Efficient Algorithms for the Numerical Solution of Differential Equations

PROEFSCHRIFT

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door

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Summary of *Efficient Algorithms for the Numerical Solution of Differential Equations*

The efficiency of a numerical integration method is defined to be the production, measured in digits of accuracy or qualitative agreement of the numerical result, divided by the cost, measured in computational effort or waiting time, of producing the result. The actual efficiency is often difficult to measure, but relative efficiencies can be compared in many cases. Three independent techniques yielding efficient methods are considered:

1. simplification of implicit relations by using implicit-explicit splittings,
2. exploitation of parallelism across the problem space,
3. exploitation of parallelism across the method parameters.

The advantage of implicit-explicit (IMEX) methods is that problems with both stiff and nonstiff terms can be integrated with only the stiff terms being treated implicitly. In some cases this leads to simplified implicit relations, for example, if the stiff terms are linear whereas the nonstiff terms are nonlinear—the case for the Navier-Stokes equations. All implicit-explicit linear multistep methods can be seen as the combination of an implicit multistep methods with an extrapolated explicit counterpart. In the simplest case of the IMEX Euler method, the stability condition is satisfied if the eigenvalues of the explicit terms are contained in the stability region of the related explicit method. This does not hold for general IMEX linear multistep methods, however. We consider two questions: 1) under what additional restriction on the eigenvalues of the explicit part does the method remain A-stable for the implicit part, and 2) under what restriction on the A(α)-stability of the implicit part do we maintain the stability region of the explicit part.

Parallelism across the problem space is especially accessible when solving partial differential equations, for which the coupling is often sparse and localized, allowing for easy load balancing and a small volume of communication. Since these problems are also frequently scalable, the resulting methods are suitable for implementation on massively parallel distributed memory computers. We consider parallel implementation of a simple block-diagonal preconditioner based on domain decomposition, in which the blocks are approximately solved using inner iterations. The global iteration method is GCR, and we discuss alternative orthogonalization methods to reduce the number of required global communications. Parallel timings are given for a Poisson equation,

arising as a result of the pressure correction method, and the Bousinesq equations for natural convection in a heated cavity.

The degree of parallelism across the method parameters is proportional to the number of stages, and therefore is especially practical when high order accuracy is required. We consider a number of approaches to parallelizing the extended backward differentiation formulas, a class of general linear methods maintaining high accuracy in the presence of stiffness. We develop, analyze and test 3- and 4-processor implementations for up to 6th order L-stable methods. The two most promising approaches for parallelizing the (lower triangular) stage coefficient matrix are: 1) approximation by the diagonal and 2) staggering of the abscissa to obtain a diagonalizable matrix.

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