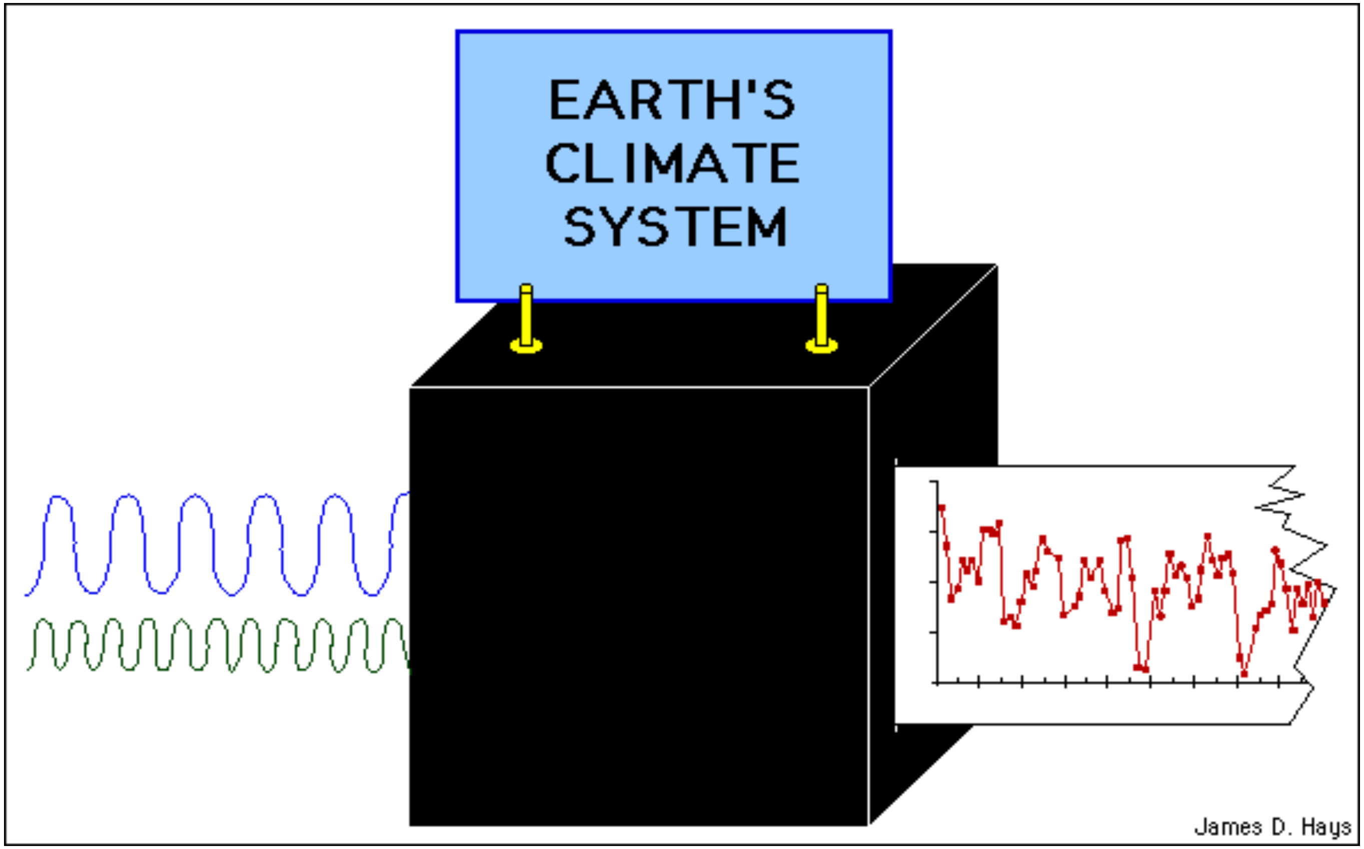
Stochastic Climate Dynamics



Henk Dijkstra, IMAU & CCSS
Physics Department, Utrecht University, Utrecht, Netherlands

https://webspace.science.uu.nl/~dijks101/styled-6/

Summary 29/9 + 6/10

Stochastic linear dynamical systems

$$dX_t = -\gamma X_t dt + \sigma dW_t$$

Hasselmann, null-hypothesis (SST)

Deterministic nonlinear systems

$$\frac{dx}{dt} = \lambda - x^2$$

bifurcations, attractors

Interaction noise and multiple equilibria

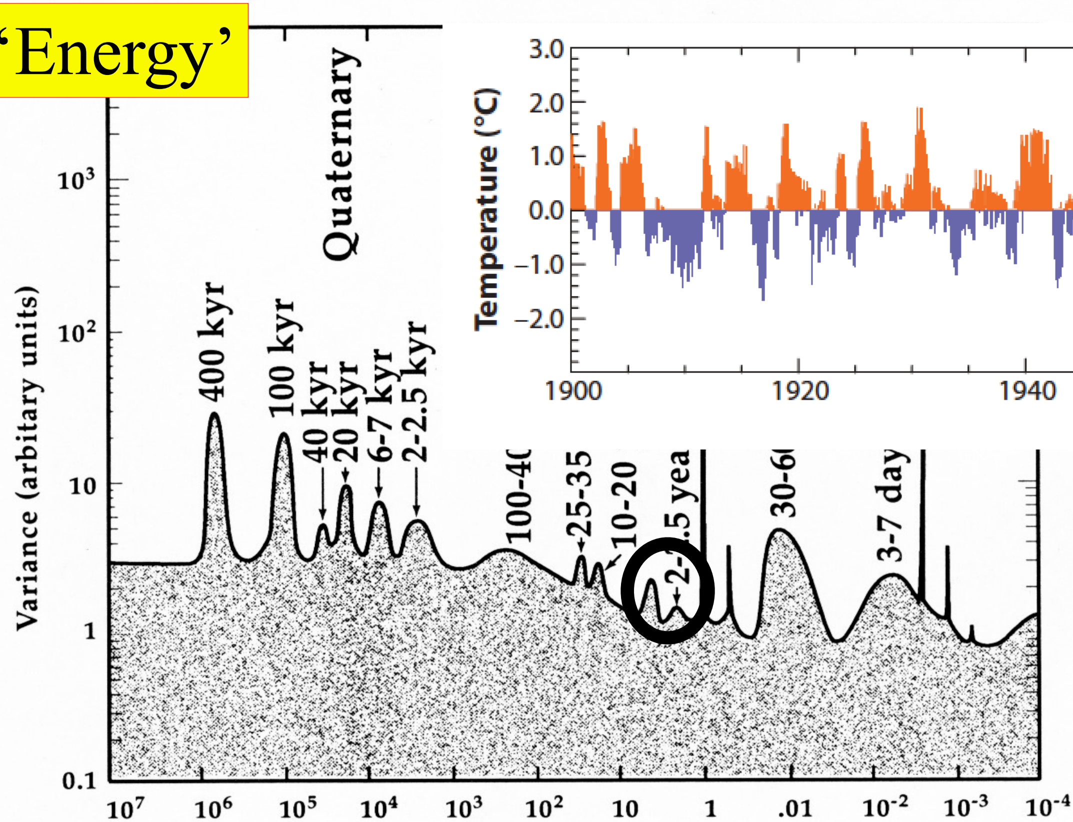
$$\begin{aligned} \frac{dx}{dt} &= -\alpha(x-1) - x(1+\mu^2(x-y)^2), \\ \frac{dy}{dt} &= F - y(1+\mu^2(x-y)^2), \end{aligned}$$

$$F(t) = \bar{F} + \sigma\xi(t)$$

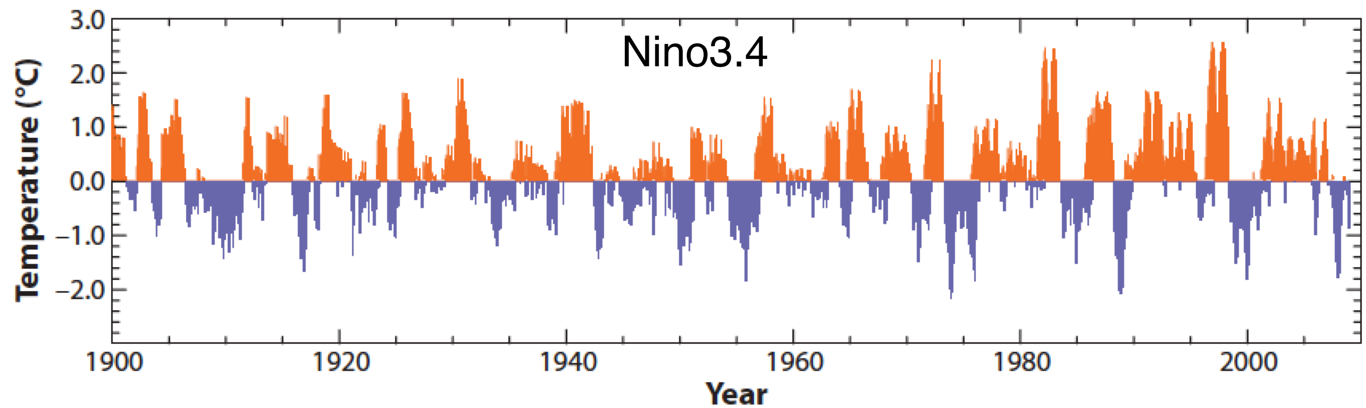
noise induced transitions

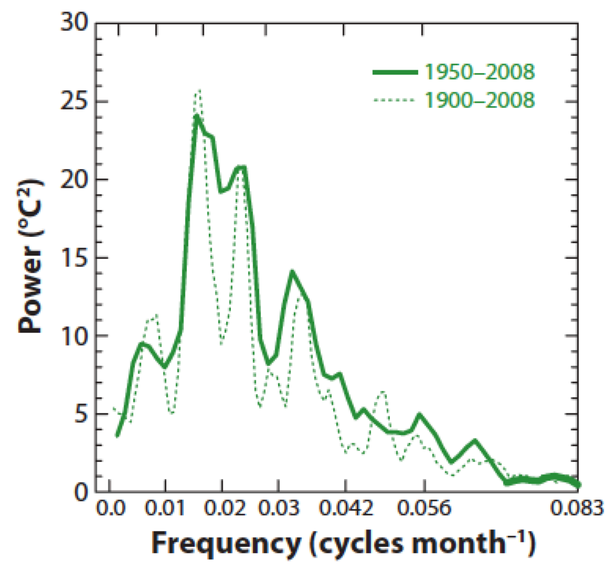
Artist's view of Climate Variability

‘Energy’



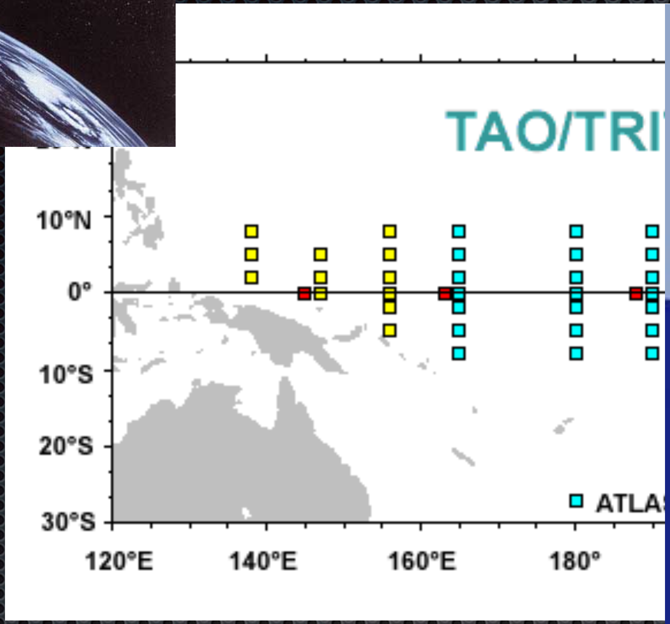
Period (years)

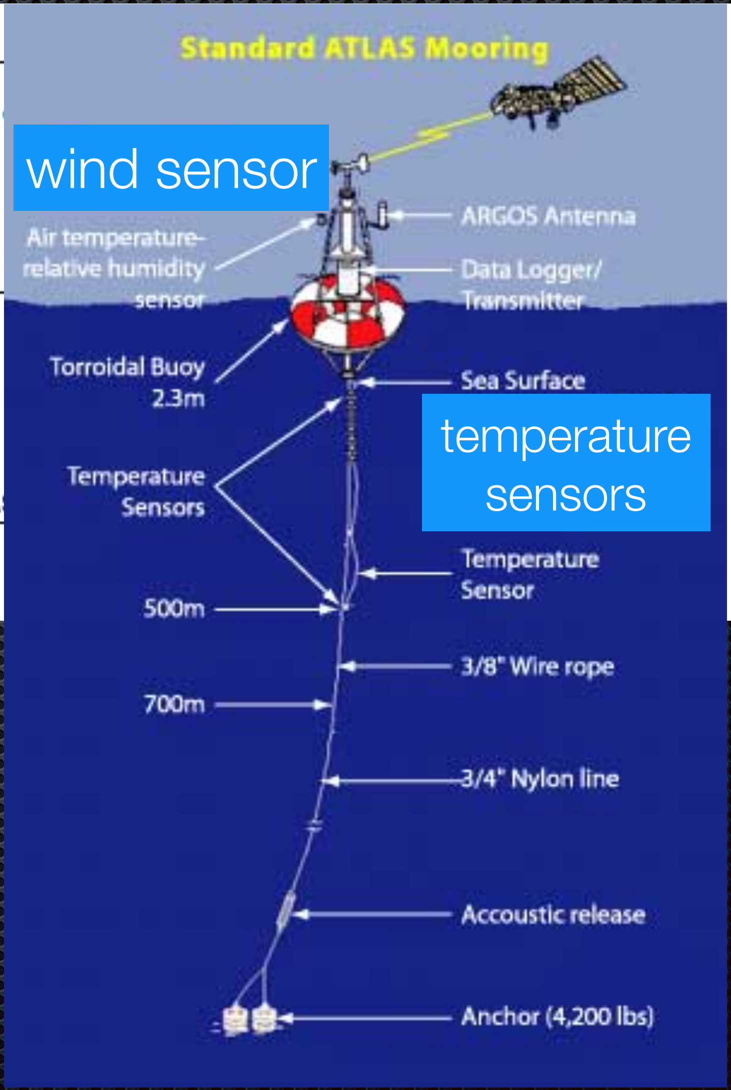




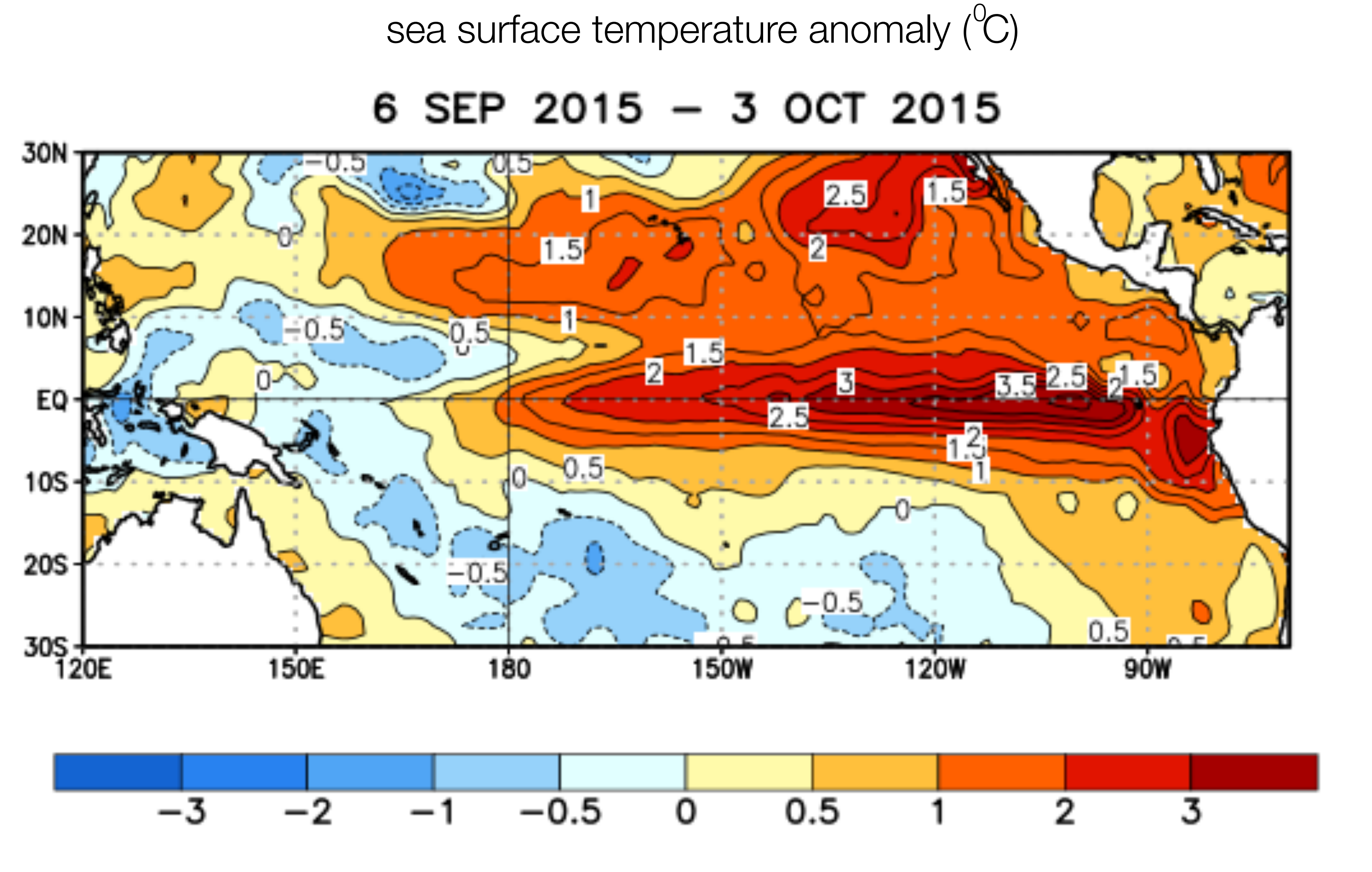
Mitchell, 1978

Observations

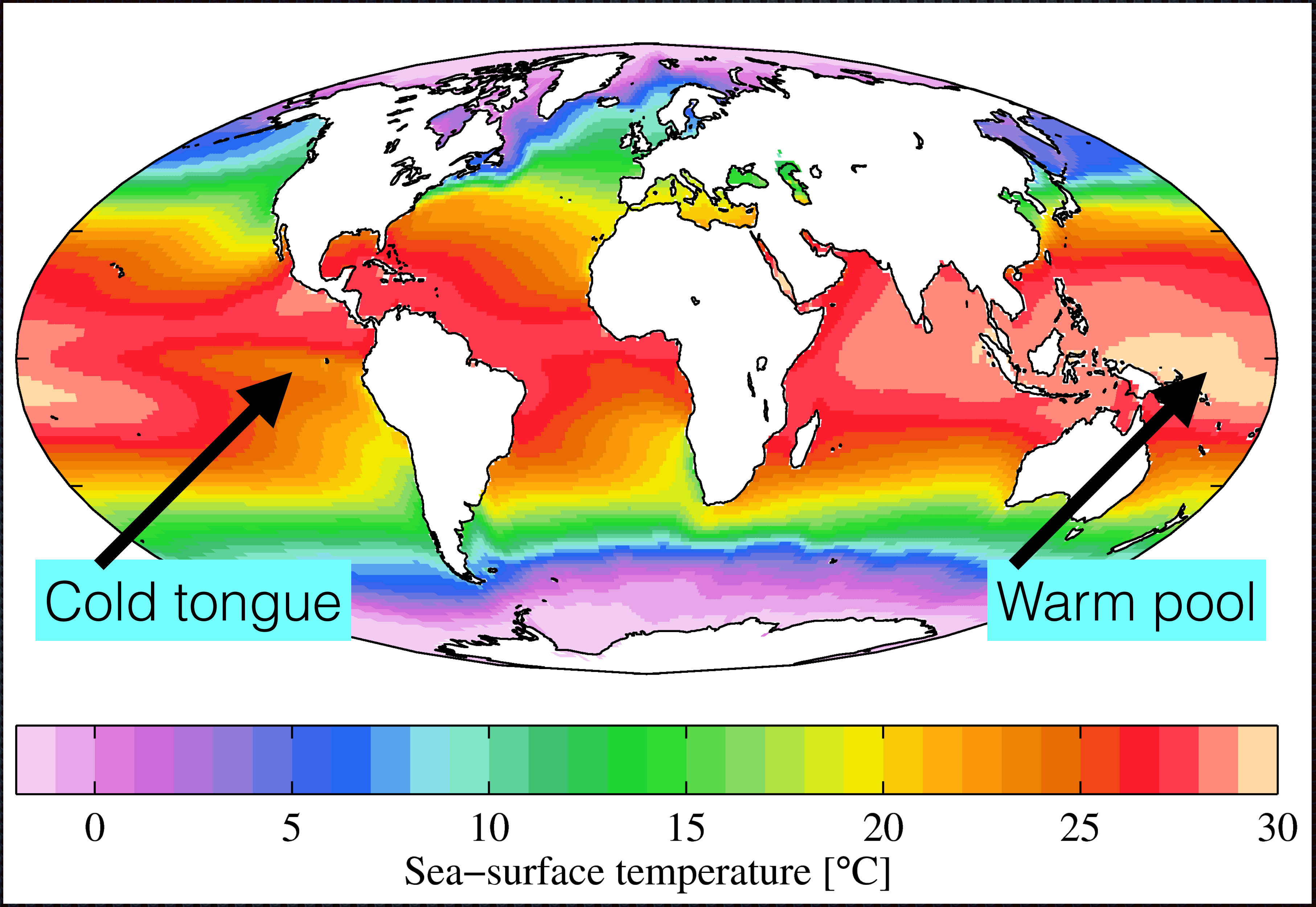




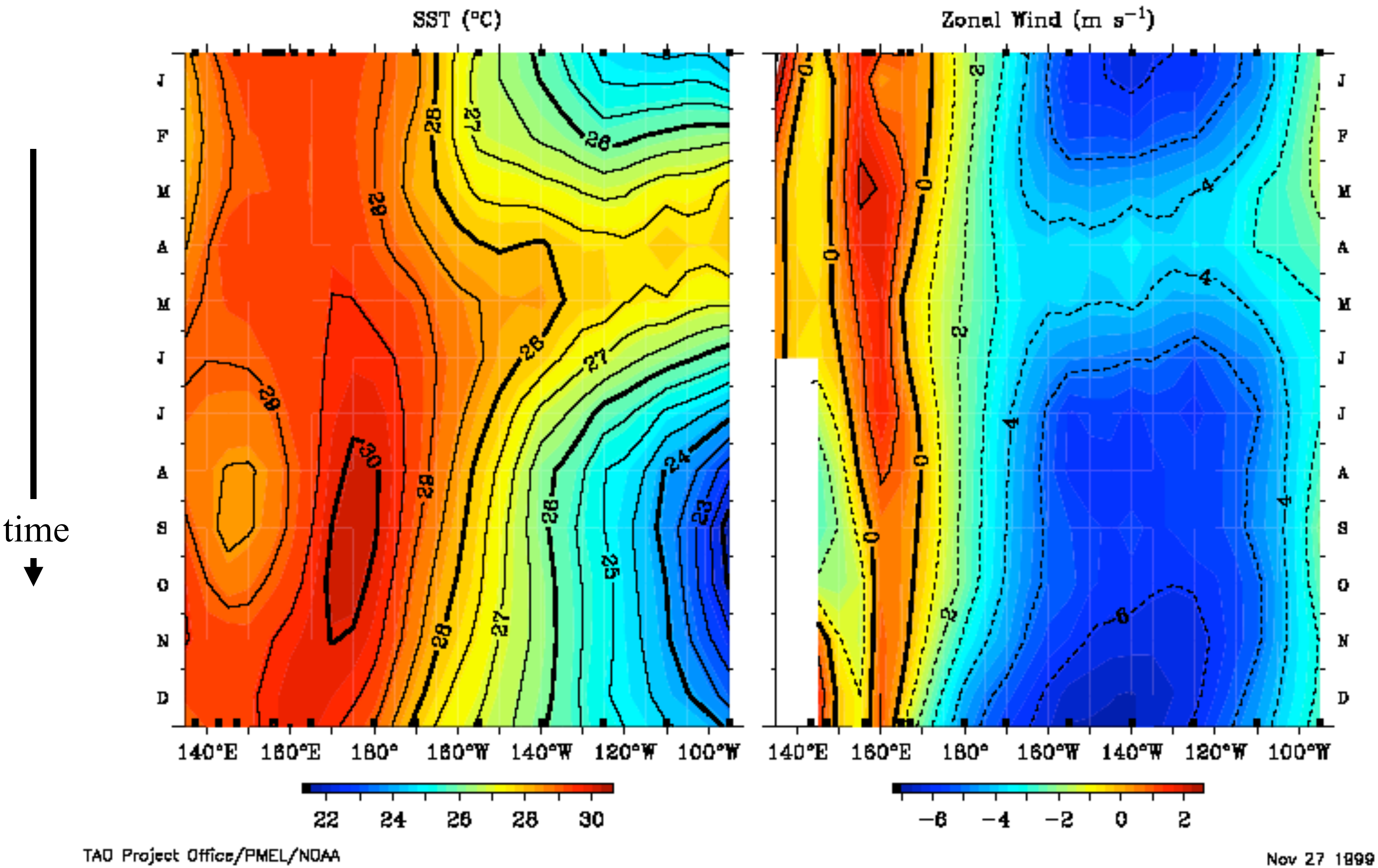
El Niño variability



Mean Sea Surface Temperature



The mean seasonal cycle

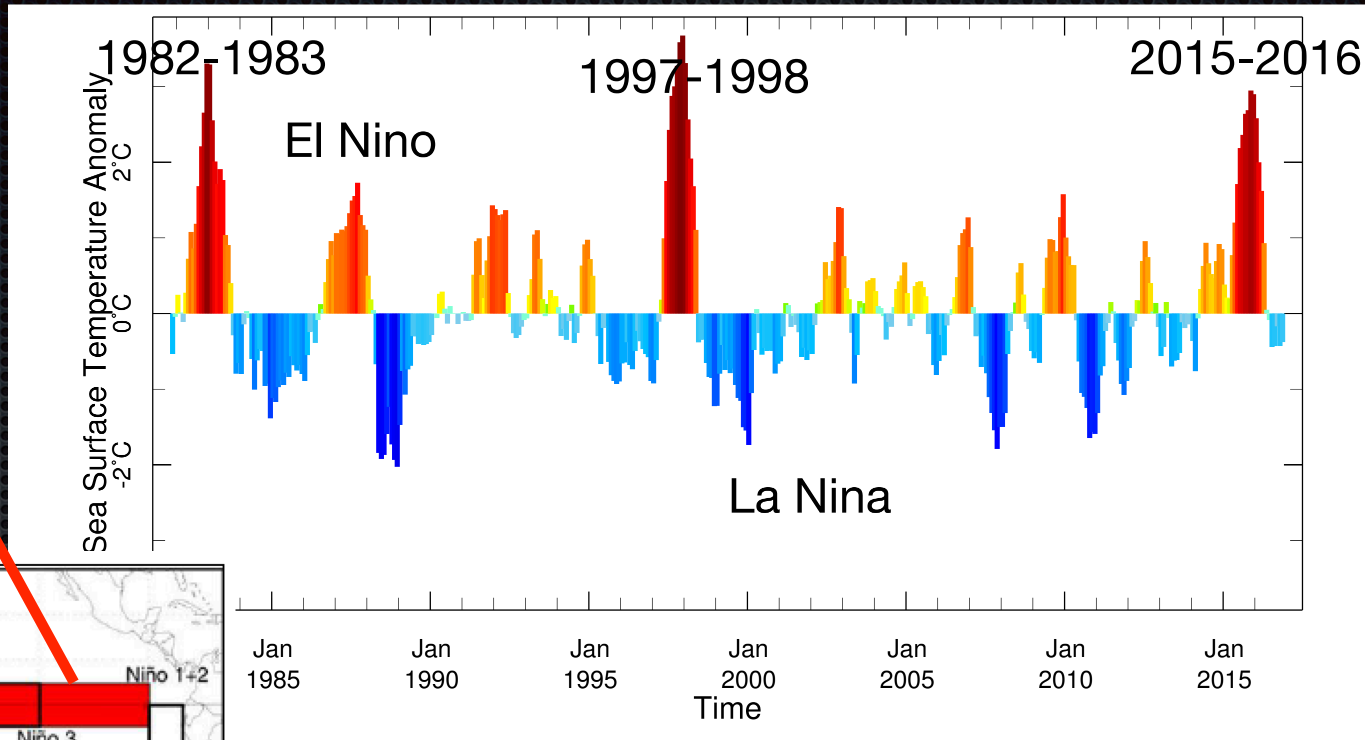


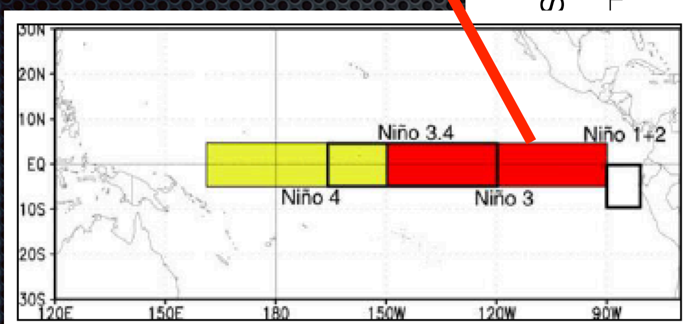
SST

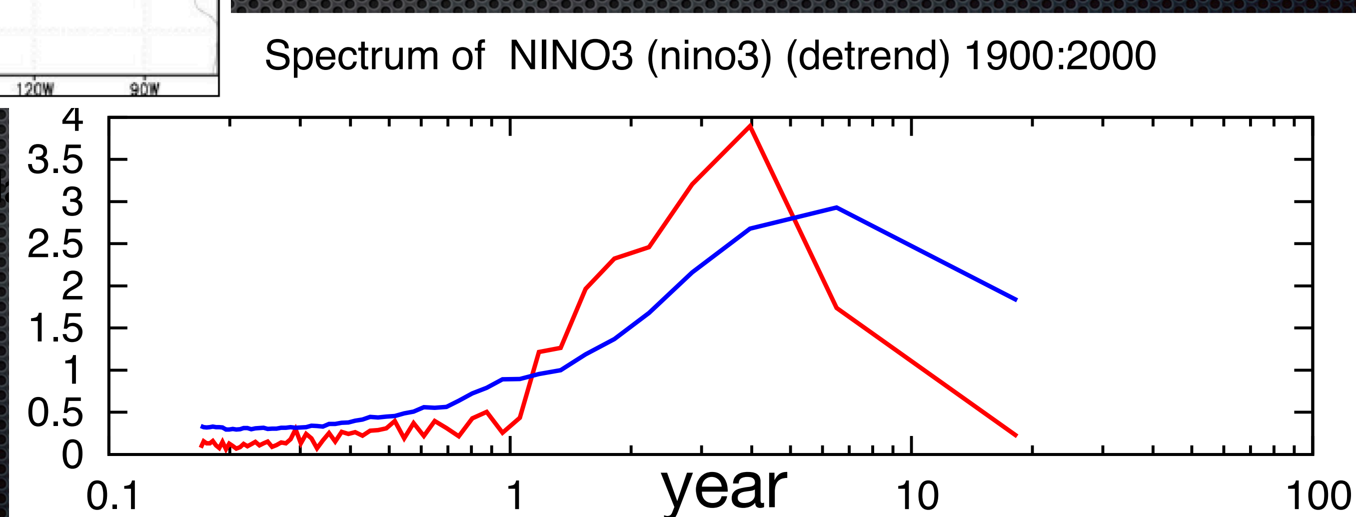
Zonal wind

Temporal variability

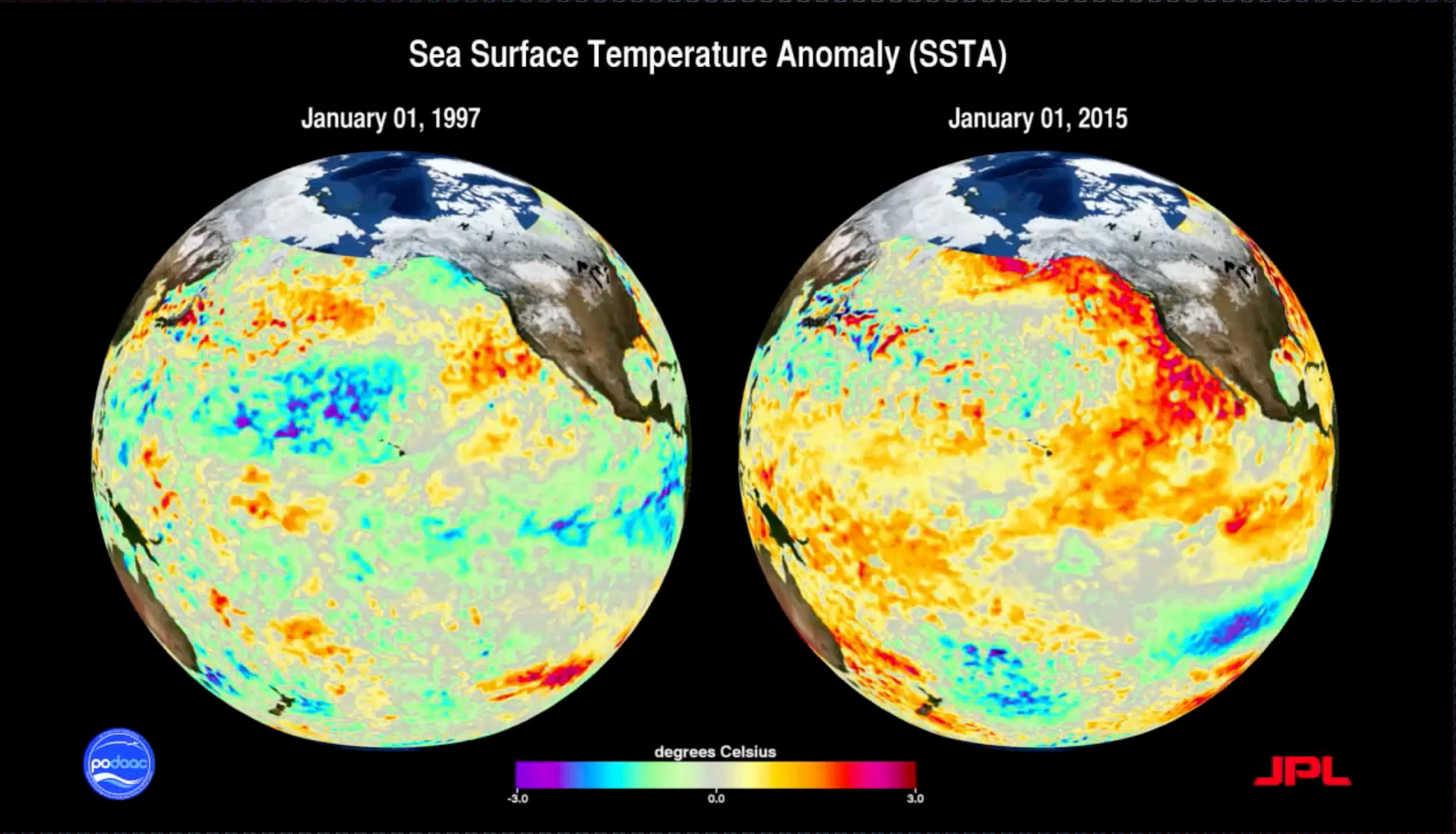
NINO3



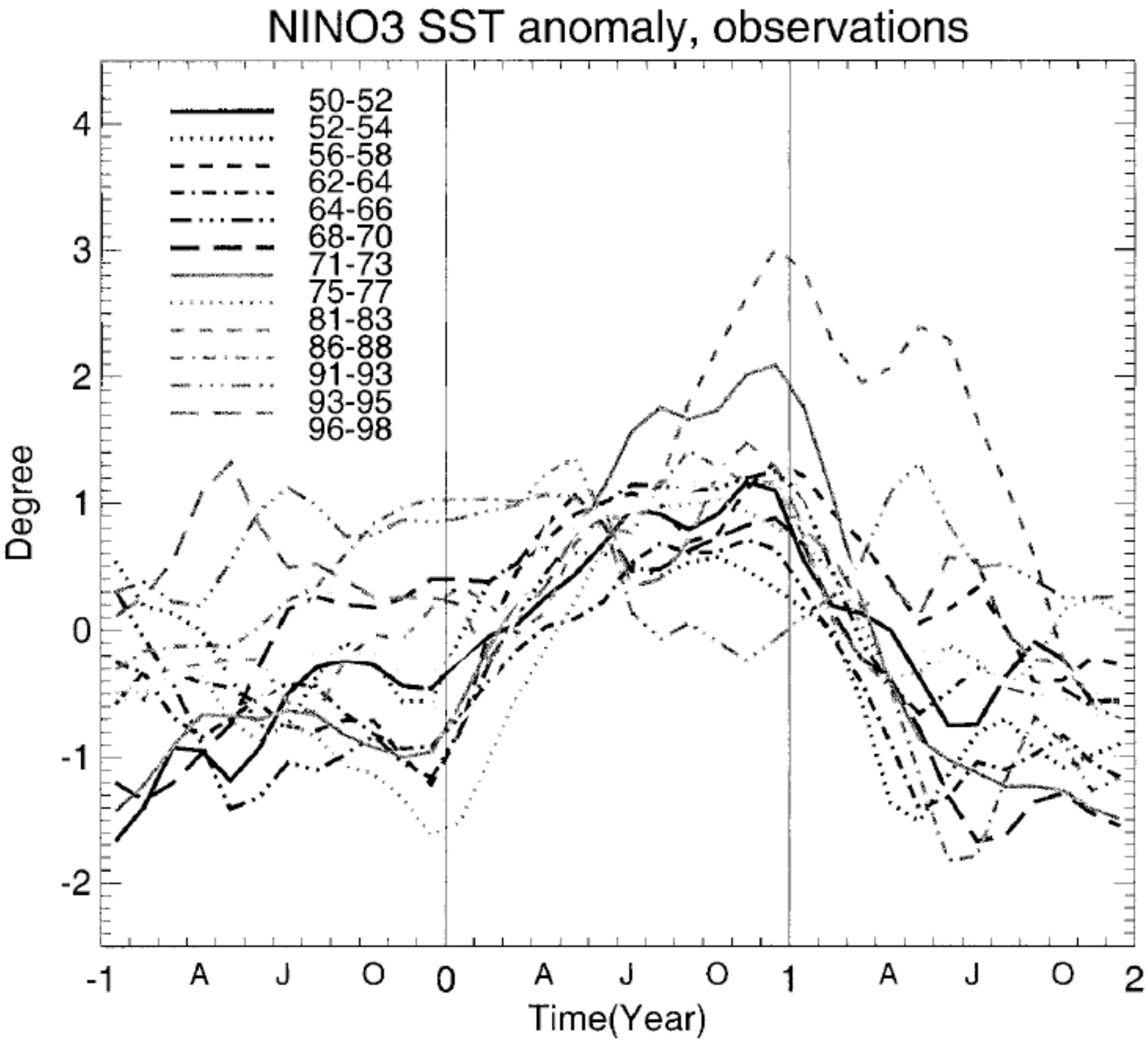




1997-1998 vs 2015-2016



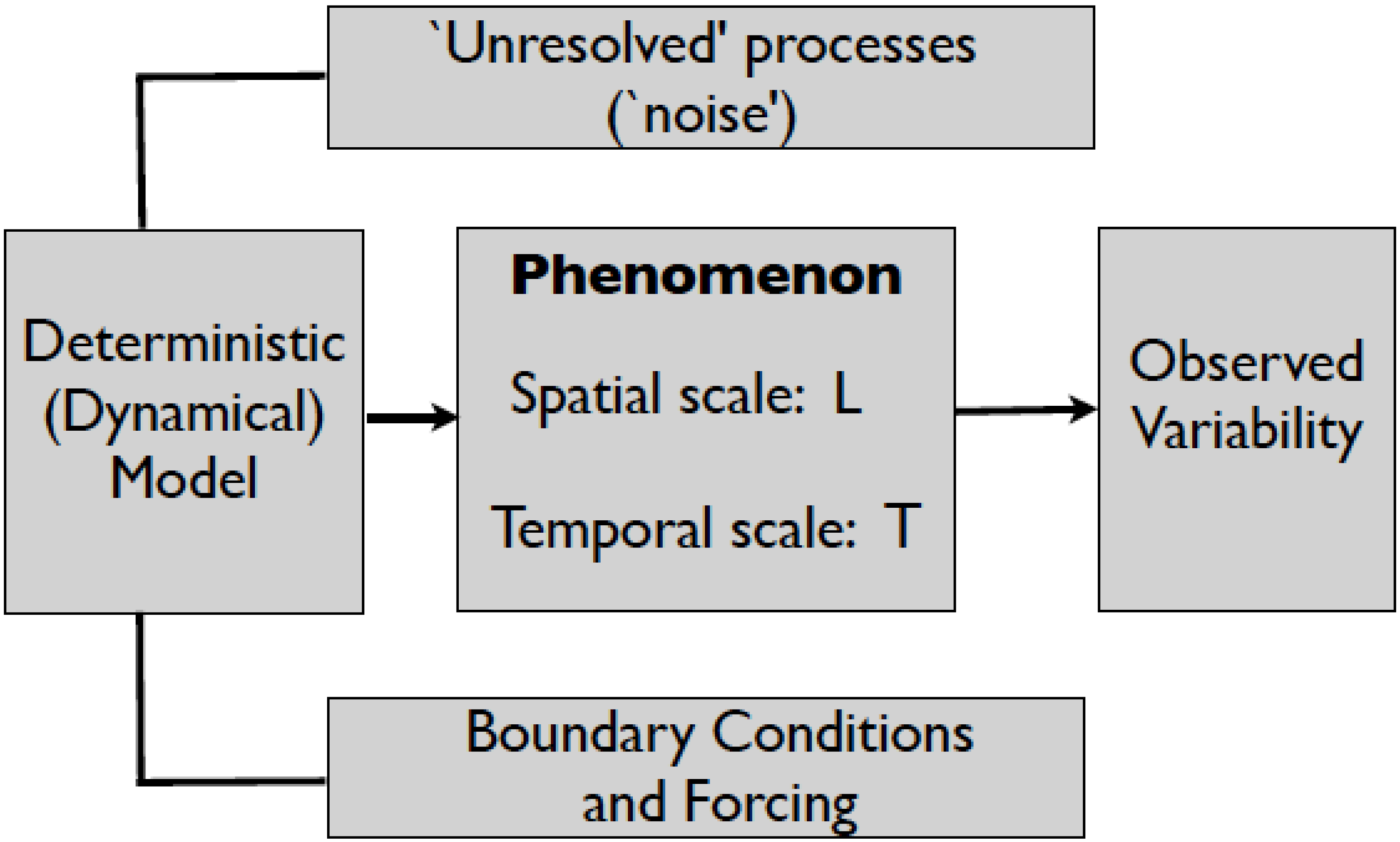
Phase locking of ENSO to the seasonal cycle



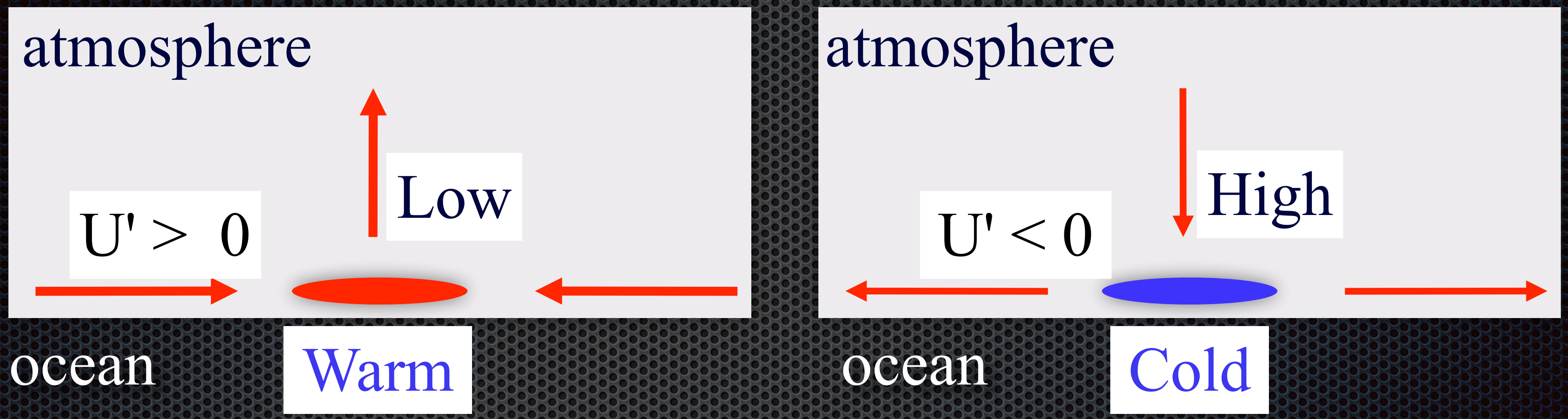
Neelin et al. (1997)

Question time

Stochastic dynamical systems approach

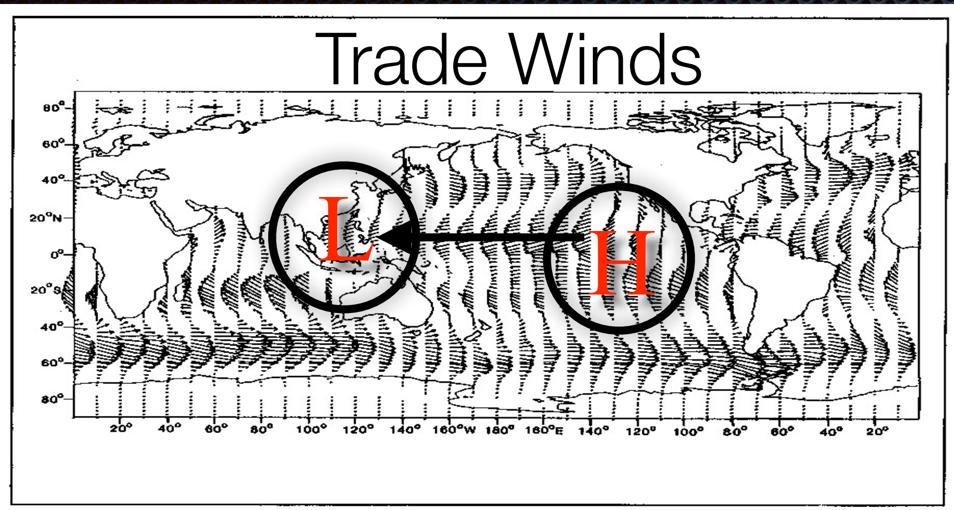


Ingredient 1: Wind response to sea surface temperature anomalies

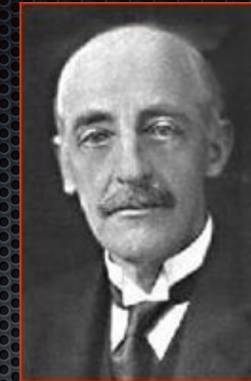


A positive sea surface temperature anomaly induces a westerly (towards the east) wind anomaly west of the sea surface temperature anomaly

The Southern Oscillation

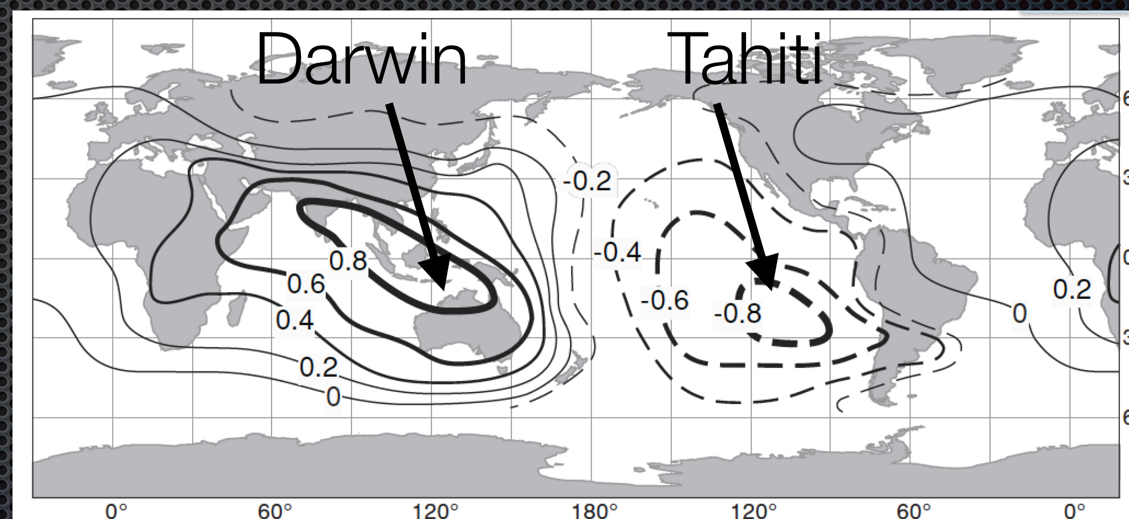


Annual mean surface winds



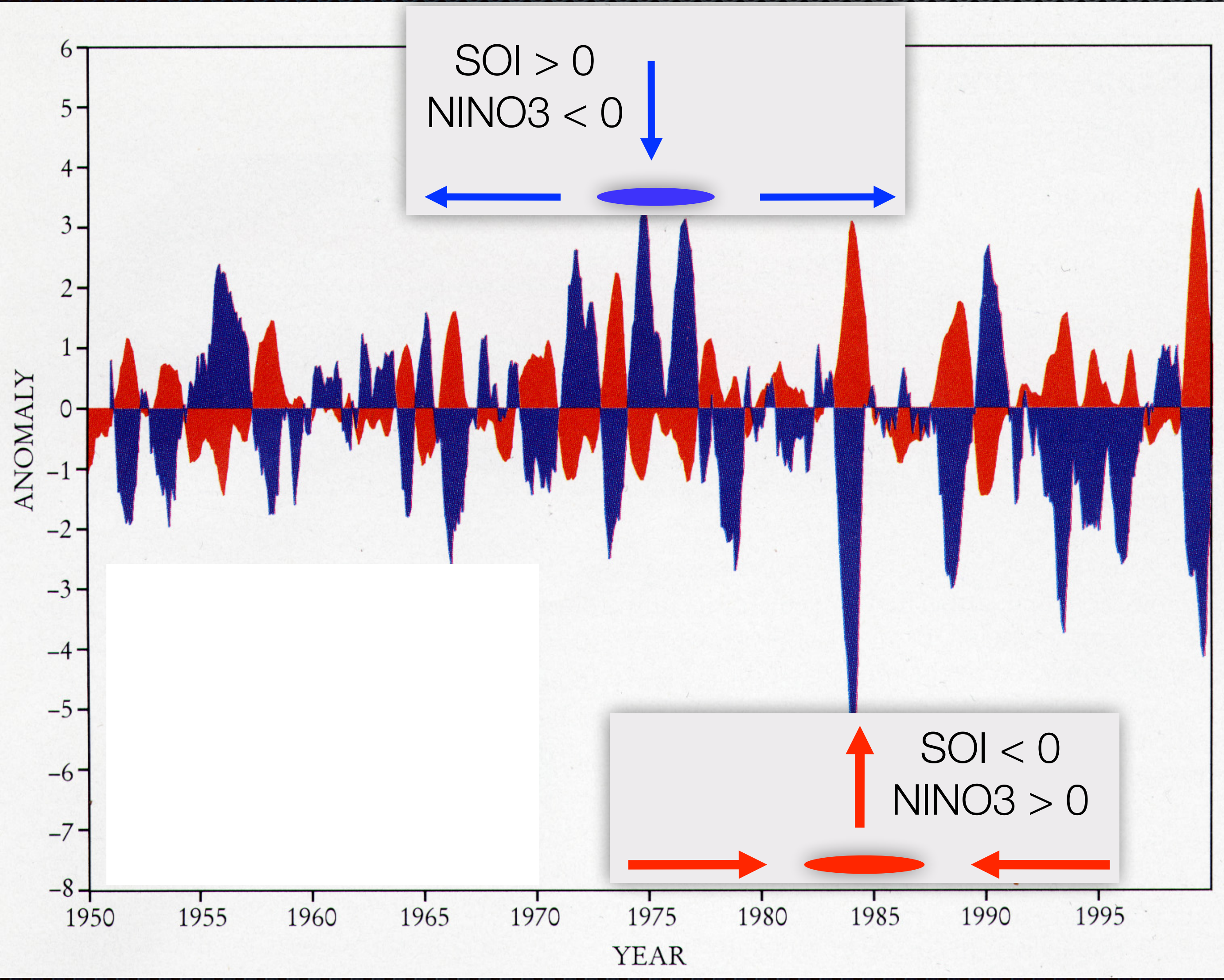
Sir Gilbert Walker
1868-1958

Correlation of Sea Level Pressure anomalies with those in Darwin



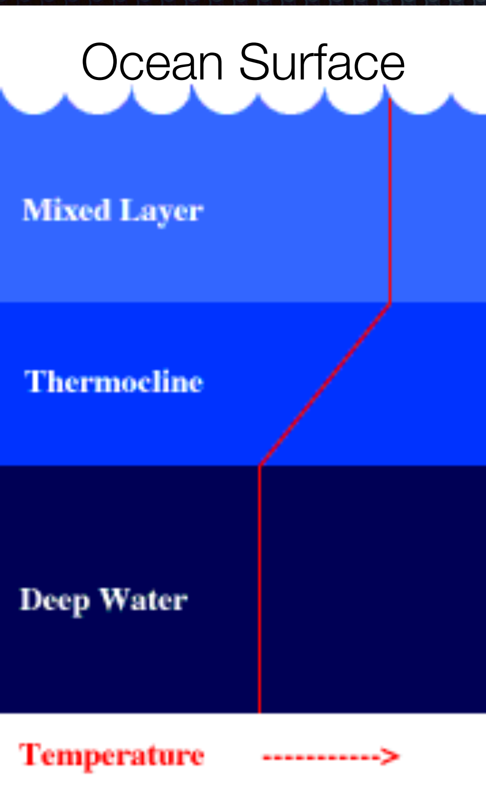
SOI = pressure anomaly (Tahiti - Darwin)

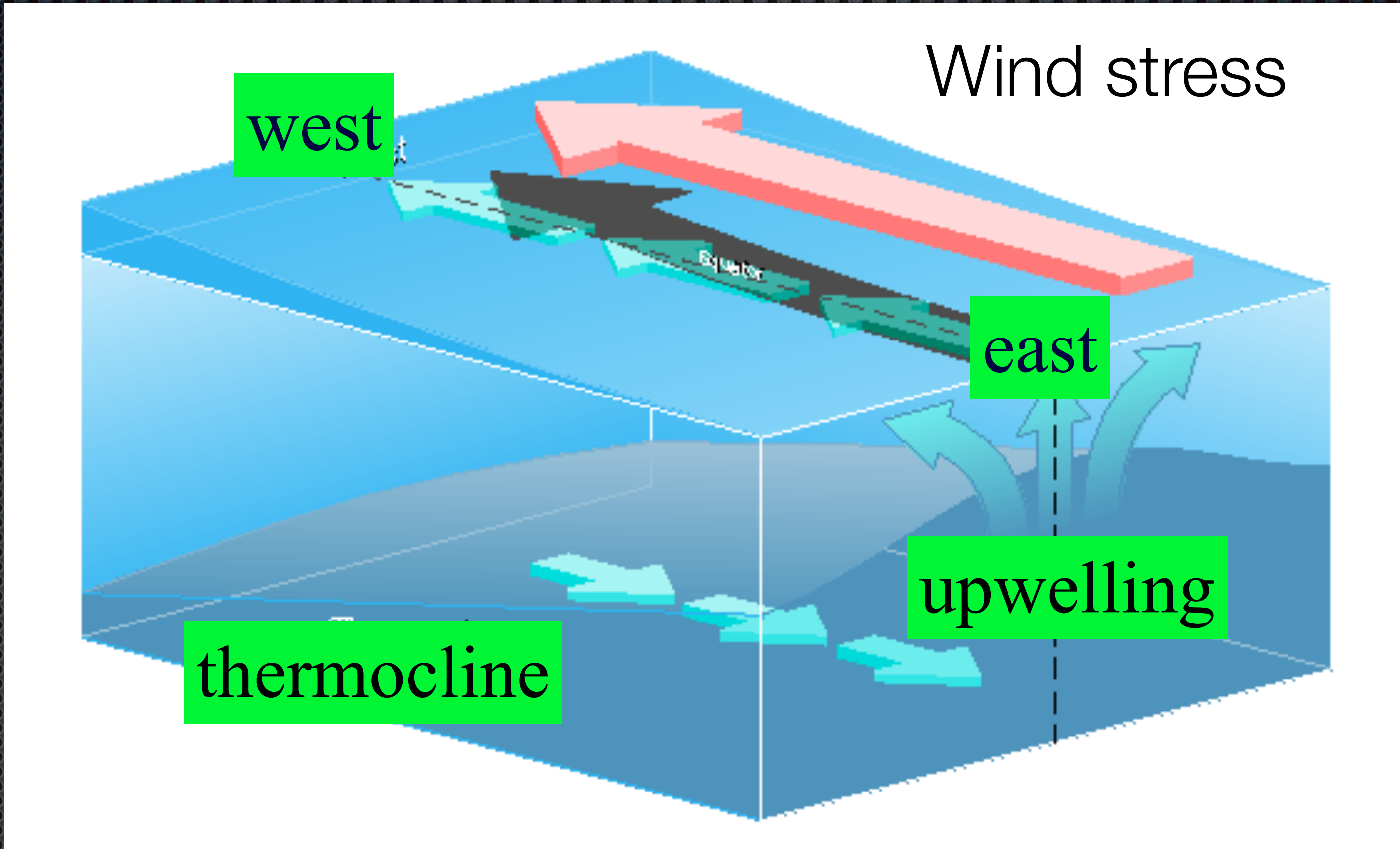
El Nino and the Southern Oscillation are one phenomenon: ENSO



Jacob Bjerknes
1897-1975

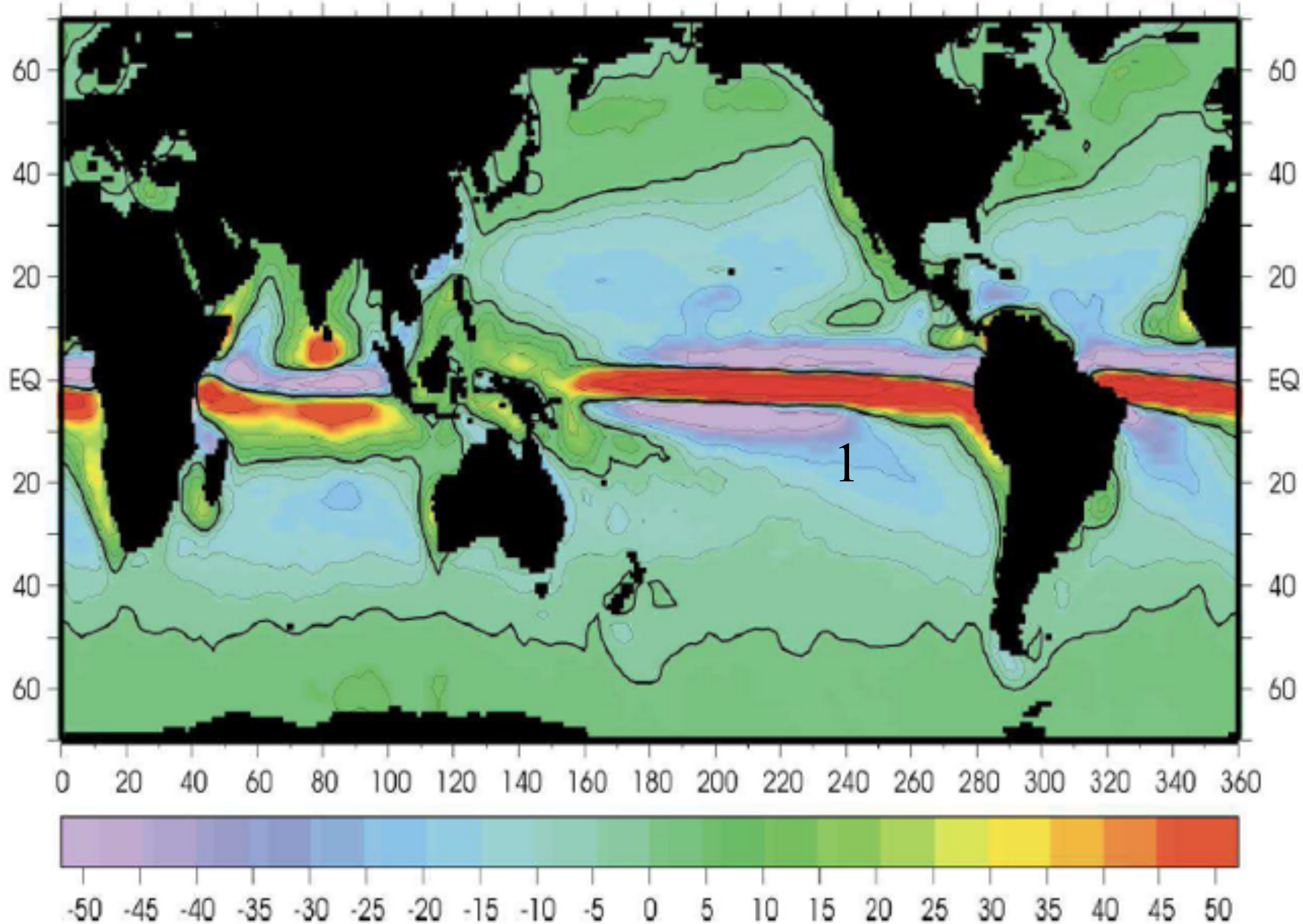
Ingredient 2:
Effect of winds on ocean upwelling & thermocline slope





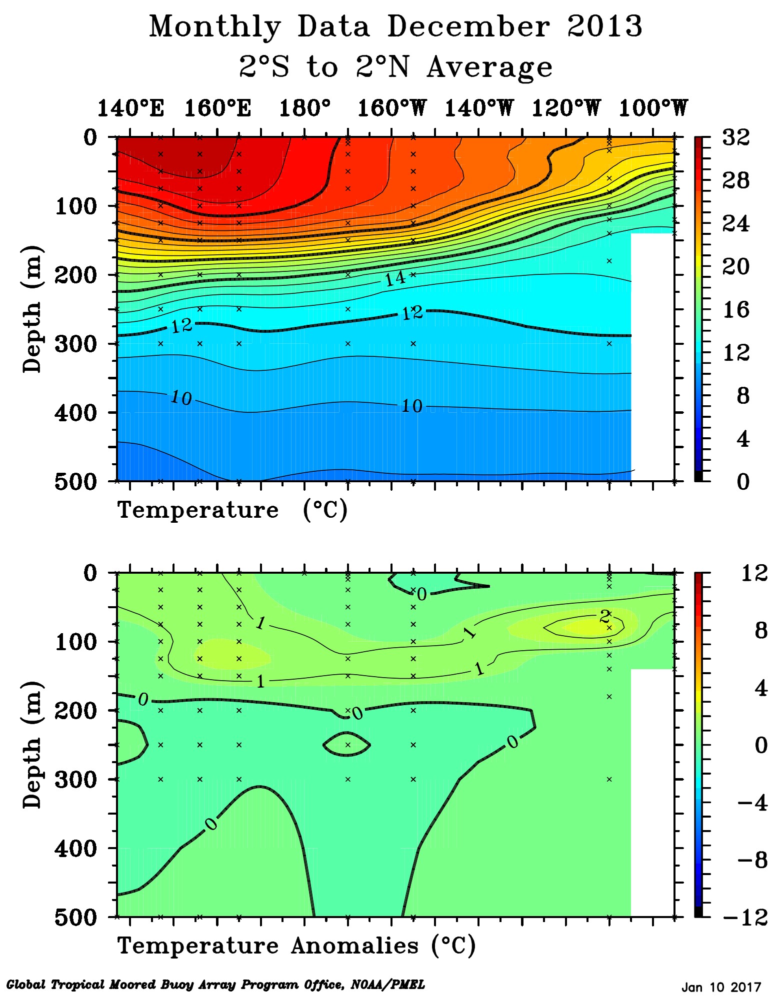
A westerly wind anomaly causes a:
- reduction in upwelling
- smaller thermocline slope

Annual mean upwelling (cm/day)

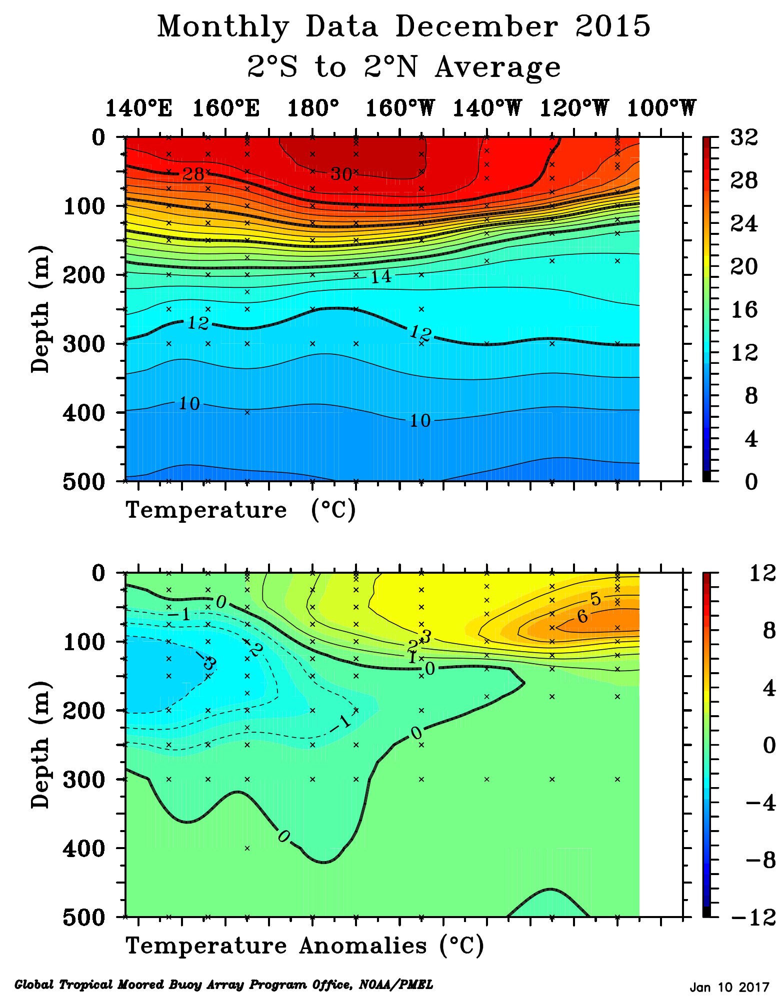


Subsurface ocean observations

`Normal'



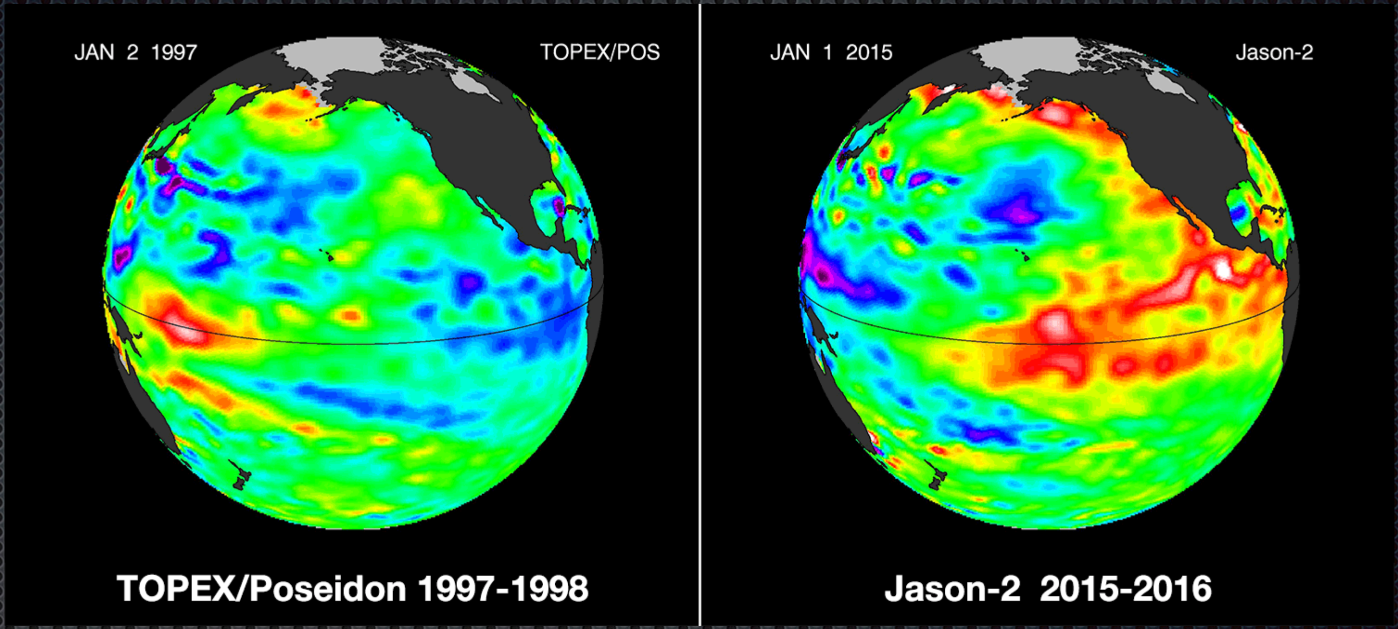
El Nino



Ingredient 3:

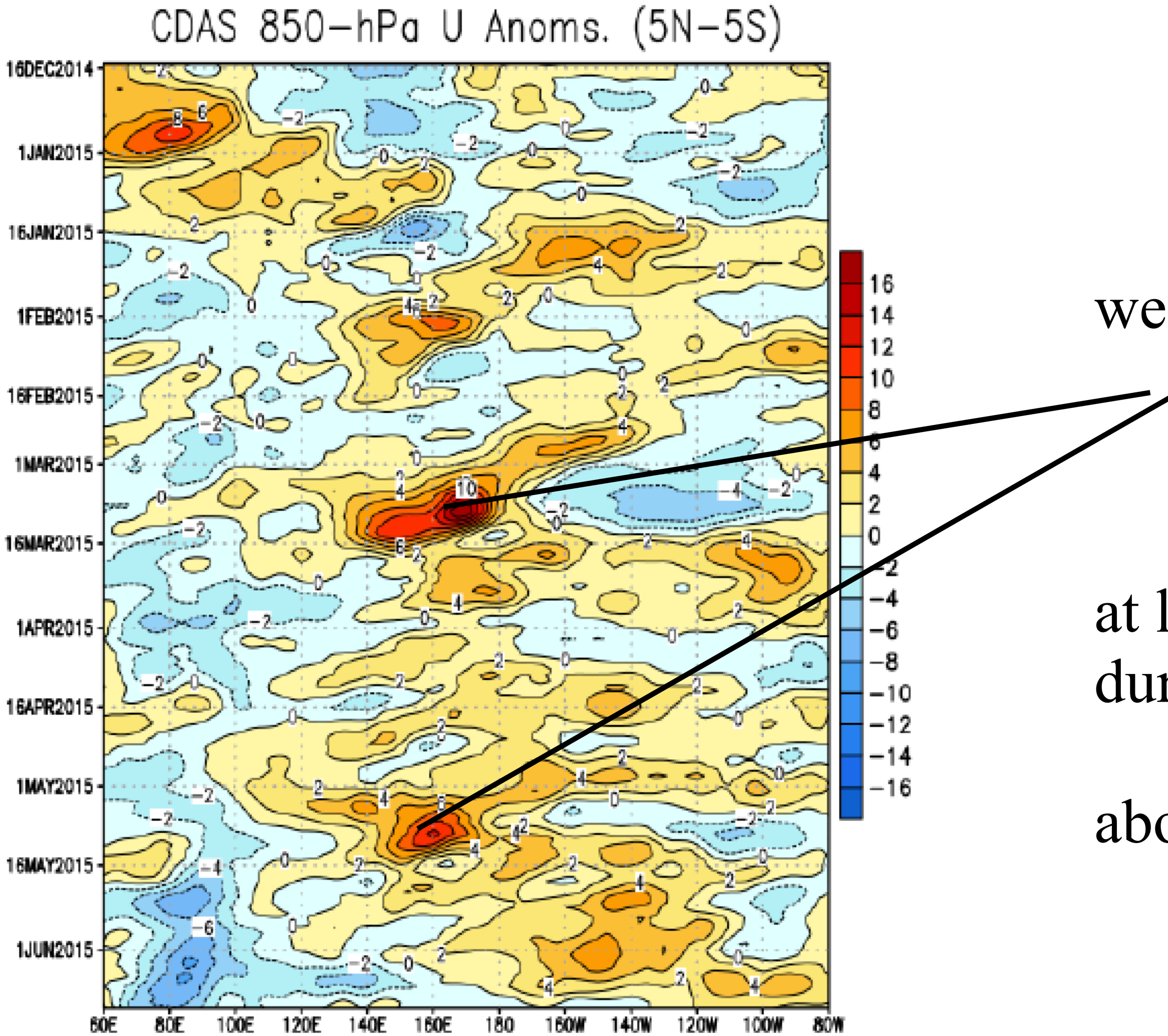
Equatorial ocean wave dynamics

Sea level anomalies



Ingredient 4: `Unresolved' processes

equatorial zonal wind anomaly 850 hPa

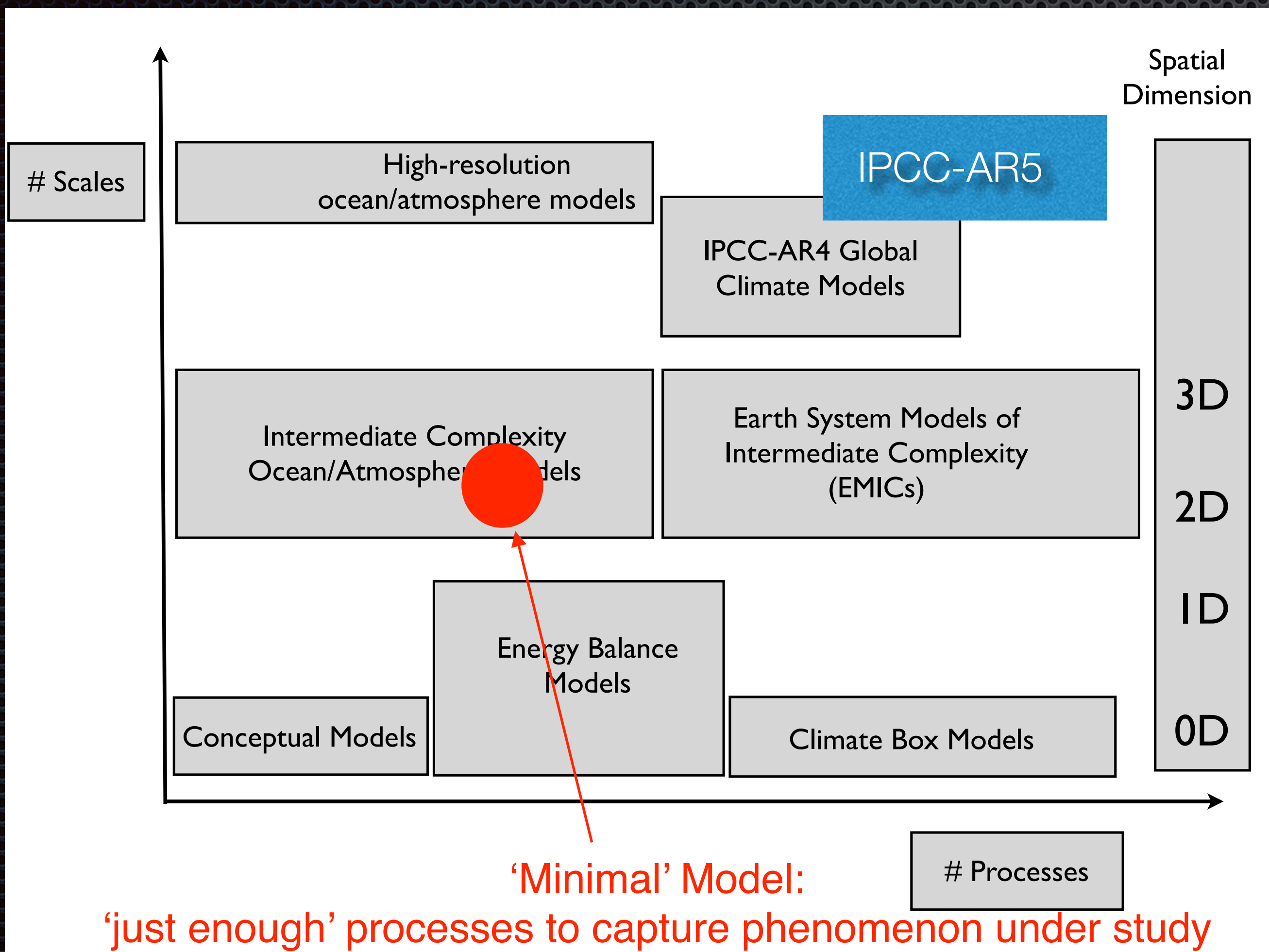


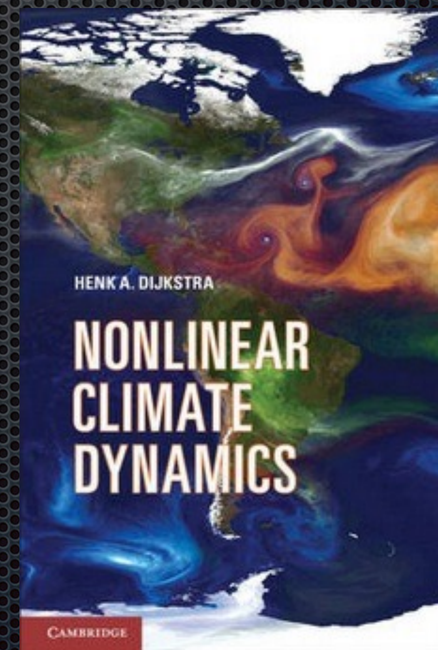
westerly wind burst

at least 7 m/s
duration 5-20 days

about 3 times per year

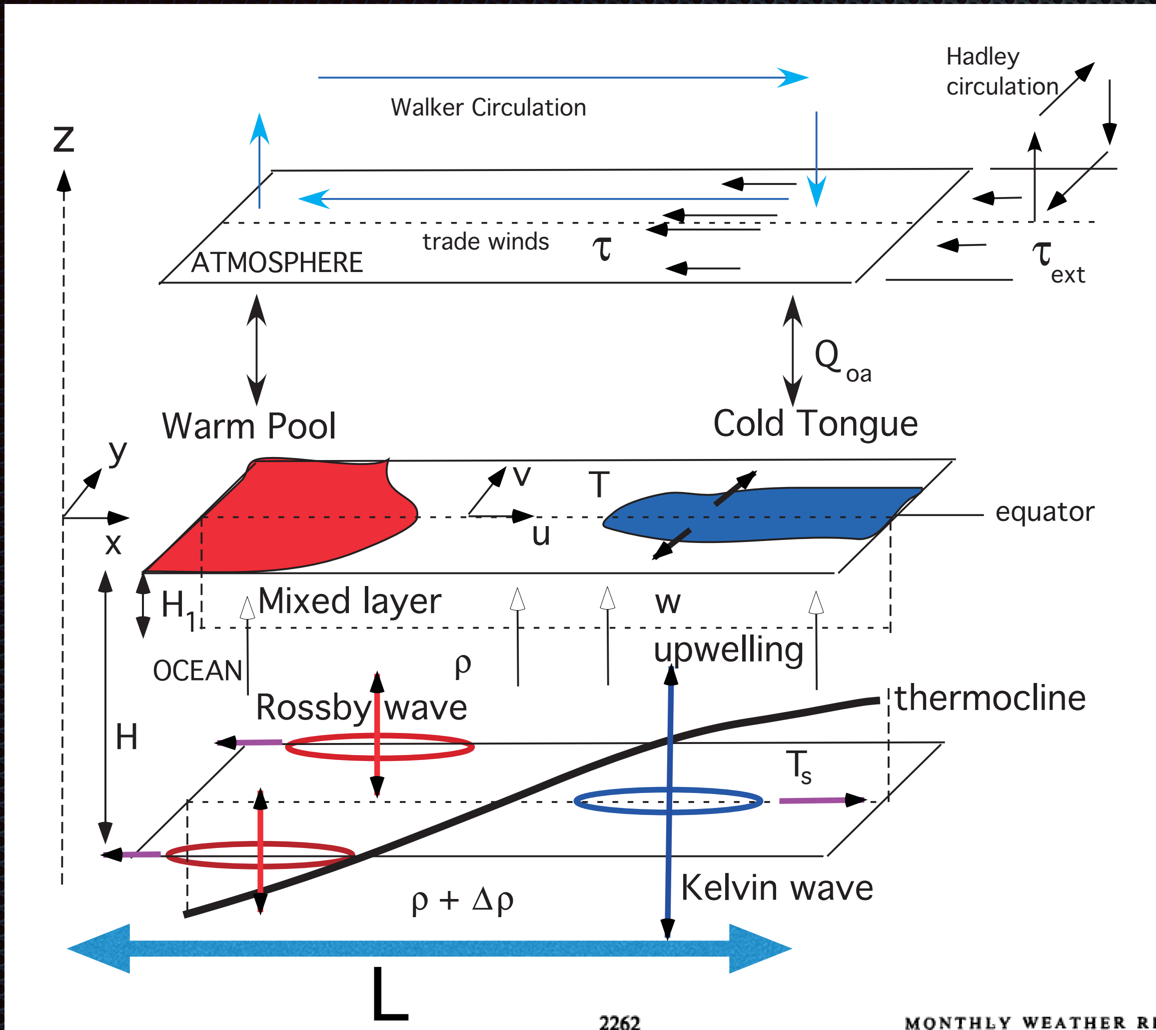
Hierarchy of Models





chapter 6

Zebiak - Cane model (1987)



Steve Zebiak Mark Cane

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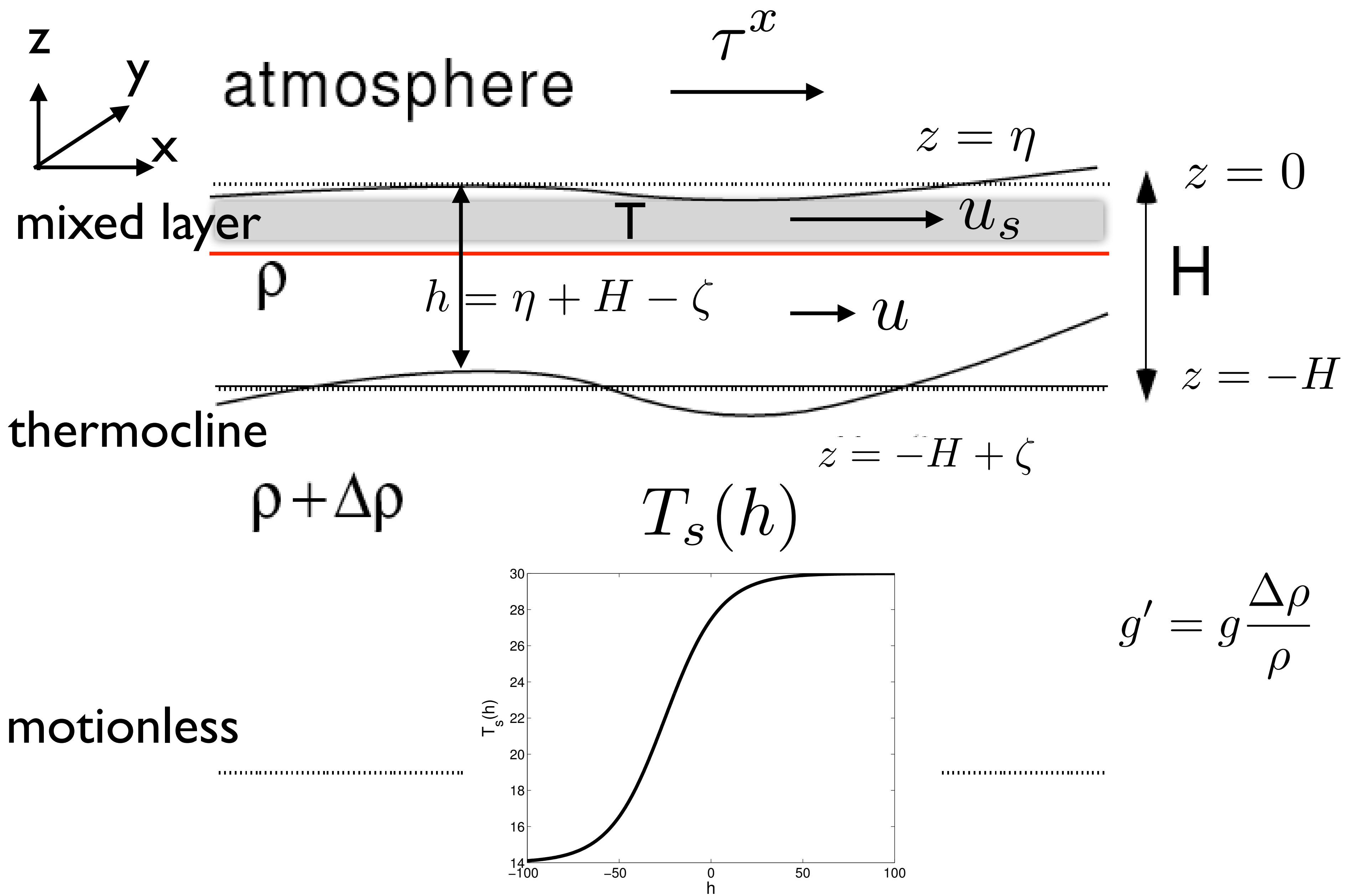
A Model El Niño–Southern Oscillation*

STEPHEN E. ZEBIAK AND MARK A. CANE
Lamont-Doherty Geological Observatory of Columbia University, Palisades, NY 10964

(Manuscript received 1 December 1986, in final form 23 March 1987)

Ocean Component of the ZC model



Equations: SST

SST dynamics

$$\frac{\partial T}{\partial t} + u_1 \frac{\partial T}{\partial x} + v_1 \frac{\partial T}{\partial y} + w_1 \mathcal{H}(w_1) \frac{T - T_s(h)}{H}$$
$$+ a_T (T - T_0) - K_H \nabla^2 T = 0$$

$$u_1 = u + u_s \; ; \; v_1 = v + v_s \; ; \; w_1 = w + w_s$$

boundary conditions

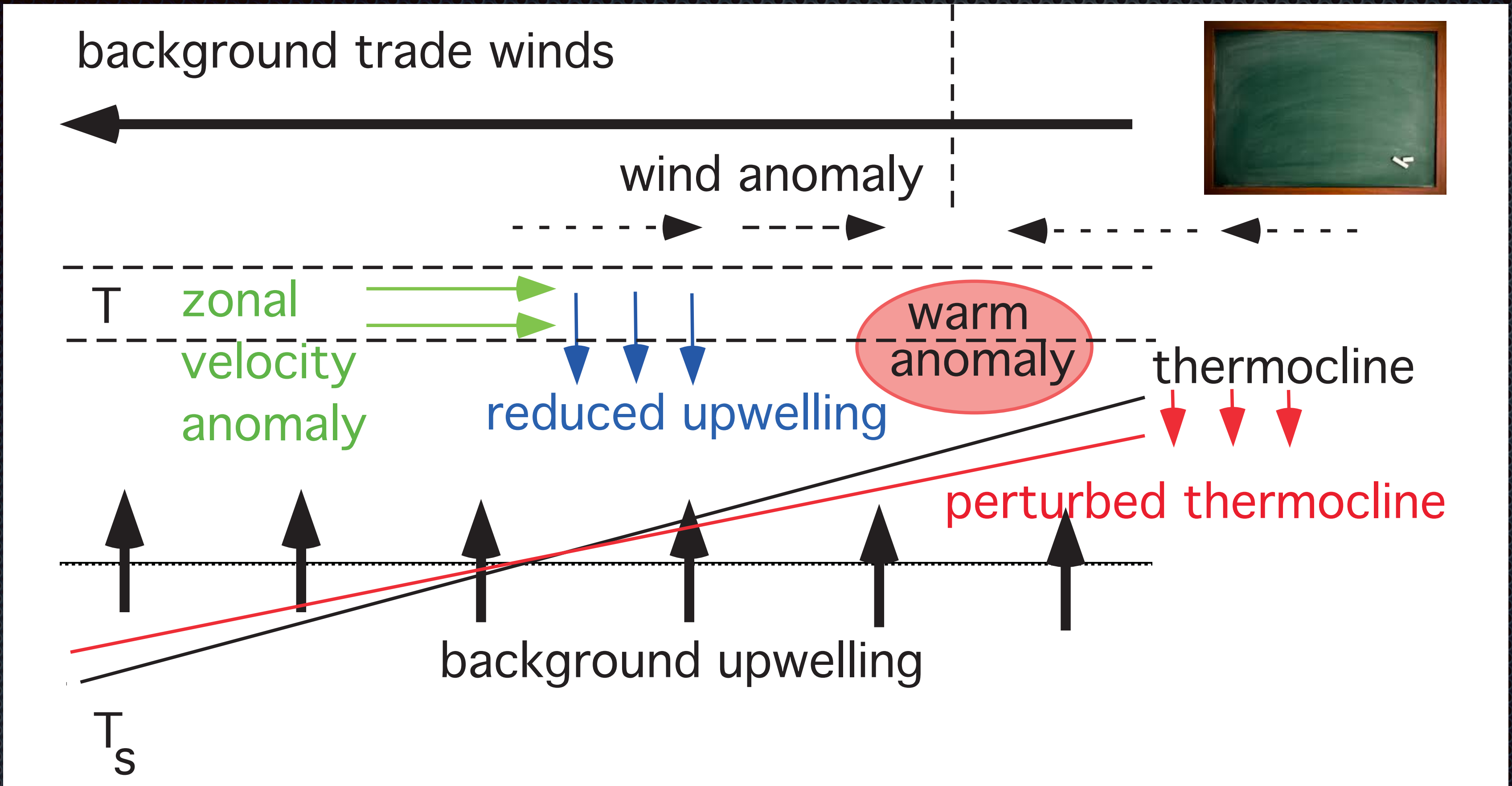
$$x = 0, L : \frac{\partial T}{\partial x} = 0$$
$$y \to \pm\infty : T \text{ bounded}$$

u_s, v_s, w_s follow directly from τ^x

A: atmospheric operator

$$\tau^x = \tau^x_{ext} + \gamma A(T - T_0)$$

Coupled (Bjerknes') feedbacks



Thermocline feedback

Upwelling feedback

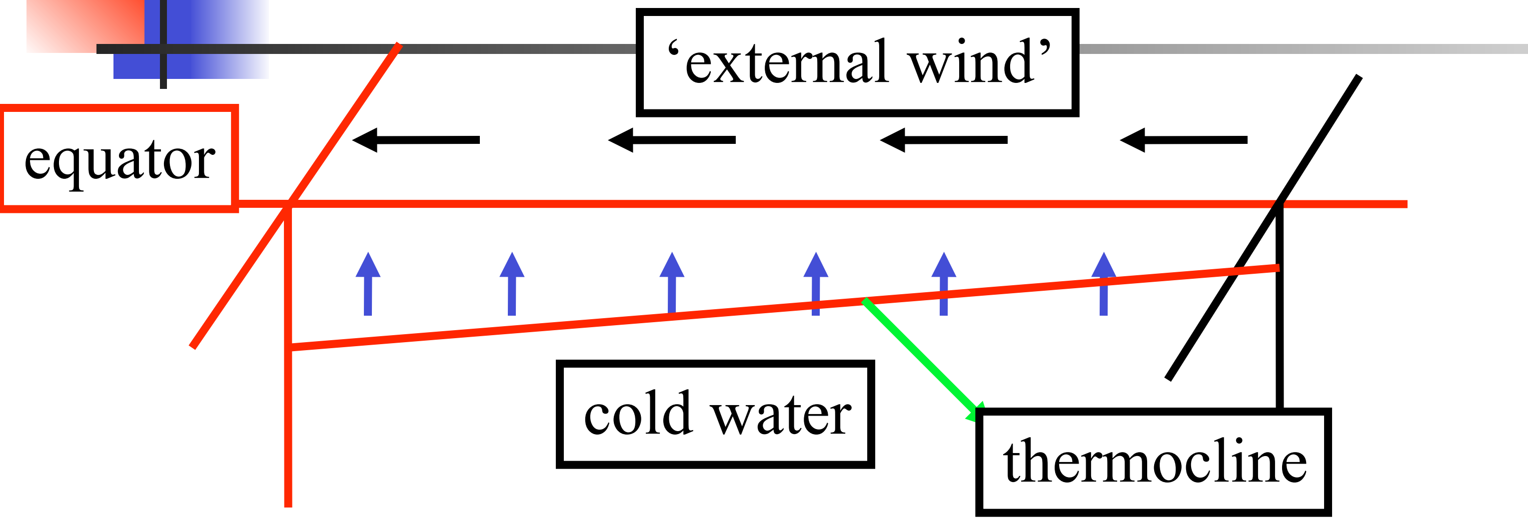
Zonal advection feedback

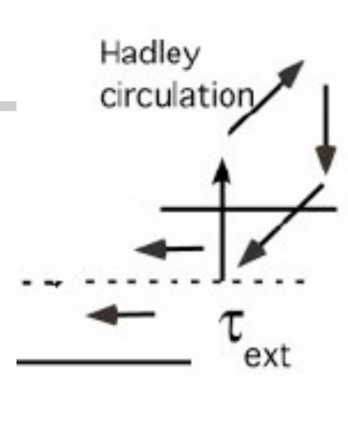
Feedbacks measured by coupling strength:
ocean heat flux per SST anomaly

$$\mu \sim L^2$$

Question time

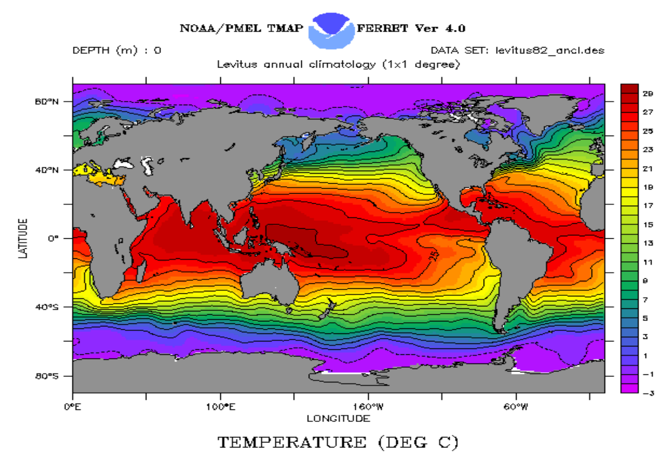
Annual mean state



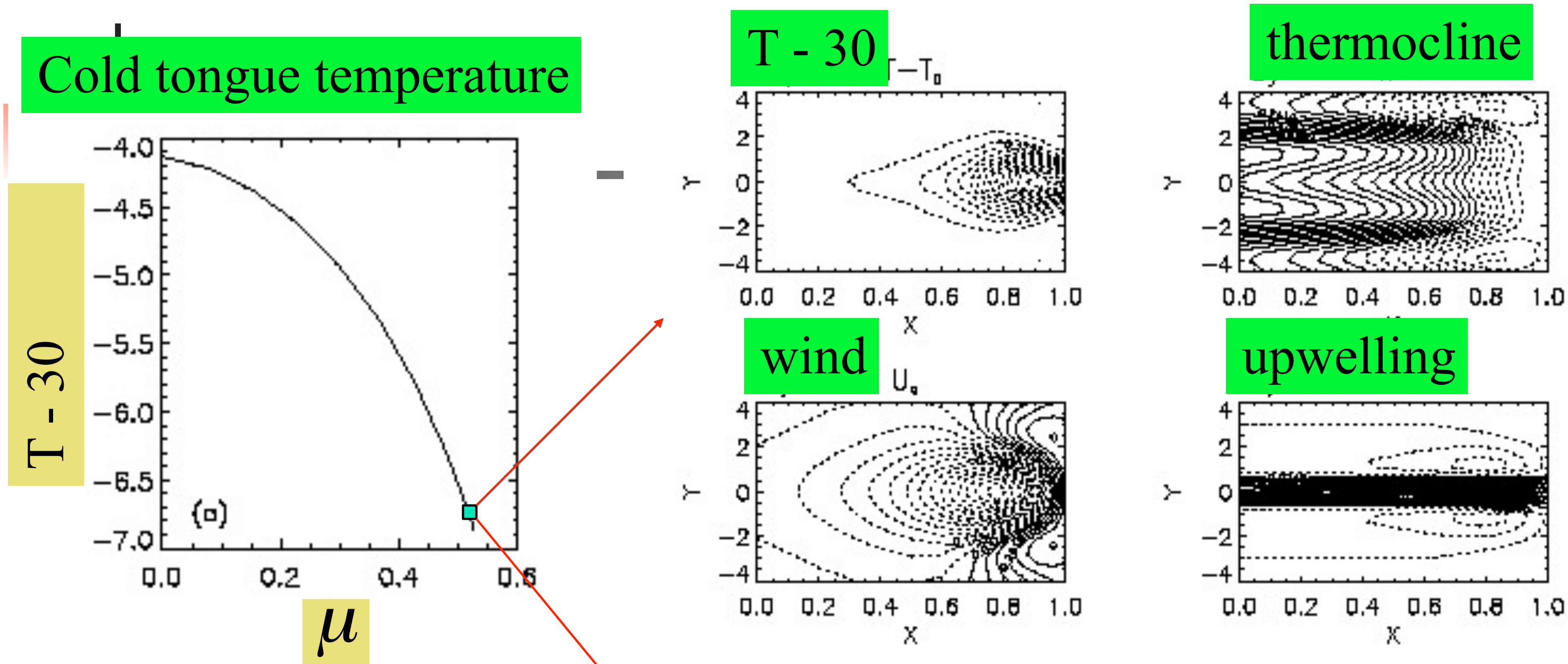


- External wind induces weak upwelling and slight slope in the thermocline

- Coupled feedbacks generate the cold tongue/warm pool structure



Stability of the annual mean state



coupling strength:

Hopf bifurcation: ENSO mode

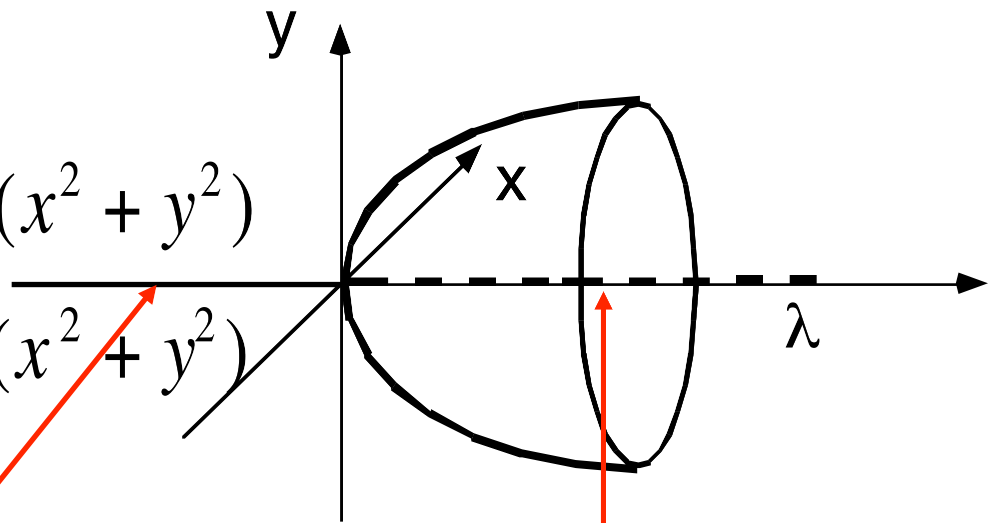
period ~ 4 years

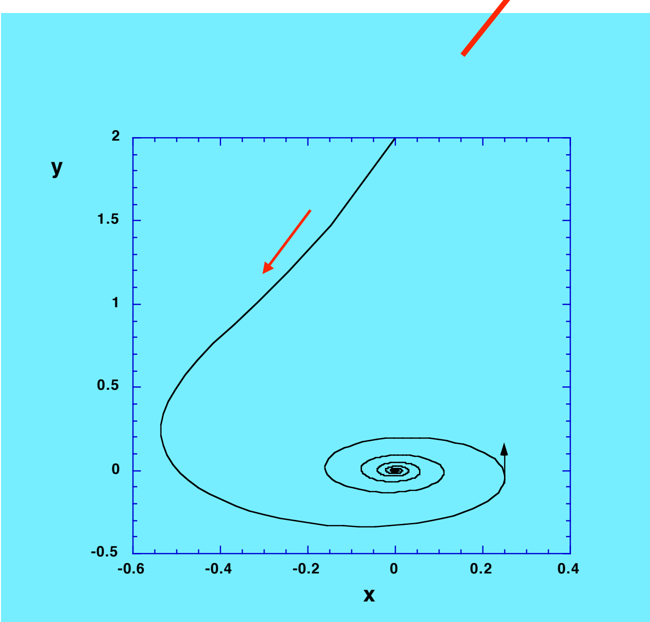
Annual mean state becomes unstable to oscillatory perturbations

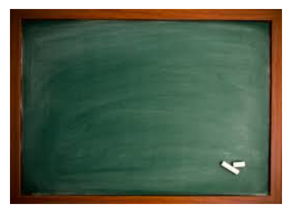
Hopf bifurcation

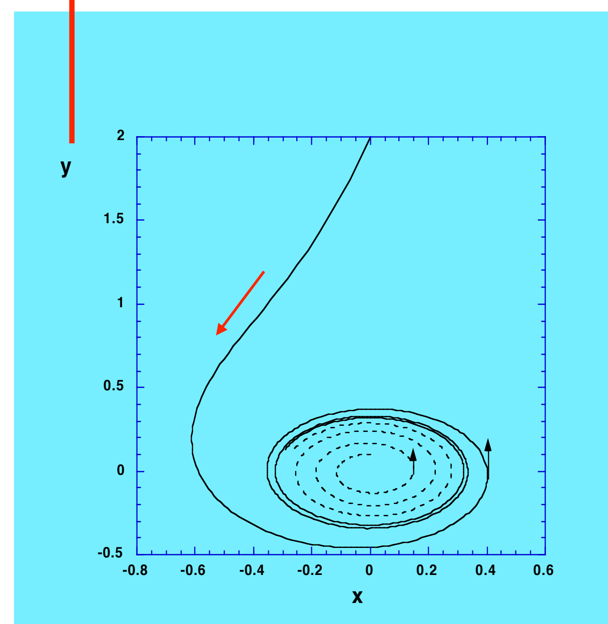
$$\dot{x} = \lambda x - \omega y - x(x^2 + y^2)$$

$$\dot{y} = \lambda y + \omega x - y(x^2 + y^2)$$



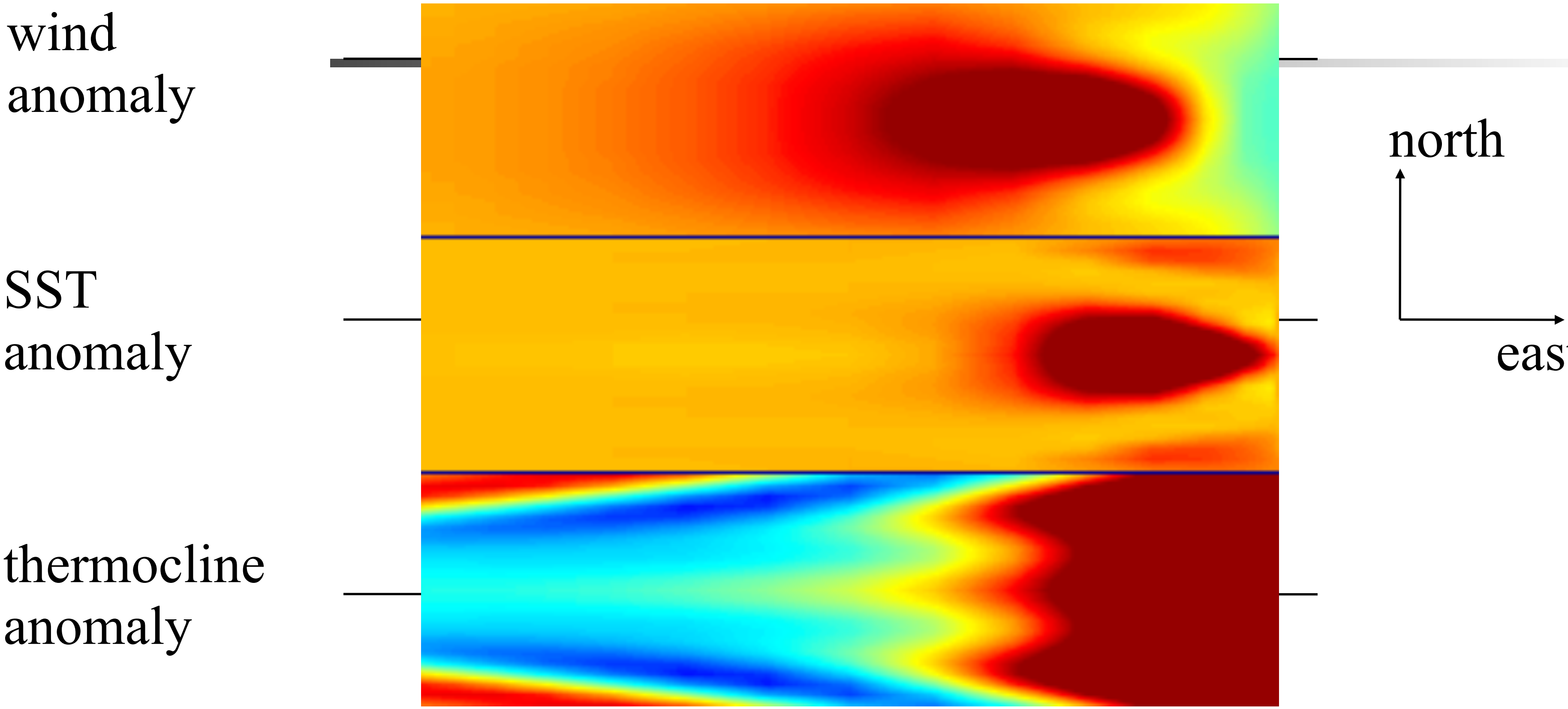






Ex: Hopf bifurcation

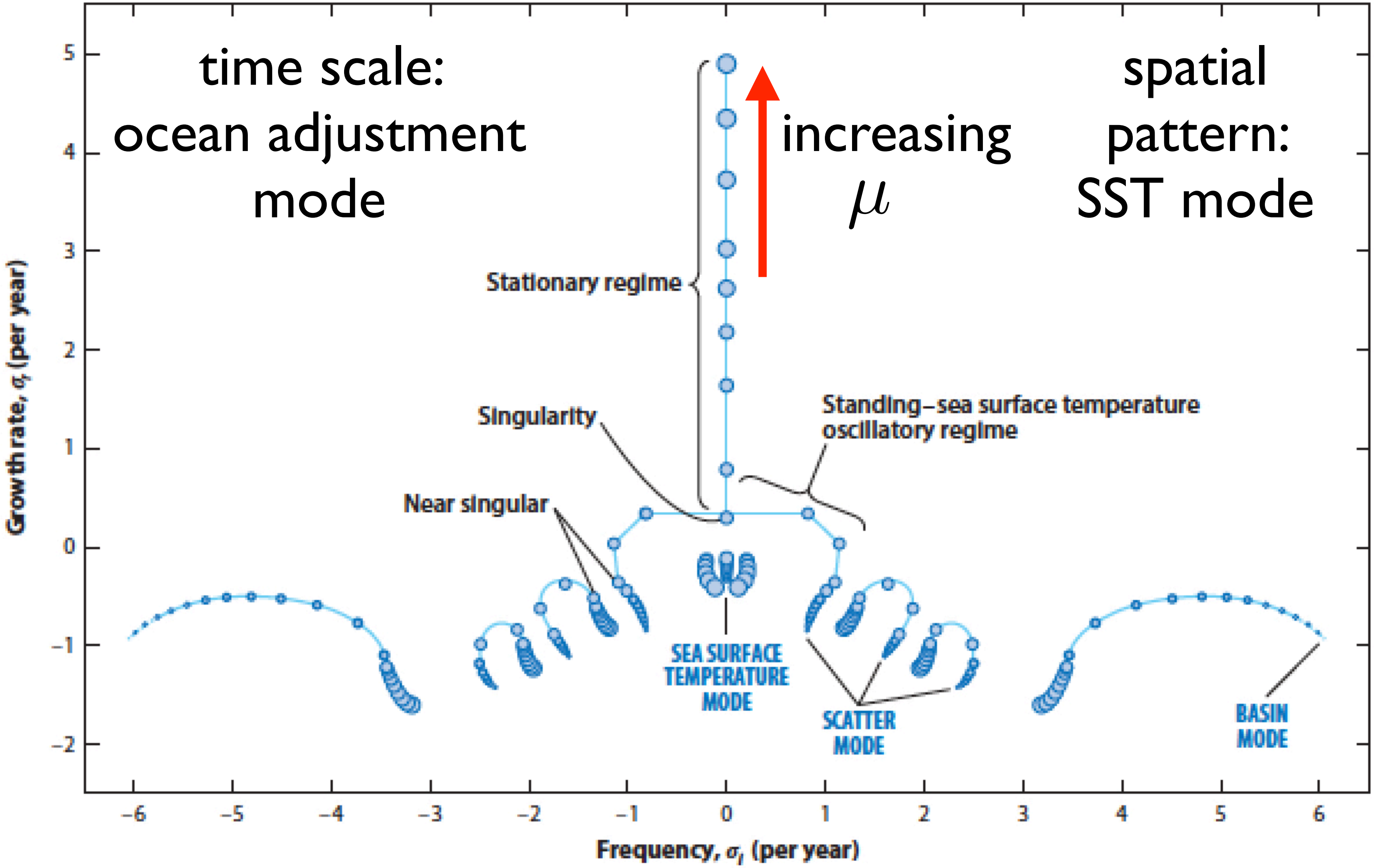
The ENSO mode



Spatial patterns: background state

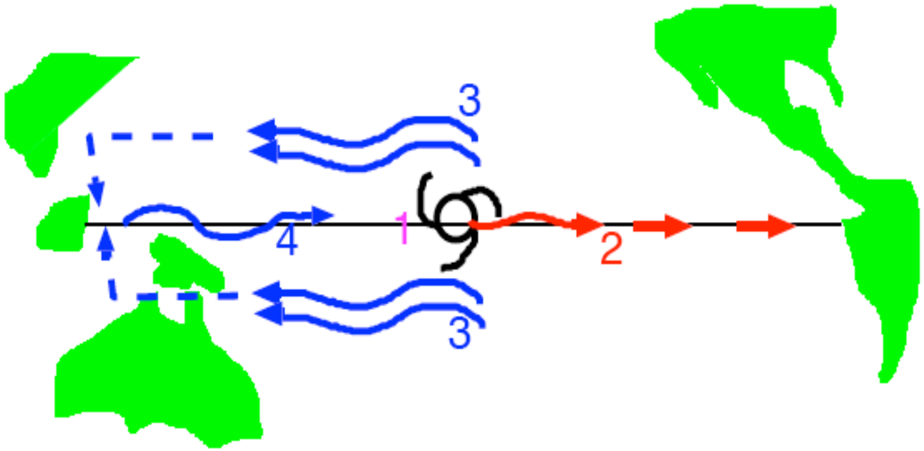
Period: ocean wave dynamics + SST adjustment

Spectral origin of the Hopf bifurcation



Jin & Neelin, JAS, 1993

Mechanism: wave oscillator

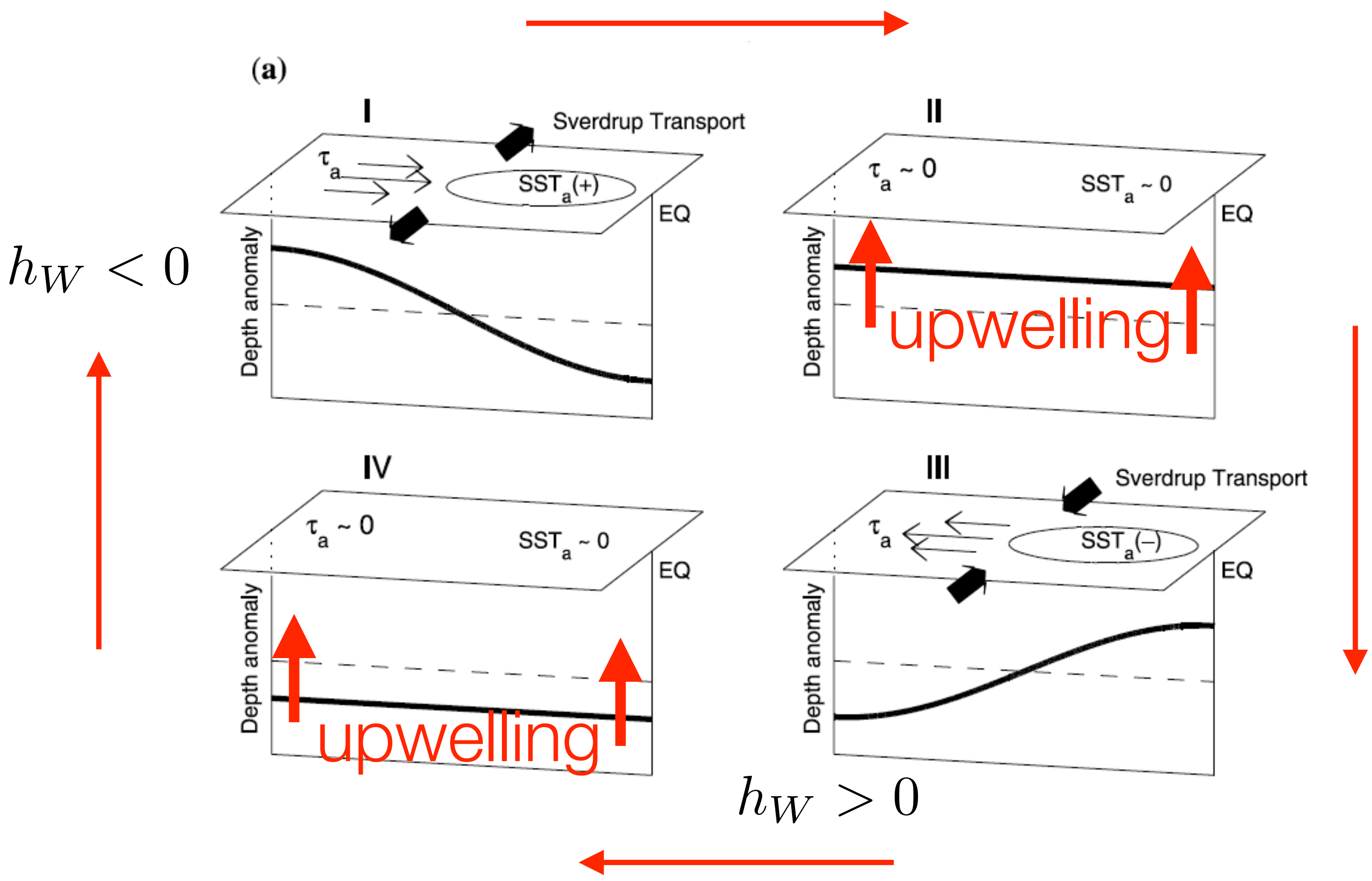


$$\frac{dT(t)}{dt} = \hat{a}h_{eq}(x_c, t - \frac{1}{2}\tau_K) + \hat{b}h_{off-eq}(x_c, t - [\frac{1}{2}\tau_R + \tau_K]) - cT(t)^3$$

~ 1 months

~5 months

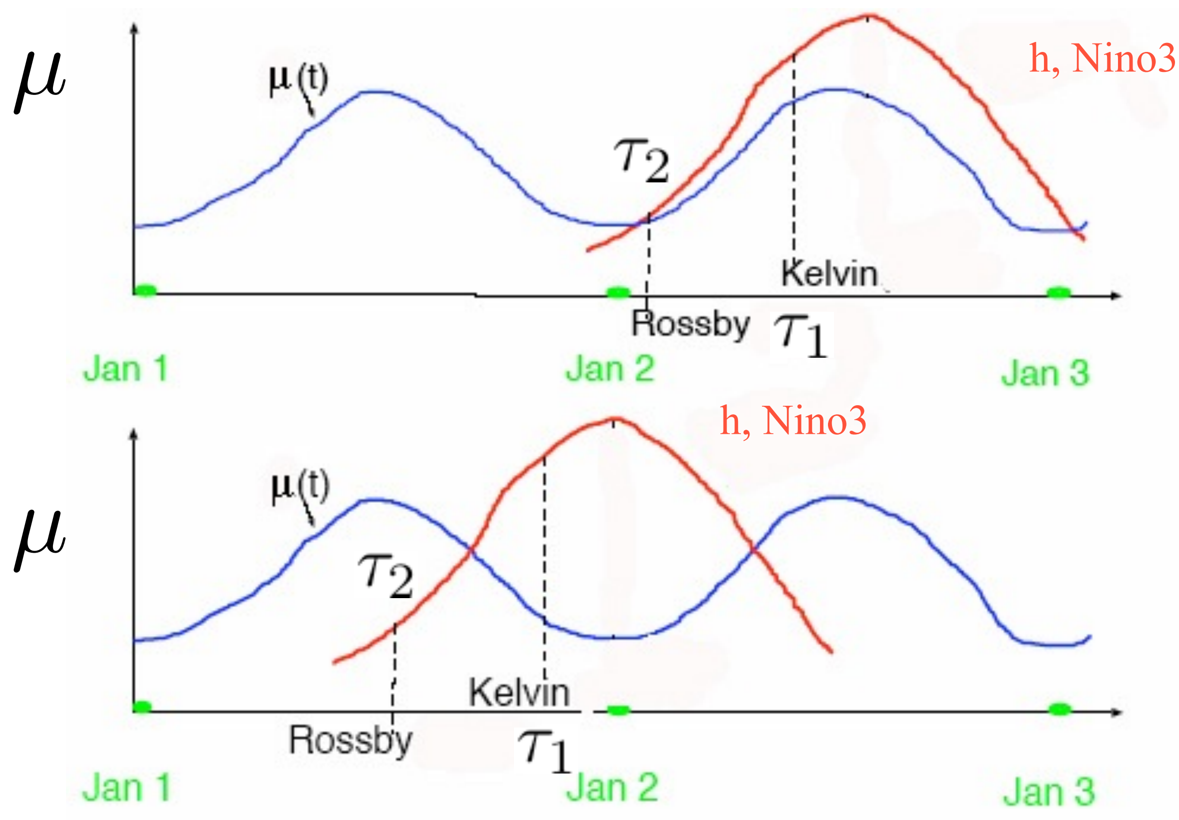
Mechanism: recharge oscillator



Phase locking to the seasonal cycle

Warm Kelvin wave | Cold Rossby wave

$$\frac{dh}{dt} = bF(\mu(t-\tau_1)h(t-\tau_1)) - cF(\mu(t-\tau_2)h(t-\tau_2)) - dh(t)$$



No peaking possible in Summer

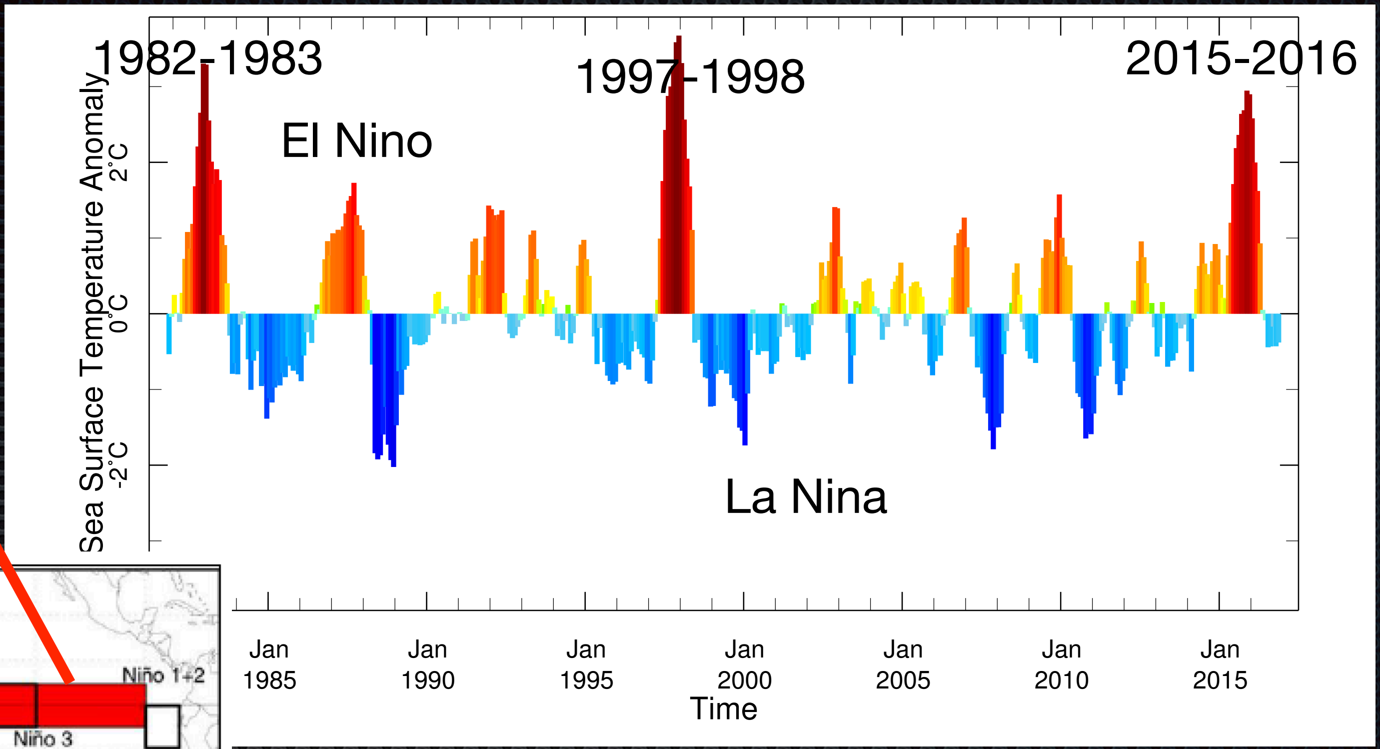
Peaking possible in Winter

Galanti & Tziperman (2001)

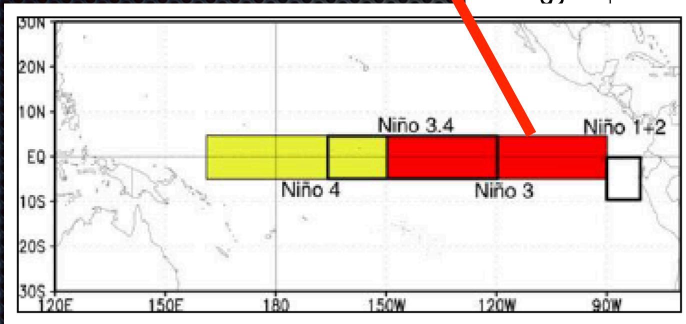
Linear mechanism: seasonal variation in coupling strength

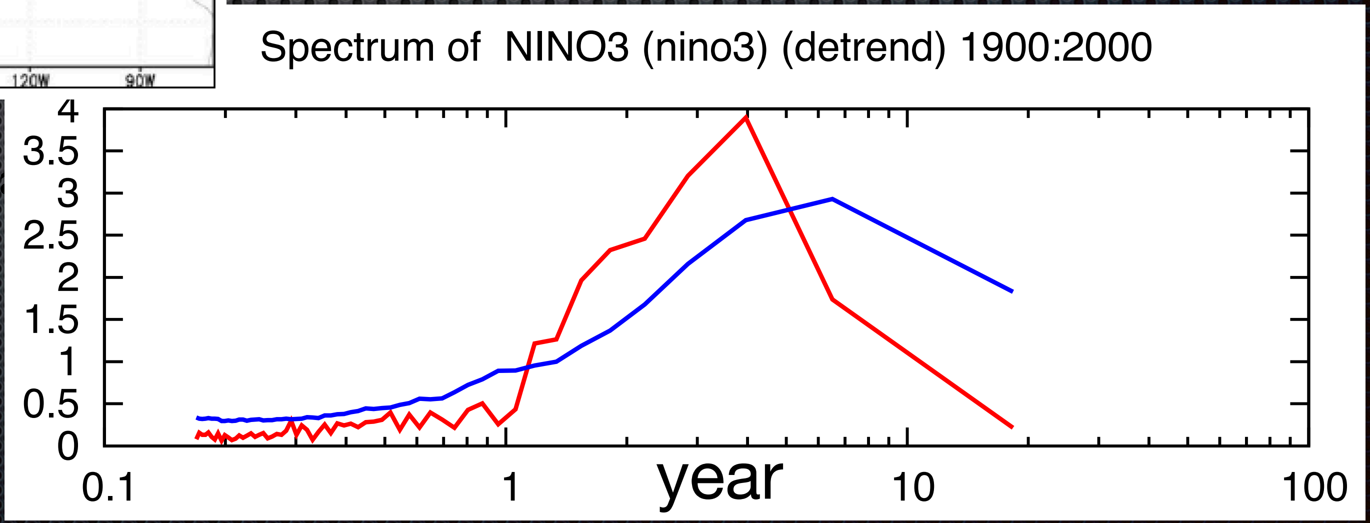
Question time

Irregularity of ENSO



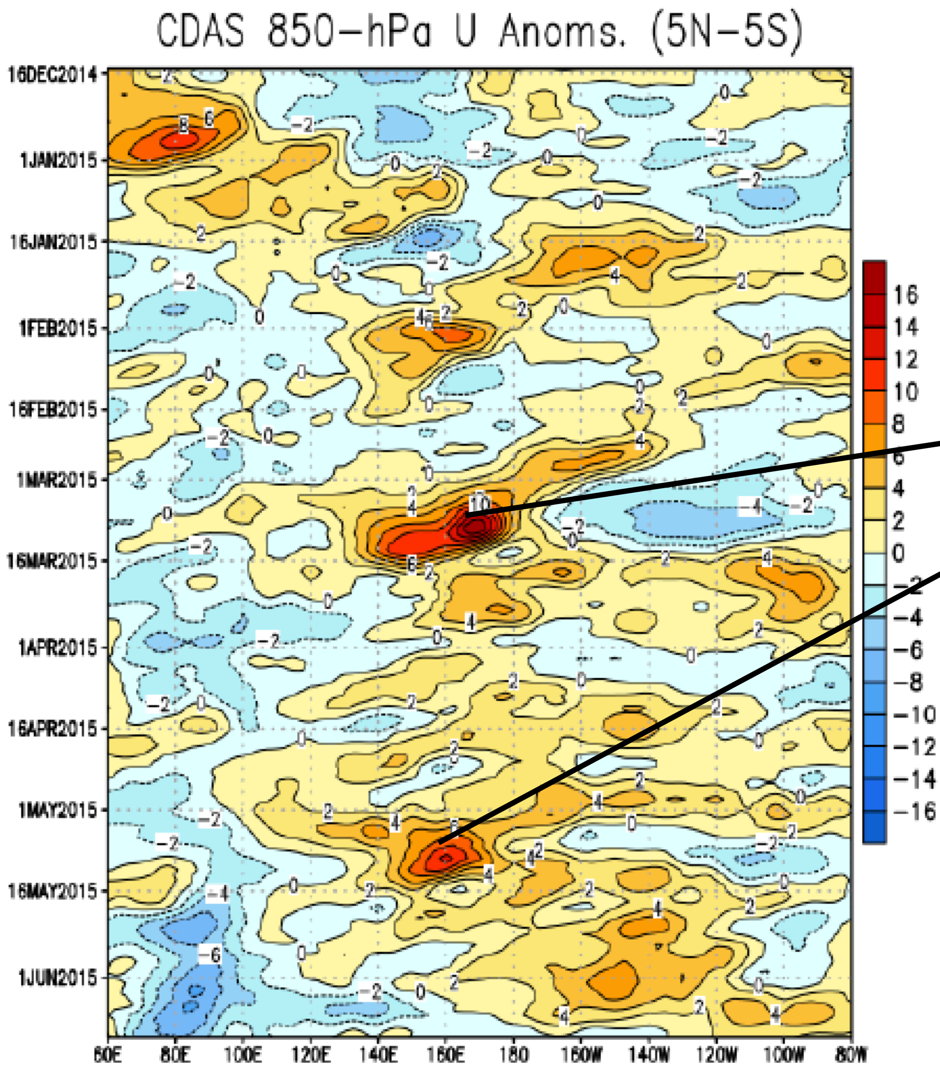
NINO3





`Unresolved' processes

time



westerly wind burst

> 7 m/s
duration 5-20 days

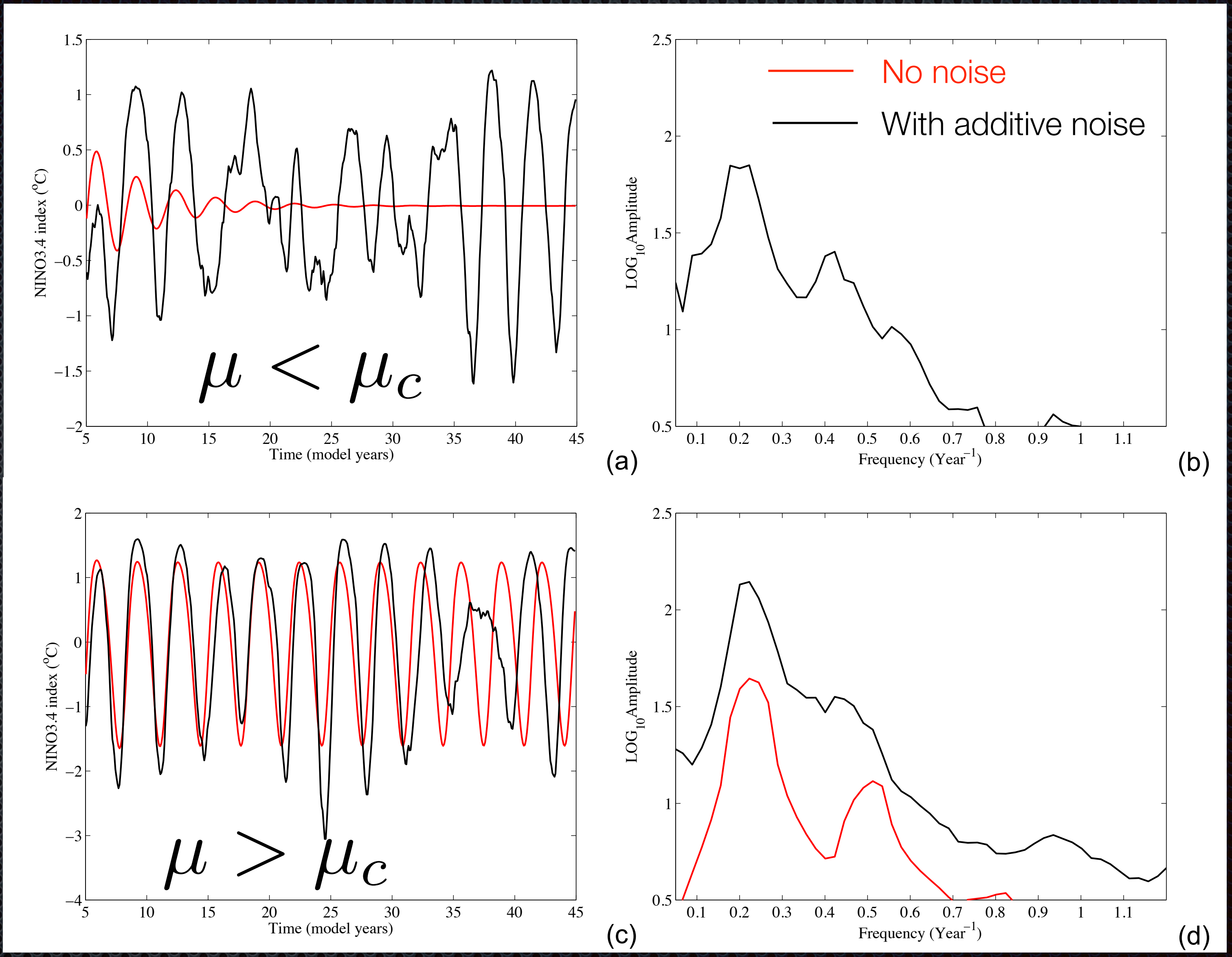
about 3-4 per year

Harrison &Vecchi, (1997)

Results: Cane-Zebiak

NINO3.4

NINO3.4



Feng & Dijkstra, Chaos, (2017)

Stochastic Hopf bifurcation

$$dX = (\lambda X - \omega Y - X(X^2 + Y^2))dt + \sigma dW_1$$

$$dY = (\lambda Y + \omega X - Y(X^2 + Y^2))dt + \sigma dW_2$$

$$r = \sqrt{x^2 + y^2} \qquad \theta = \arctan \frac{x}{y}$$

$$\frac{\partial r}{\partial x} = \frac{x}{r} \qquad \frac{\partial r}{\partial y} = \frac{y}{r} \qquad \frac{\partial \theta}{\partial x} = -\frac{y}{r^2} \qquad \frac{\partial \theta}{\partial y} = \frac{x}{r^2}$$

$$\frac{\partial^2 r}{\partial x^2} = \frac{y^2}{r^3} \qquad \frac{\partial^2 r}{\partial y^2} = \frac{x^2}{r^3} \qquad \frac{\partial^2 \theta}{\partial x^2} = \frac{2xy}{r^4} \qquad \frac{\partial^2 \theta}{\partial y^2} = -\frac{2xy}{r^4}$$

$$dR = (\lambda R - R^3 + \frac{\sigma^2}{2R})dt + \sigma(\cos\Theta dW_1 + \sin\Theta dW_2)$$

$$d\Theta = \omega dt + \frac{\sigma}{R}(-\sin\Theta dW_1 + \cos\Theta dW_2)$$

Fokker-Planck equation

$$d\mathbf{X}_t = \mathbf{f}(\mathbf{X_t}, t)dt + \mathbf{g}(\mathbf{X_t}, t)d\mathbf{W}_t$$

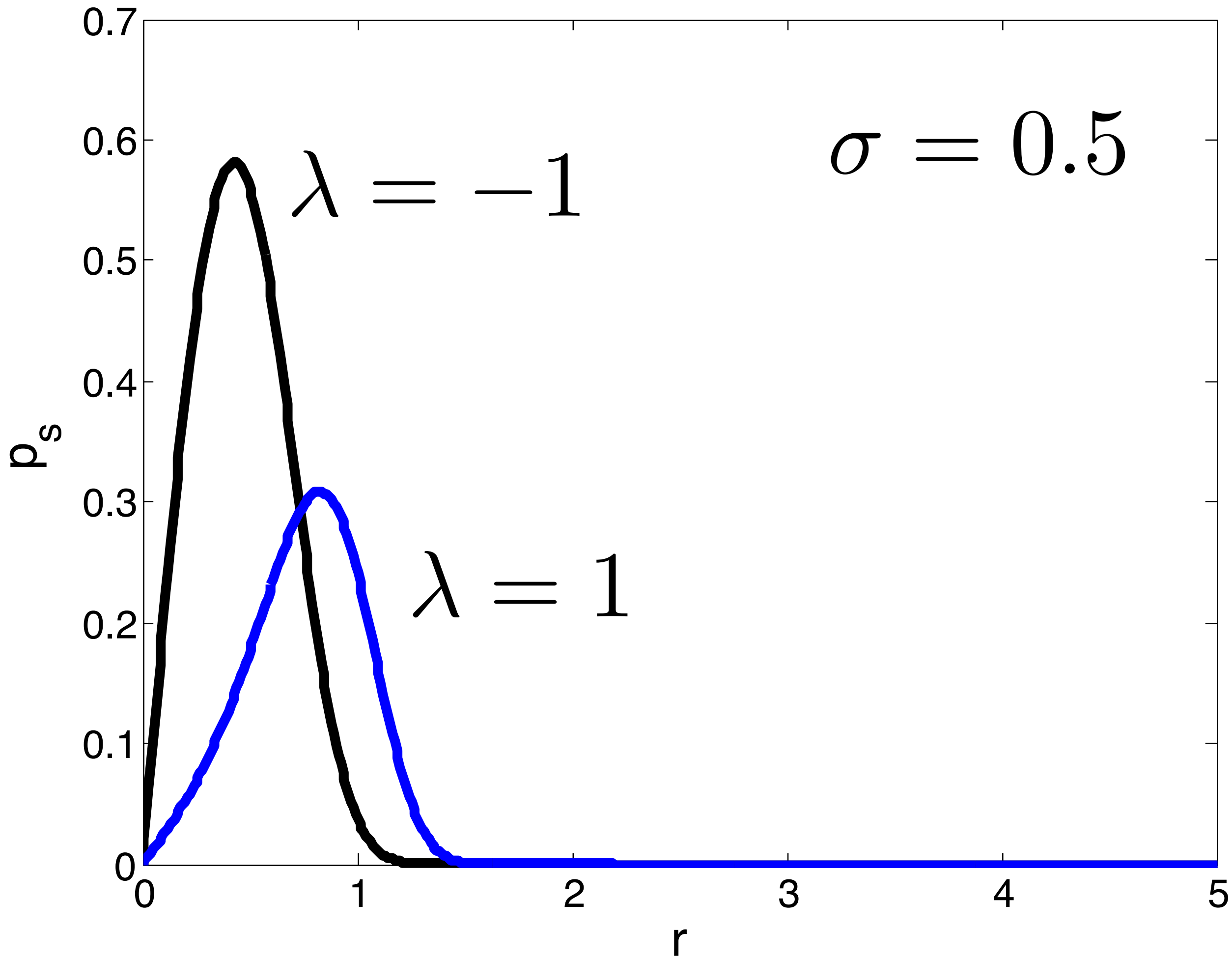
$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x_i}(f_i p) + \frac{1}{2}\frac{\partial^2}{\partial x_i \partial x_j}(D_{ij} p)$$

$$D_{ij} = g_{ik} g_{kj}.$$

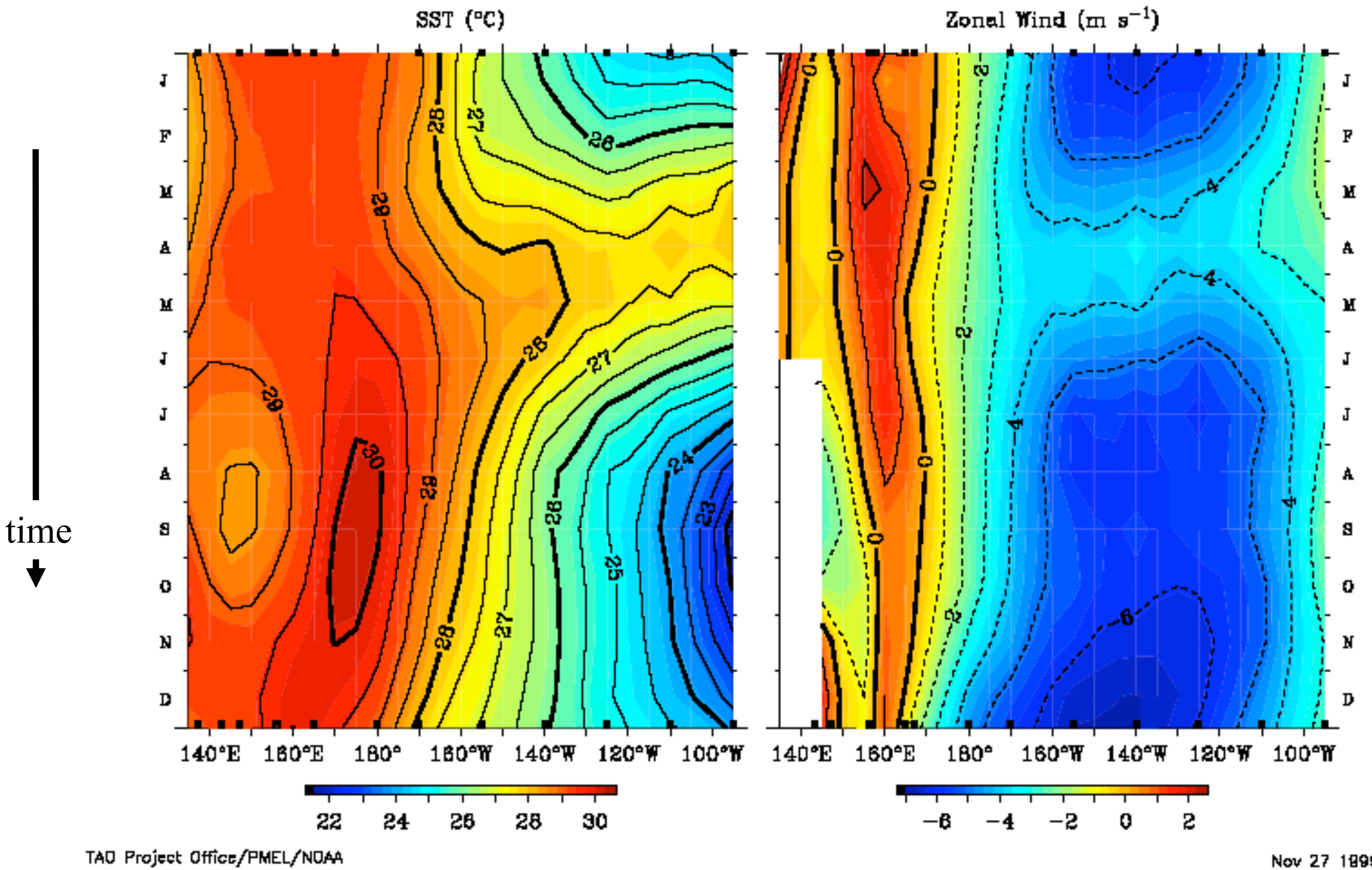
$$\frac{\partial p}{\partial t} = -\frac{\partial[(\lambda r - r^3 + \frac{\sigma^2}{2r})p]}{\partial r} - \frac{\partial(\omega p)}{\partial \theta} + \frac{\sigma^2}{2}\left(\frac{\partial^2 p}{\partial^2 r} + \frac{1}{r^2}\frac{\partial^2 p}{\partial^2 \theta}\right)$$

Stationary distribution

$$p_s(r) = Nr \ \exp\left(\frac{\lambda r^2}{\sigma^2} - \frac{r^4}{2\sigma^2}\right)$$



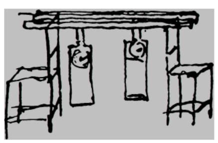
The mean seasonal cycle



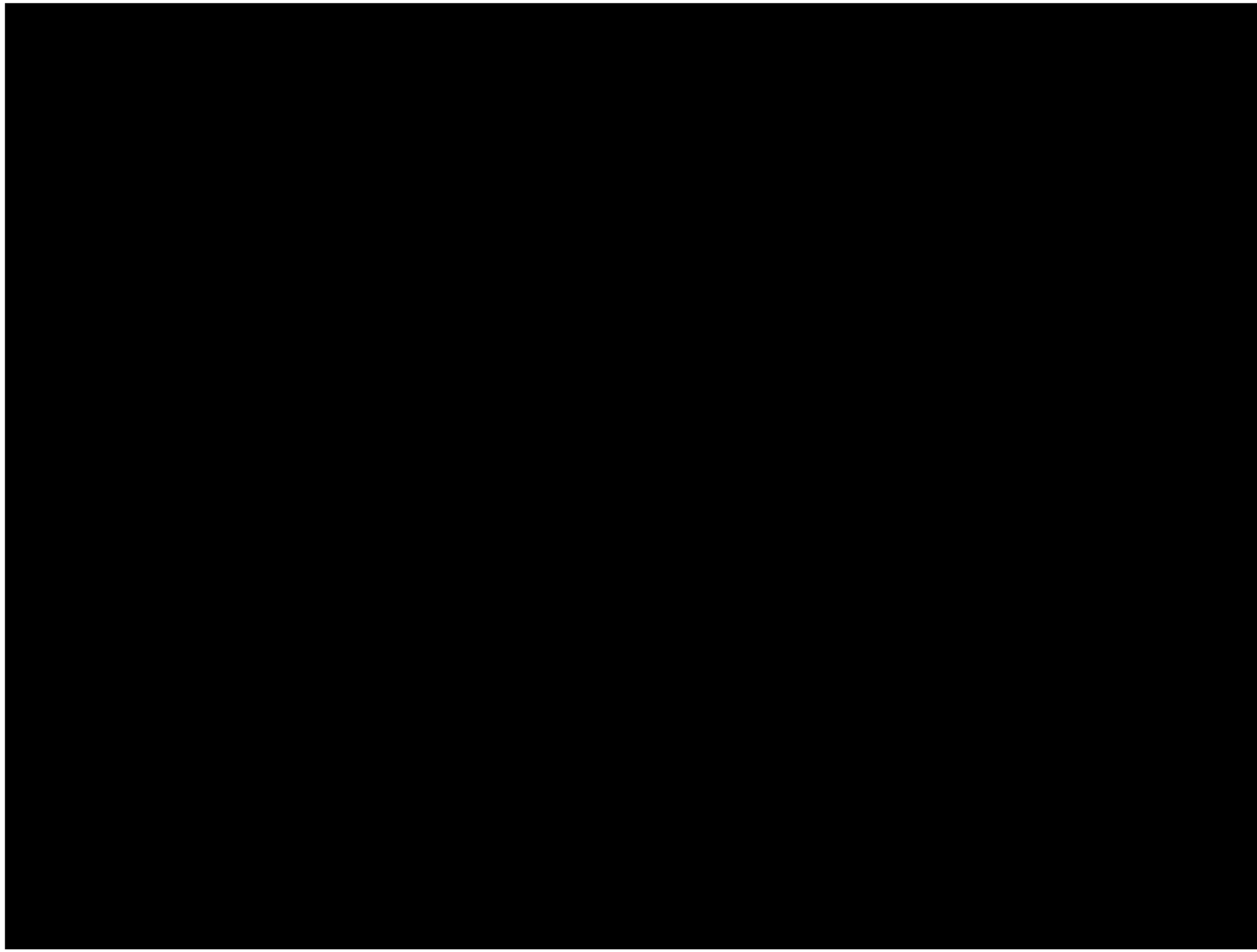
SST

Zonal wind

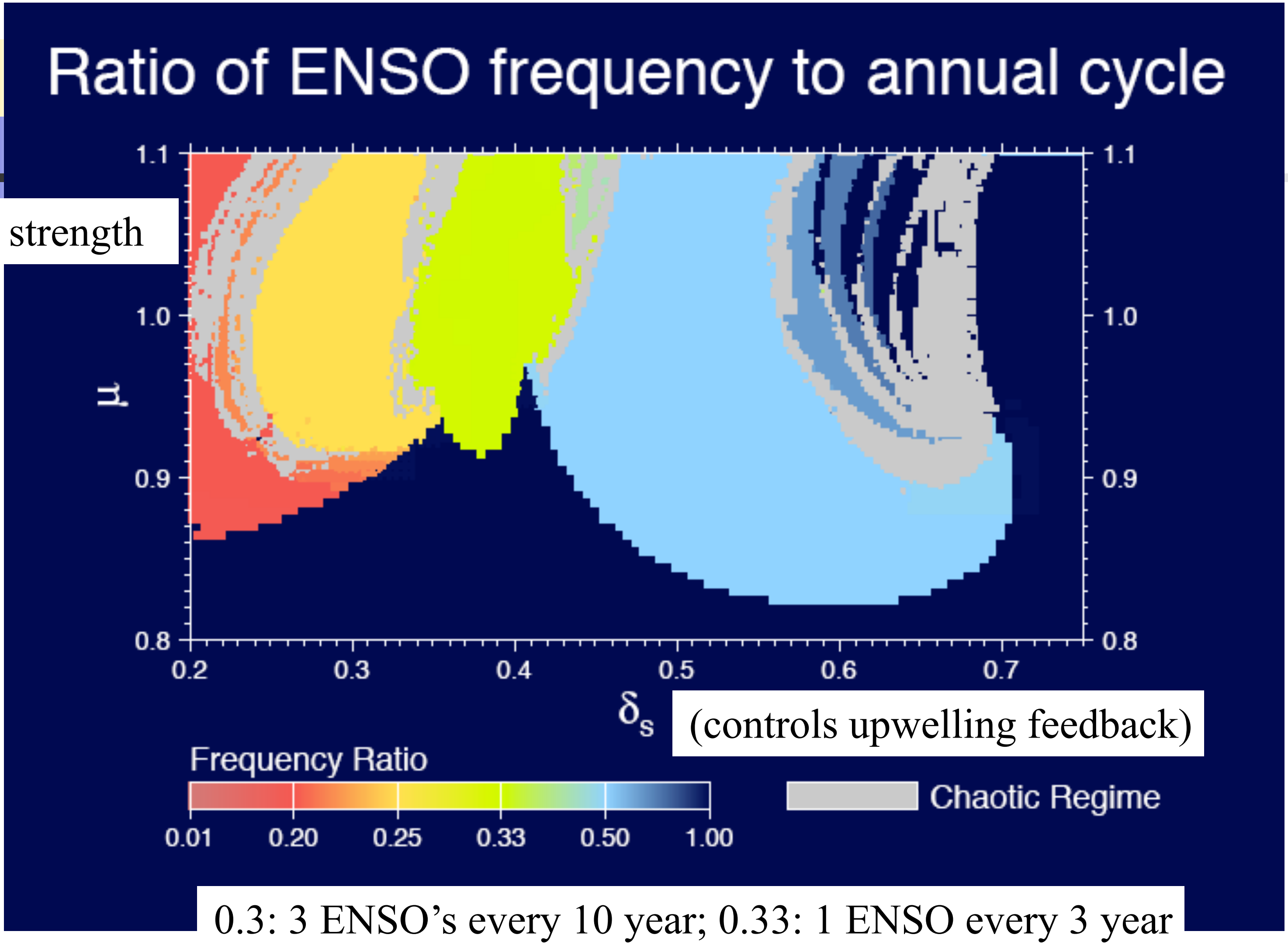
Synchronization with the seasonal cycle



Christiaan Huygens
(1629-1695)



Zebiak-Cane model results



coupling strength

(controls upwelling feedback)

0.3: 3 ENSO's every 10 year; 0.33: 1 ENSO every 3 year

Devil's Terrace

Question time

Summary

El Nino is a large-scale pattern of interannual sea surface temperature variability in the equatorial Pacific

El Nino can be understood as an oscillatory mode of variability of the coupled equatorial ocean - global atmosphere system affected by atmospheric noise

Dynamical systems framework:
Stochastic Hopf Bifurcation

Physical mechanism: Recharge oscillator