Book: Dynamical Oceanography
Author: Dijkstra, H. A.; email: H.A.Dijkstra@uu.nl
Chapter: 9, Exercise: 9.2: Rossby waves in a two-layer model
Version: 2

Small amplitude motions occur in an initially motionless two-layer system where layer i has a density ρ_{*i} and equilibrium thickness H_i, $i = 1, 2$. Consider only waves with $l_* = 0$ and use $\rho_{*2} - \rho_{*1} = 1\ kg\ m^{-3}$ and $H_1 = 500\ m$, $H_2 = 4500\ m$.

a. Calculate the dimensional phase speed C_*^x of the baroclinic and barotropic Rossby waves with a wavelength $\lambda_* = 2\pi L/k$ of 100 km at a latitude $45°N$.

For convenience, we first convert the appropriate dispersion relaton (9.19) to dimensional quantities using (9.13):

$$\sigma_* = \frac{U}{L}\sigma = \frac{-U\beta k}{L\left(\chi + k^2 + l^2\right)} = \frac{-\beta_0 k_*}{\chi/L^2 + k_*^2 + l_*^2}$$

We then take $l_* = 0$ and $\lambda_* = 2\pi/k_*$= 100 km. The latitude is 45°N, so $f_0 = 1.03 \cdot 10^{-4}$ s^{-1} and $\beta_0 = 1.6 \cdot 10^{-11}$ m^{-1} s^{-1}. The dimensional phase speed for the barotropic wave ($\chi = 0$) is:

$$C_*^x = \frac{\sigma_*}{k_*} = -\frac{\beta_0}{k_*^2} = -\frac{\beta_0 \lambda_*^2}{4\pi^2} = 4.1 \cdot 10^{-3}\mathrm{ms}^{-1}$$

For the baroclinic phase speed ($\chi = F_1 + F_2$) we find:

$$C_*^x = \frac{-\beta_0}{\frac{f_0^2}{g'}\left(\frac{1}{H_1} + \frac{1}{H_2}\right) + k_*^2} = \frac{-\beta_0}{\frac{f_0^2 \rho_0}{g\triangle\rho}\left(\frac{1}{H_1} + \frac{1}{H_2}\right) + \frac{4\pi^2}{\lambda_*^2}} = 2.5 \cdot 10^{-3} ms^{-1}$$

b. Sketch the velocity distributions as a function of depth for both types of Rossby waves.

If we sketch the velocity distributions for both types of Rossby waves we see for the barotropic mode that in both layers the speeds are equal to each other. For the baroclinic mode we see that the speeds in both layers are opposite (and unequal in magnitude) to each other.