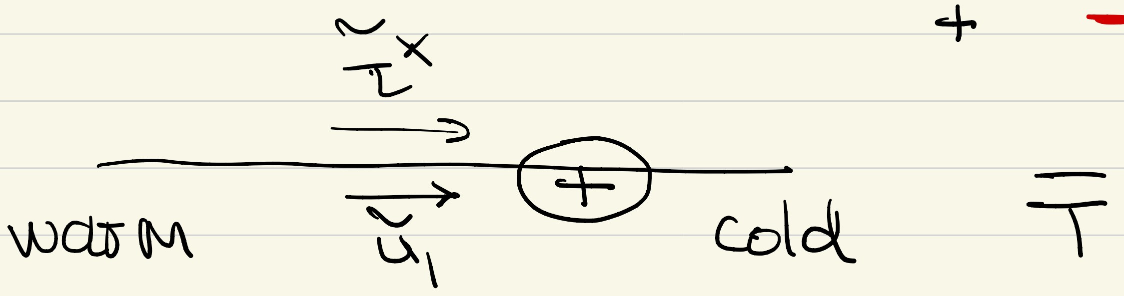
ZAF

$$\frac{\partial T}{\partial t} = -u_1 \frac{\partial T}{\partial x}$$

$$T = \bar{T} + \tilde{T}$$

$$\rightarrow \quad \frac{\partial(\bar{T} + \tilde{T})}{\partial t} = -\left(\bar{u}_1 + \tilde{u}_1\right) * \left(\frac{\partial \bar{T}}{\partial x} + \frac{\partial \tilde{T}}{\partial x}\right)$$

$$\rightarrow \quad \frac{\partial \tilde{T}}{\partial t} = -\left(\bar{u}_1 \frac{\partial \tilde{T}}{\partial x} + \underbrace{\tilde{u}_1 \frac{\partial \bar{T}}{\partial x}}_{+ \quad -}\right)$$



$$\frac{\partial \bar{T}}{\partial x} < 0$$

$\rightarrow x$

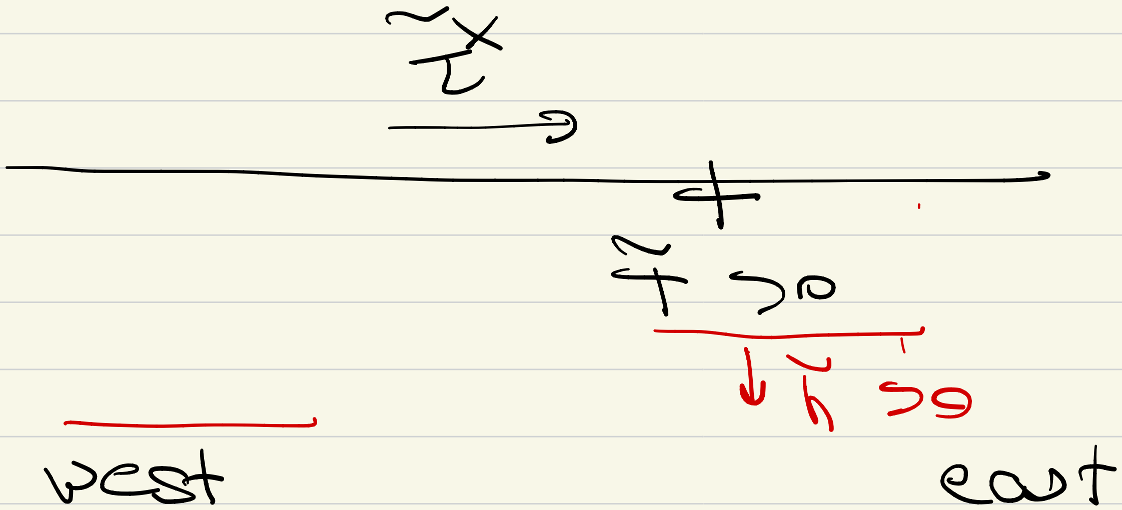
$$\rightarrow \quad \frac{\partial \tilde{T}}{\partial t} > 0$$

Thermocline feedback

$$T_t = -w_1 \frac{T - T_s(h)}{H}$$

$$\bar{T}_t + \tilde{T}_t = -\left(\bar{w}_1 + \tilde{w}_1\right)\left(\frac{\bar{T} + \tilde{T} - T_s(\bar{h} + \tilde{h})}{H}\right)$$

$$\tilde{T}_t = -\underset{+}{\bar{w}_1}\left(\tilde{T} - \underset{+}{T_s'}(\bar{h})\,\underset{+}{\tilde{h}}\right)$$



$$\tilde{T}_t > 0$$

ω fixed $\qquad \lambda$ control

fixed point; $\bar{x} = \bar{y} = 0$

Stability: $\quad x = \bar{x} + \tilde{x}$

$y = \bar{y} + \tilde{y}$

$$\rightarrow \quad \dot{\tilde{x}} = \lambda \tilde{x} - \omega \tilde{y}$$

$$\dot{\tilde{y}} = \omega \tilde{x} + \lambda \tilde{y}$$

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{pmatrix} = J \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$$

$$J = \begin{pmatrix} \lambda & -\omega \\ \omega & \lambda \end{pmatrix}$$

e.v. $\quad \det \left| J - \sigma I \right| = 0$

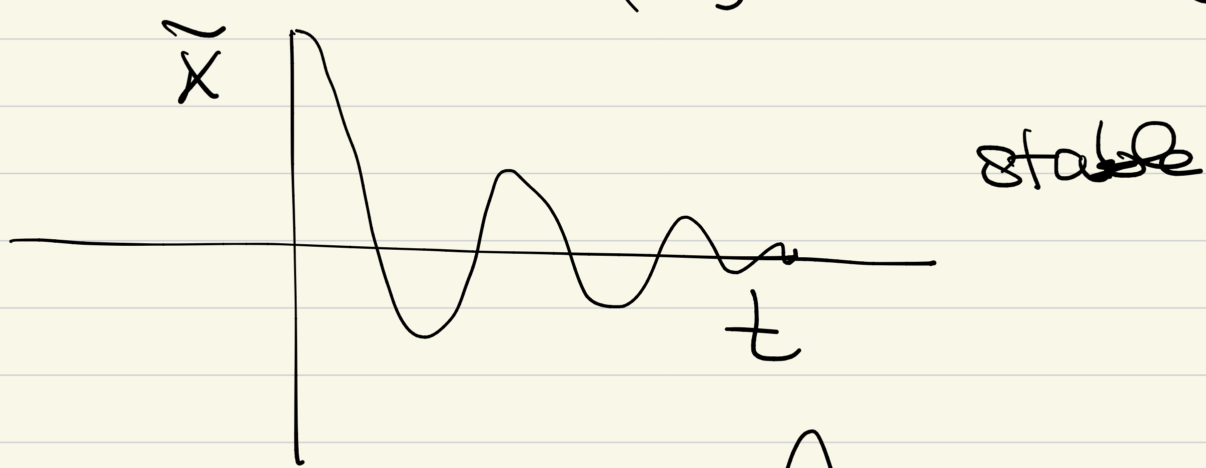
$$\begin{vmatrix} \lambda - \sigma & -\omega \\ \omega & \lambda - \sigma \end{vmatrix} = 0$$

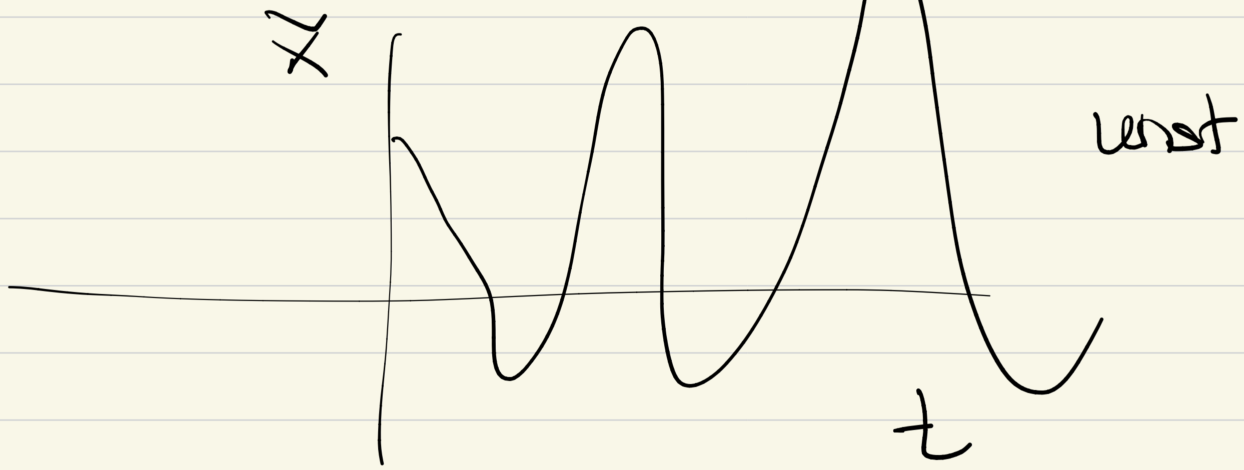
$$(\lambda - \sigma)^2 + \omega^2 = 0$$

$$\sigma_{1,2} = \lambda \pm i\omega$$

$\lambda > 0$: $\mathrm{Re}(\sigma) > 0$ growth

$\lambda < 0$: $\mathrm{Re}(\sigma) < 0$ decay





$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\dot{x} = \dot{r}\cos\theta - \dot{\theta} r \sin\theta$$

$$\dot{y} = \dot{r}\sin\theta + \dot{\theta} r \cos\theta$$

$$x^2 + y^2 = r^2$$

$$\Rightarrow \quad \dot{r}\cos\theta - \dot{\theta} r \sin\theta^{\;\dot{x}} =$$

$$= \lambda r \cos\theta - \omega r \sin\theta$$

$$- r^3 \cos\theta \qquad (1)$$

$$\dot{r}\sin\theta + \dot{\theta} r \cos\theta^{\;\dot{y}} =$$

$$= \omega r \cos\theta + \lambda r \sin\theta$$

$$- r^3 \sin\theta \qquad (2)$$

$$\cos\theta\,(1) + \sin\theta\,(2) \Rightarrow$$

$$\dot{r} = \lambda r - r^3$$

$$-\sin\theta\,(1) + \cos\theta\,(2) \Rightarrow$$

$$\cancel{r}\dot{\theta} = -\omega \cancel{r}$$

$$\begin{cases} \dot{r} = \lambda r - r^3 \\ \dot{\theta} = \omega \end{cases}$$

$$dR = \frac{\partial r}{\partial x} dX + \frac{\partial r}{\partial y} dY +$$

$$\frac{1}{2}\left(\frac{\partial^2 r}{\partial x^2} (dX)^2 + \frac{\partial^2 r}{\partial y^2} (dY)^2 + \dots \right)$$

$$dX = \sigma dW_1 \qquad dY = \sigma dW_2$$

$$(dX)^2 = \sigma^2 dt$$

$$dR = \frac{x}{r} dX + \frac{y}{r} dY +$$

$$\frac{1}{2}\left(\frac{y^2}{r^3}\sigma^2 dt + \frac{x^2}{r^3}\sigma^2 dt \right) + \dots$$

$$= \cos\theta \, \sigma \, dW_1 + \sin\theta \, \sigma \, dW_2$$

$$+ \frac{\sigma^2}{2}\frac{1}{R}$$