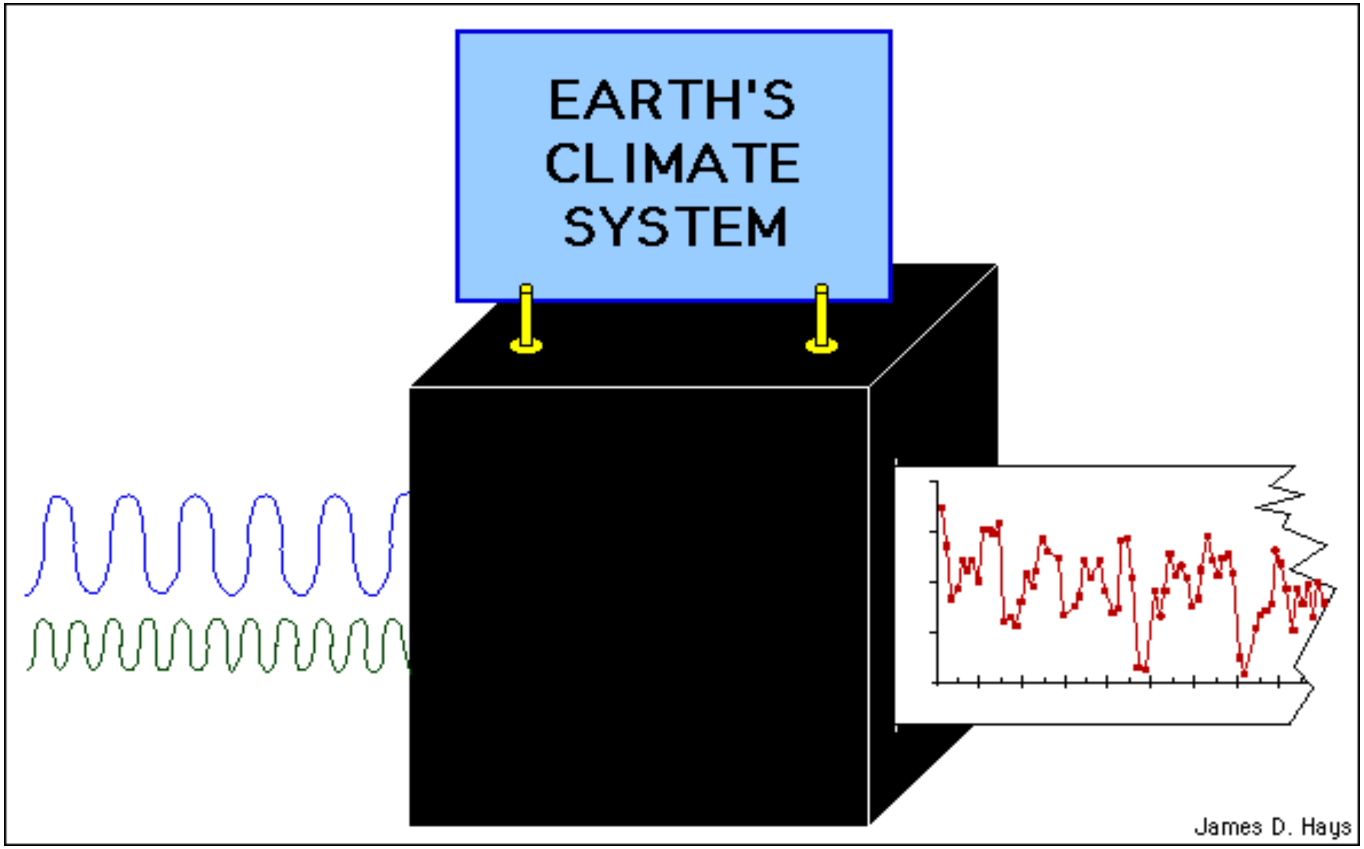
Stochastic Climate Dynamics



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https://webspace.science.uu.nl/~dijks101/styled-6/

Summary day 1 + 2 + 3 + 4

Stochastic linear dynamical systems

$$dX_t = -\gamma X_t dt + \sigma dW_t$$

Hasselmann, null-hypothesis (SST)

Interaction noise and multiple equilibria

$$\frac{dy}{dt} = F - y(1 + \mu^2(1-y)^2)$$

$$F(t) = \bar{F} + \sigma\xi(t)$$

Interaction noise and internal oscillations

$$dX = (\lambda X - \omega Y - X(X^2 + Y^2))dt + \sigma dW_1$$

$$dY = (\lambda Y + \omega X - Y(X^2 + Y^2))dt + \sigma dW_2$$

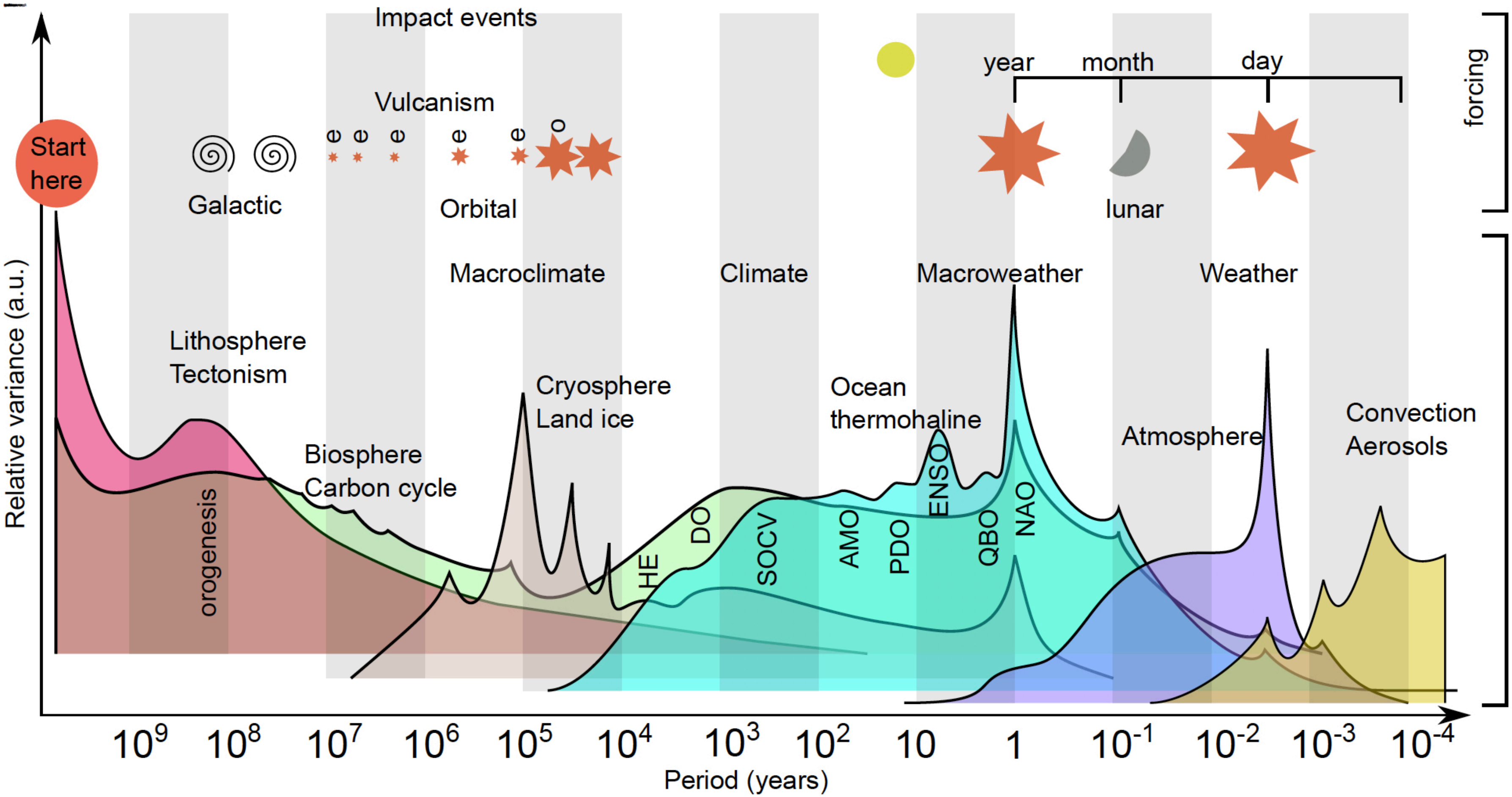
Weather & ENSO Prediction
(Chaos + noise)

$$\dot{x} = s(y - x)$$

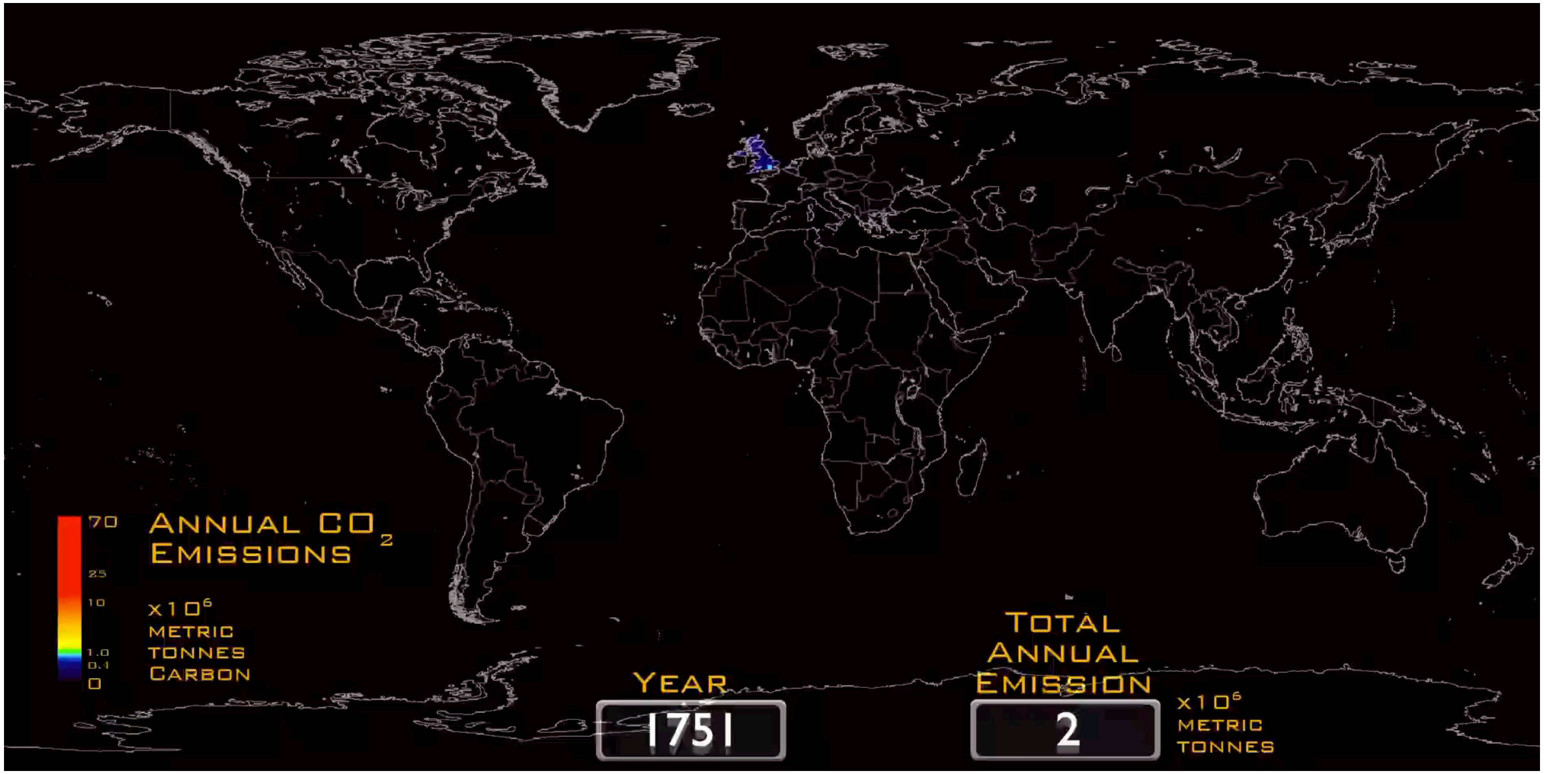
$$\dot{y} = -xz + rx - y$$

$$\dot{z} = xy - bz$$

Natural Climate Variability

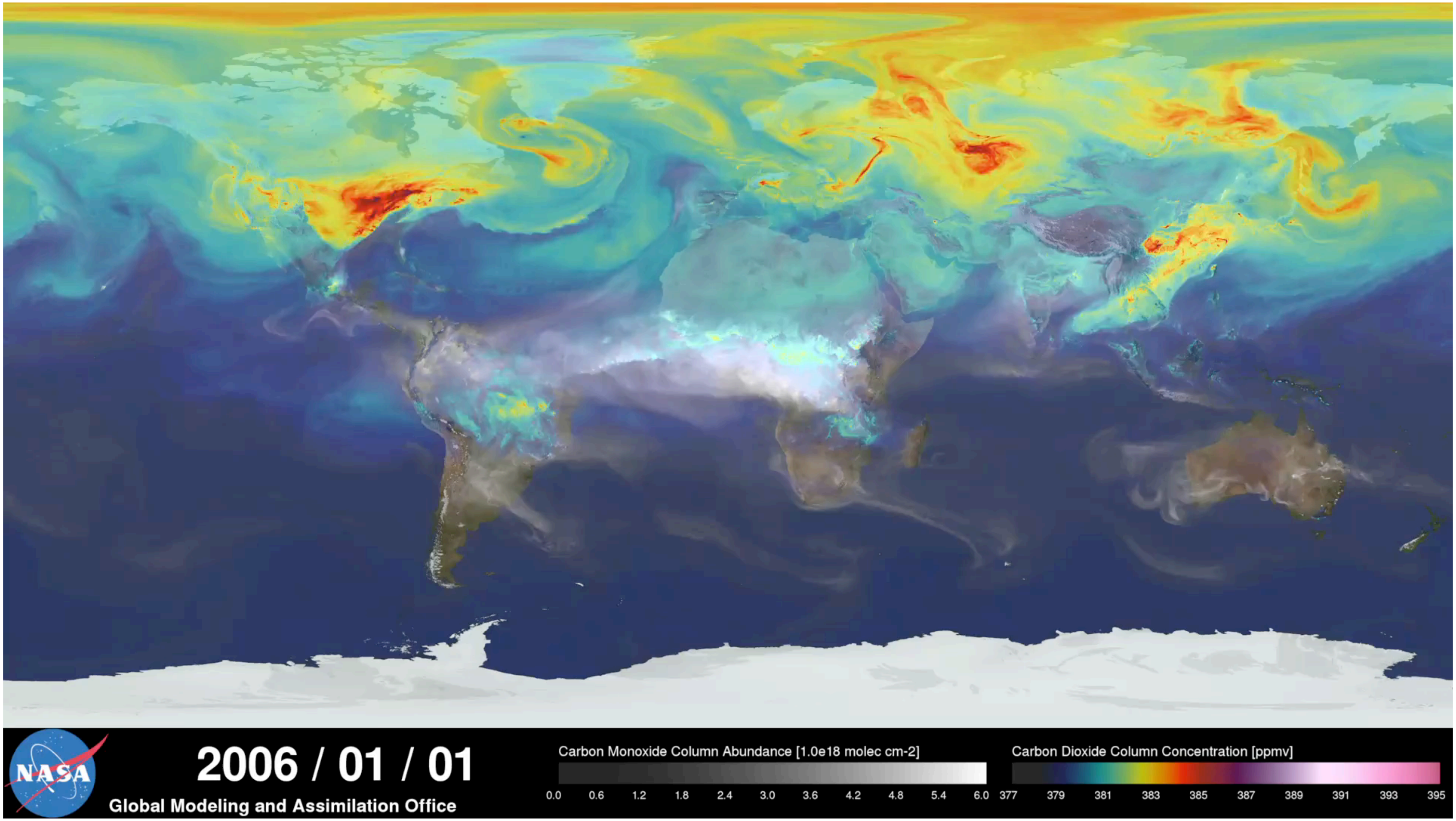


Anthropogenic Climate Change

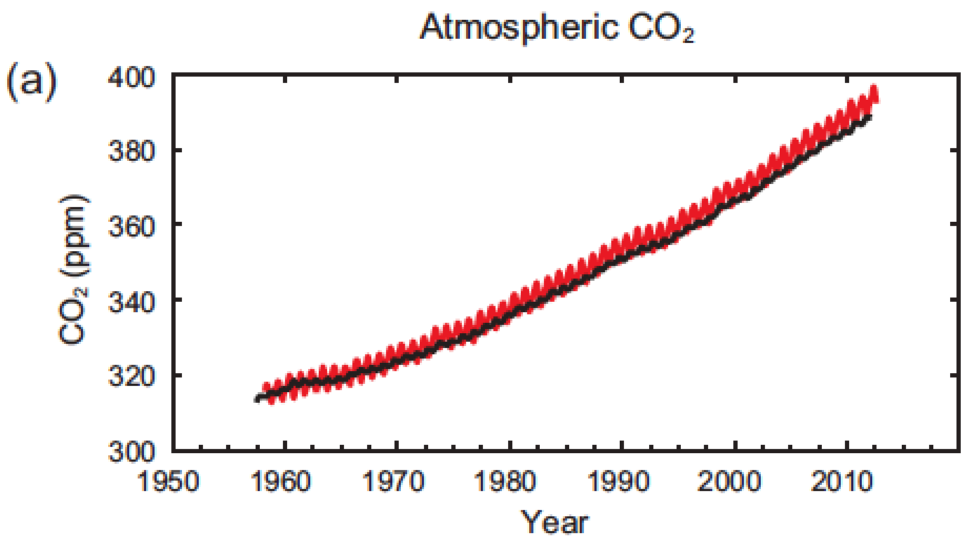


From 1870 to 2014, cumulative carbon emissions totaled about 545 GtC. Emissions were partitioned among the atmosphere (approx. 230 GtC or 42%), ocean (approx. 155 GtC or 28%) and the land (approx. 160 GtC or 29%).

A year of CO and CO2 emissions

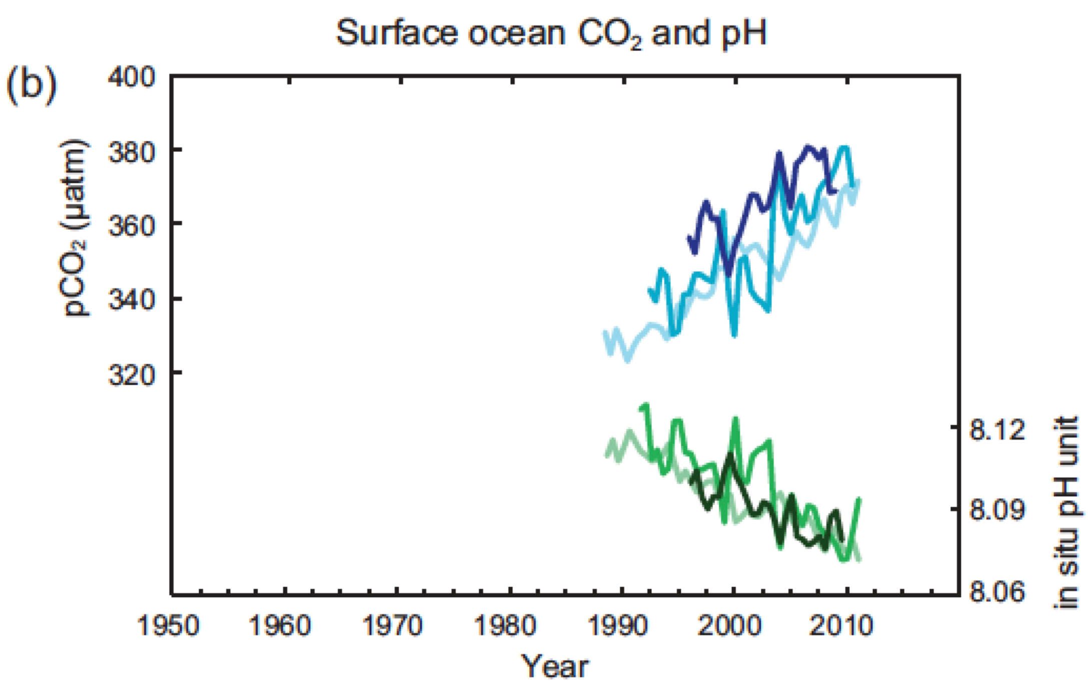


Changes in the global carbon cycle



IPCC, 2013

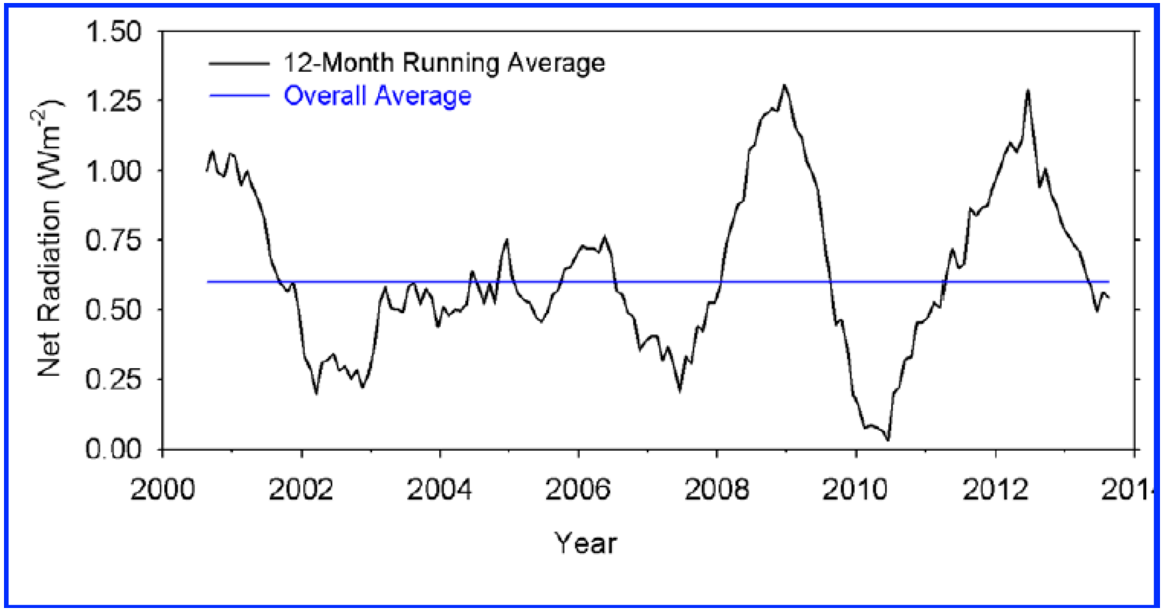
About 45% increase since preindustrial times (280 ppmv)



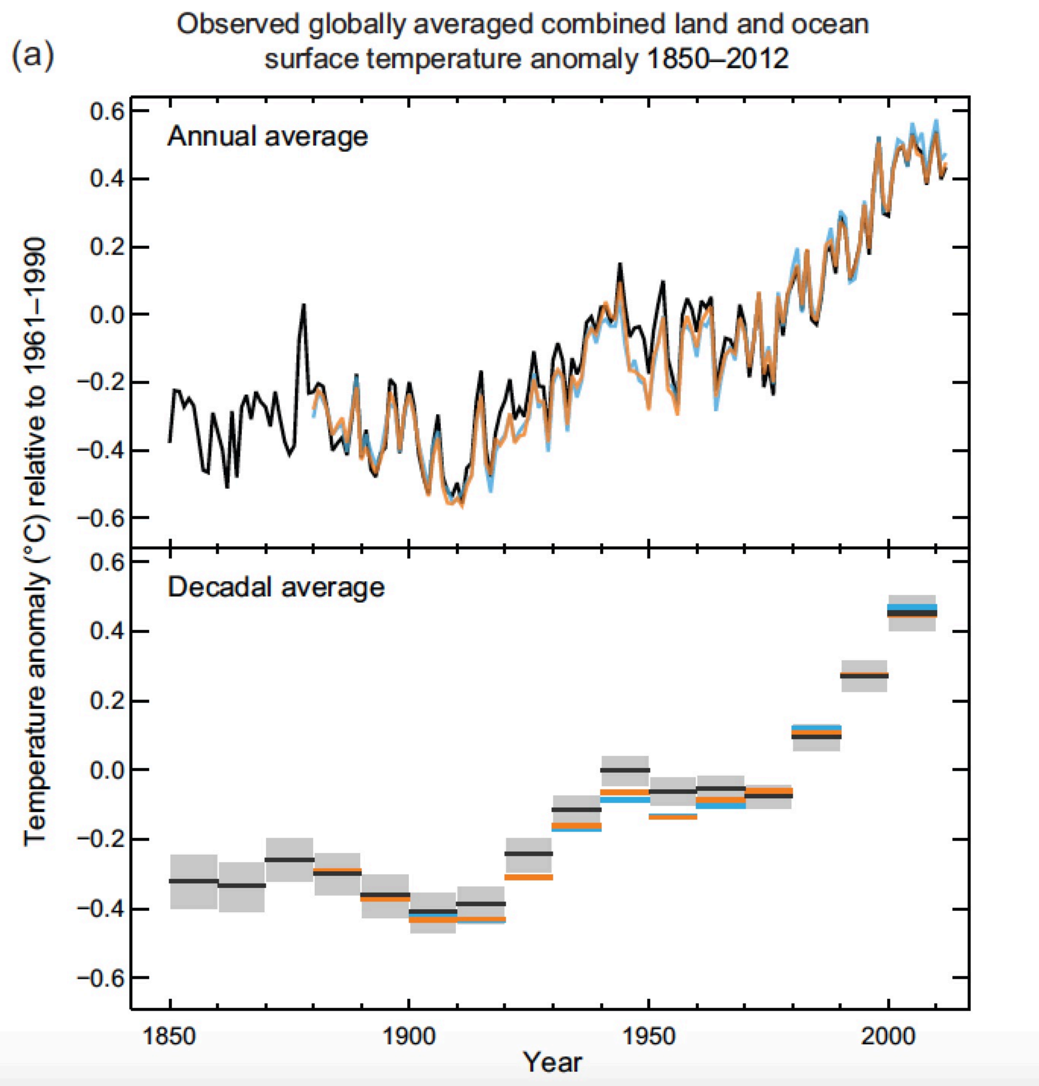
Ocean acidification: the other CO2 problem

Earth's energy (im)balance

CERES net downward radiation (W/m^2)



Climate Change: observations



IPCC, 2013

Question time

Main questions

How warm is it going to be in 2100?

Are the changes going to be ‘smooth’ or ‘bumpy’?

When is it too late to act to prevent dangerous climate change?

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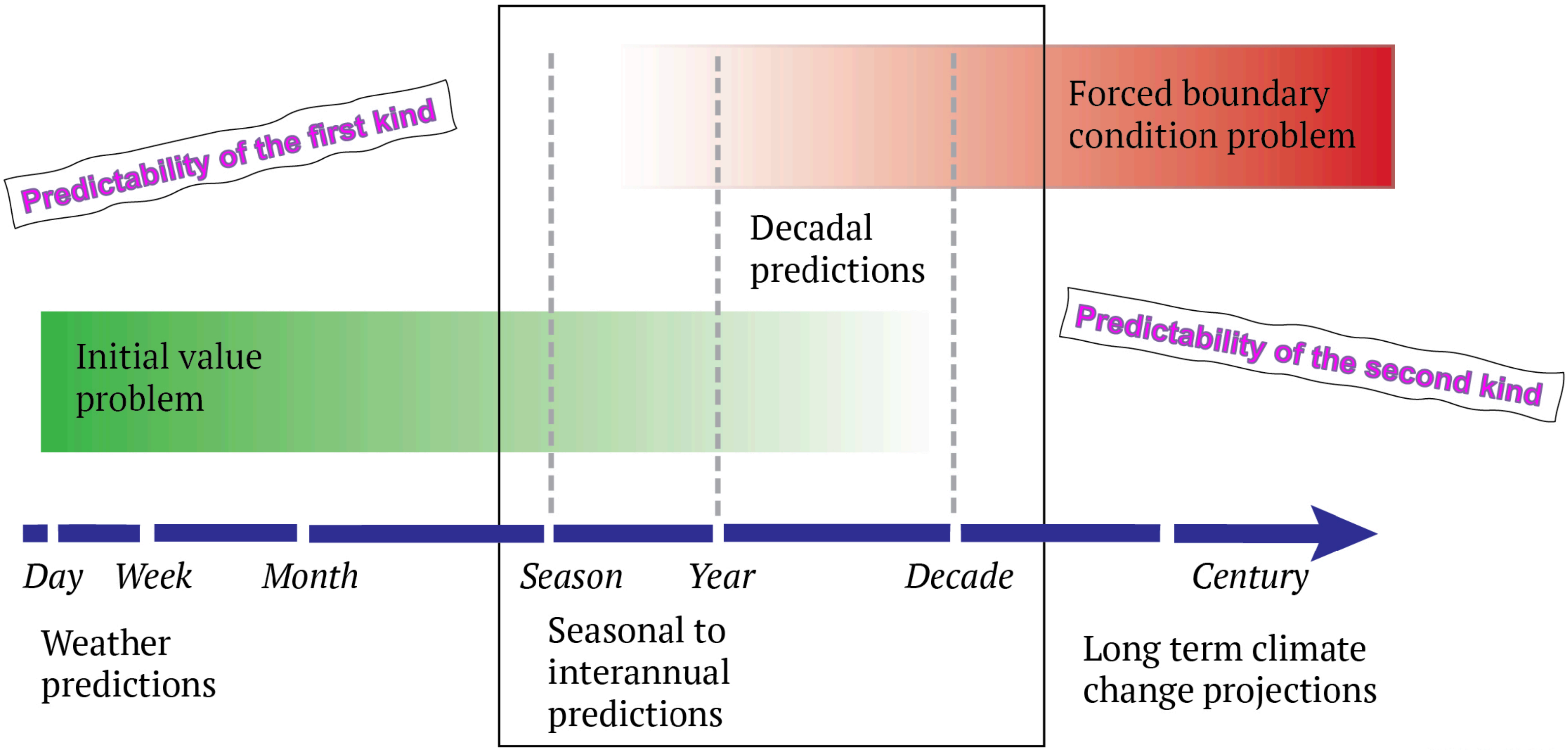
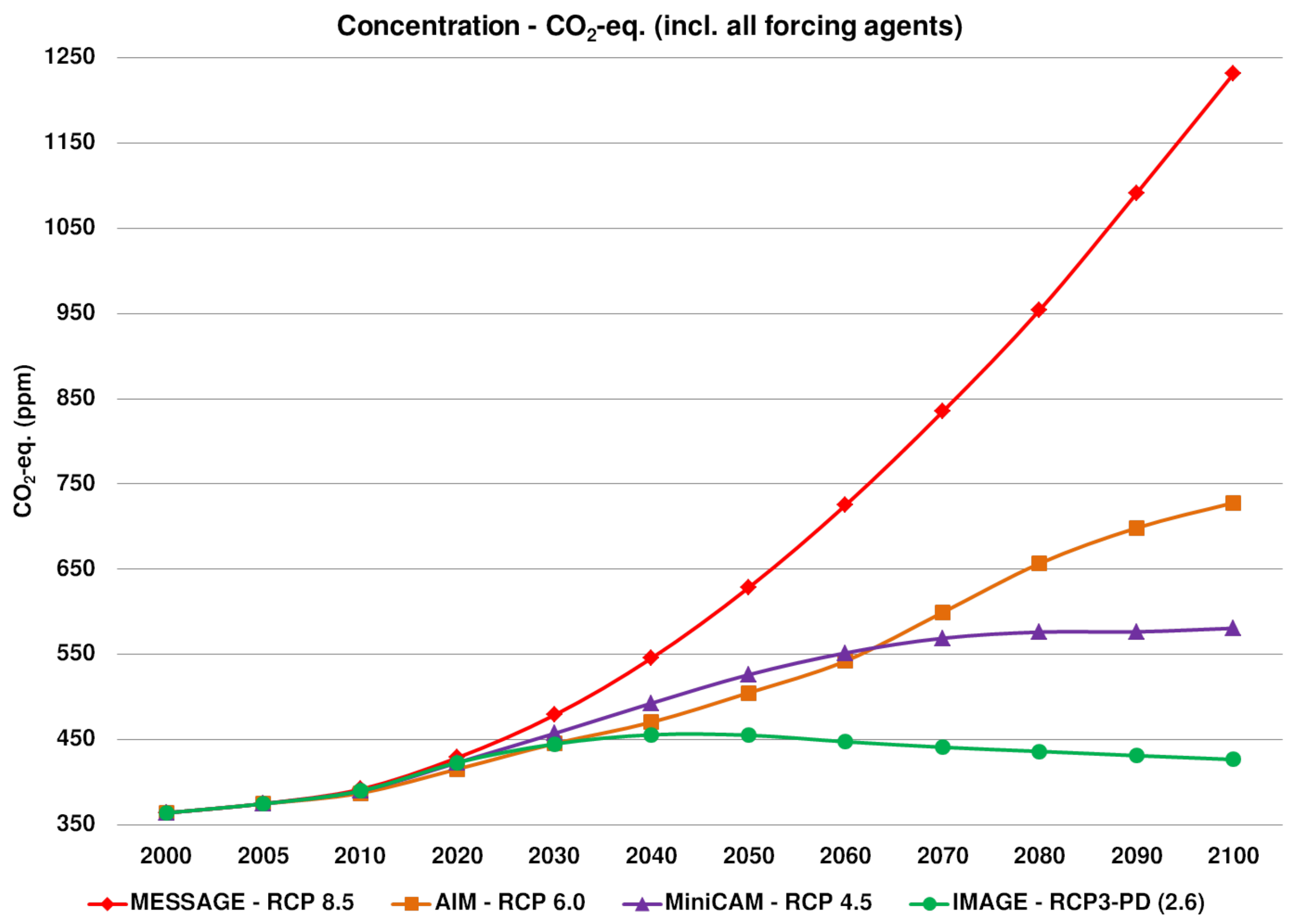
Future climate change as a predictability problem



IPCC AR5

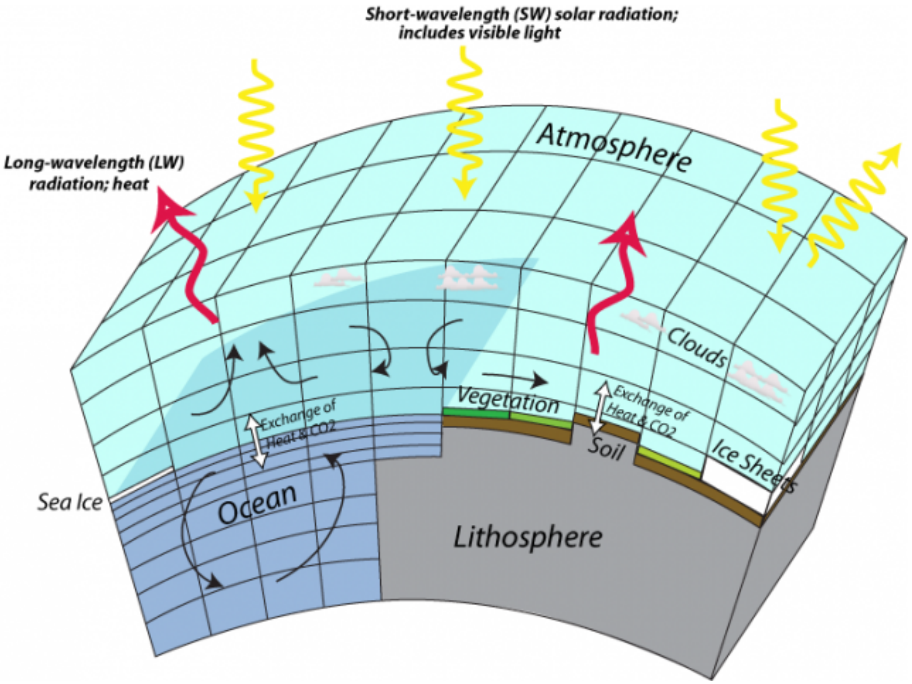
Future Climate Change:
Representative concentration pathways



IPCC, Chapter 11, 2013

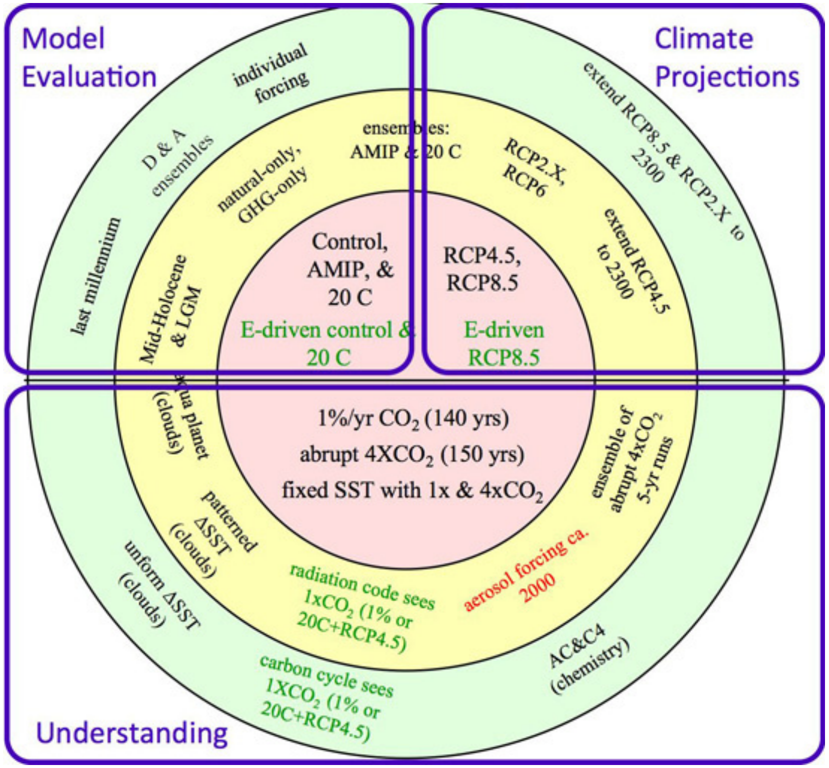
Climate projections

GCM (Global Climate Model)



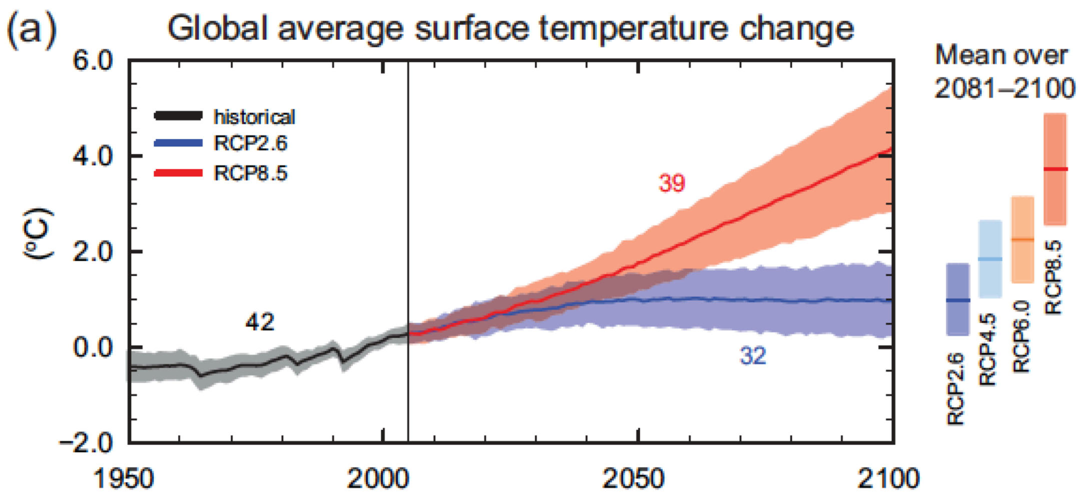
Typical horizontal resolution:
ocean, sea-ice: 0.5-1 degree
atmosphere, land: 1-2 degree

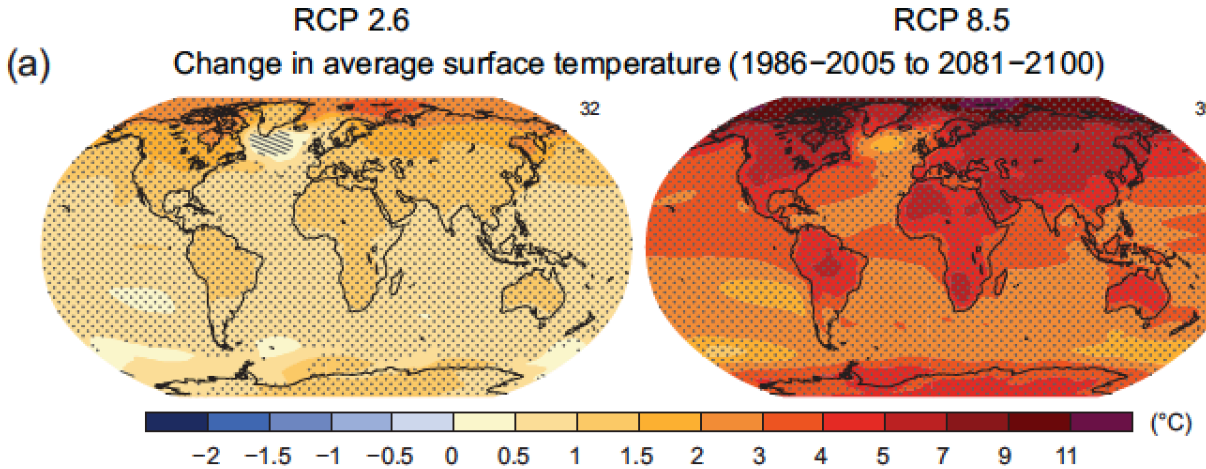
CMIP



~ 20 modeling groups
~ 50 different models
Ensemble of realizations

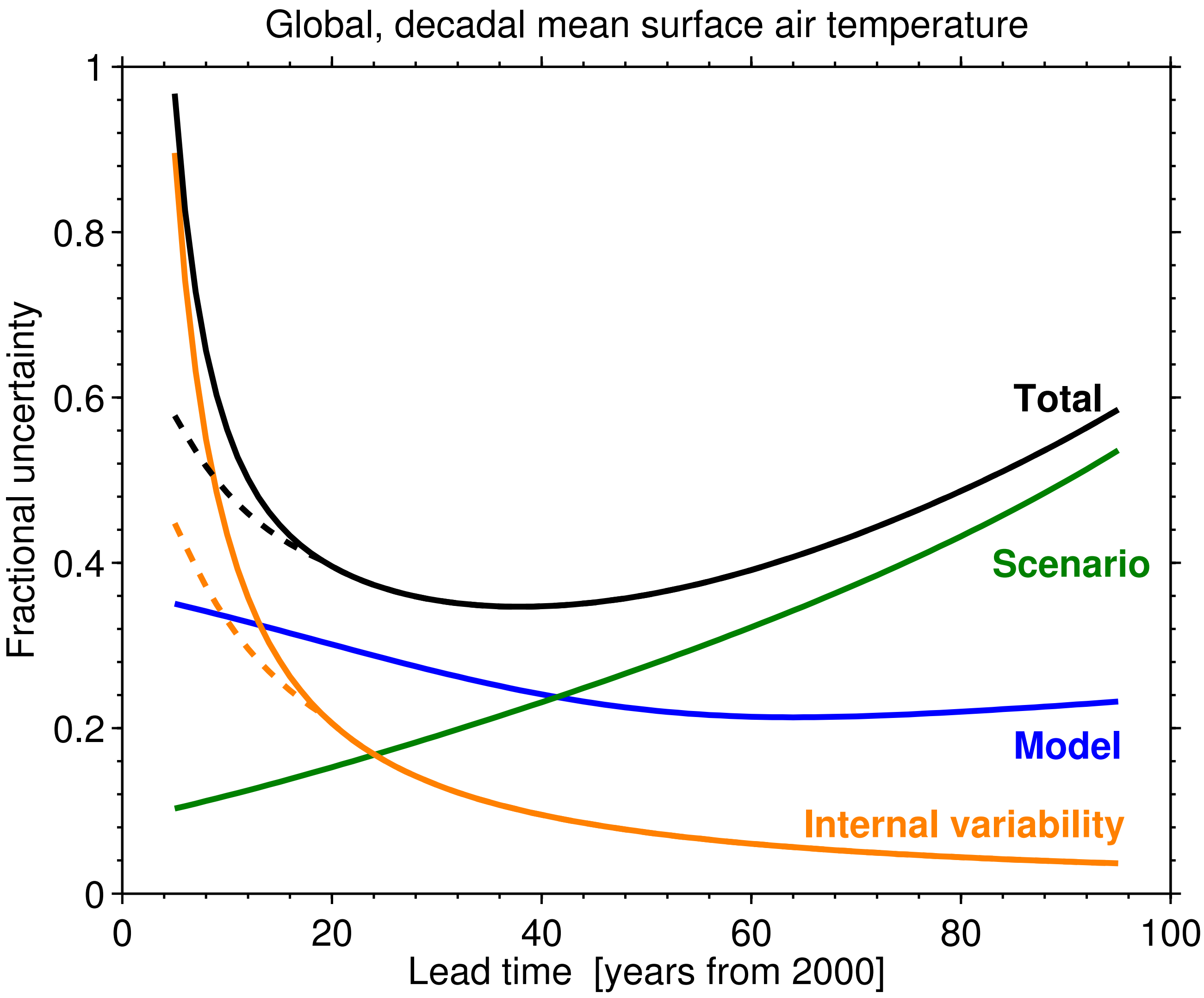
Results: Typical projection (CMIP5)





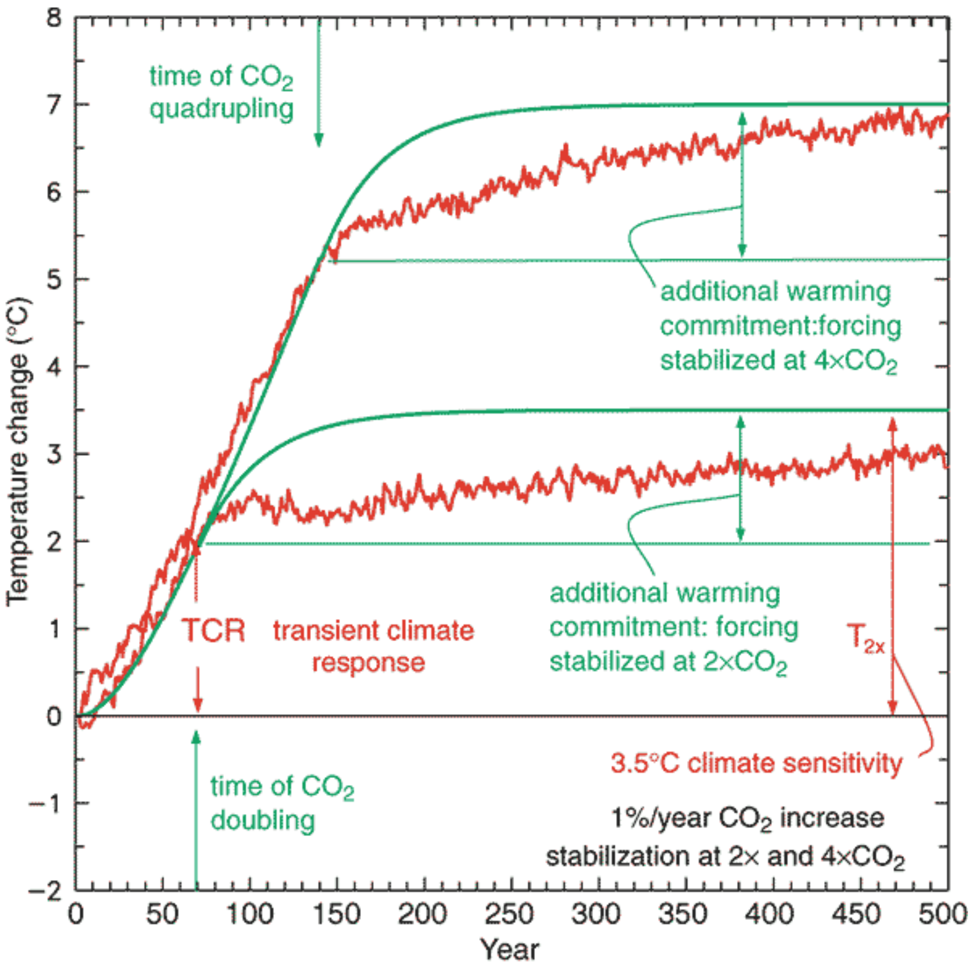
Question time

Projection uncertainties



Hawkins and Sutton, 2009

Climate Sensitivity



Equilibrium (or Charney) climate sensitivity S:
Equilibrium increase in GMST due to a doubling of CO_2

Approach

Radiative forcing: $R(T, \alpha(T))$

$$\Delta R = \left(\frac{\partial R}{\partial T} + \frac{\partial R}{\partial \alpha}\frac{\partial \alpha}{\partial T}\right)\Delta T + a(\Delta T)^2 + \mathcal{O}(\Delta T)^3$$

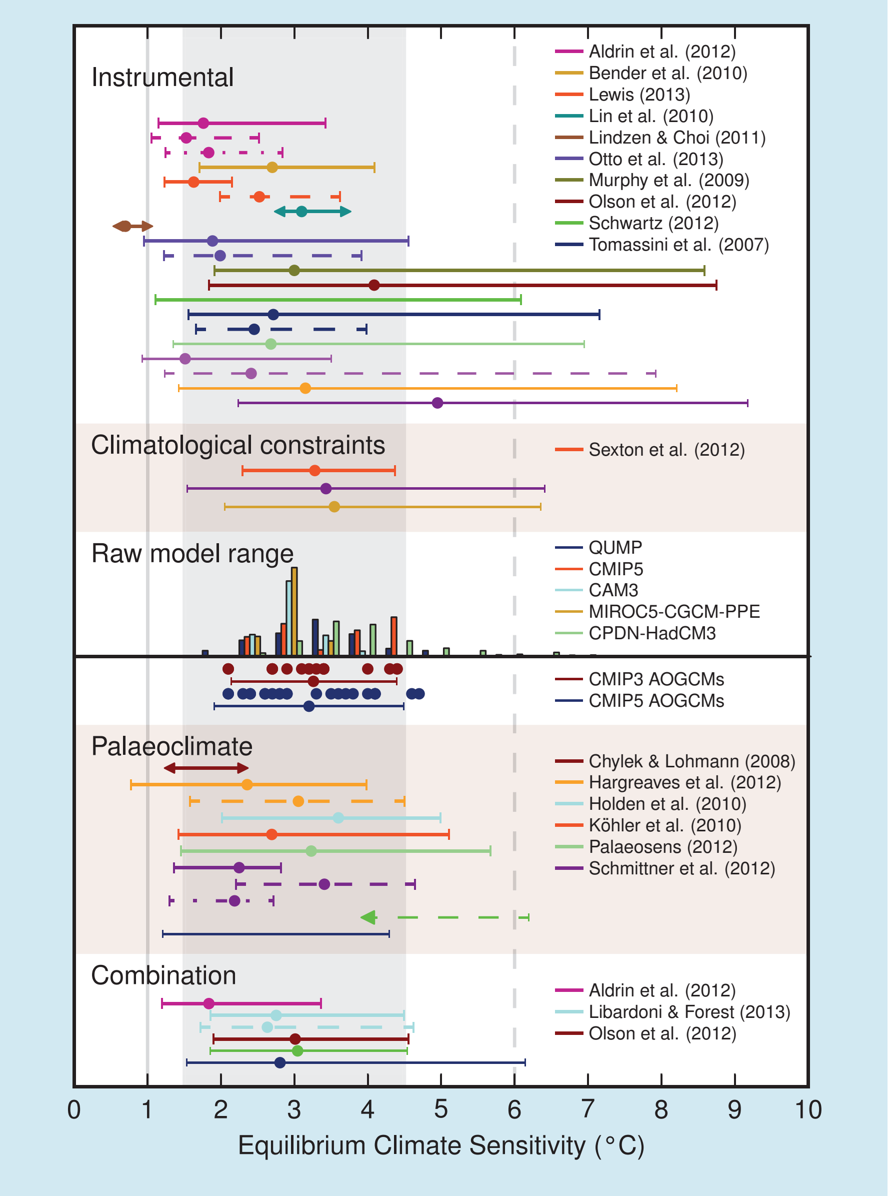
$$\frac{1}{S_0} = \frac{\partial R}{\partial T} \; ; \; f = -S_0 \frac{\partial R}{\partial \alpha}\frac{\partial \alpha}{\partial T}$$

Small amplitude:

$$\Delta T = \frac{S_0}{1-f}\Delta R \rightarrow S = \frac{\Delta T}{\Delta R} = \frac{S_0}{1-f}$$

$S_0 = 0.3 \; K/(Wm^{-2})$ Planck response

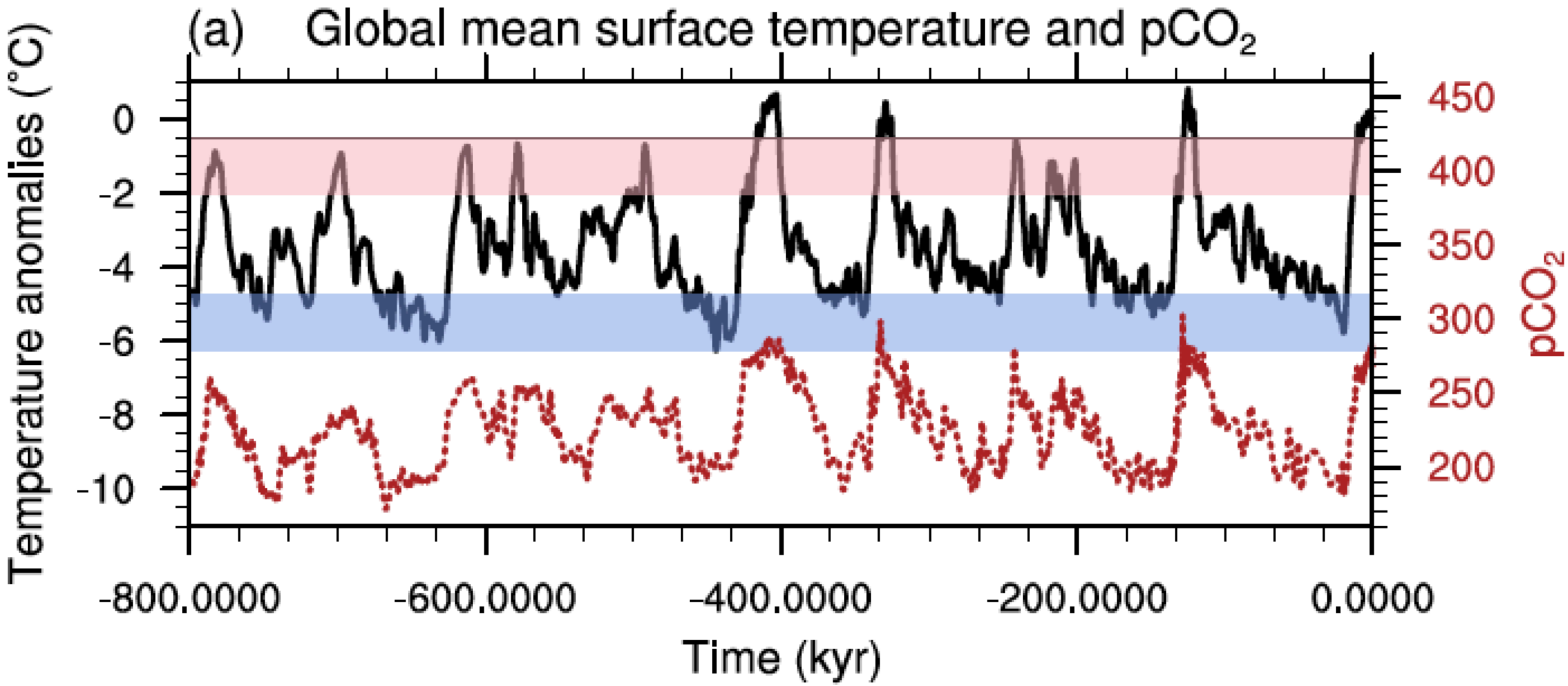
Equilibrium climate sensitivity



S = 0.4-1.2 K/(Wm^{-2})
or 1.5 - 4.5°C for doubling CO_2

Chapter 12 of IPCC-AR5 (2013)

Proxy data example: glacial cycles

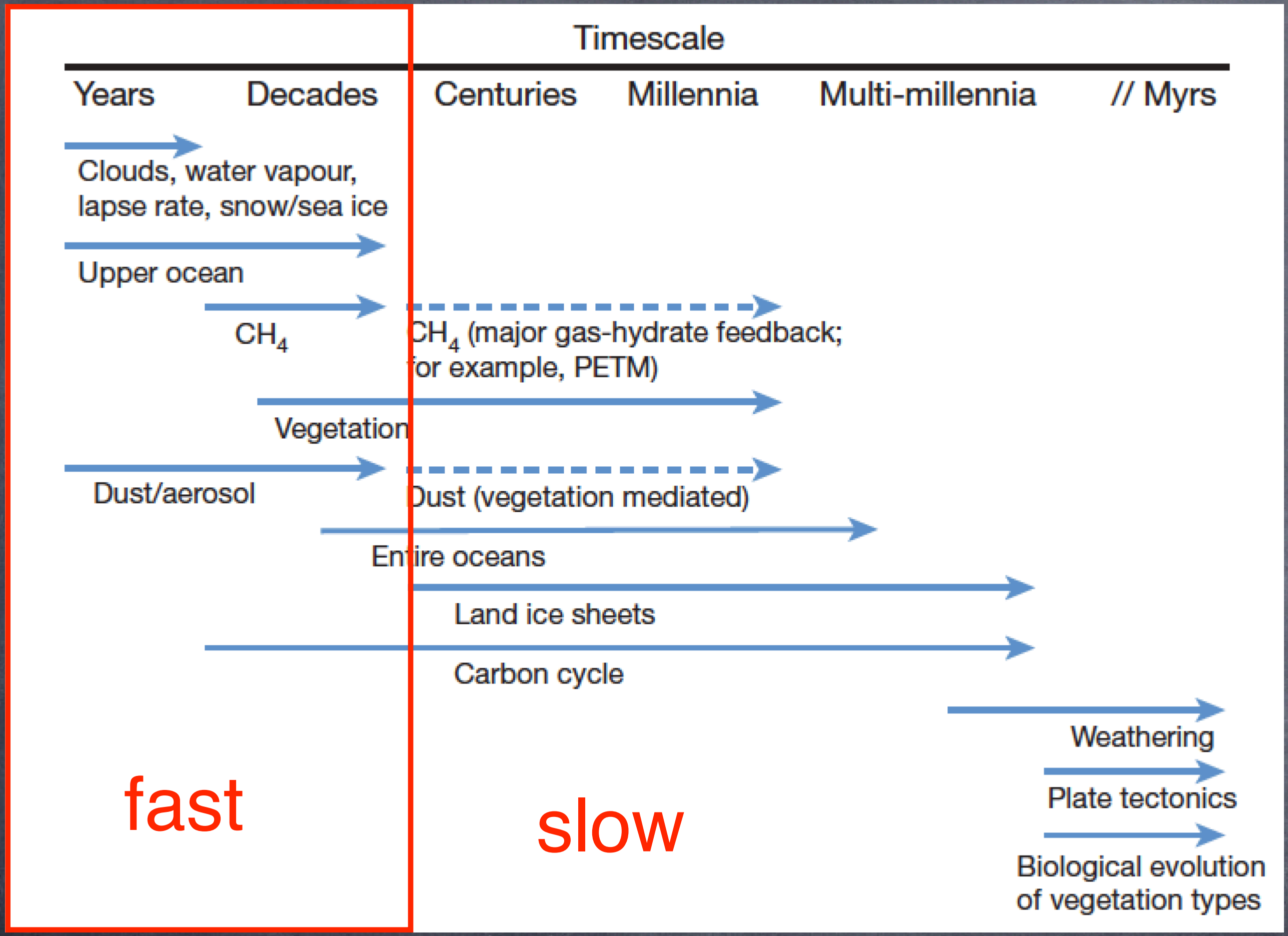


Result: S ~ 2 - 2.5 K/(W/m2)

Was climate much more sensitive than today or is this just a different type of sensitivity?

Data: Koehler et al. (2010)

Feedbacks on different time scales



$$\lambda_P = -\frac{1}{S_0}\, Wm^{-2}K^{-1}$$

$$S^a = \frac{\Delta T}{\Delta R} = \frac{-1}{\lambda_P + \sum_{i=1}^{N} \lambda_i^f}$$

Charney sensitivity: only fast feedbacks

How to determine Charney climate sensitivity from proxy data?

$$S^a = S_{[CO_2]} = \frac{\Delta T}{\Delta R_{[CO_2]}} = \frac{-\Delta T}{\Delta R_{[OLW]} + \Delta R_{[SI]}}, \quad = \frac{-1}{\lambda_P + \lambda_\alpha};$$

Charney sensitivity

$$S^p = S_{[CO_2]} = \frac{\Delta T}{\Delta R_{[CO_2]}} = \frac{-\Delta T}{\Delta R_{[OLW]} + \Delta R_{[SI]} + \Delta R_{[LI]}} = \frac{-1}{\lambda_P + \lambda_\alpha + \lambda_L},$$

Earth system sensitivity

$$S^a = S^p\left(1 + \frac{\lambda_L}{\lambda_P + \lambda_\alpha}\right),$$

SI: sea ice

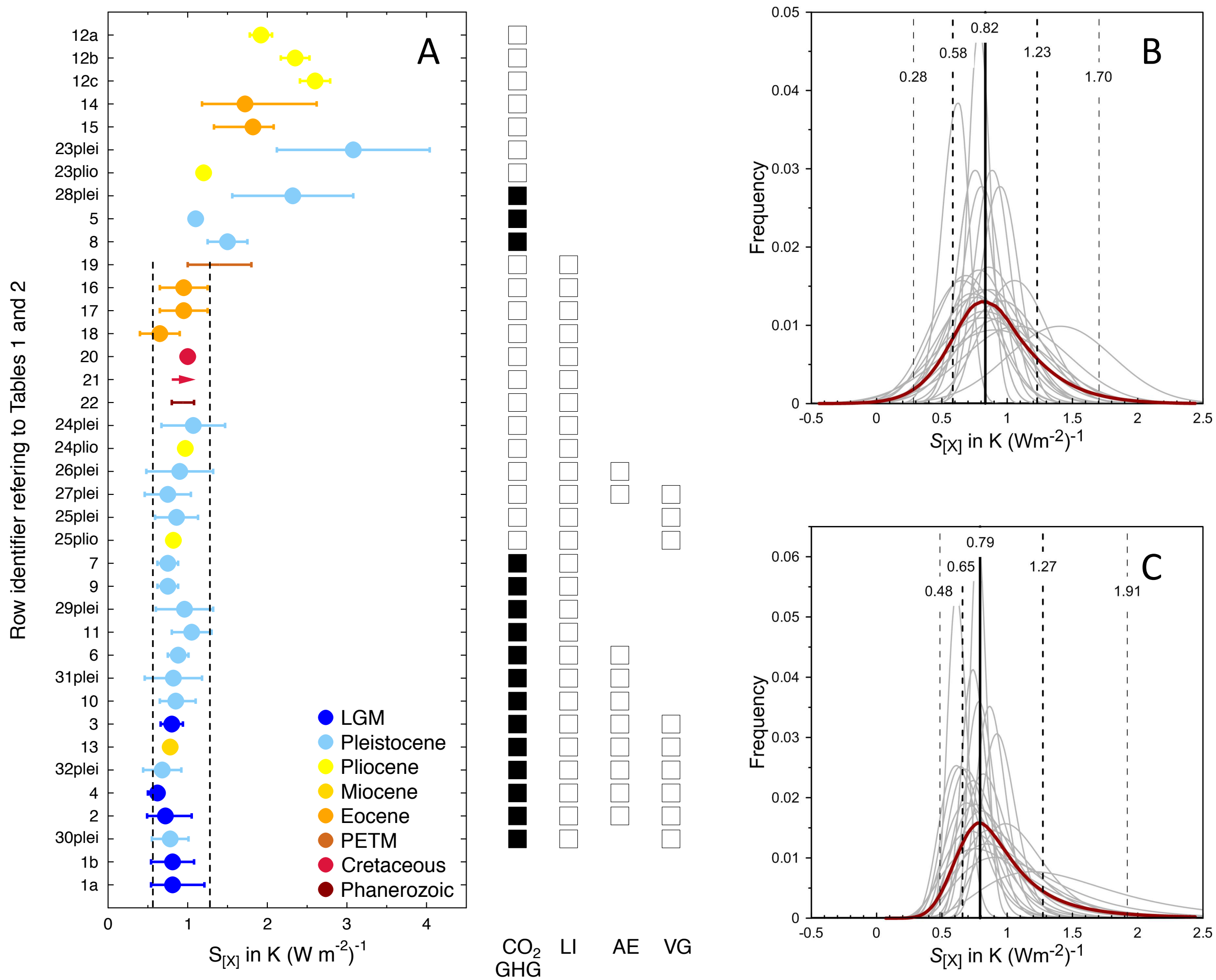
L: land ice

More general:

$$S^a = S^p \left(1 + \frac{\sum_{j=1}^{M} \lambda_j^{s}}{\lambda_P + \sum_{i=1}^{N} \lambda_i^{f}} \right)$$

Results

slow feedback correction



LI: land ice
AE: aerosol
VG: vegetation

Palaeosense (Rohling et al.) (2012)

Main questions?

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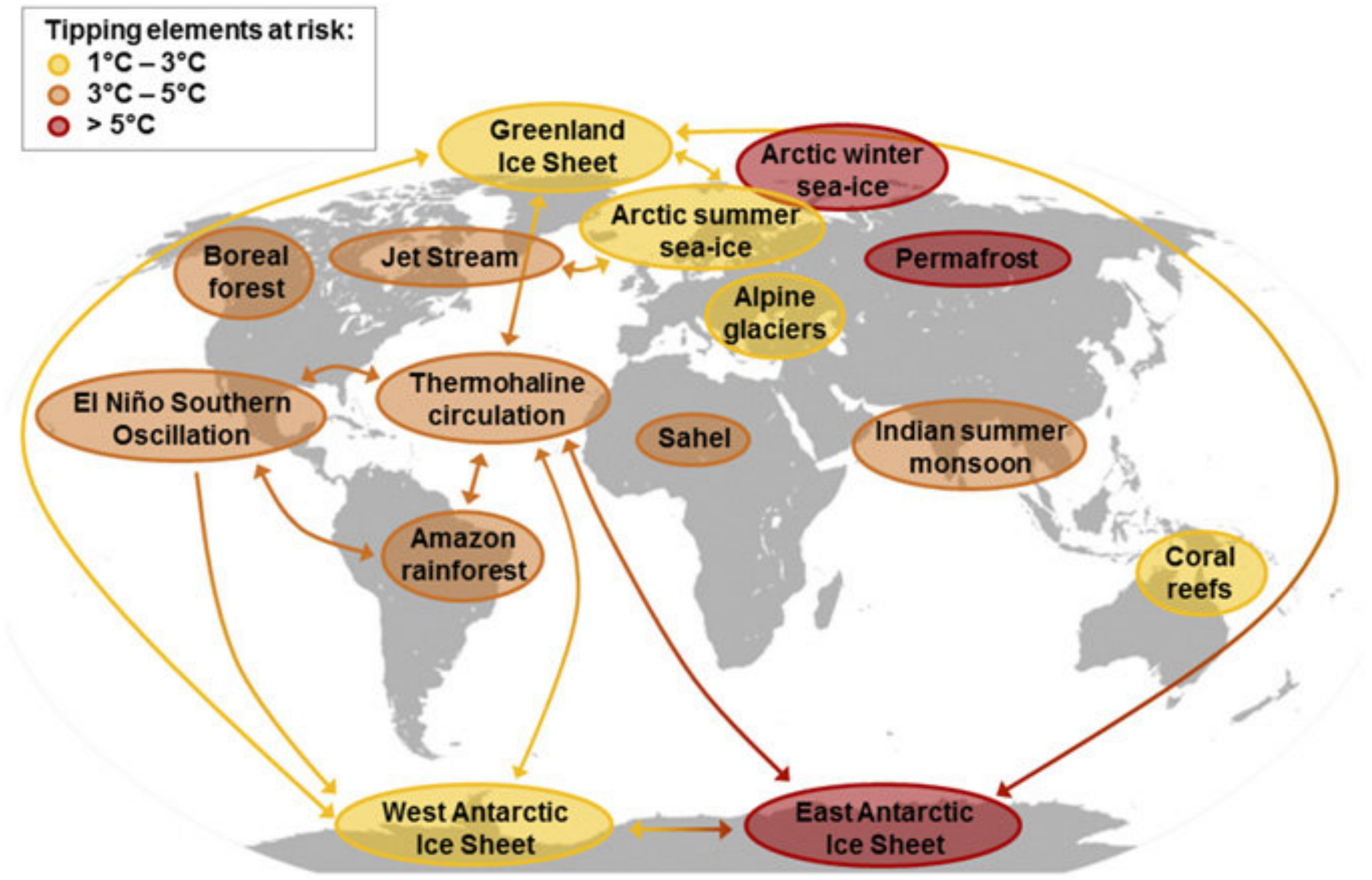
Tipping elements



Steffen et al. (2018)

Non-autonomous fast-slow dynamical systems

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{y}),$$

$$\frac{d\mathbf{y}}{dt} = \epsilon \mathbf{g}(\mathbf{x}, \mathbf{y}),$$

x: state vector

y: parameter vector (e.g. forcing)

$$\tau = \epsilon t \quad \Rightarrow \quad \epsilon \frac{d\mathbf{x}}{d\tau} = \mathbf{f}(\mathbf{x}, \mathbf{y}), \qquad \frac{d\mathbf{y}}{d\tau} = \mathbf{g}(\mathbf{x}, \mathbf{y}),$$

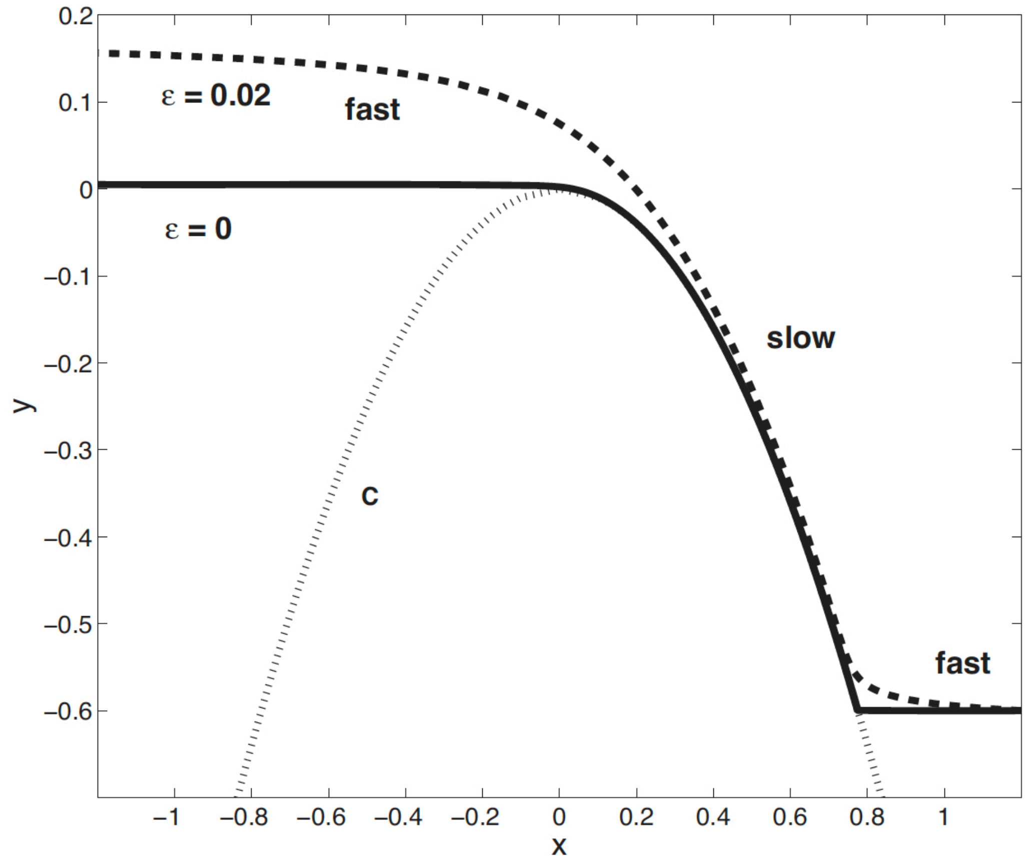
$$\epsilon = 0 : \quad \left\{ C = \left\{(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^{d+p} : \mathbf{f}(\mathbf{x}, \mathbf{y}) = 0\right\} \right.$$

critical manifold

Fast-slow saddle-node bifurcation

$$\epsilon \frac{dx}{d\tau} = -y - x^2,$$

$$\frac{dy}{d\tau} = 1.$$



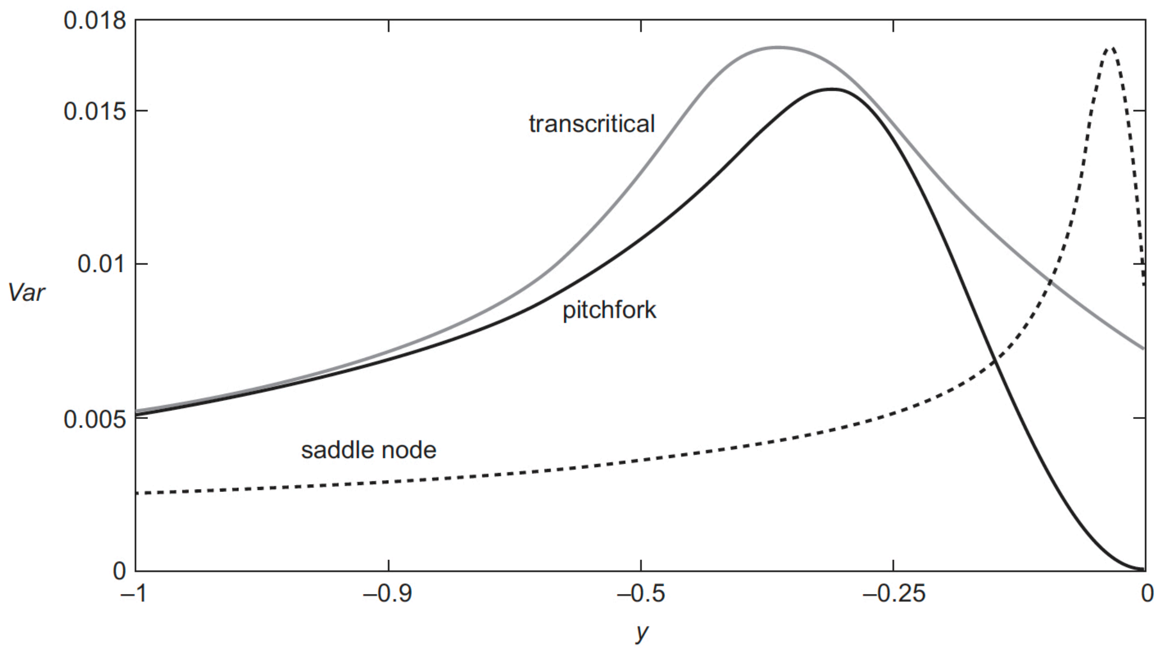
Variance in fast-slow systems

$$dX_t = f(X_t, Y_t)dt + \sigma dW_t,$$

$$dY_t = \epsilon \, dt,$$

$\epsilon = 0$

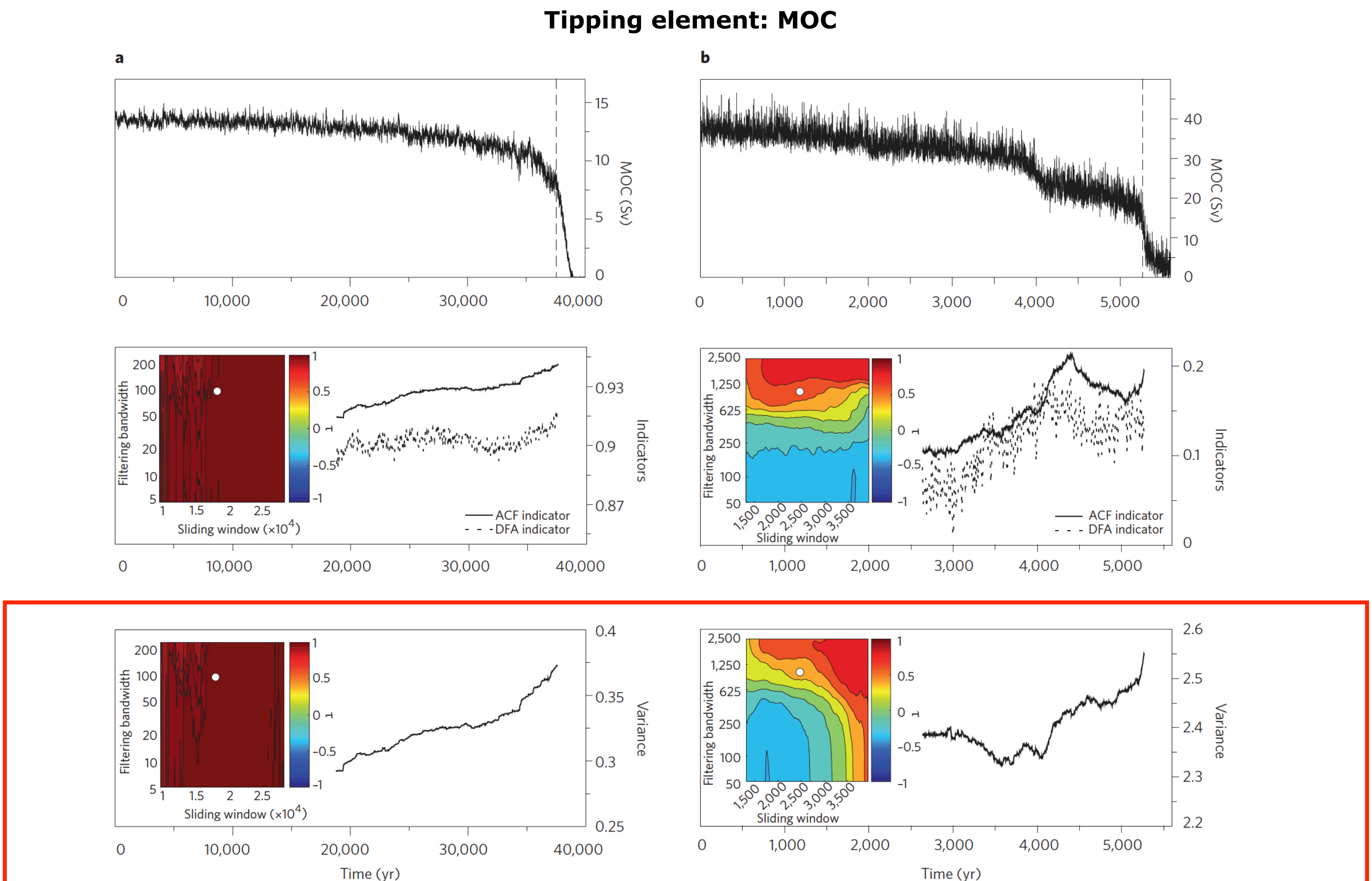
$$\frac{\partial p^y}{\partial t} = -\frac{\partial (f(x,y)p^y)}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2 p^y}{\partial x^2}, \quad \longrightarrow \quad \bar{p}^y(x) = \frac{1}{N} e^{\int_a^x \frac{2}{\sigma^2} f(s,y)\, ds},$$



Variance increases when approaching critical conditions!

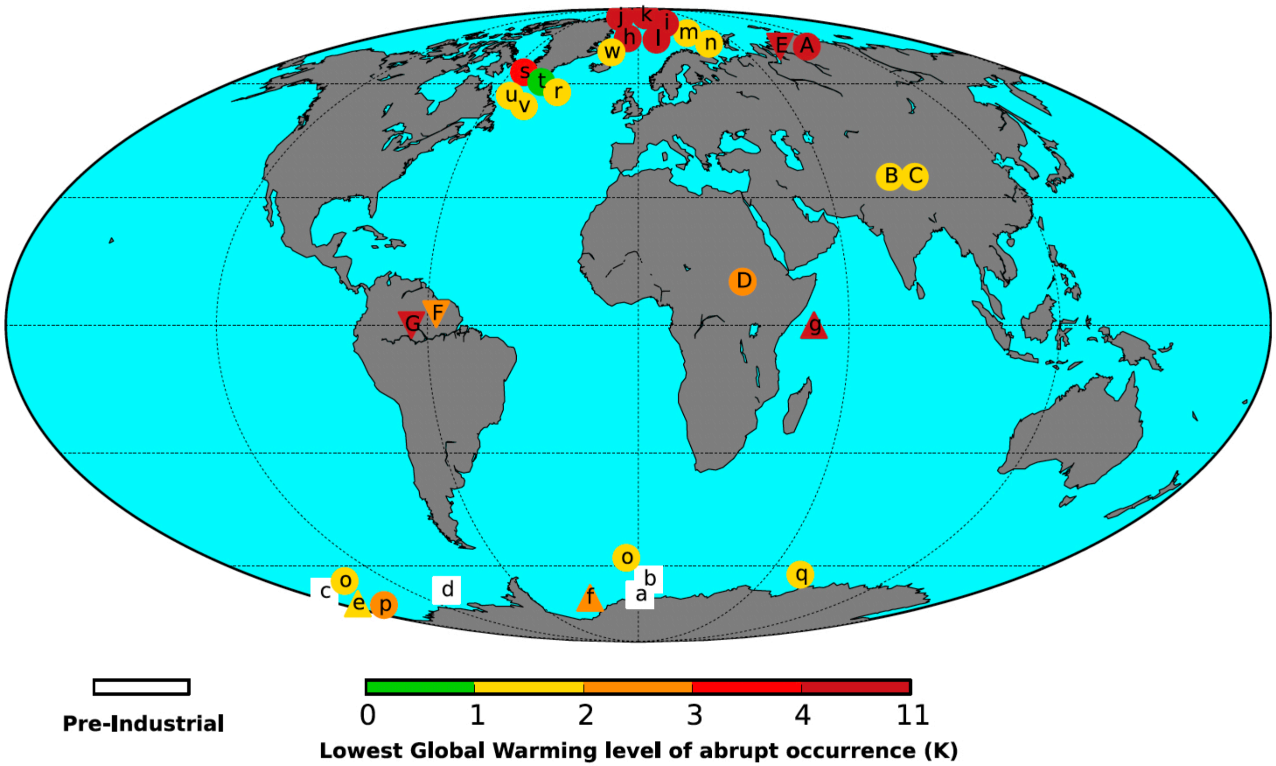
Kuehn (2011)

Early Warning Indicators



Lenton (2011)

Detected transitions in CMIP5



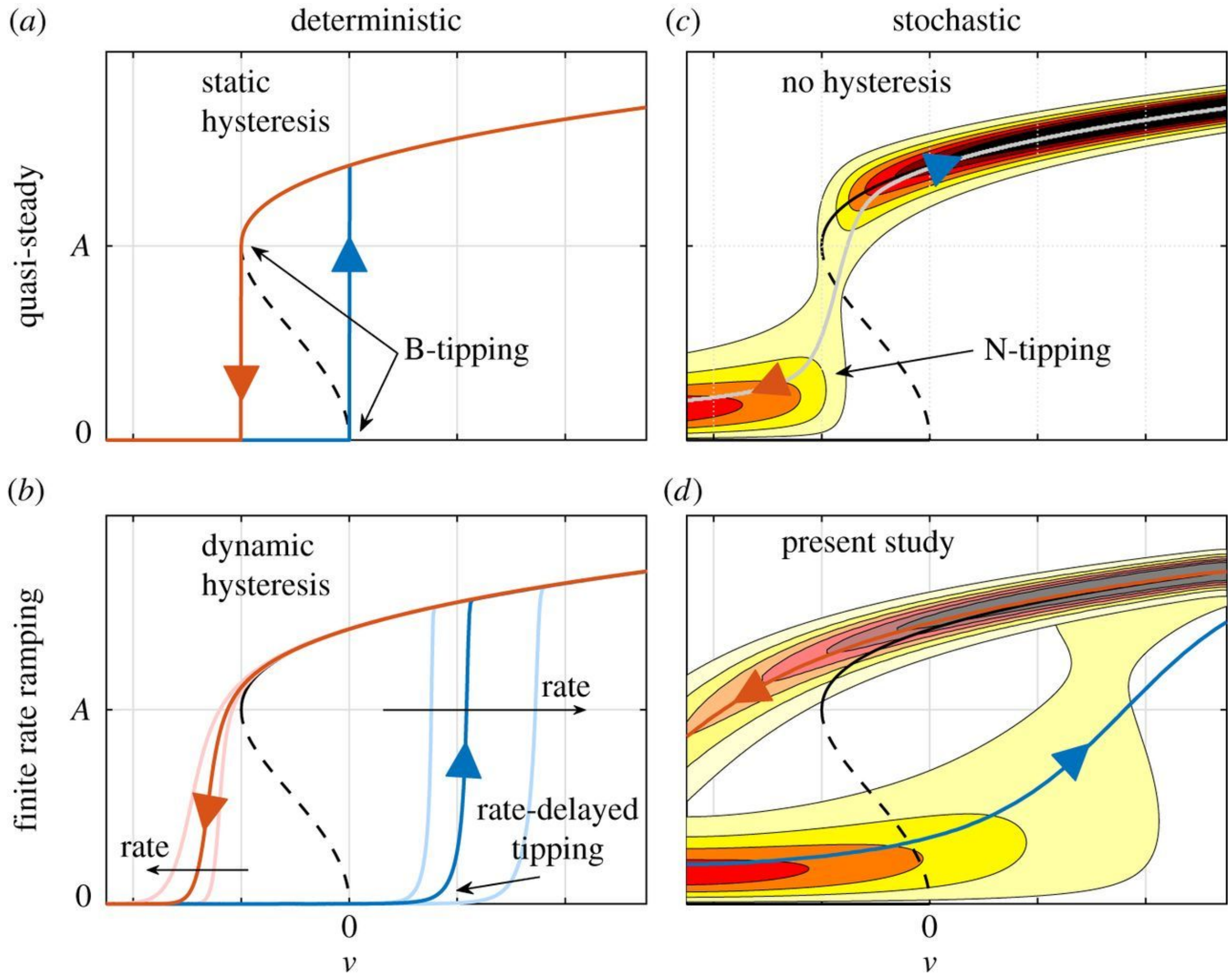
Sea ice bimodality: (a):bcc-csm1-1, (b):bcc-csm1-1-m, (c):GFDL-CM3, (d): IPSL-CM5A-LR, (e):GISS-E2-R, (f):GISS-E2-H
Upwelling change: (g):IPSL-CM5A-LR
Arctic sea ice collapse: (h):CCSM4, (i):CNRM-CM5, (j):CSIRO-Mk3-6-0, (k):MPI-ESM-LR, (l):HadGEM2-ES
Abrupt sea ice loss: (m):CanESM2, (n):CMCC-CESM, (o):MRI-CGCM3, (p):FGOALS-g2
Abrupt sea ice increase: (q):MRI-CGCM3
Convection collapse: (r):GISS-E2-R, (s):CESM-CAM5, (t):GFDL-ESM2G, (u):MIROC5, (v):CSIRO-Mk3-6-0
AMOC-induced collapse: (w):FIO-ESM

Permafrost collapse: (A):HadGEM2-ES
Vegetation composition change: (D):BNU-ESM
Forest Dieback: (F):HadGEM2-ES, (G):IPSL-CM5A-LR
Snow melt: (B):GISS-E2-H, (C):GISS-E2-R
Forest expansion: (E):HadGEM2-ES

Category I, Category II, Category III, Category IV

Drijfhout et al. (2015)

Rate-dependent tipping: beyond fast-slow



Bonciolino et al. (2018)

Question time

Main questions?

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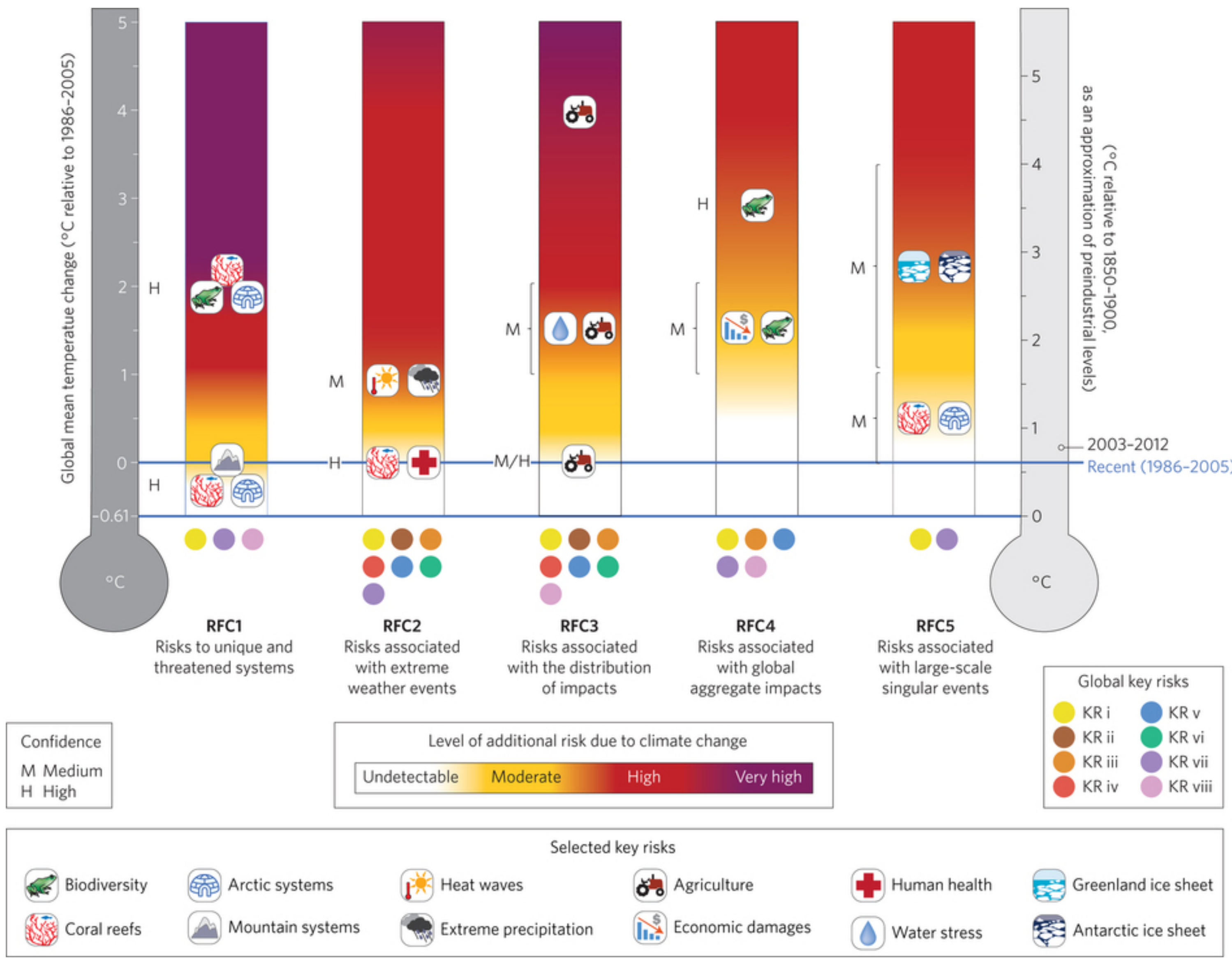
Dangerous Anthropogenic Interference



IPCC (2013)

How to avoid dangerous climate change?

- Determining efficiently what happens under different emission scenario's

- Evaluate effects of policy decisions affecting emissions

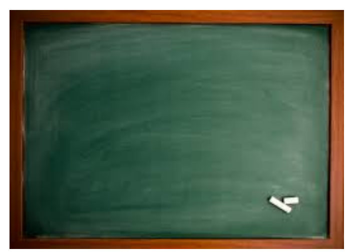
Need reduced model capturing warming behavior for different actions

Linear Response Theory

$$dX_t = (f(X_t) + f_e(X_t, t))dt + \sigma dW_t$$

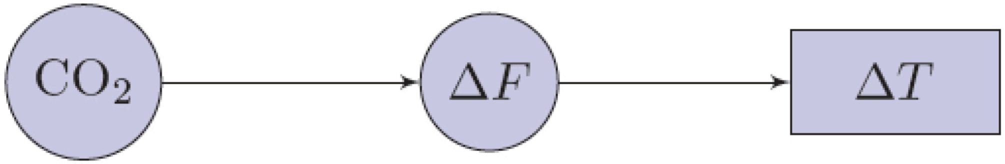
$f_e(X_t, t) = 0$ → $\dfrac{\partial p}{\partial t} = \bar{L}(p)$ → $\bar{L}(\bar{p}) = 0$

$f_e(X_t, t)) \neq 0$ → $p = \bar{p} + \bar{p}_e$ $\quad L_e(p) = -\dfrac{\partial(f_e p)}{\partial x}$



LRT → $p_e(x, t) = \int_{-\infty}^{t} e^{\bar{L}(t-s)} L_e(\bar{p}) ds$

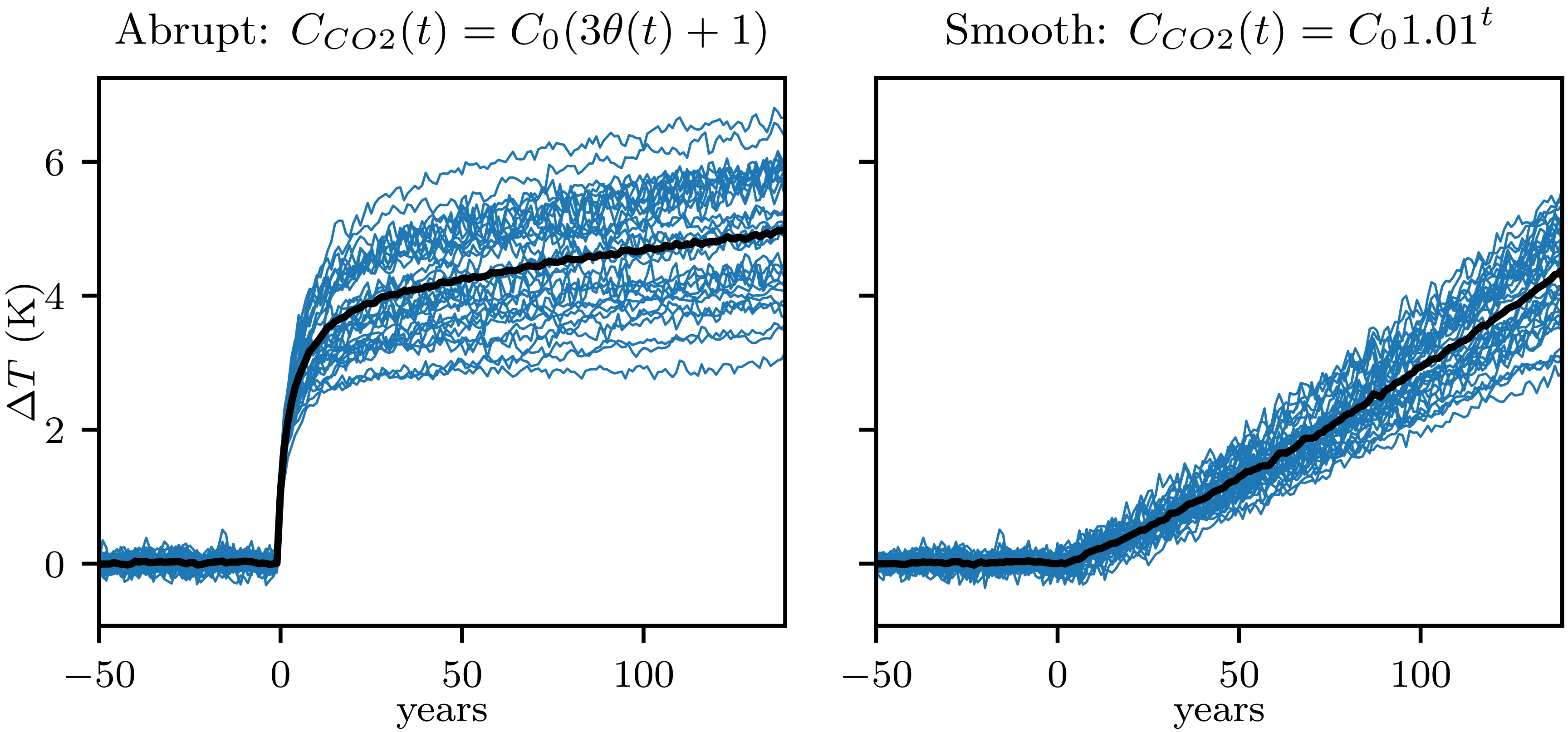
Linear Response Theory (LRT)



$$\Delta T_{\Delta F} = \int_0^t G_T(t-s)\Delta F(s)ds$$

Using LRT one can determine the response to any forcing!

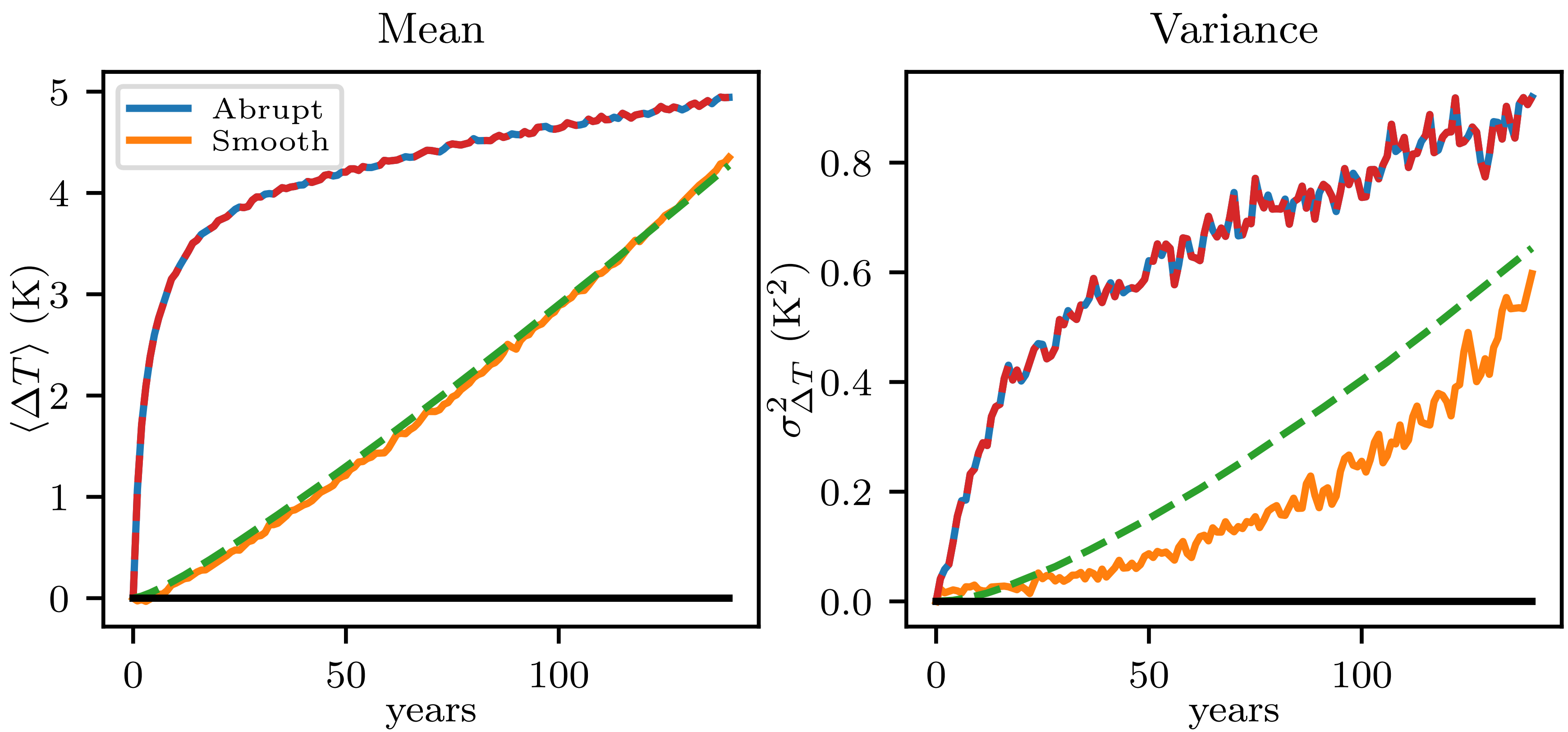
CMIP5 simulations



Abrupt forcing

Smooth forcing

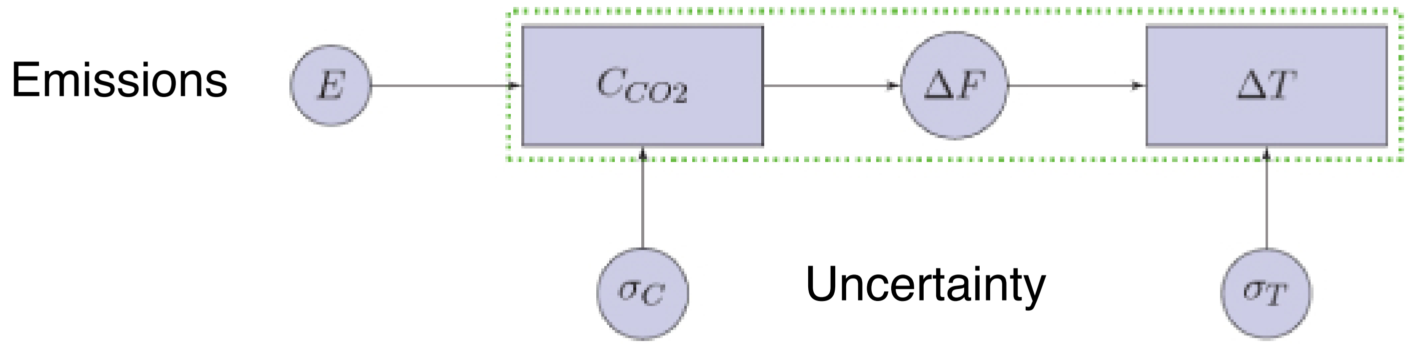
Results Linear Response Theory



solid: CMIP5

dashed: LRT

Coupling a Carbon Model



- Carbon Model (Joos et al., 2013):

$$G_{CO2}(t) = a_0 + \sum_{i=1}^{3} a_i e^{-\frac{t}{\tau_i}}$$

- Full Reponse Function Model

$$C_{CO2}(t) = C_{CO2,0} + \int_0^t G_{CO2}(\tau)\ E_{CO2}(t-\tau)\ d\tau$$

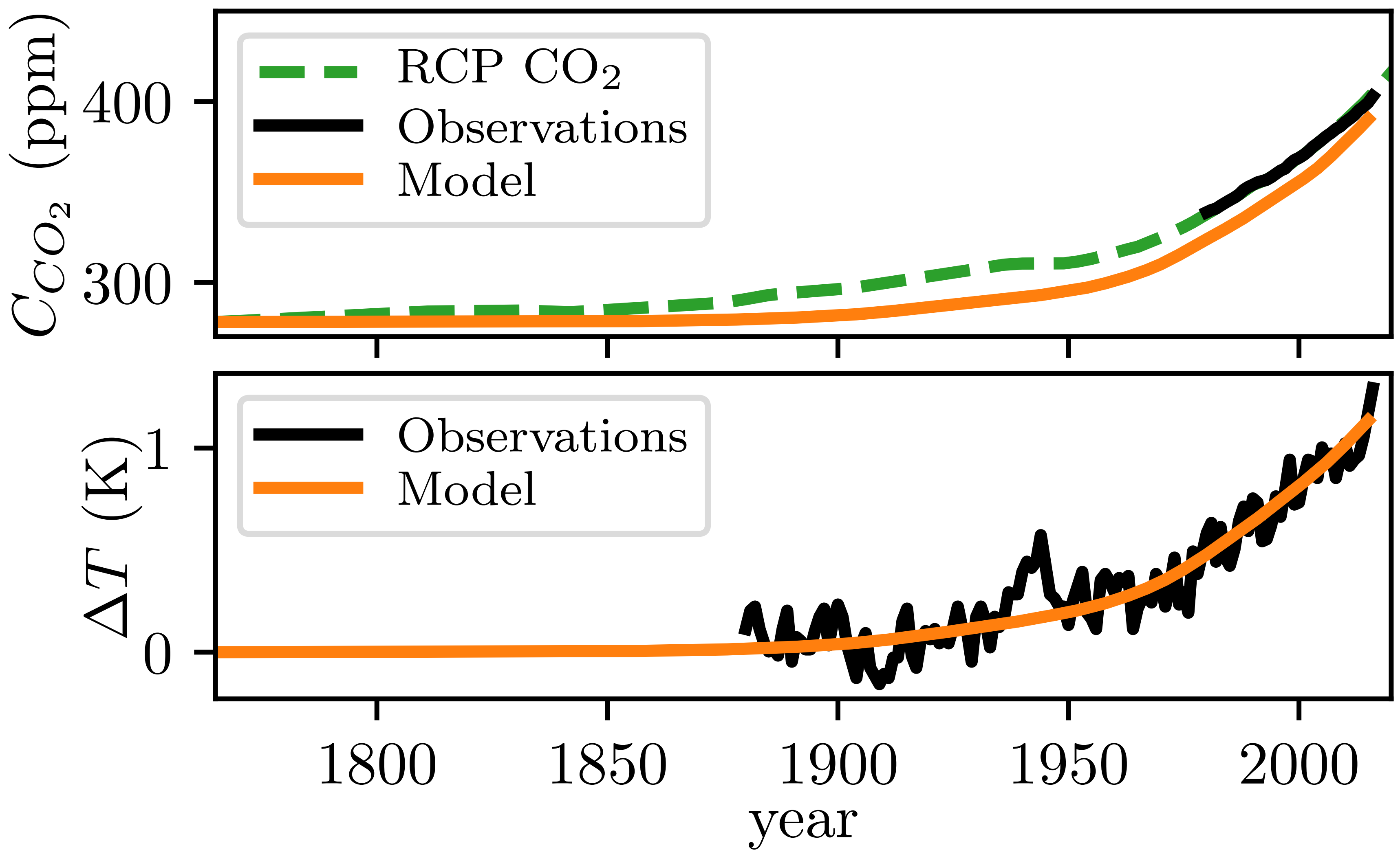
$$\Delta F_{CO2} = A\ \alpha_{CO2} \ln(C/C_0)$$

$$\Delta T(t) = \int_0^t G_T(\tau) \Delta F_{CO2}(t-\tau) d\tau$$

- We also find:

$$G_T(t) = \sum_{i=0}^{2} b_i e^{-t/\tau_{bi}}$$

Carbon Model Performance



Stochastic State Space Model

Carbon

$$dC_P = a_0 E dt$$

$$dC_1 = \left(a_1 E - \frac{1}{\tau_1} C_1 \right) dt$$

$$dC_2 = \left(a_2 E - \frac{1}{\tau_2} C_2 \right) dt$$
$$+ \sigma_{C2} \, dW_t$$

$$dC_3 = \left(a_3 E - \frac{1}{\tau_3} C_3 \right) dt$$

$$C_{CO2} = C_P + \sum_{i=1}^{3} C_i$$

Temperature

$$\Delta F = A \, \alpha \ln(C_{CO2}/C_0)$$

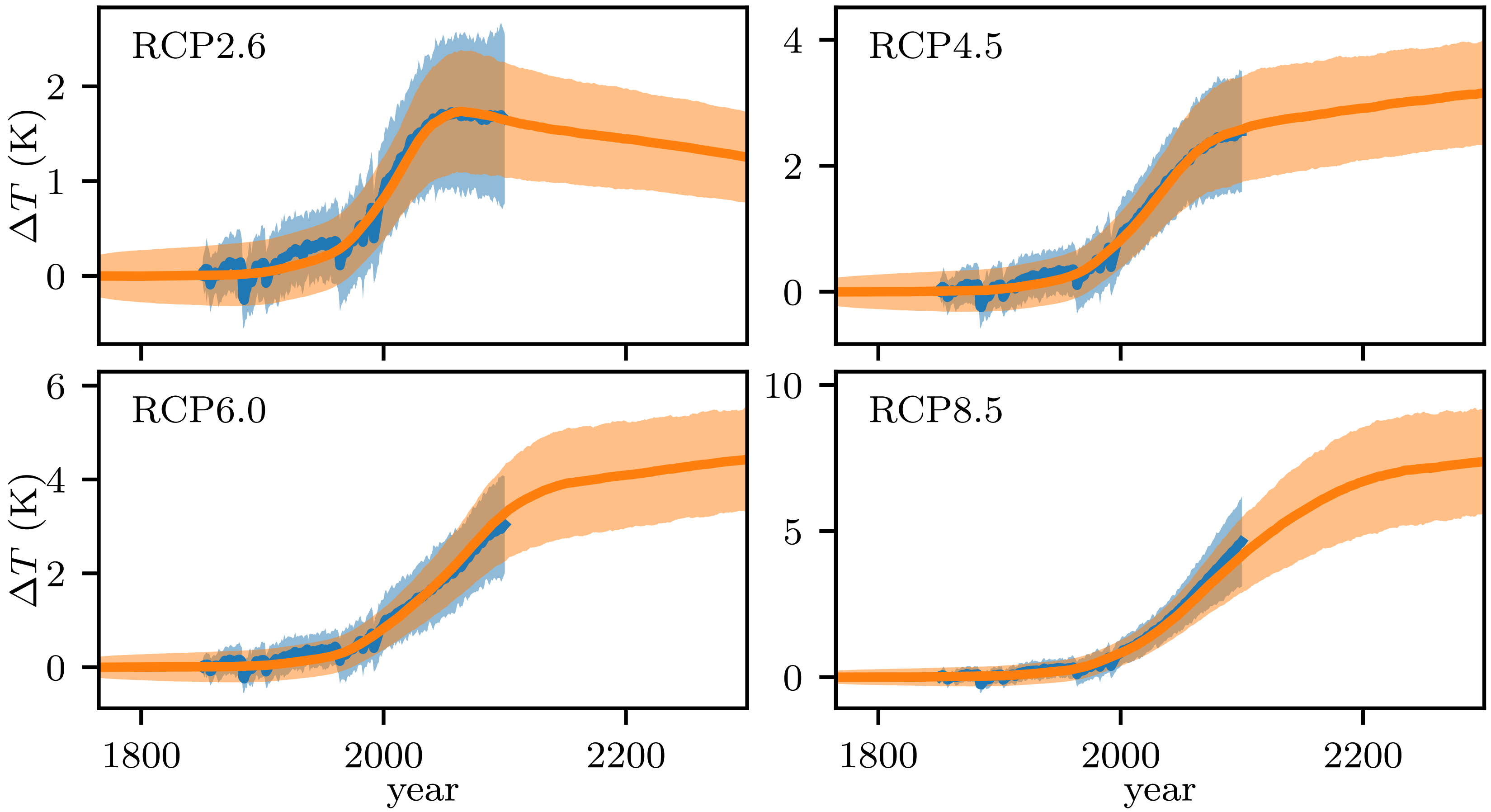
$$d\Delta T_0 = \left(b_0 \Delta F - \frac{1}{\tau_{b0}} \Delta T_0 \right) dt$$
$$+ \sigma_{T0} \, dW_t$$

$$d\Delta T_1 = \left(b_1 \Delta F - \frac{1}{\tau_{b1}} \Delta T_1 \right) dt$$

$$d\Delta T_2 = \left(b_2 \Delta F - \frac{1}{\tau_{b2}} \Delta T_2 \right) dt$$
$$+ \sigma_{T2} \Delta T_2 \, dW_t$$

$$\Delta T = \sum_{i=0}^{2} \Delta T_i$$

Results: RCP reconstruction



envelopes: 2 standard deviation

Save Carbon Budget

Maximum Cumulative Emissions that reach a certain warming target

$$p(\Delta T \leq T_{max}) = \beta$$

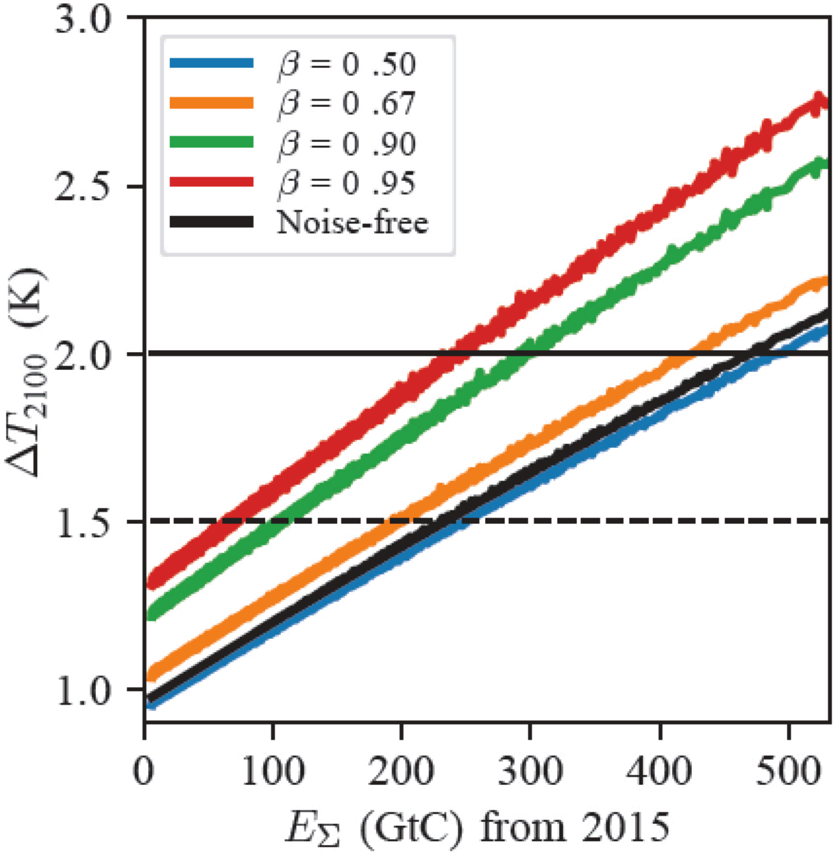


Table 3. Safe Carbon Budget (in GtC since 2015) as a function of threshold and safety probability β.

β	0.5	0.67	0.9	0.95	Noise-free
$T_{max} = 1.5$ K	247	198	107	69	233
$T_{max} = 2$ K	492	424	298	245	469

IPCC-AR5: 377 - 517 GtC to likely stay below 2 K

Millar et al. (2017): 200 GtC to likely stay below 1.5 K

Point of No Return

- Use economical assumptions to determine emissions
  $\Rightarrow$ baseline 'business-as-usual' scenario
- Control emissions by mitigation $m(t)$ and abatement $a(t)$ $\Rightarrow$ actions on climate change modify 'business-as-usual' scenario

Definition (Point of No Return)

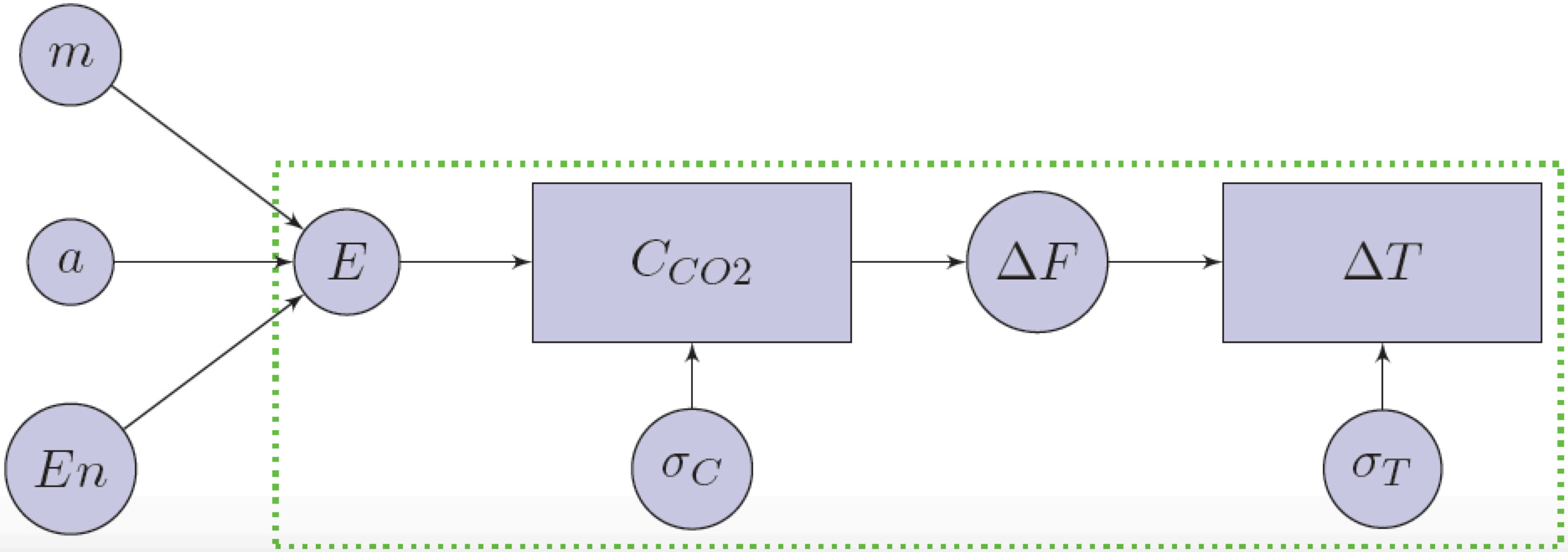
The Point of No Return (PONR) is the time t_P from which on no allowed $[a(t), m(t)]$ such that $0 \leq a(t), m(t) \leq 1, t_P \leq t \leq t_f$ can be chosen to fulfill

$$p(\Delta T(t_f) \leq T_{max}) \geq \beta \tag{9}$$

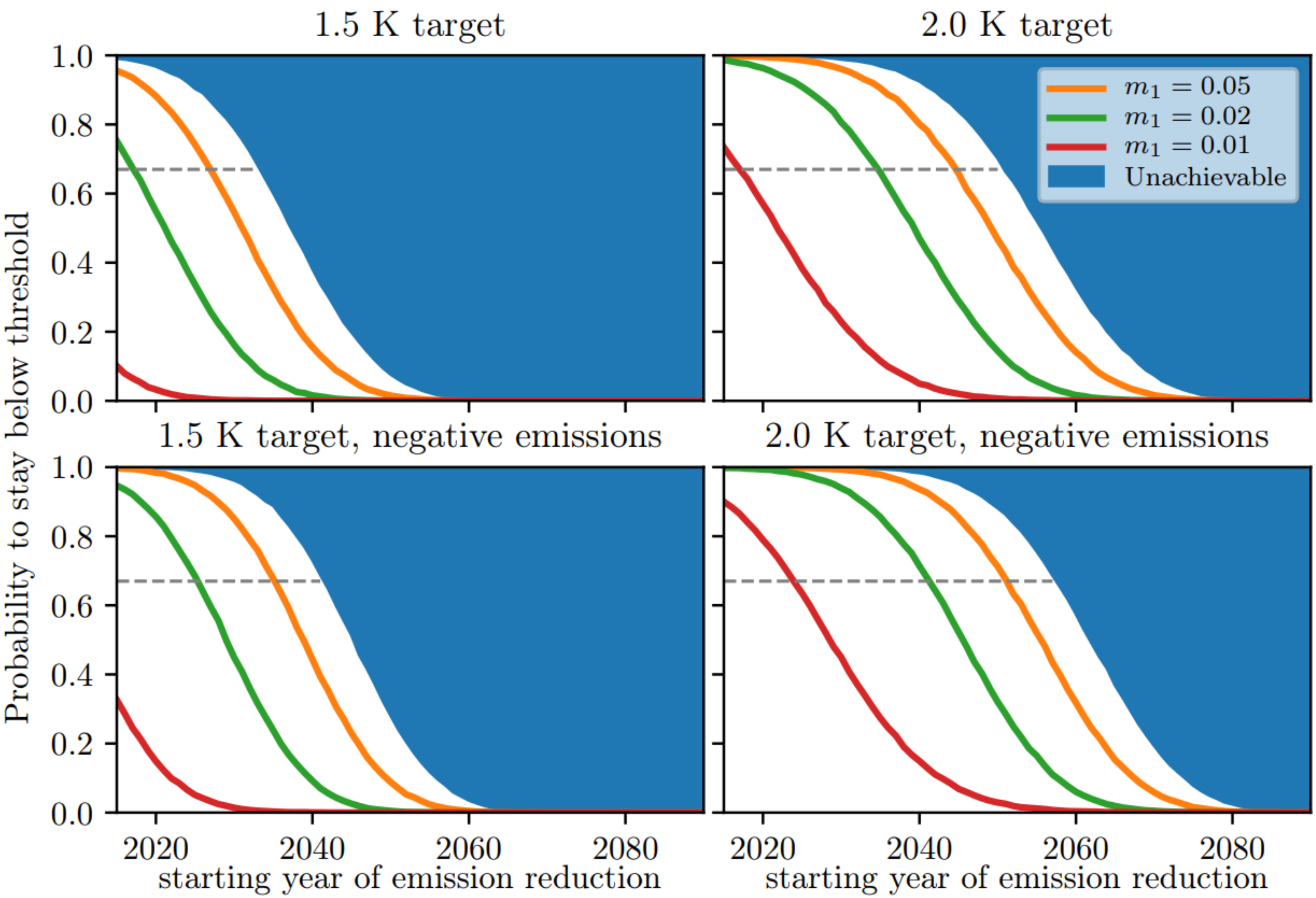
Economy & Transition pathways

$$Y = Y_0 e^{gt}$$
$$En = \gamma_0 e^{-r_\gamma t} Y$$
$$E = (1-a)(1-m)En$$

1. Extreme Mitigation (EM): From time t_a on, we set $m = 1$, i.e. $E = 0$.
2. Fast Mitigation (FM): From time t_a on, both a, m increase by 0.05 per year.
3. Ambitious Mitigation (AM): As FM, but the increase is 0.02 per year.



PNR: results



Probability of not exceeding 'safe' temperature thresholds in 2100

PONR: results

1. Extreme Mitigation (EM): At time t_a, we set $m = 1$, so $E = 0$ from then onward.
2. Fast Mitigation (FM): From time t_a onwards, both a, m increase by 0.05 per year.
3. Ambitious Mitigation (AM): As FM, but the increase is 0.02 per year.

Table 4. Point of no return as a function of threshold and safety probability β without and with strong negative emissions.

	β	0.5		0.67		0.9		0.95		noise-free	
	E_{neg}	none	strong	none	strong	none	strong	none	strong	none	strong
EM	$T_{max} = 1.5\,K$	2038	2046	2034	2042	2026	2035	2022	2032	2037	2045
	$T_{max} = 2\,K$	2056	2062	2051	2058	2042	2049	2038	2046	2055	2061
FM	$T_{max} = 1.5\,K$	2032	2039	2027	2036	2020	2028	2016	2025	2030	2038
	$T_{max} = 2\,K$	2050	2056	2045	2052	2036	2043	2032	2039	2048	2055
MM	$T_{max} = 1.5\,K$	2022	2029	2018	2026	–	2019	–	–	2021	2029
	$T_{max} = 2\,K$	2040	2046	2035	2042	2026	2033	2022	2030	2038	2045

Aengenheyster et al. (2018)

Question time

Summary

- Without drastic emission reductions, the Earth will have warmed ~ 3C by the year 2100.
- CMIP5/6 GCMs are not fit for purpose to capture possible bumpy transitions , so unpleasant surprises may be ahead
- Easy to communicate metrics:
    - SCB: “We cannot reach target X when emitting more than SCB”
    - PONR: “We cannot reach target X when starting after year PONR”

PONR with realistic action pathways is close (2035 for 67%) for the 2K target and already passed for the 1.5K target.

URGENT action is needed to avoid dangerous climate change!