Notes

6 - 10 - 2020

Slide 21

$$\Delta T = T_e - T_p$$

$$\Delta S = S_e - S_p$$

$$\rightarrow \quad \frac{d\,\Delta T}{dt} = -\frac{1}{t_r}\left(\Delta T - \theta\right) - Q(\Delta\rho)\,\Delta T$$

$$\frac{d\,\Delta S}{dt} = \frac{F_s}{H} S_0 - Q(\Delta\rho)\,\Delta S$$

$$\begin{cases} \Delta T = \theta\, x & \Delta S = y\,\dfrac{\alpha_T}{\alpha_S}\,\theta \\ \tilde{t} = t/t_d \end{cases}$$

$$Q(\Delta\rho) = \frac{1}{t_d} + \frac{q}{\rho_0^2 V}\left(\Delta\rho\right)^2$$

$$\begin{aligned} \Delta\rho &= \rho_e - \rho_p \\ &= \left(-\alpha_T\,\Delta T + \alpha_S\,\Delta S\right)\rho_0 \\ &= \left(-\alpha_T\,\theta\,x + \cancel{\alpha_S}\,\frac{\alpha_T}{\cancel{\alpha_S}}\,\theta\,y\right)\rho_0 \\ &= \rho_0\,\alpha_T\,\theta\,(y - x) \end{aligned}$$

$$\dot{x} = \frac{dx}{dt}$$

$$\frac{\theta}{t_d}\,\dot{x} = -\frac{1}{t_r}\left(x\theta - \theta\right) + x\theta\left(\frac{1}{t_d} + \frac{q}{\rho_0^2}\,\theta^2 v_T^2\left(y - x\right)^2\right)$$

$$\longrightarrow \quad \dot{x} = -\overbrace{\frac{t_d}{t_r}}^{\alpha}\left(x - 1\right) + x\left(1 + \mu^2\left(y - x\right)^2\right)$$

$$\longrightarrow \quad \dot{x} = -\alpha\left(x - 1\right) + x\left(1 + \mu^2\left(y - x\right)^2\right)$$

$$\alpha = \frac{t_d}{t_r}$$

Slide 27

$$\begin{cases} \dot{x} = -x + t \\ x(s) = x_0 \end{cases}$$

HS. $x_H(t) = e^{-t}$

PS $x_p(t) = \alpha(t)\, e^{-t}$ *

$$\dot{\alpha} e^{-t} - \cancel{\alpha e^{-t}} = -\cancel{\alpha e^{-t}} + t$$

$$\dot{\alpha} = t e^{t}$$

$$\alpha(t) = \int e^{t'} t' \, dt'$$

$$= t e^{t} - \int e^{t'} \, dt'$$

$$= e^{t} (t - 1)$$

$$\begin{cases} x(t) = C e^{-t} + t - 1 \\ x(s) = x_0 \end{cases}$$

$$\rightarrow \quad s - 1 + C e^{-s} = x_0$$

$$\rightarrow \quad C = (x_0 + 1 - s)\, e^{s}$$

$$x(t) = t - 1 + e^{-t} e^{s} \left(x_0 + 1 - s \right)$$

$$= x_0 e^{s-t} + t - s e^{s-t} - \left(1 - e^{s-t} \right)$$

Pullback: fix t, let $s \to -\infty$

$$\underbrace{x(t) \xrightarrow{s \to -\infty} t - 1}$$

Slide 29

$$\frac{dx}{dt} = \lambda - x^2 \quad , \quad \lambda \in \mathbb{R}$$

$$\frac{dx}{dt} = 0 \longrightarrow \text{fixed points } \bar{x}$$

$$\lambda - \bar{x}^2 = 0 \qquad (*)$$

$$\bar{x} = \pm\sqrt{\lambda} \quad , \quad \lambda \geq 0$$

stability : $x = \bar{x} + \tilde{x}$

$$|\tilde{x}| \ll |\bar{x}|$$

$$\frac{d}{dt}\left(\bar{x} + \tilde{x}\right) = \lambda - \left(\bar{x} + \tilde{x}\right)^2$$

$$\longrightarrow \quad \frac{d\tilde{x}}{dt} = \underbrace{\lambda - \left(\bar{x}^2\right.}_{0} + \underbrace{2\bar{x}\tilde{x}}_{} \left. + \cancel{\tilde{x}^2}\right)$$

$$= \underbrace{\left(-2\bar{x}\right)}_{\sigma}\tilde{x}$$

$$\tilde{x}(t) = \tilde{x}_0 e^{\sigma t}$$

$$\bar{x} = \pm\sqrt{\lambda} \qquad \tilde{x}_0 e^{\sigma t} = \tilde{x}(t)$$

$$\sigma = -2\bar{x}.$$

$\bar{x} = \sqrt{\lambda}$: $\sigma = -2\sqrt{\lambda} < 0$

$\tilde{x} \to 0, \; t \to \infty$

$\bar{x} = -\sqrt{\lambda}$: $\sigma = 2\sqrt{\lambda} > 0$

$\tilde{x} \to \infty \; t \to \infty$