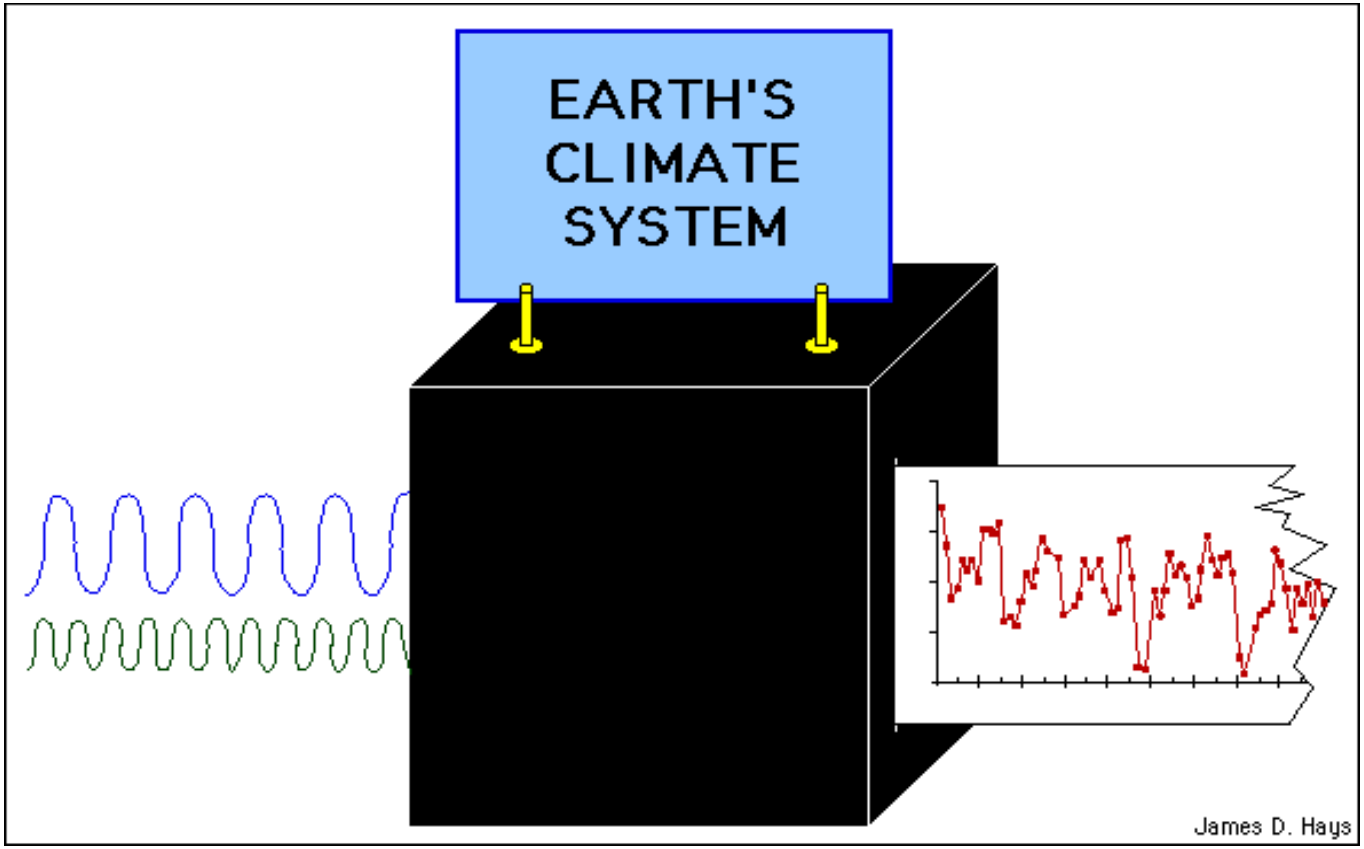
Stochastic Climate Dynamics



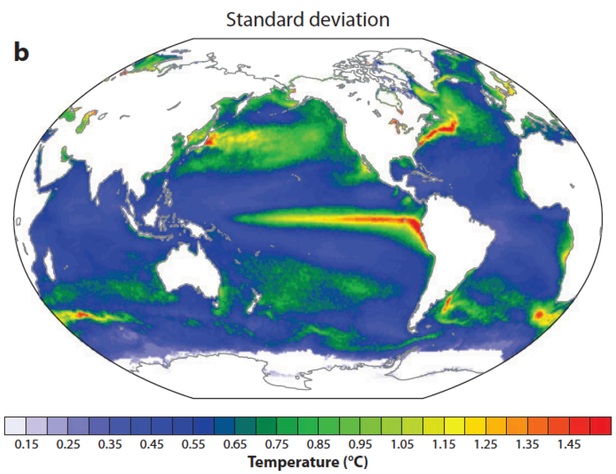
Henk Dijkstra, IMAU & CCSS
Physics Department, Utrecht University, Utrecht, Netherlands

https://webspace.science.uu.nl/~dijks101/styled-6/

Summary 29-9-2020: SST variability

Ocean mixed layer temperature: $$dX_t = -\gamma X_t dt + \sigma dW_t$$

Wiener process

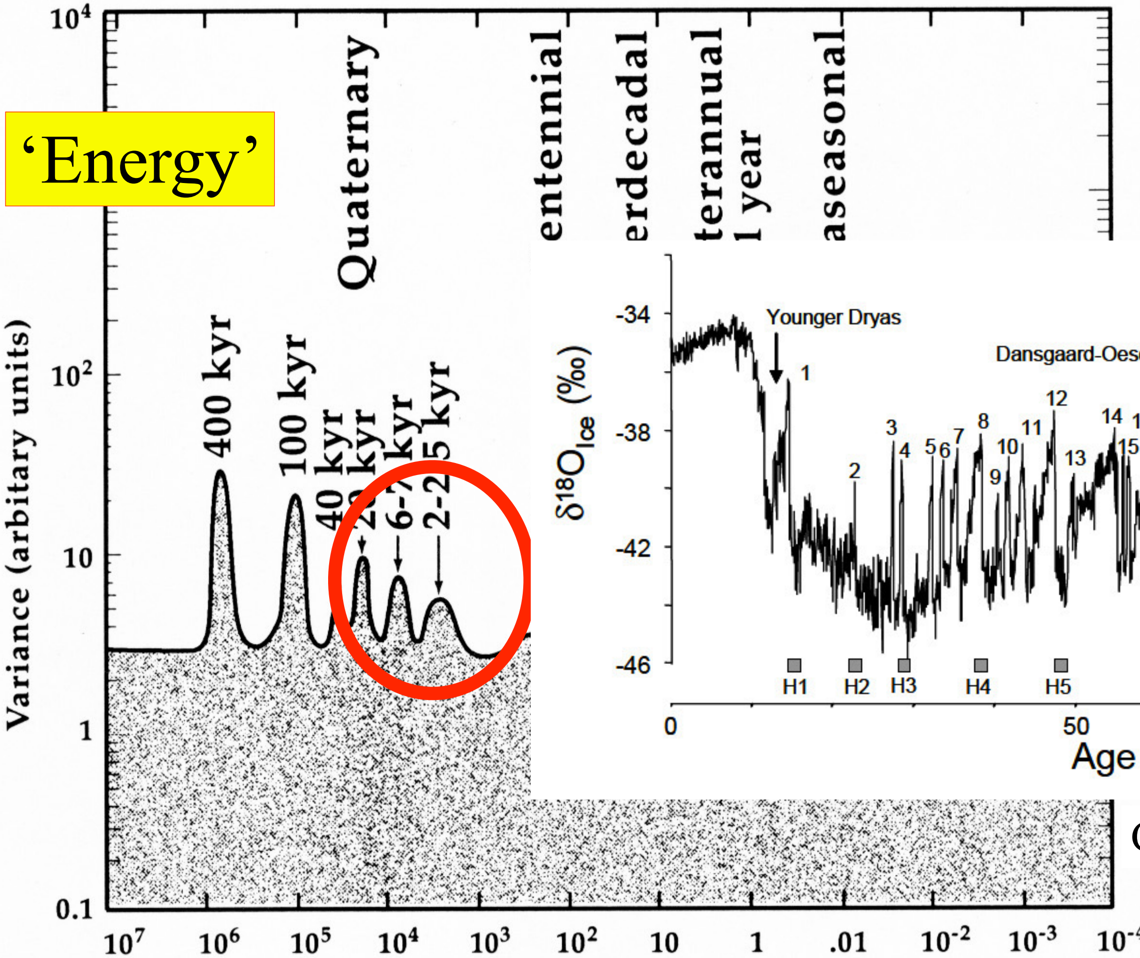


$$X_t = e^{-\gamma t}(X_0 + \sigma \int_0^t e^{\gamma s} dW_s)$$

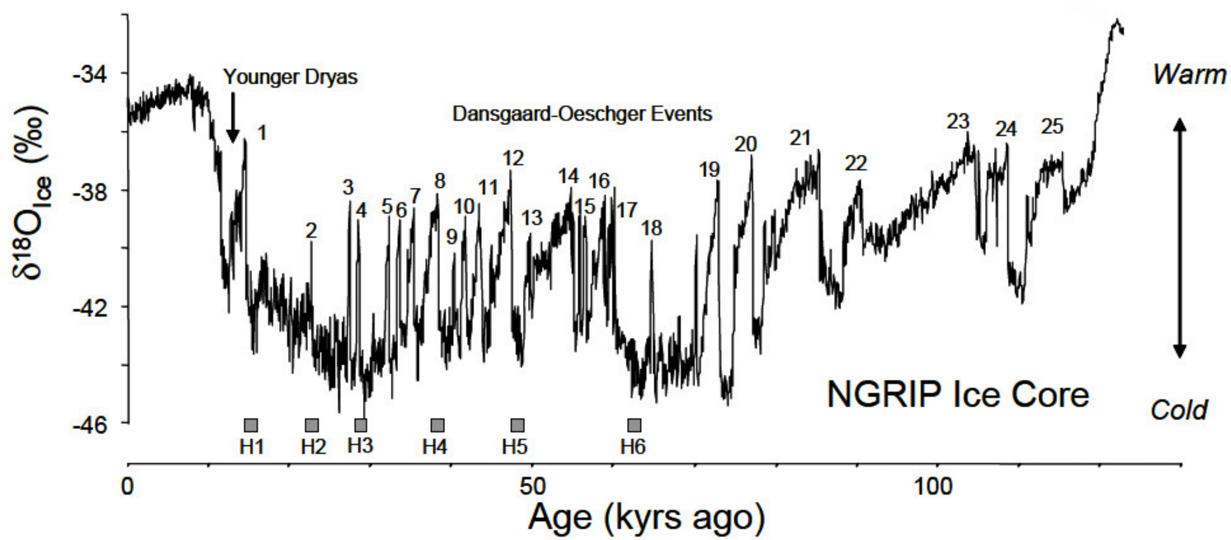
Continuous: Ornstein-Uhlenbeck process
Discrete: red noise or AR(1) process

The red noise spectrum serves as a null-hypothesis for SST variability!

Understanding Dansgaard-Oeschger events?

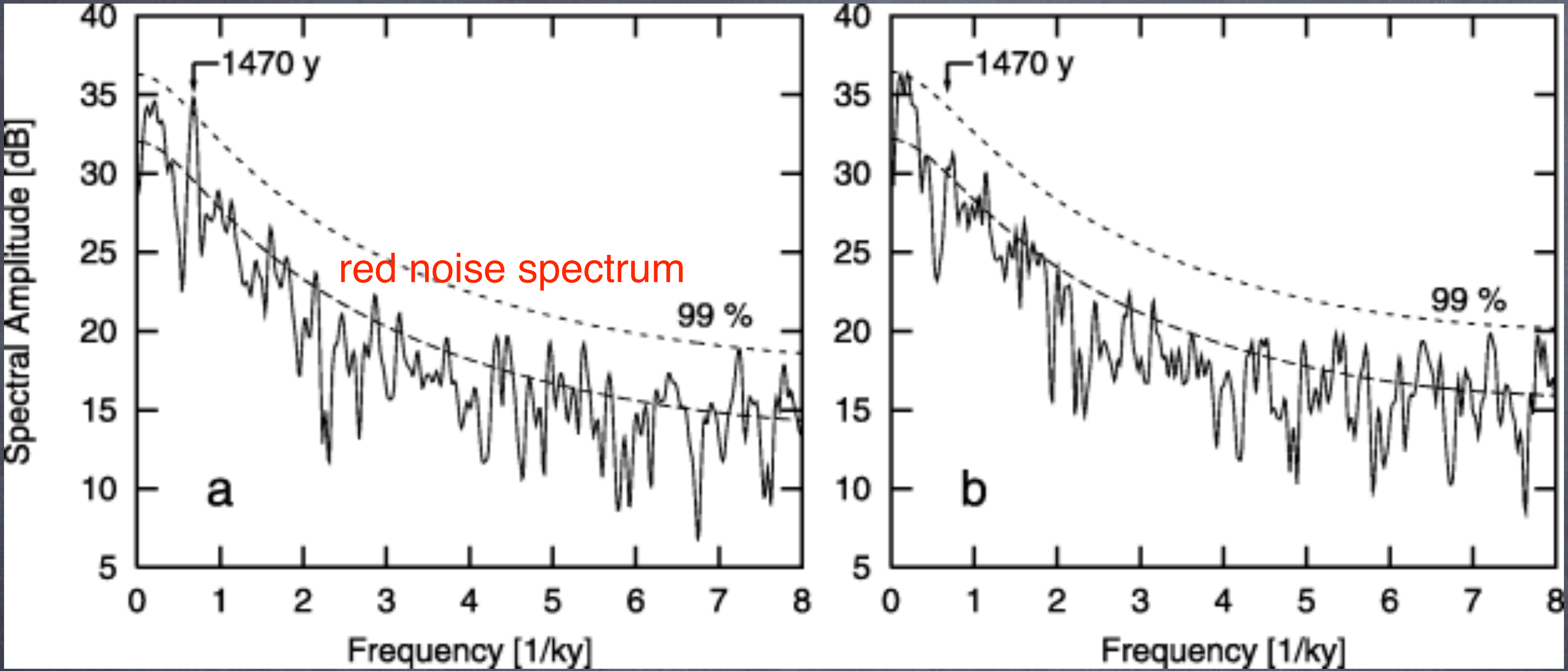


Mitchell, 1978



Clement et al., Rev. Geophysics (2008)

Spectral characteristics



13-50 kyr BP

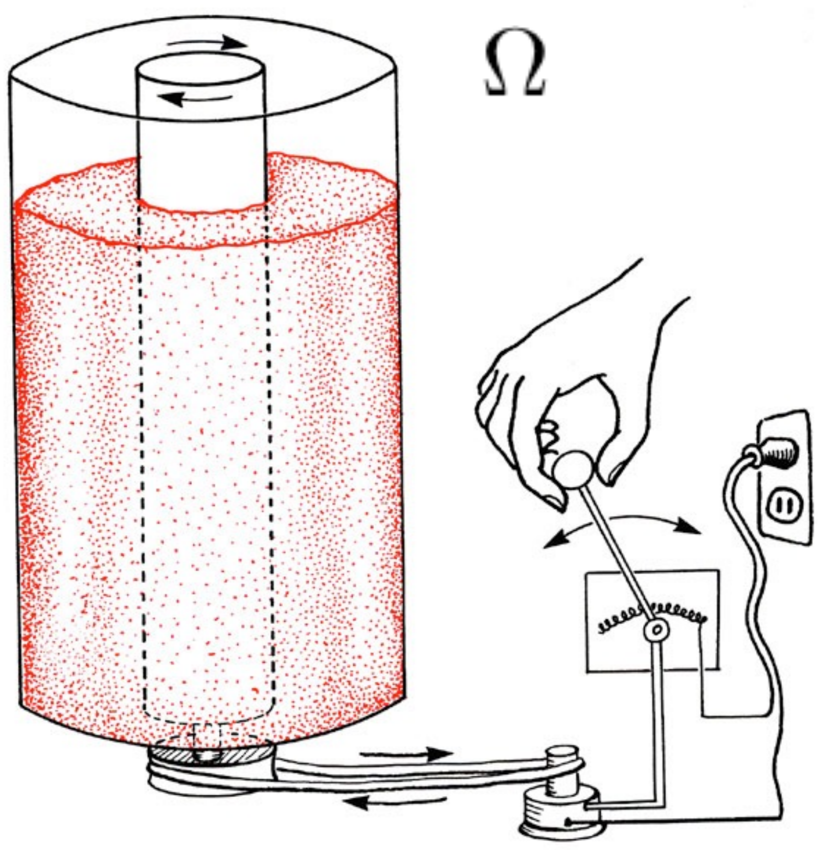
same but omitting 31-36 kyr BP

Which processes determine the preference for the millennial time scale?

Schultz, Paleoceanography (2002)

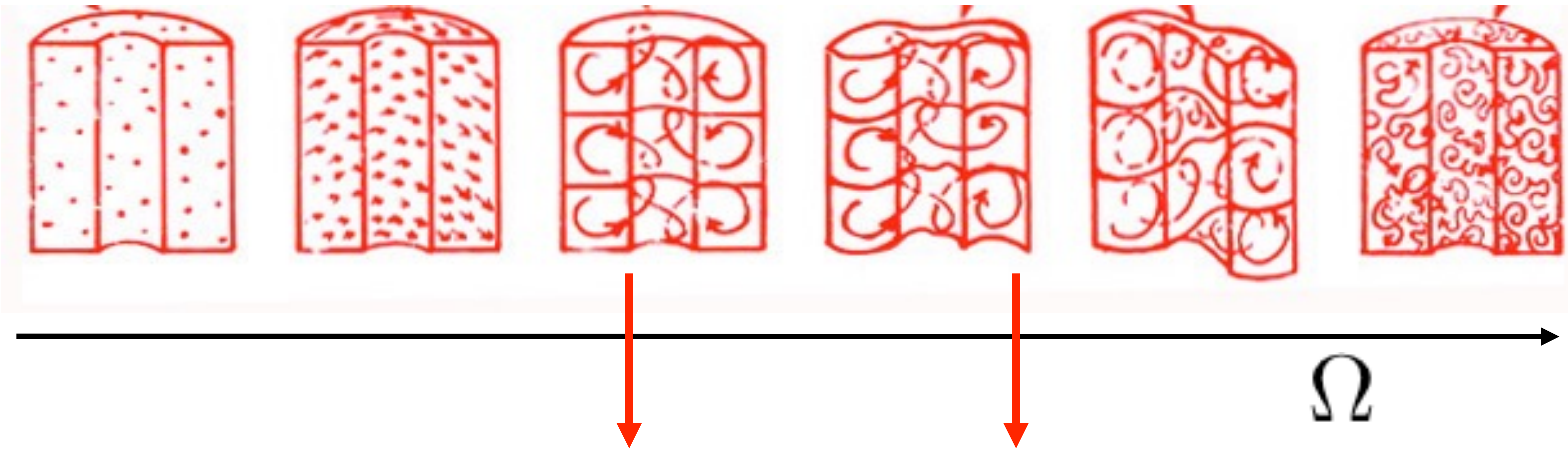
Dynamical systems approach

The Taylor-Couette Flow

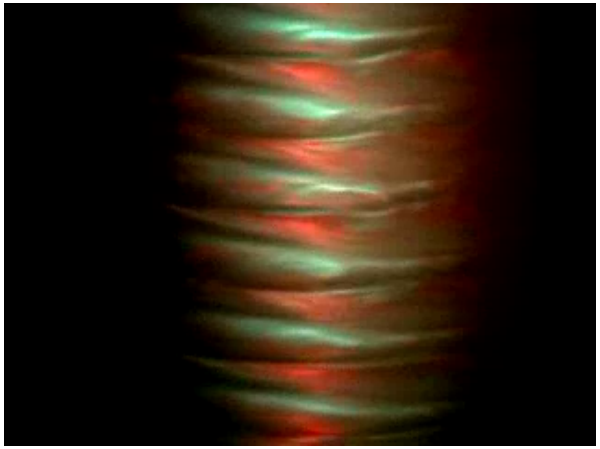


Abraham, R. H. and Shaw, C. D.,
Dynamics, the Geometry of Behavior,
(1988)

Transition behavior



Taylor vortices



Wavy vortices

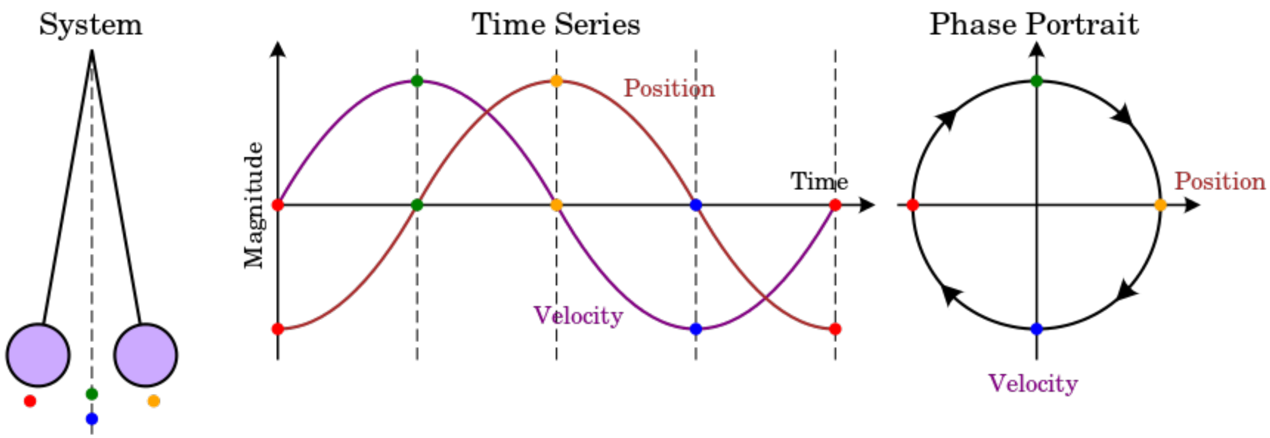
Phase/State space

$$x = \theta \quad ; \quad y = \frac{d\theta}{dt}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

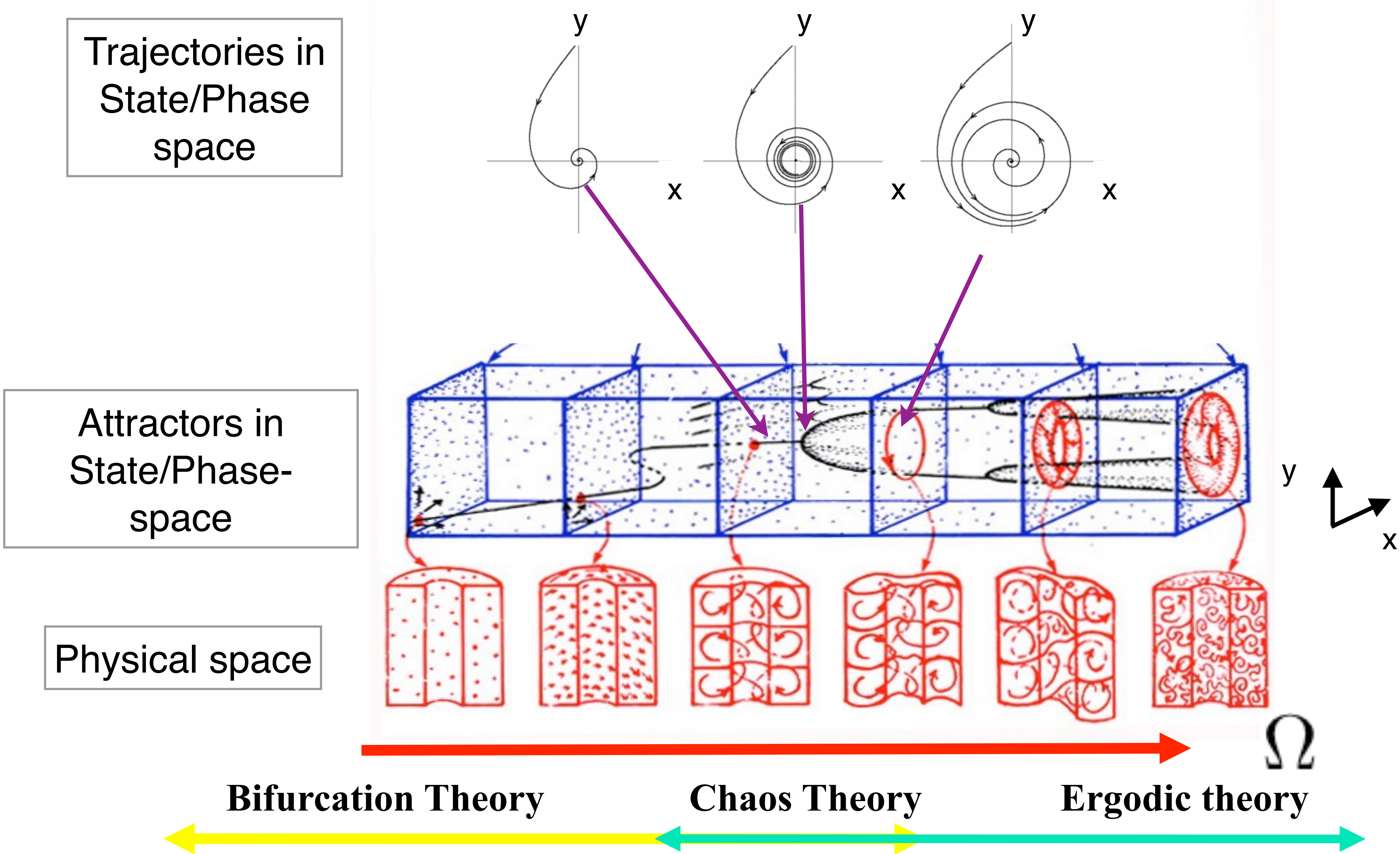
$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -\frac{g}{L}x$$

degrees of freedom d = 2



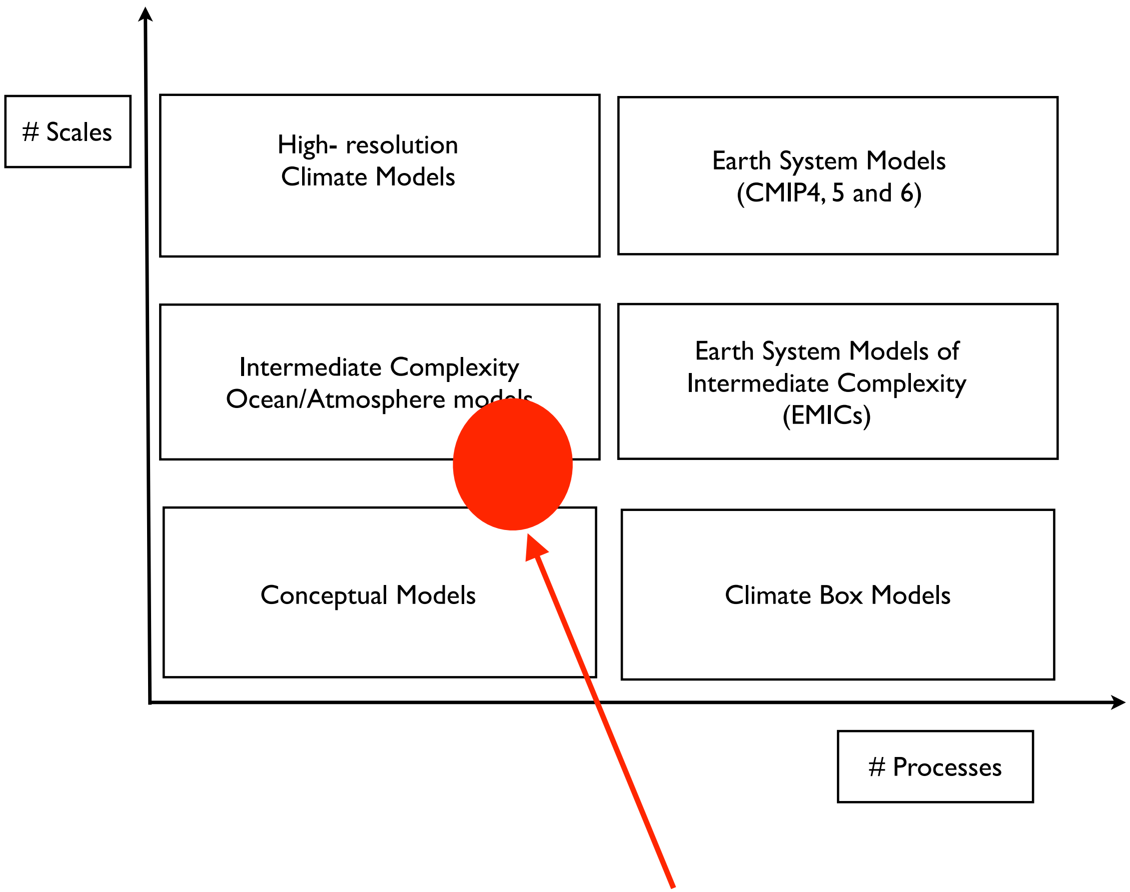
Geometry of motion!

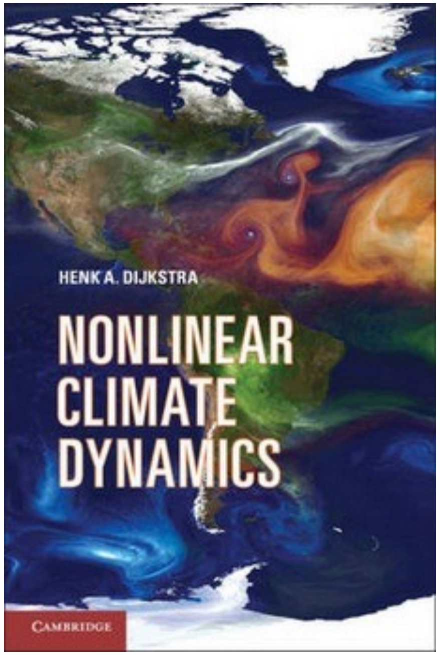
Representations



Steady --> Periodic --> Quasi-periodic --> … --> Irregular (Chaotic) … -> Turbulent

Application to Climate Variability





chapter 6

‘Minimal’ Model:
‘just enough’ processes to capture phenomenon under study

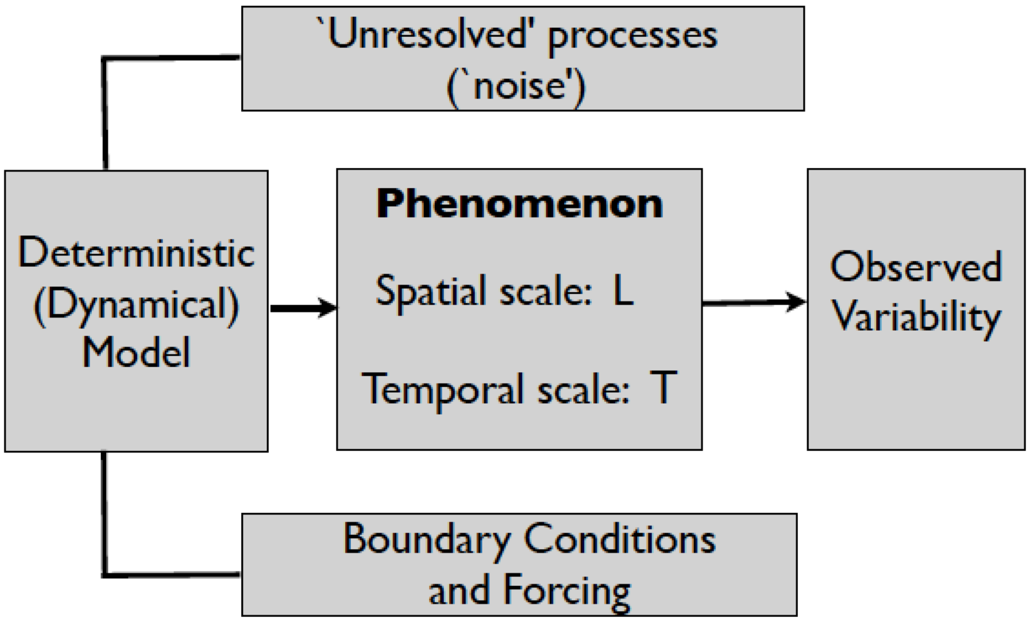
Stochastic dynamical systems approach

Hierarchy of models

Behavior (statistics) over the full parameter space

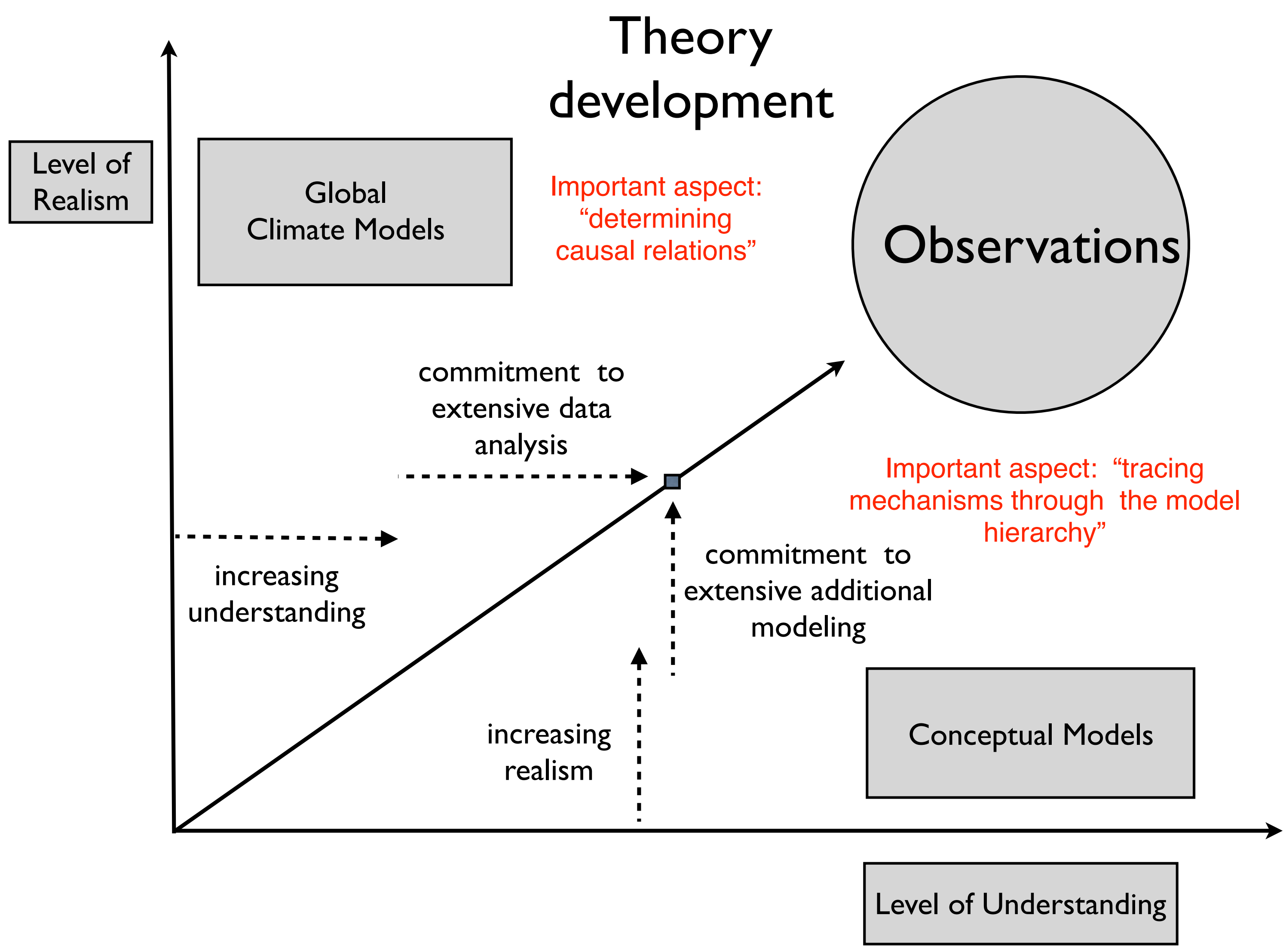
Geometrical view of motion

Dynamical system



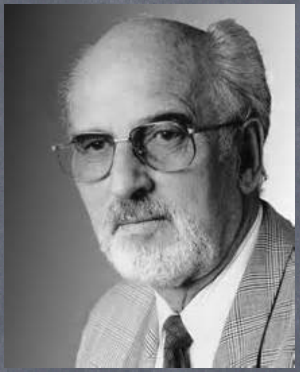
One person’s signal is another person's noise

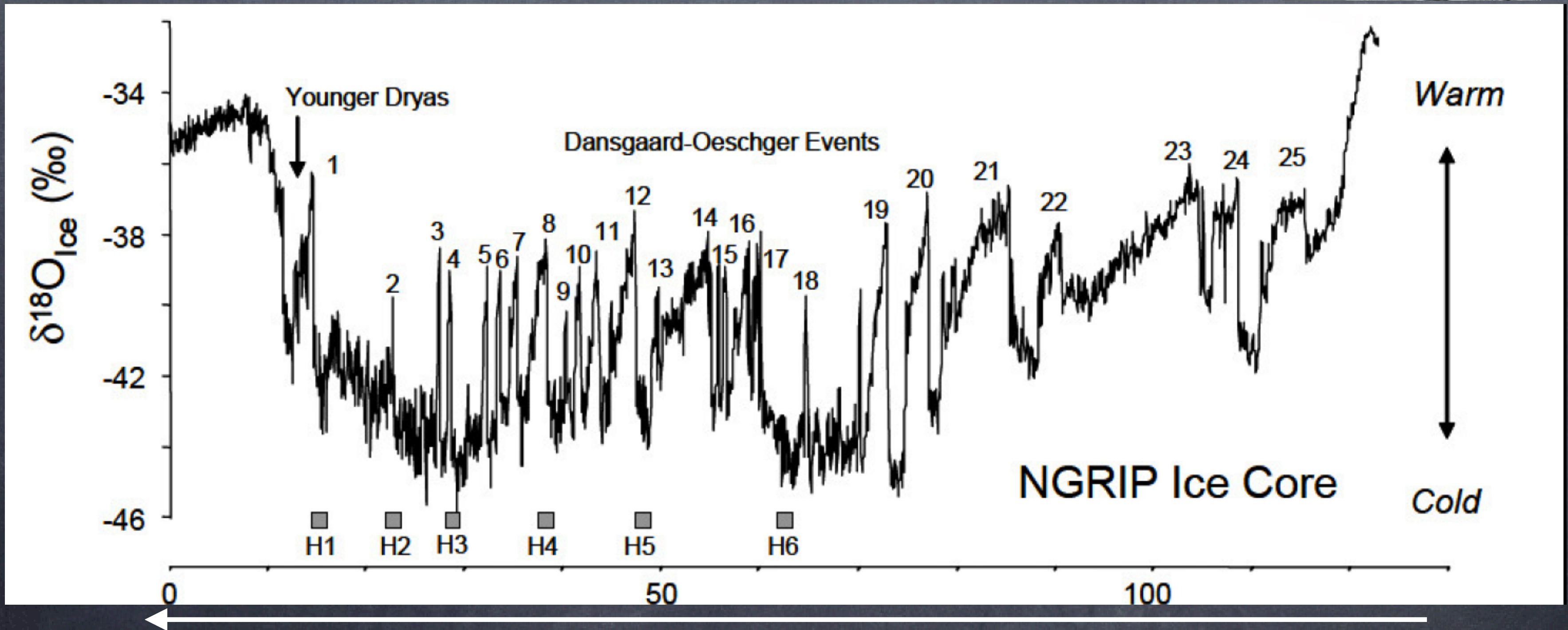
Towards understanding …



Question time

Climate variability on Greenland during the last 100,000 years

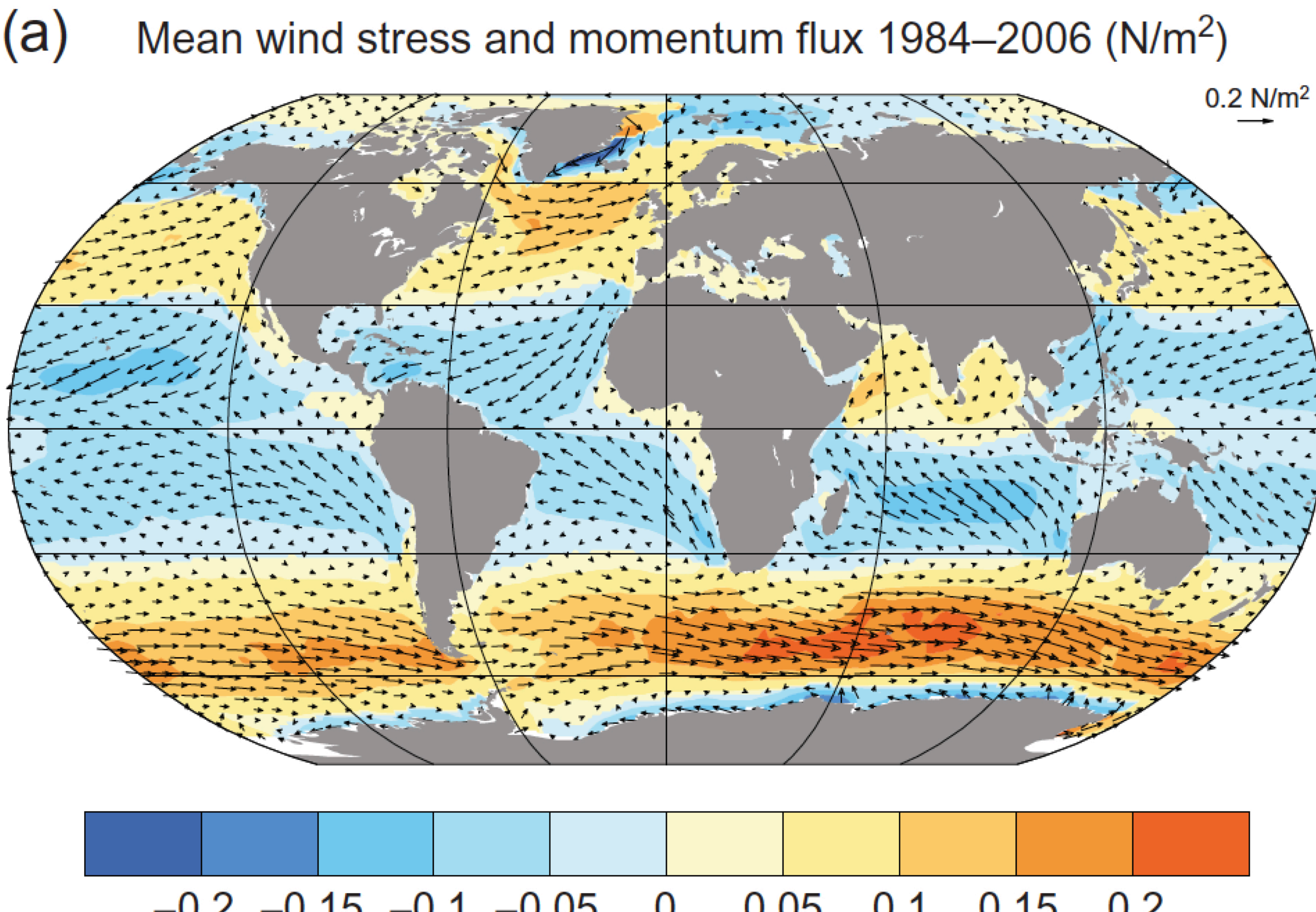




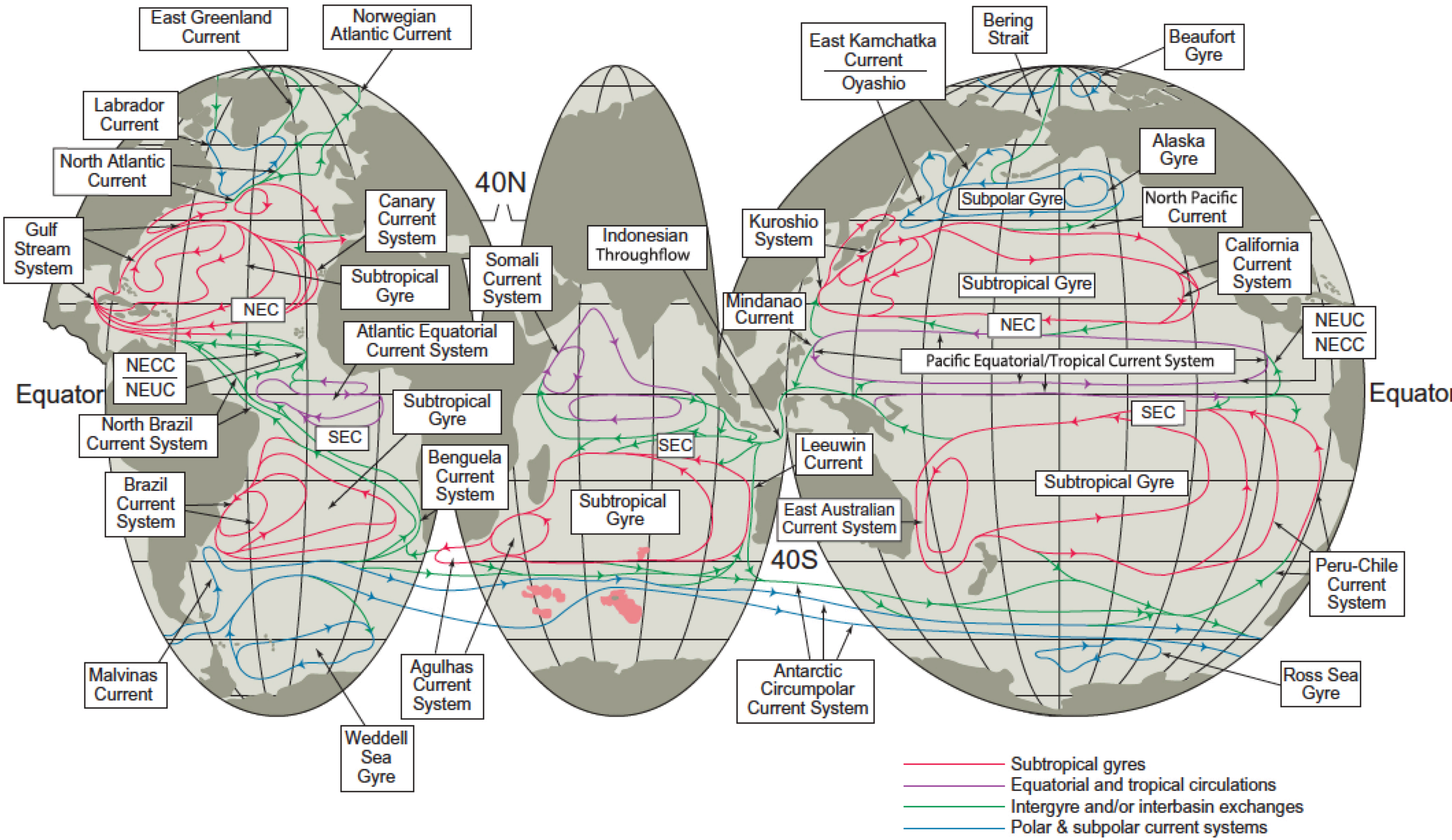
time (kyr)

Clement et al., Rev. Geophysics (2008)

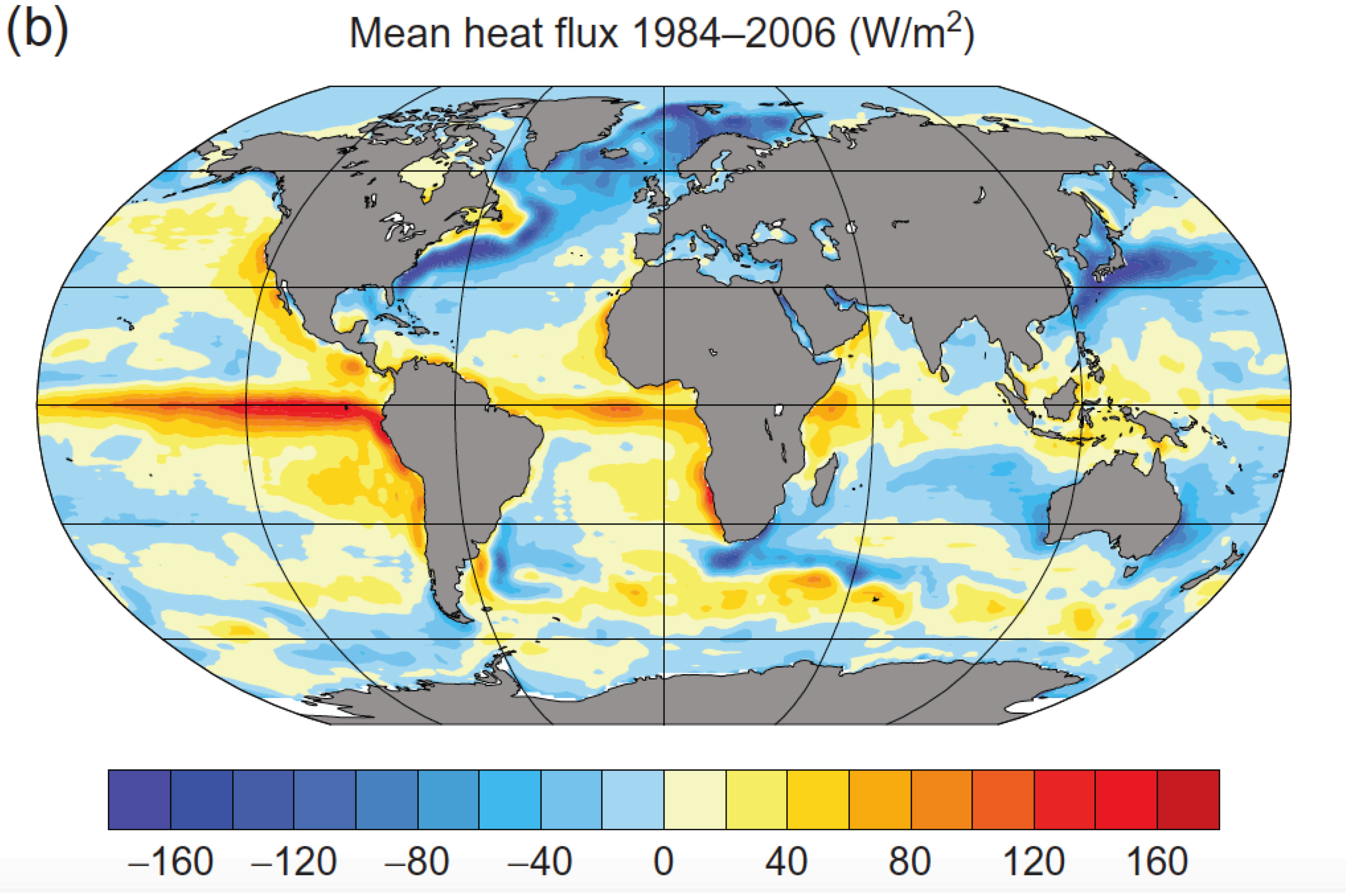
Wind stress


(a)
Mean wind stress and momentum flux 1984–2006 (N/m2)
0.2 N/m2
–0.2
–0.15
–0.1
–0.05
0
0.05
0.1
0.15
0.2

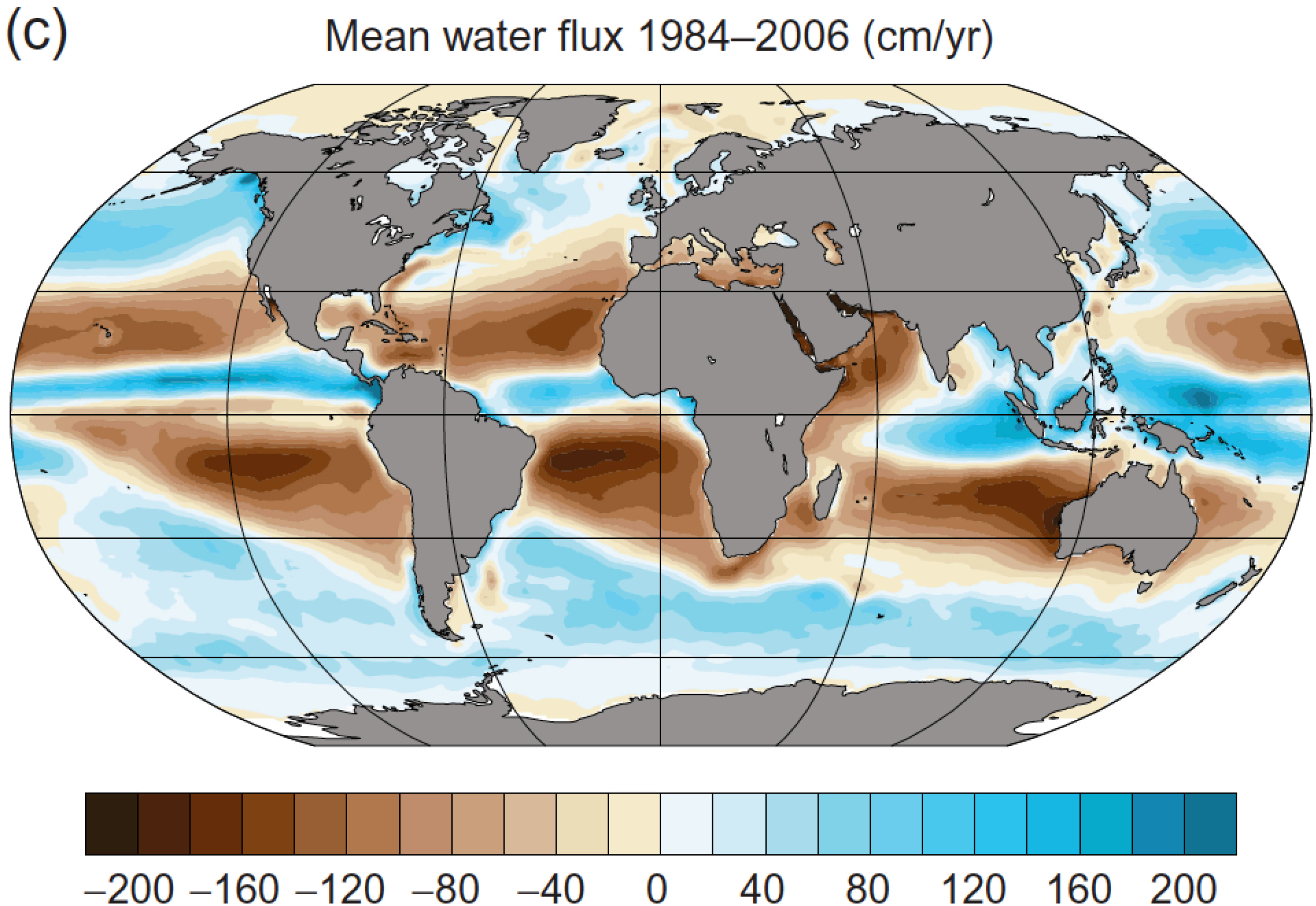
Surface Circulation



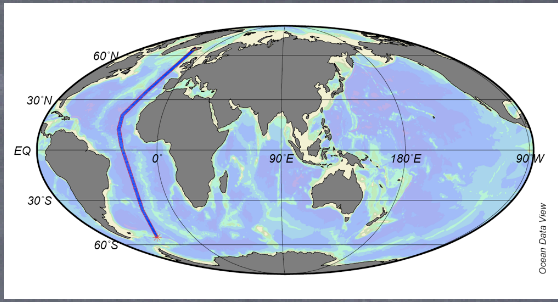
Heat Flux

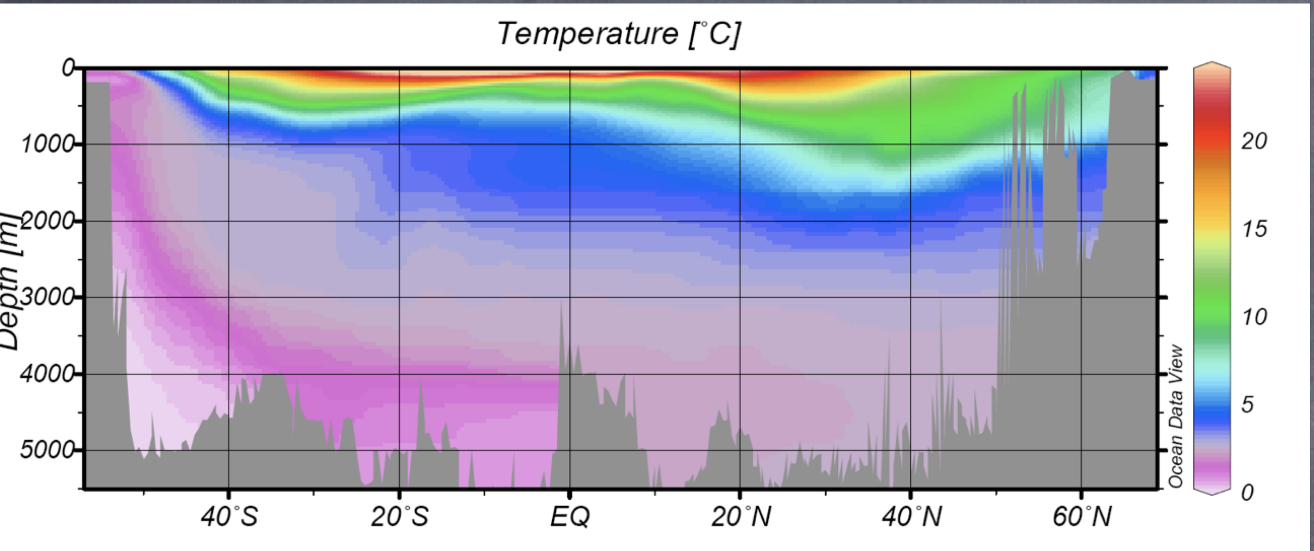


Fresh water flux



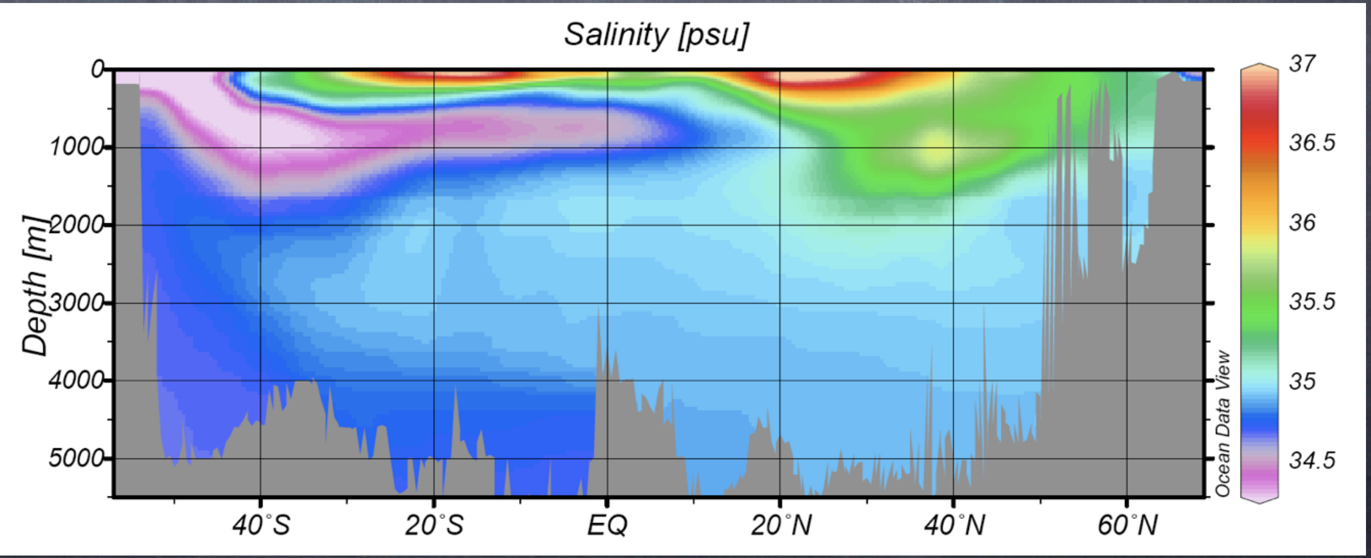
Along Atlantic N-S section:





Temperature

(C)



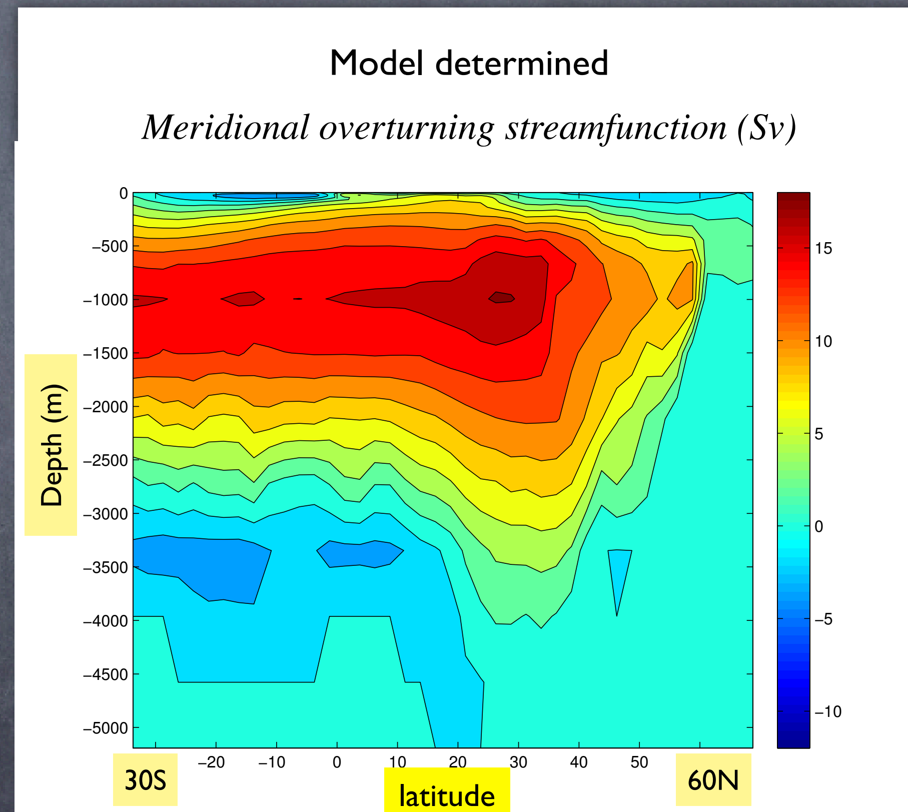
Salinity

(ppt)

Meridional Overturning Circulation (MOC)

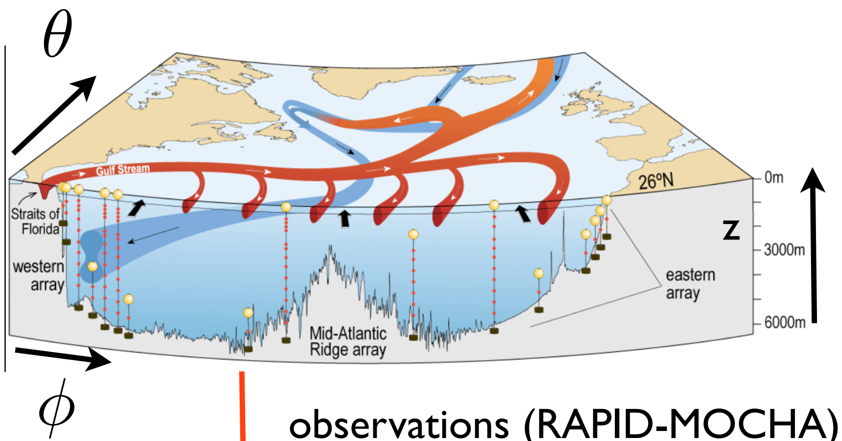
$$\Psi(y,z,t) = \int_z^0 \int_{x_w}^{x_e} v(x,y,z',t)\,dx\,dz'$$

v: meridional velocity

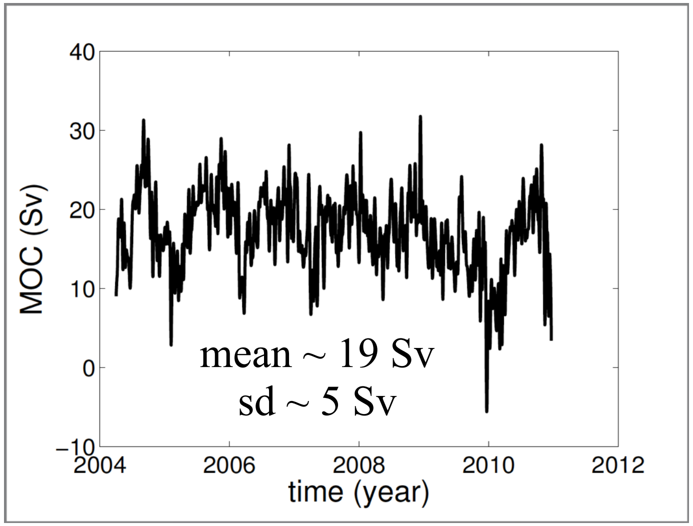


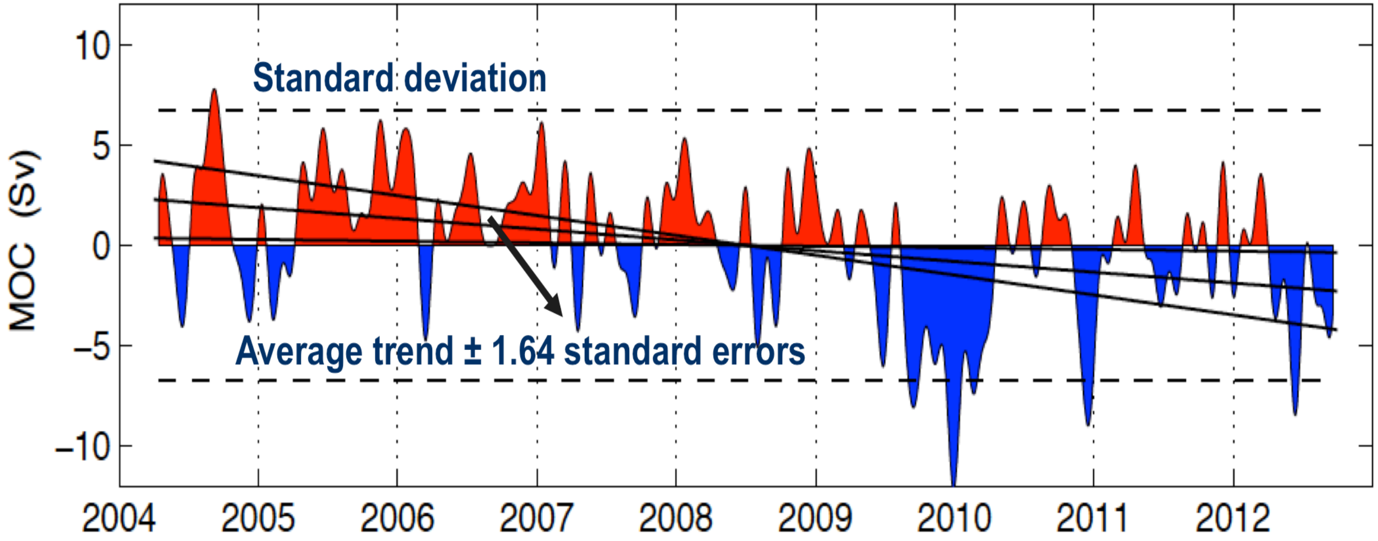
MOC: Volume transport in latitude/depth (is observable)

Changes in the MOC



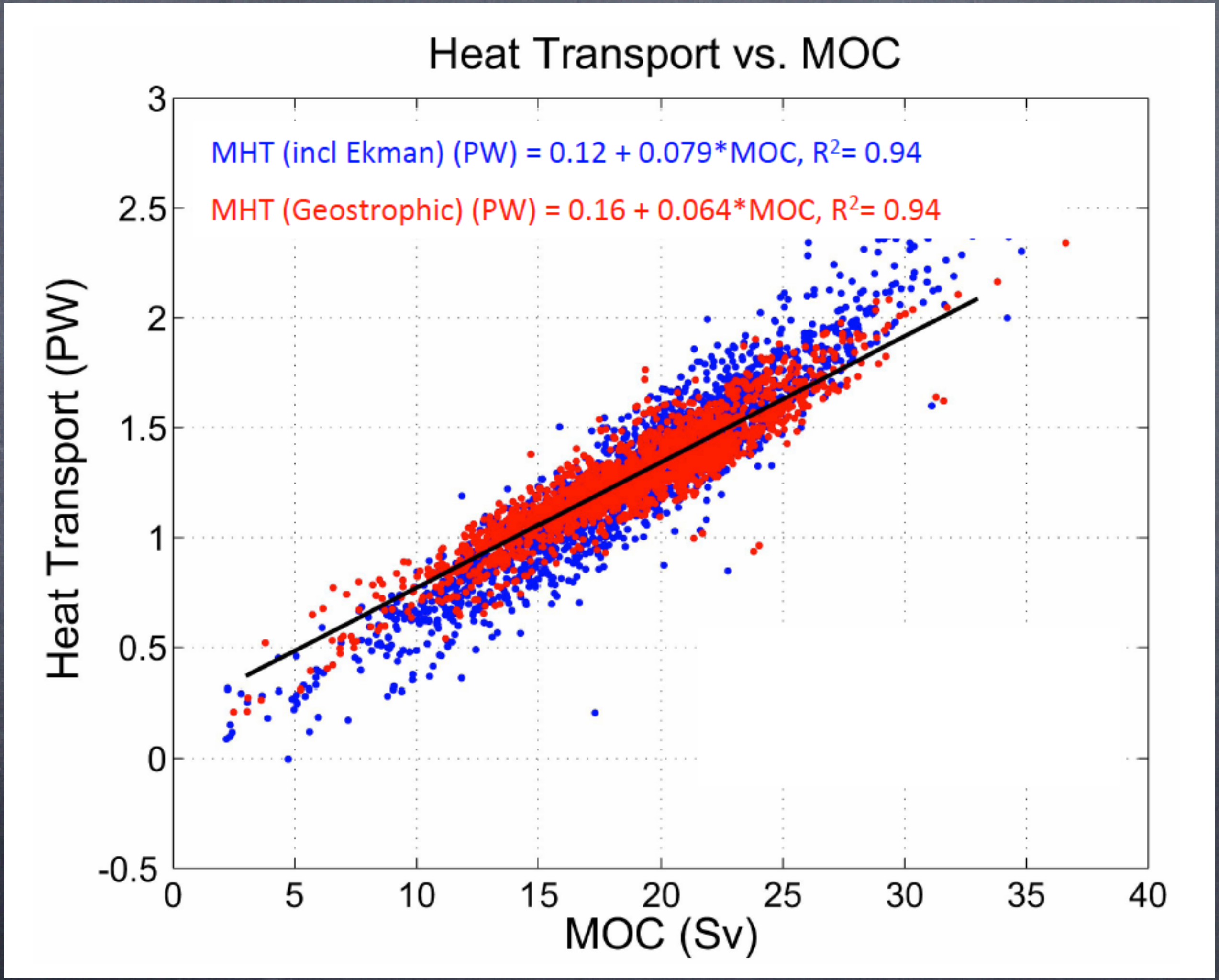
observations (RAPID-MOCHA)





Smeet et al., Ocean Science, (2014)

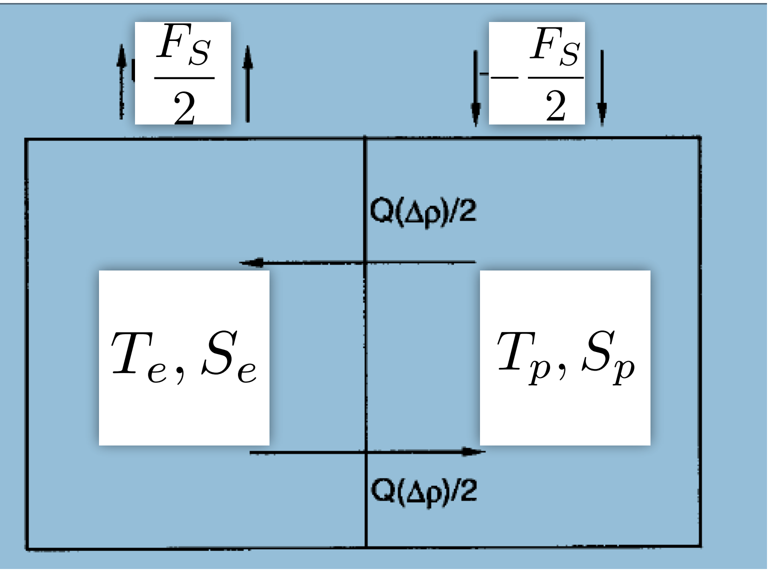
Connection MOC and meridional heat transport



Johns et al., JClim, (2010)

The Stommel model

$$\frac{dT_e}{dt} = -\frac{1}{t_r}(T_e - (T_0 + \frac{\theta}{2})) - \frac{1}{2}Q(\Delta\rho)(T_e - T_p),$$
$$\frac{dT_p}{dt} = -\frac{1}{t_r}(T_p - (T_0 - \frac{\theta}{2})) - \frac{1}{2}Q(\Delta\rho)(T_p - T_e),$$
$$\frac{dS_e}{dt} = \frac{F_S}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_e - S_p),$$
$$\frac{dS_p}{dt} = -\frac{F_S}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_p - S_e),$$



$$\rho = \rho_0(1 - \alpha_T(T - T_0) + \alpha_S(S - S_0)),$$

$$Q(\Delta\rho) = \frac{1}{t_d} + \frac{q}{\rho_0^2 V}(\Delta\rho)^2,$$

Stommel, Tellus, (1961)

Cessi, JPO, (1994)

Dimensionless model

$$\frac{dx}{dt} = -\alpha(x-1) - x(1+\mu^2(x-y)^2),$$
$$\frac{dy}{dt} = F - y(1+\mu^2(x-y)^2),$$

Parameter	Meaning	Value	Unit
t_r	temperature relaxation time scale	25	days
H	mean ocean depth	4500	m
t_d	diffusion time scale	180	years
t_a	advective time scale	29	years
q	transport coefficient	1.92×10^{12}	$m^3\ s^{-1}$
V	ocean volume	$300 \times 4.5 \times 8200$	km^3
α_T	thermal expansion coefficient	10^{-4}	K^{-1}
α_S	haline contraction coefficient	$7.6 \quad 10^{-4}$	-
S_0	reference salinity	35	$g\ kg^{-1}$
θ	meridional temperature difference	25	K

$$\mu^2 = \frac{q t_d (\alpha_T \theta)^2}{V}$$

$$\alpha = t_d / t_r$$

$$F = \frac{\alpha_S S_0 t_d}{\alpha_T \theta H} F_S.$$

Question time

Dynamical systems: theory

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t, \lambda)$$

$\mathbf{x}$: state vector

λ : parameter

$\mathbf{f}$: vector field

t : time

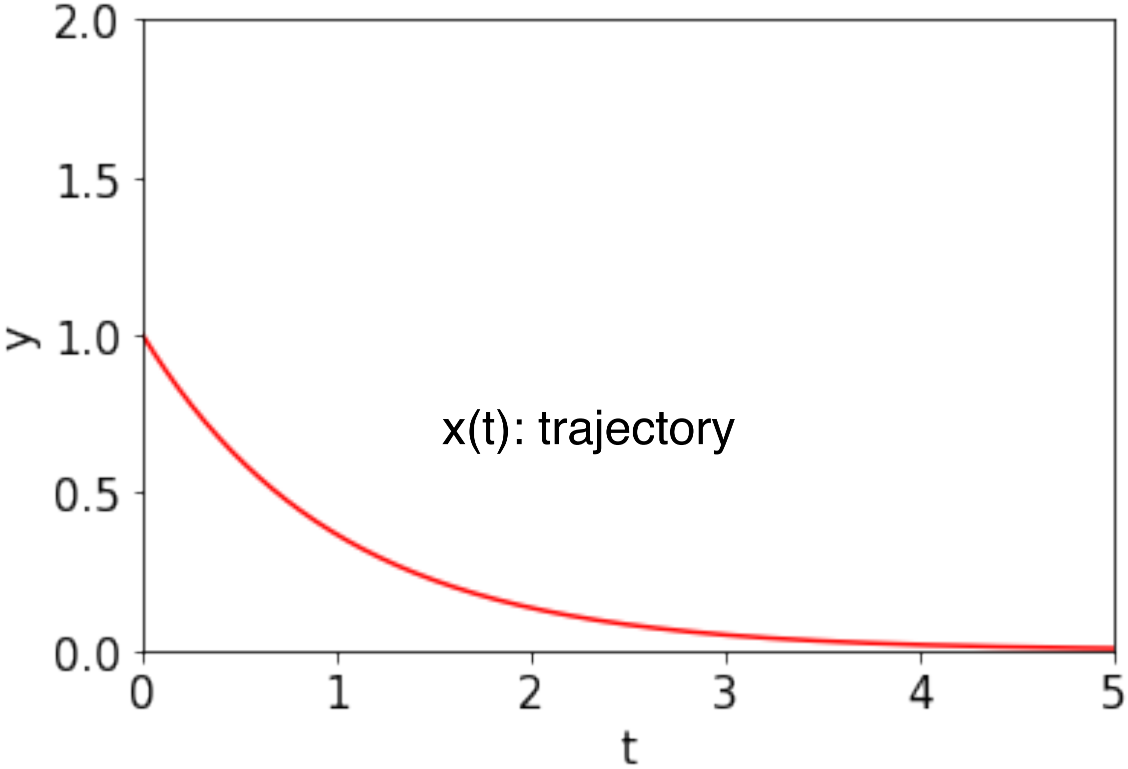
$\mathbf{x} \in \mathbb{R}^d$; d : dimension

autonomous $\mathbf{f}(\mathbf{x}, \lambda)$

non-autonomous $\mathbf{f}(\mathbf{x}, t, \lambda)$

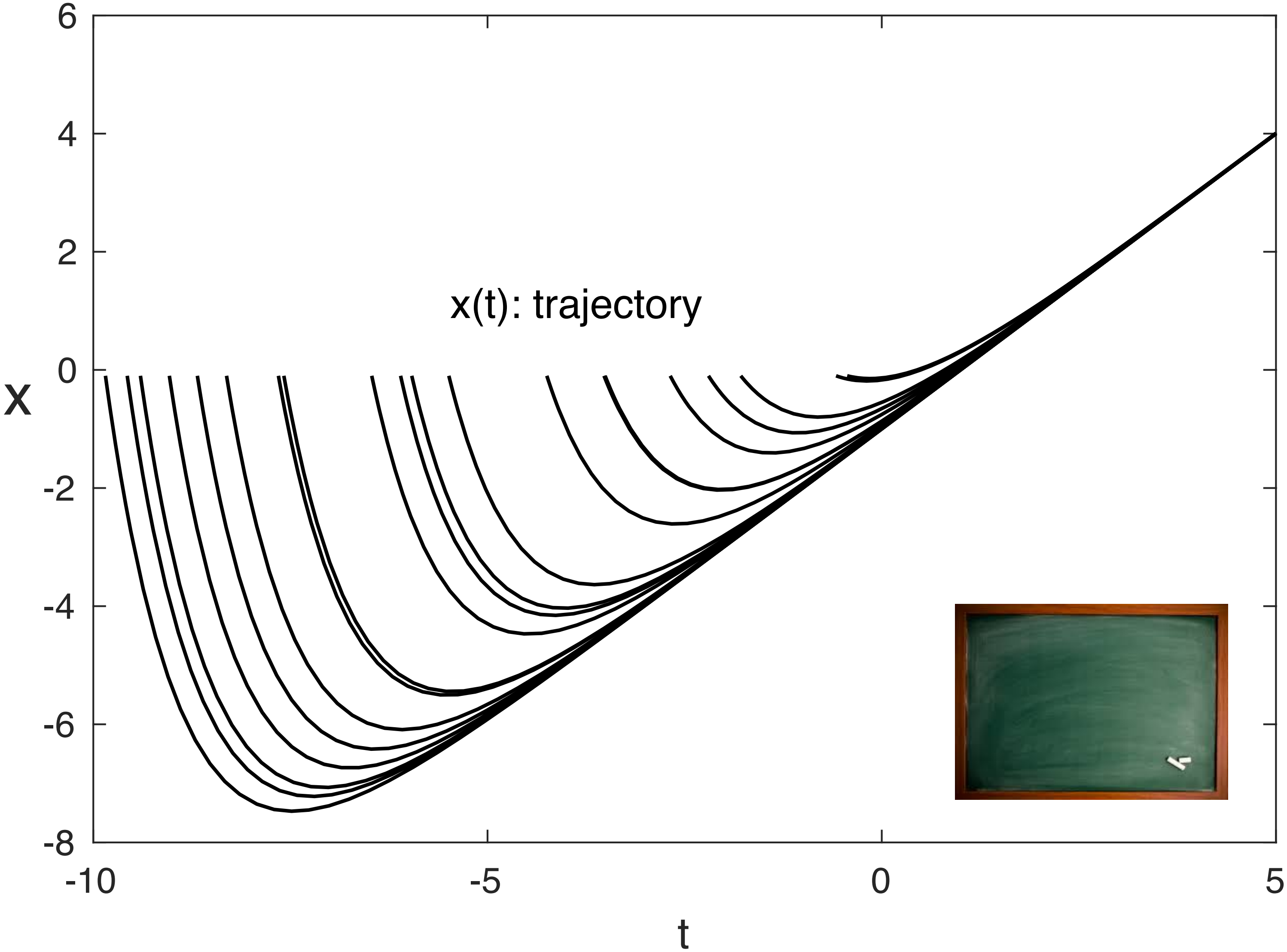
Attractor

$$\frac{dx}{dt} = -x$$



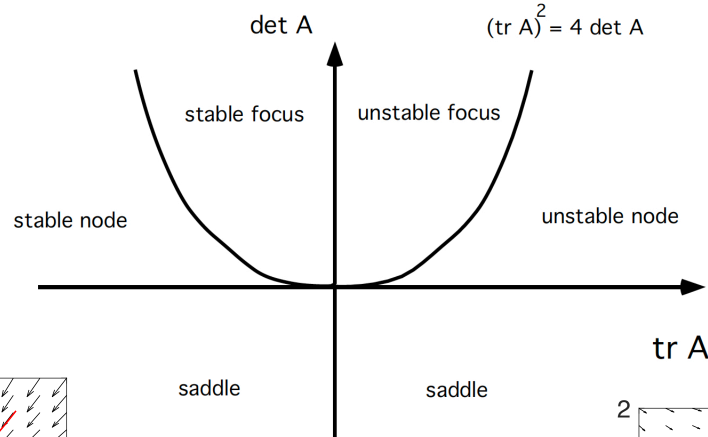
Pullback Attractor

$$\frac{dx}{dt} = -x + t$$

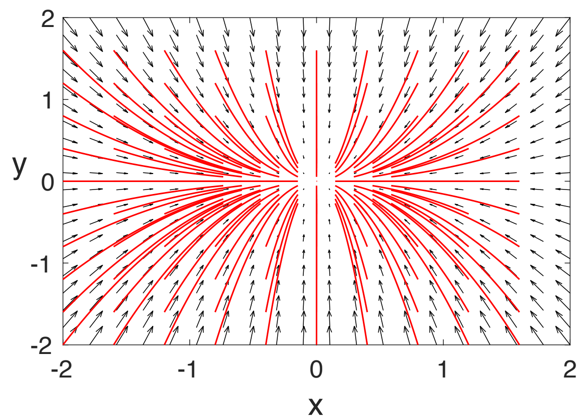


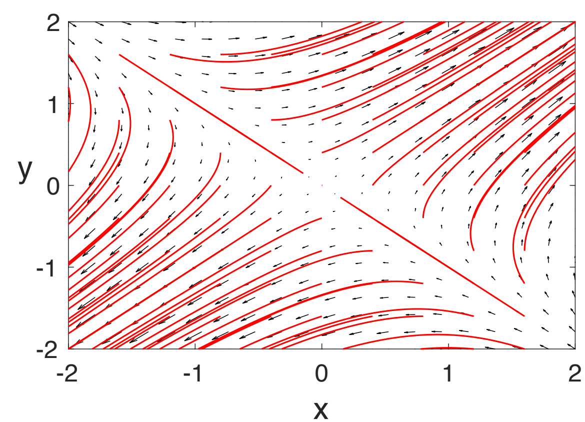
Phase portraits, 2D linear dynamical systems

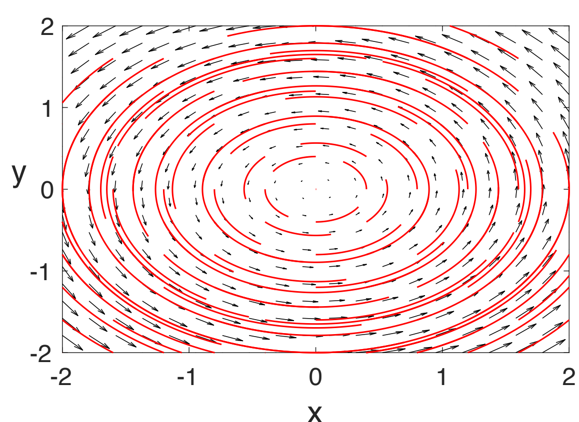
$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$

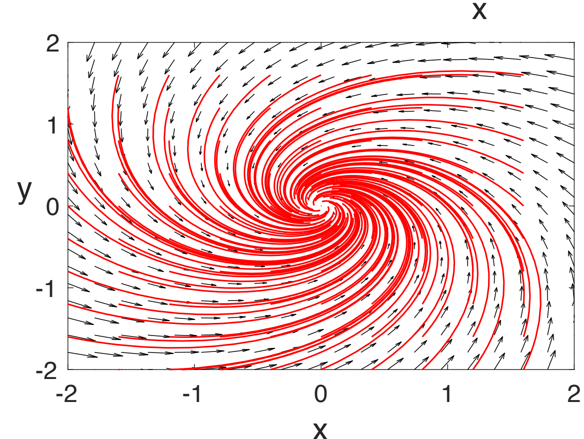


$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$



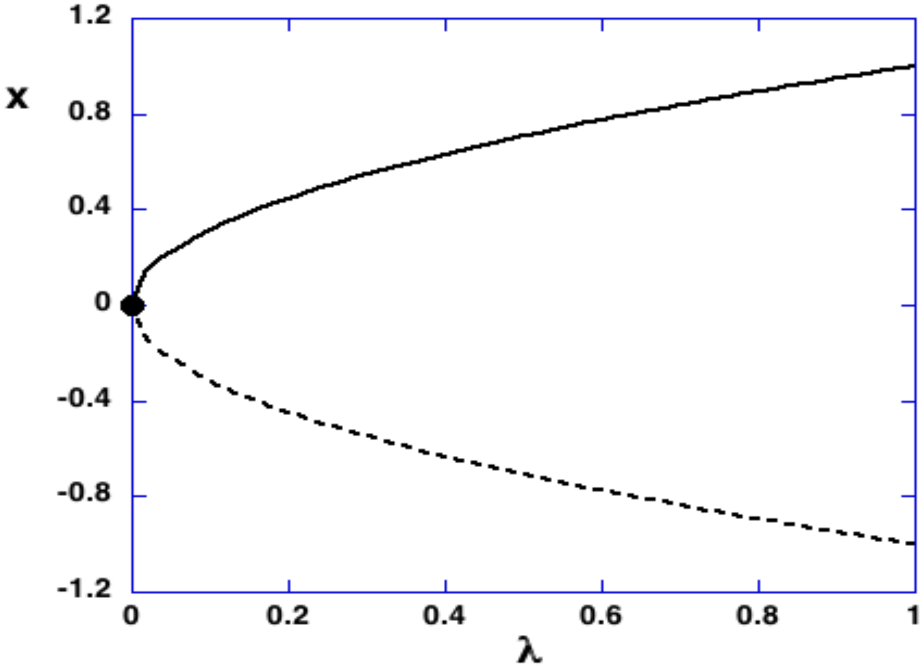




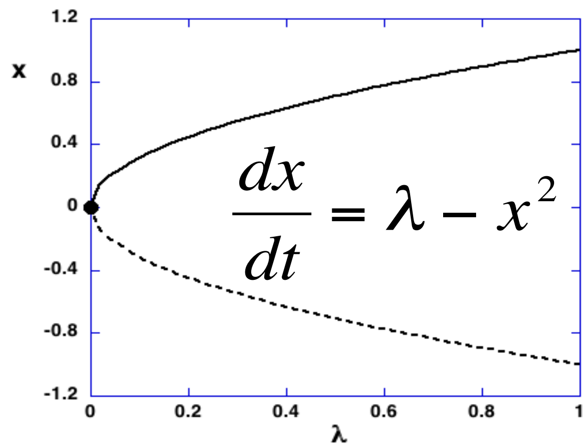


Elementary transitions (co-dim 1 bifurcations)

$$\frac{dx}{dt} = \lambda - x^2$$



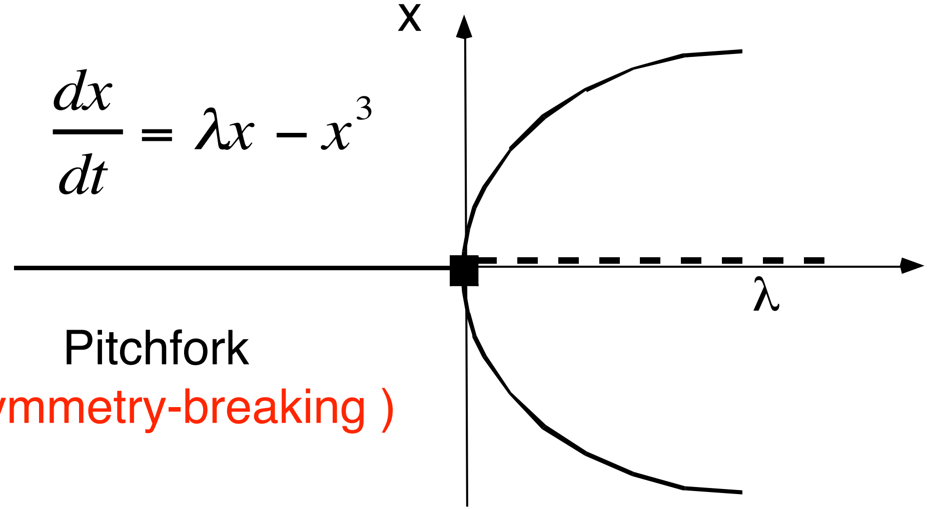
Elementary transitions (co-dim 1 bifurcations)



$$\frac{dx}{dt} = \lambda - x^2$$

Saddle-node (non-existence)

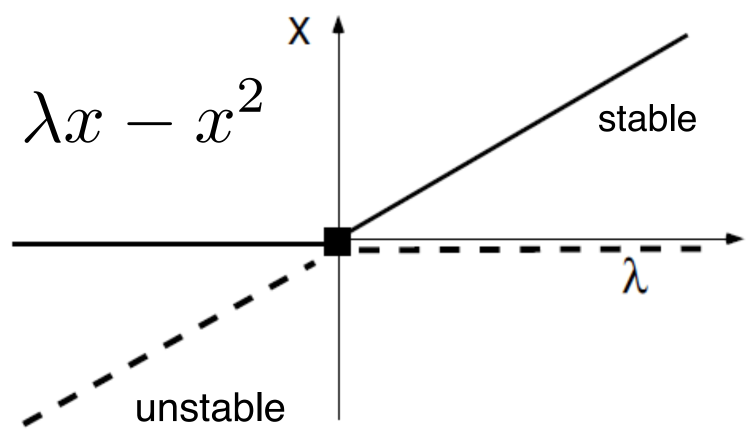
$$\frac{dx}{dt} = \lambda x - x^3$$



Pitchfork

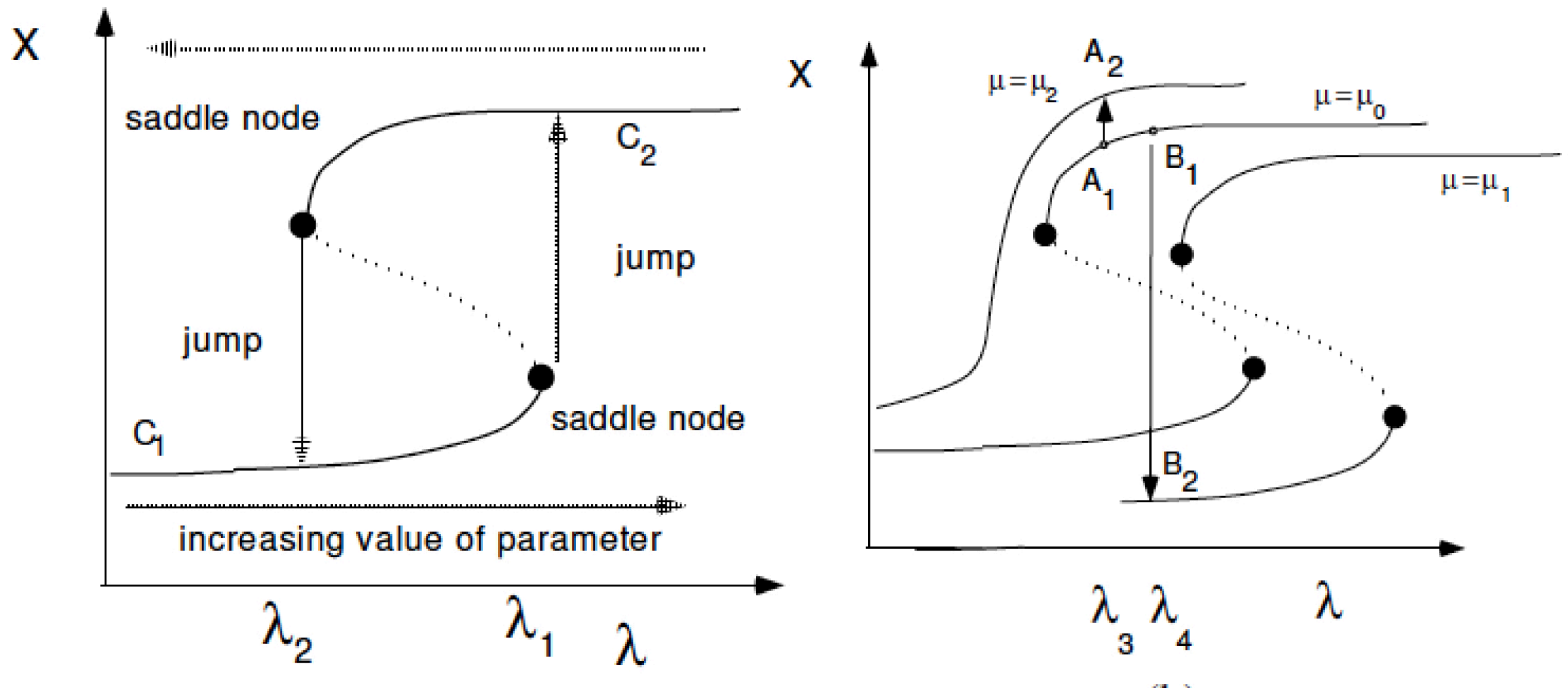
(symmetry-breaking)

$$\frac{dx}{dt} = \lambda x - x^2$$



Transcritical (exchange of stability)

Why determine full bifurcation diagrams?



Interpretation sensitivity studies

Software

d < 10 : Matcont, Content, PyDSTool, and several more

10 < d < 100 : AUTO

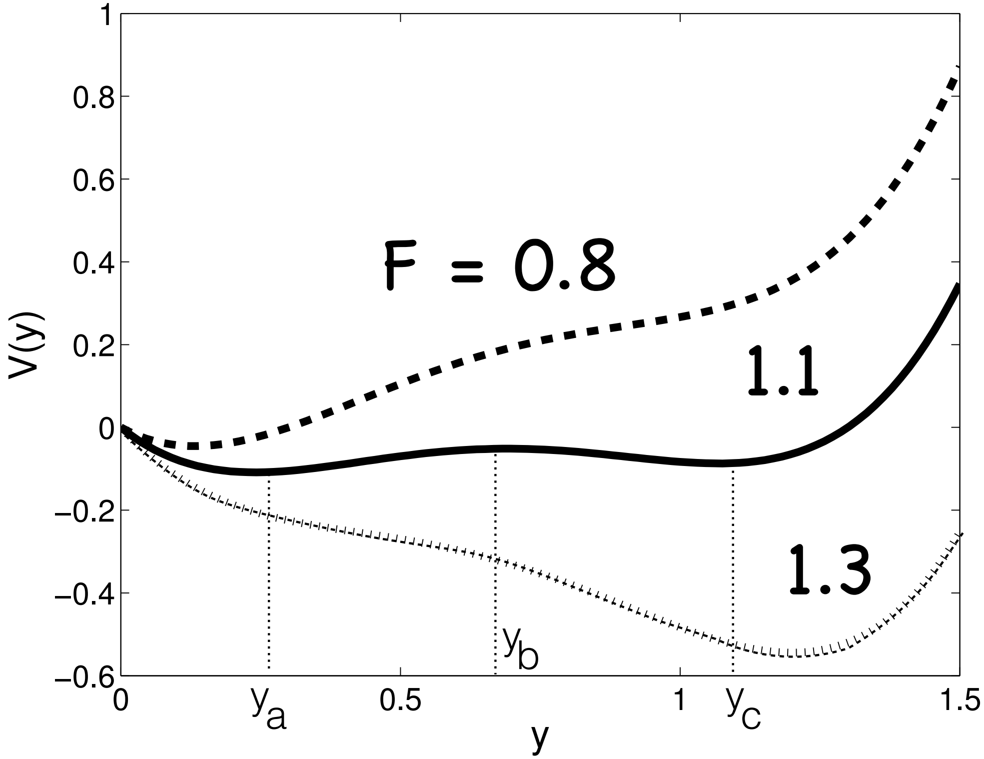
http://indy.cs.concordia.ca/auto/

100< d < 10^7 : Specialized, tailored codes
(similar setup as AUTO, but with different solvers)

Equilibria dimensionless Stommel model

$$\frac{dx}{dt} = -\alpha(x-1) - x(1+\mu^2(x-y)^2),$$
$$\frac{dy}{dt} = F - y(1+\mu^2(x-y)^2),$$

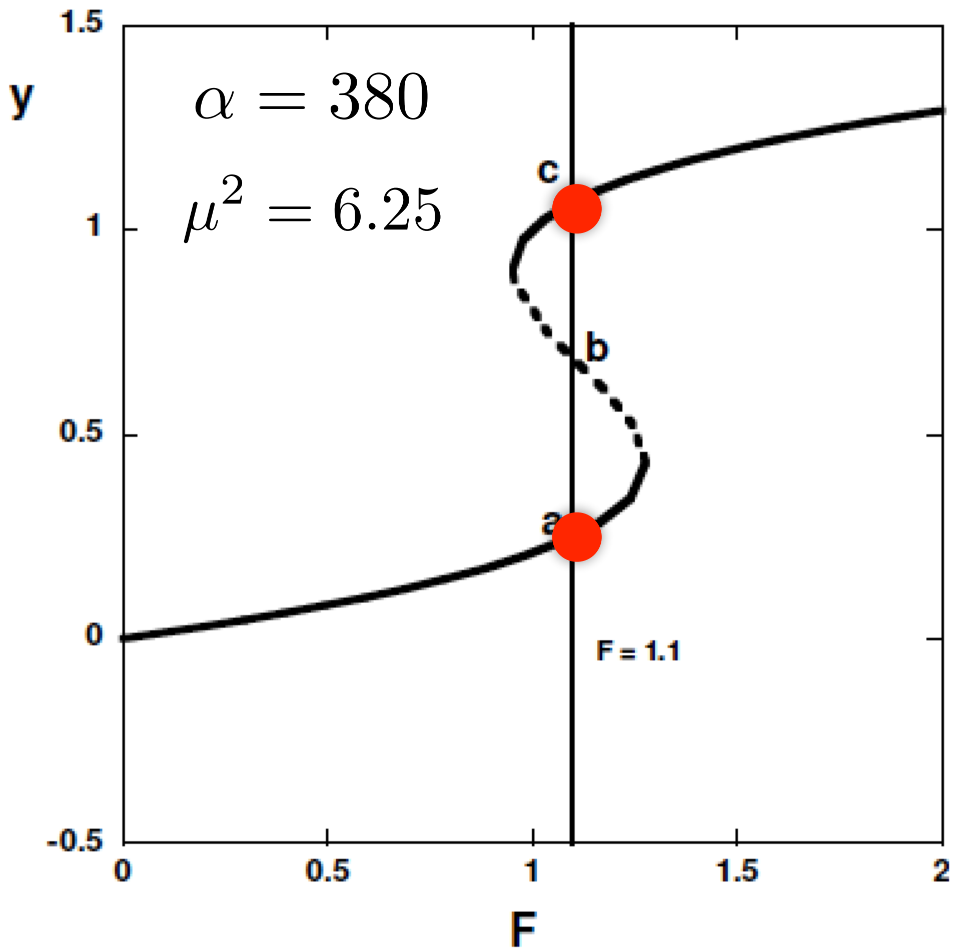
$$\alpha = 380$$
$$\mu^2 = 6.25$$



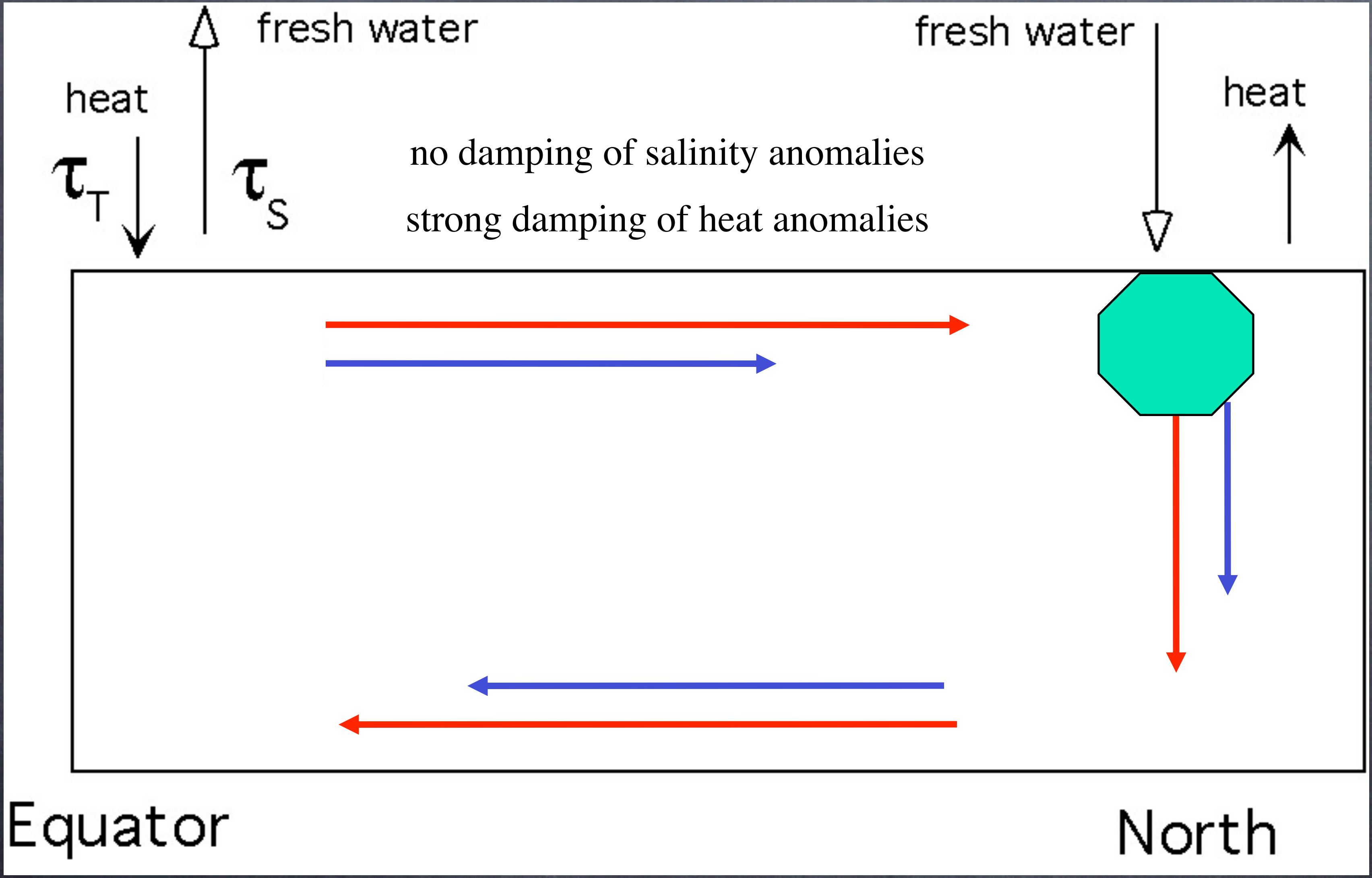
$$\alpha \to \infty \quad : \qquad \dot{y} = -V'(y) \; ; \; V(y) = -(Fy - \frac{y^2}{2} - \mu^2(\frac{y^2}{2} - \frac{2y^3}{3} + \frac{y^4}{4}))$$

Equilibria dimensionless Stommel model

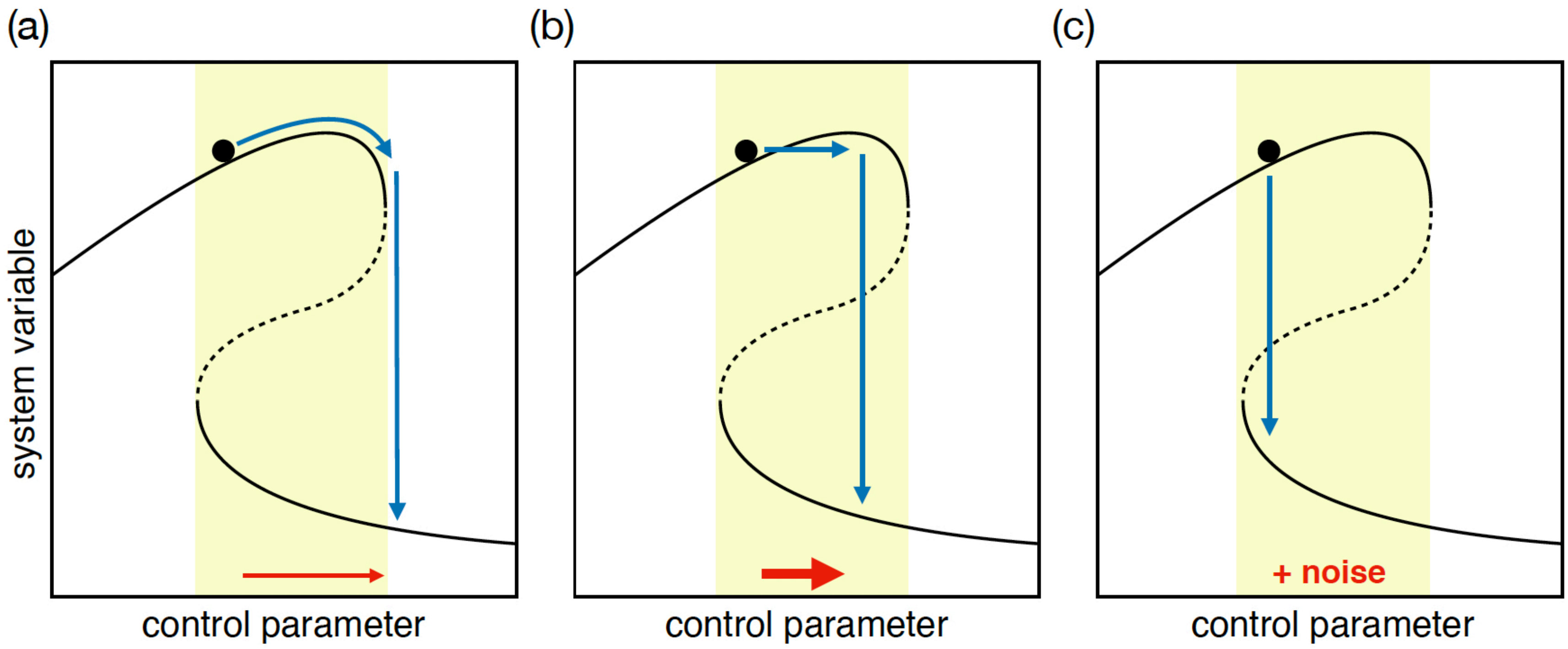
$$\frac{dy}{dt} = F - y(1 + \mu^2(1-y)^2)$$



The salt-advection feedback



Tipping phenomena



Bifurcation tipping

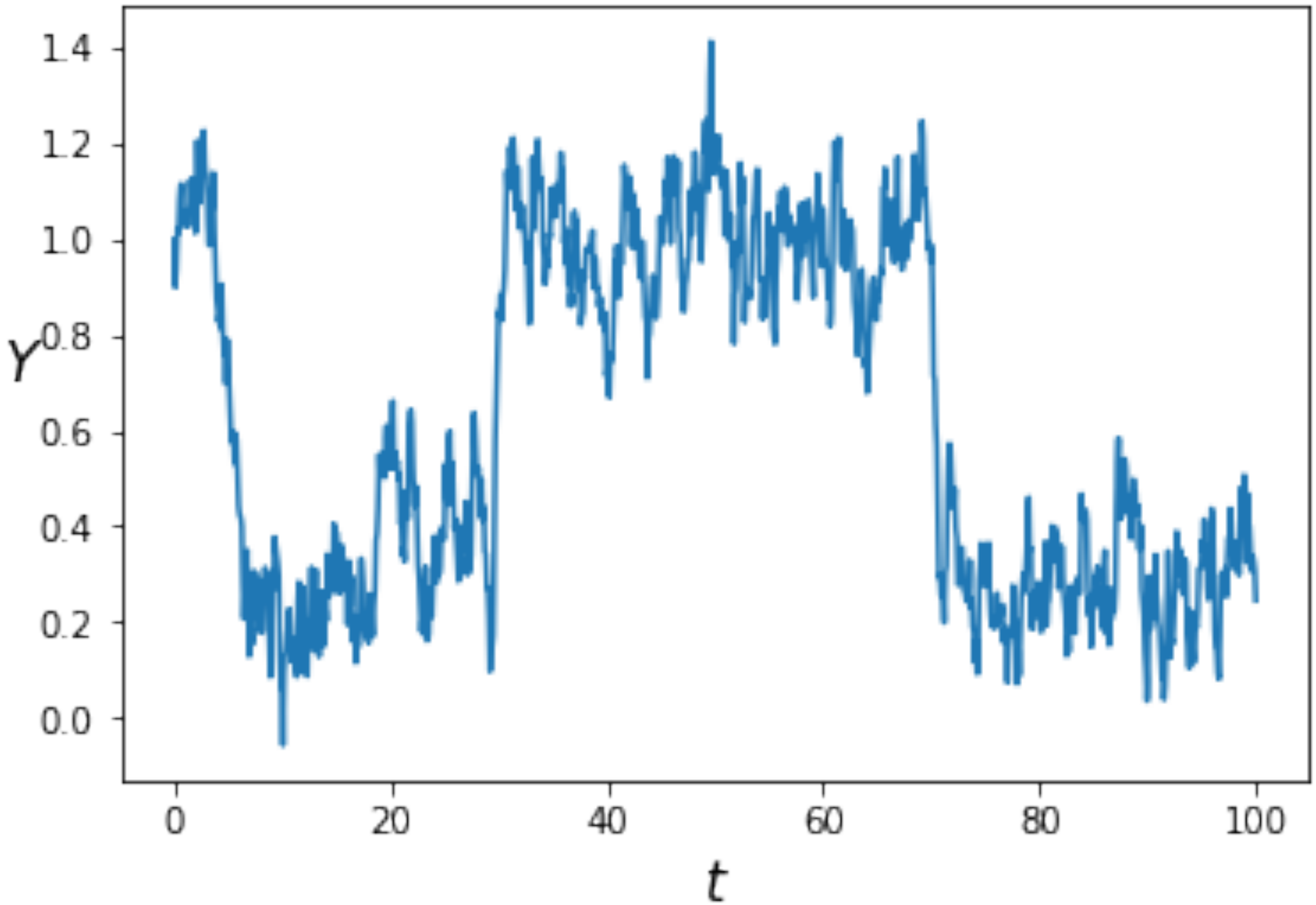
Rate-induced tipping

Noise-induced tipping

Question time

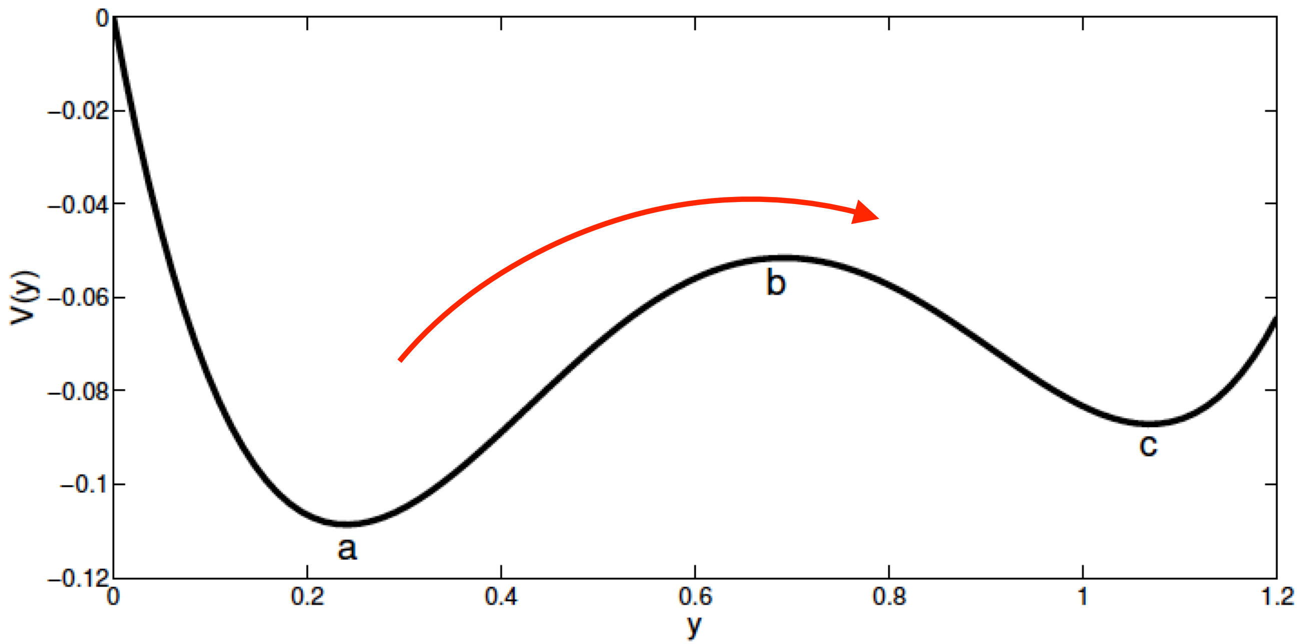
Transitions and probability density function

Effects of noise: $F(t) = \bar{F} + \sigma\xi(t)$



Mean escape time from a to c

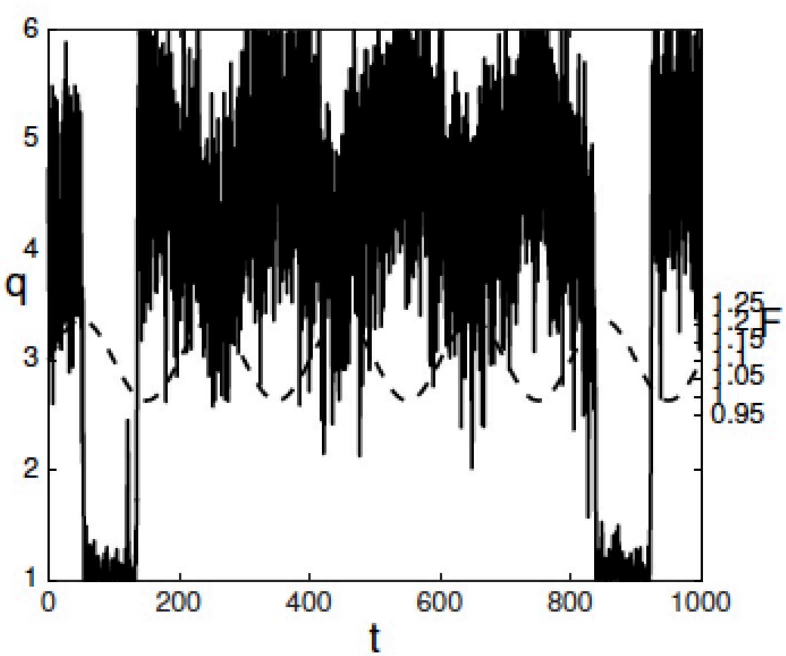
$$\bar{T}(y_a) = 2\pi \sqrt{\frac{1}{|V''(y_a)|\,|V''(y_b)|}} \exp\left(\frac{2}{\sigma^2}\left[V(y_b) - V(y_a)\right]\right).$$



Synchronization

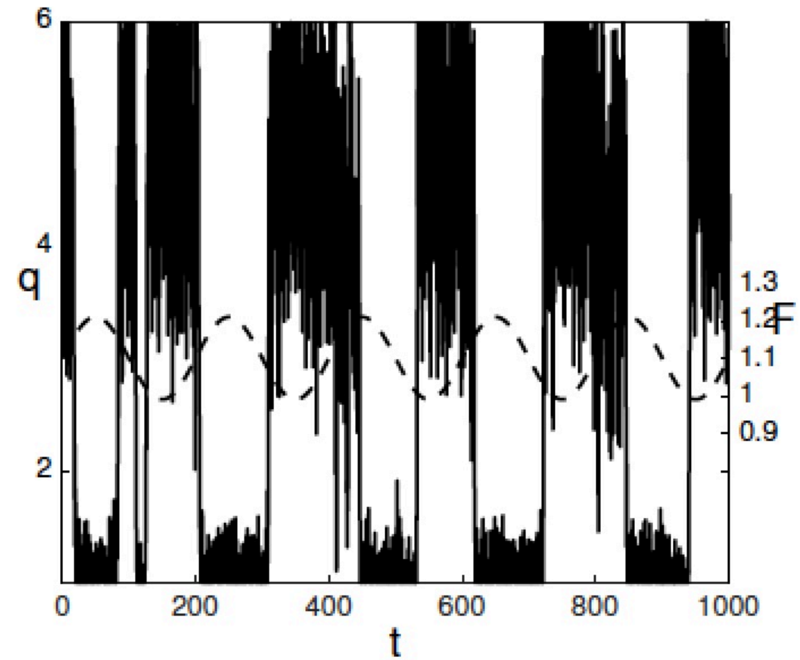
Effects of noise + periodic forcing:

$$F(t) = \bar{F} + A\sin(\omega t) + \sigma\xi(t)$$



(a)

$\sigma = 0.0125$



(b)

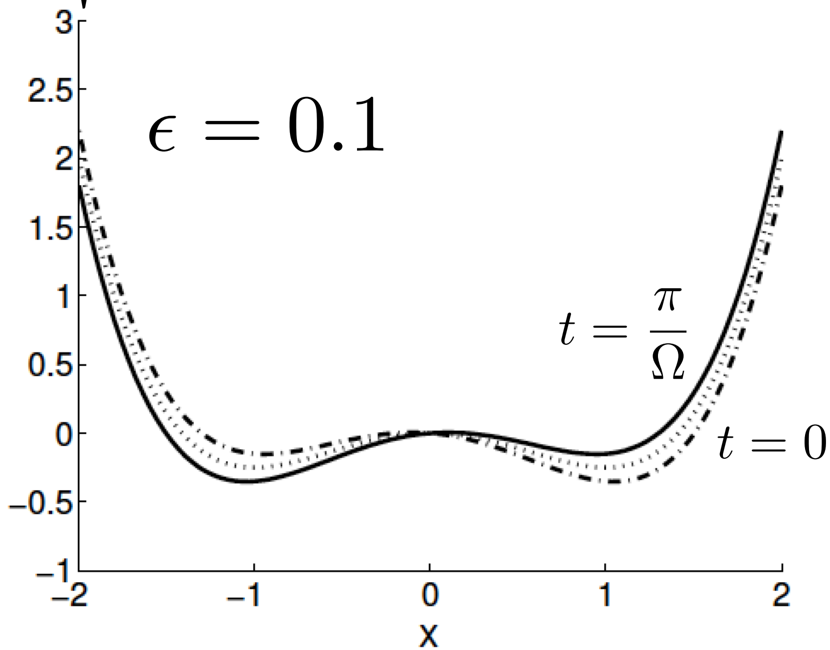
$\sigma = 0.0175$

Stochastic resonance: mechanism

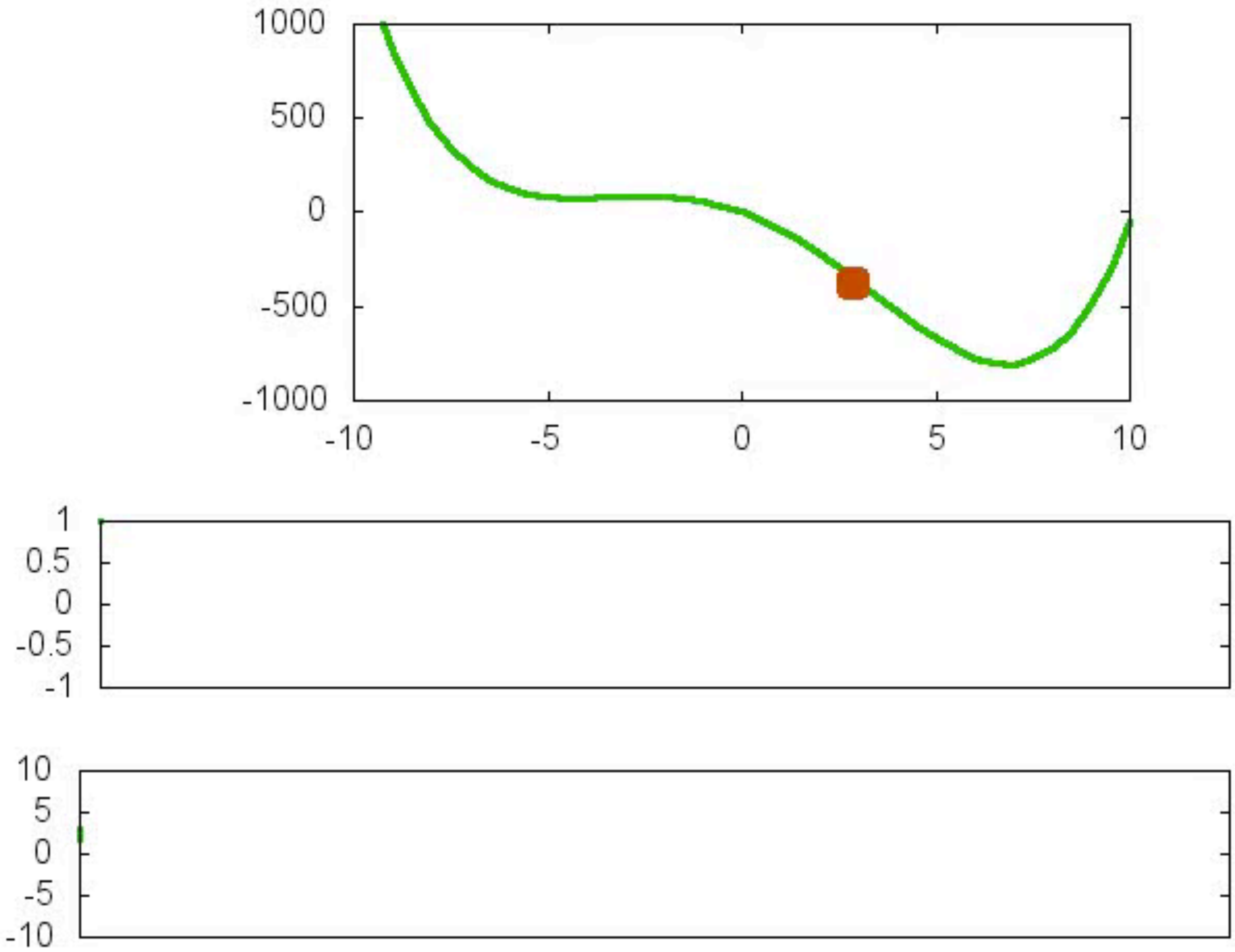
$$V(x,t) = -\frac{x^2}{2} + \frac{x^4}{4} - \epsilon x \cos \Omega t$$

$$< \tau(-1 \to 1) > \approx 2\pi \sqrt{\frac{1}{|V''(-1)V''(0)|}} e^{\frac{2(V(0)-V(-1))}{\sigma^2}} = \pi\sqrt{2}\ e^{\frac{1-\epsilon \cos \Omega t}{2\sigma^2}}$$

$$< \tau(1 \to -1) > \approx 2\pi \sqrt{\frac{1}{|V''(1)V''(0)|}} e^{\frac{2(V(0)-V(1))}{\sigma^2}} = \pi\sqrt{2}\ e^{\frac{1+\epsilon \cos \Omega t}{2\sigma^2}}$$



Stochastic resonance



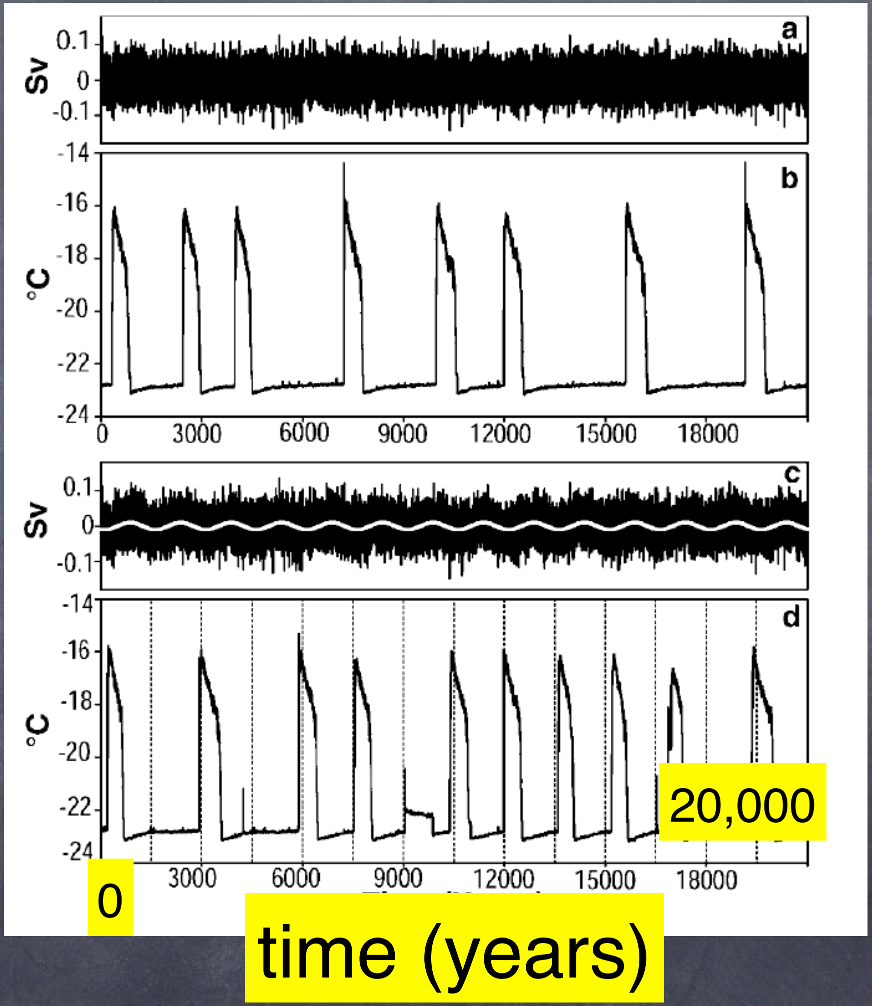
Liz Gonzales (You Tube)

Results in an EMIC

freshwater forcing ‘noise’

temperature

+ periodic forcing 1500 year period

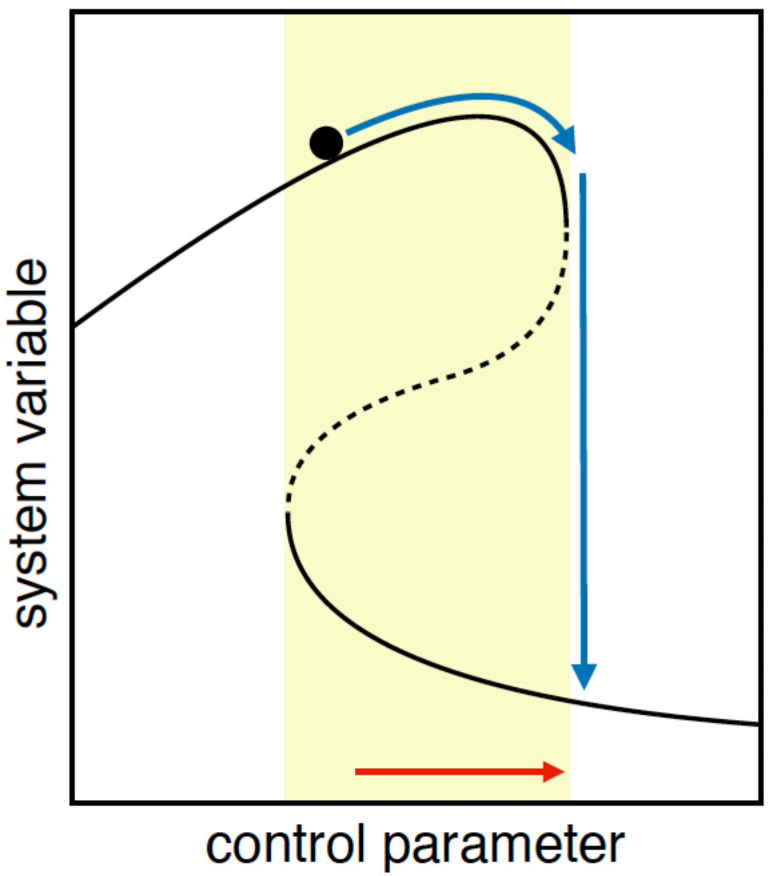


Ganopolski & Rahmstorf, PRL, (2002)

Question time

Early warning of saddle node bifurcations?

$$X_t = e^{-\gamma t}(X_0 + \sigma \int_0^t e^{\gamma s} dW_s)$$



AR(1):

$$X_{n+1} = \alpha X_n + Z_{n+1}$$

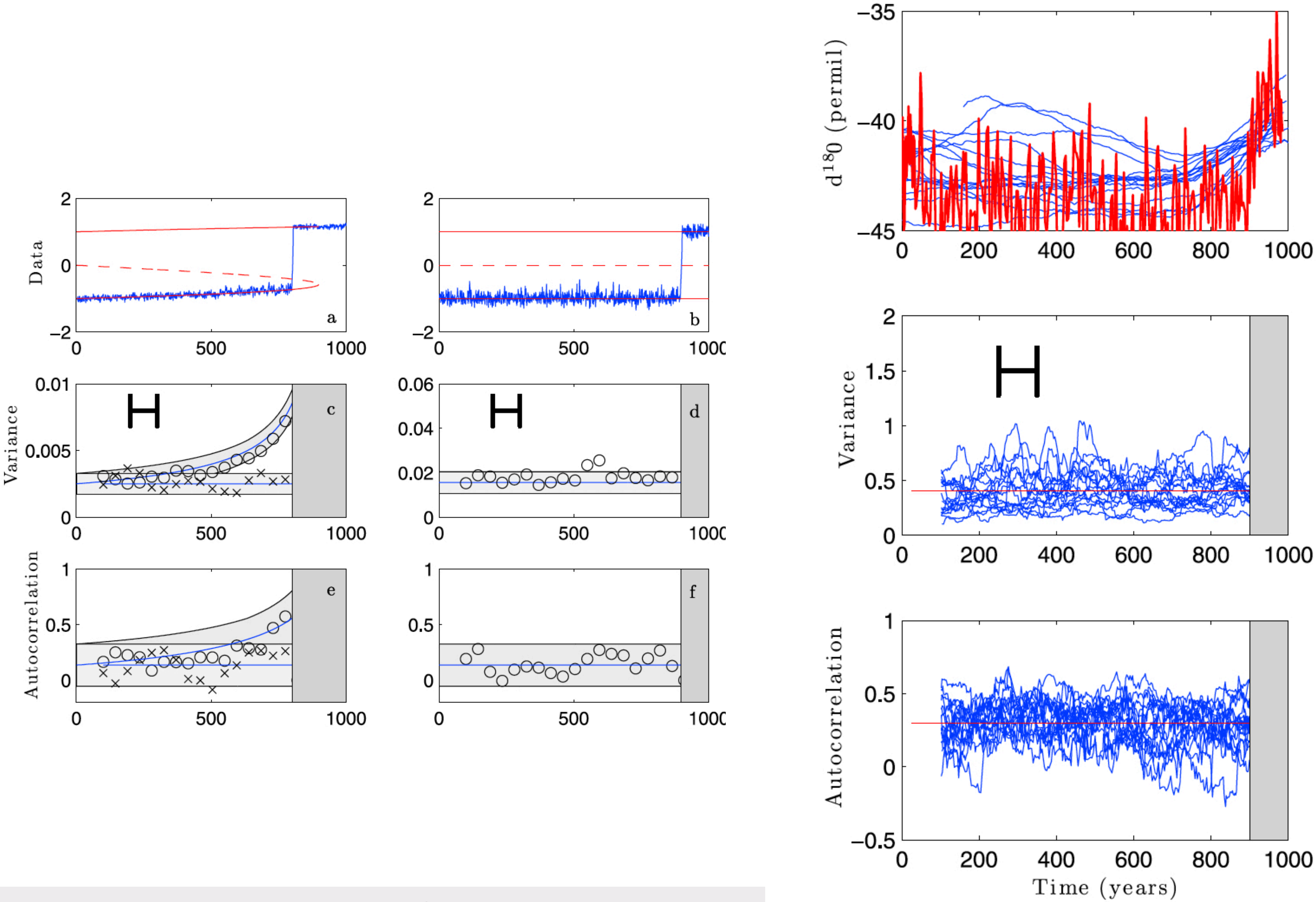
$$\alpha = e^{-\gamma \Delta t}, Z_n = \sigma dW_n$$

$$Var[X] = \frac{\sigma^2}{1-\alpha^2}$$

$$\rho_n = \alpha^n$$

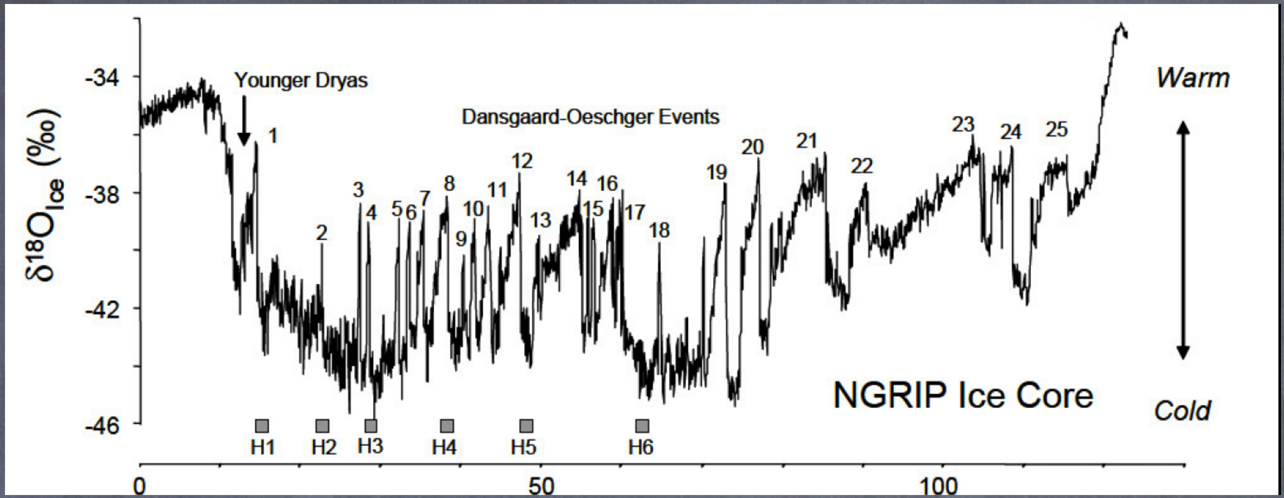
At saddle node: $\gamma \to 0$ $Var[X] \to \infty \; ; \; \rho_n \to 1$

Observations?



Ditlevsen & Johnson , GRL, (2010)

Summary



Stochastic resonance has been proposed as a mechanism for the Dansgaard-Oeschger events, but this theory has several problems

Possibly just noise induced transitions in the AMOC (Occam's Razor)