being a complex amplitude and with the dispersion relation

$$\sigma = \frac{-\beta k}{\chi + k^2 + l^2}$$

Consider now an incoming Rossby wave with wavenumbers, angular frequency and streamfunction $(k_i, l_i, \sigma_i, \Psi_i)$ which moves to the western boundary at $x = 0$.

a. Give the relation between k_i of the incoming wave and σ_i.

For the incoming wave, the dispersion relation can be rewritten as

$$(k_i + \frac{\beta}{2\sigma_i})^2 = -(\chi + l_i^2) + \frac{\beta^2}{4\sigma_i^2}$$

and solving for k_i we have for the wave with a westward group velocity (the long Rossby wave)

$$k_i = -\frac{\beta}{2\sigma_i} - \sqrt{-(\chi + l_i^2) + \frac{\beta^2}{4\sigma_i^2}}$$

Consider now also the reflected wave; this wave can be represented by quantities $(k_r, l_r, \sigma_r, \Psi_r)$.

b. What are the boundary conditions for ψ at $x = 0$.

The total streamfunction ψ is a superposition of the incoming and outgoing waves, i.e.

$$x = 0 : \psi = \Psi_i + \Psi_r$$

No normal flow conditions on the western boundary imply

$$-u = \frac{\partial \psi}{\partial y} = 0.$$

This leads to

$$l_i \Psi_{0i} \exp[i(l_i - \sigma_i t)] = -l_r \Psi_{0r} \exp[i(l_r - \sigma_r t)]$$

c. Determine the streamfunction of the total flow consisting of the incoming and the reflected Rossby wave.

From the boundary condition at $x = 0$, it follows that $\sigma_i = \sigma_r = \sigma$, $l_i = l_r = l$ and $\Psi_r = -\Psi_i = -\Psi$. As the group speed of the reflected wave needs to be eastward, the wavenumber k_r is determined by

$$k_r = -\frac{\beta}{2\sigma} + \sqrt{-(\chi + l^2) + \frac{\beta^2}{4\sigma^2}}$$

and hence $k_r > k_i$. The total streamfunction ψ is therefore

$$\psi(x, y, t) = \Psi(\exp\left[i(k_i x + ly - \sigma t)\right] - \exp\left[i(k_r x + ly - \sigma t)\right])$$