Project 2: Bifurcation analysis

In this project, you will practice with the solution of nonlinear stochastic differential equations using a conceptual model of the AMOC. Make a Python notebook for all computations below.

Consider the extended nonlinear stochastic Stommel model given by the equations

$$dY_t = (\bar{F} - Y_t(1 + \mu(1 - Y_t)^2))dt + \bar{F} dZ_t \tag{1a}$$

$$dZ_t = g'(t)dt + \sigma dW_t, \tag{1b}$$

where $g(t)$ describes a time-dependence of the freshwater forcing, with $Y_0 = 0.1$ and $Z_0 = 0$.

(i) Consider first the deterministic system with $\sigma = g(t) = 0$. Compute the bifurcation diagram and specifically the values of F at the saddle-node bifurcations for fixed $\mu = 6.2$ numerically using the PyDSTool program.

(ii) Write a Python program to integrate the system of SDEs (1) using the Euler-Maruyama (EM) scheme. Determine a solution of the equations for $F = 1.1$, $\sigma = 0.1$, $\mu = 6.2$ and $g(t) = 0$. Make a plot of the probability density function for this case.

(iii) Determine the equilibrium solution of the Fokker-Planck equation for this case (so $g(t) = 0$) analytically, and compare the result with that in (ii).

(iv) Next, consider the stochastic case with $g(t) = \epsilon t$ and $\epsilon = 0.001$. Study the behavior of the model for increasing noise amplitude (again with $F = 1.1$ and $\mu = 6.2$). What type of tipping occurs when σ increases?