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Dynamics of networks if everyone strives for structural holes

Collaboration mainly with
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February 12, 2015

UCU, guest lecture Complexity

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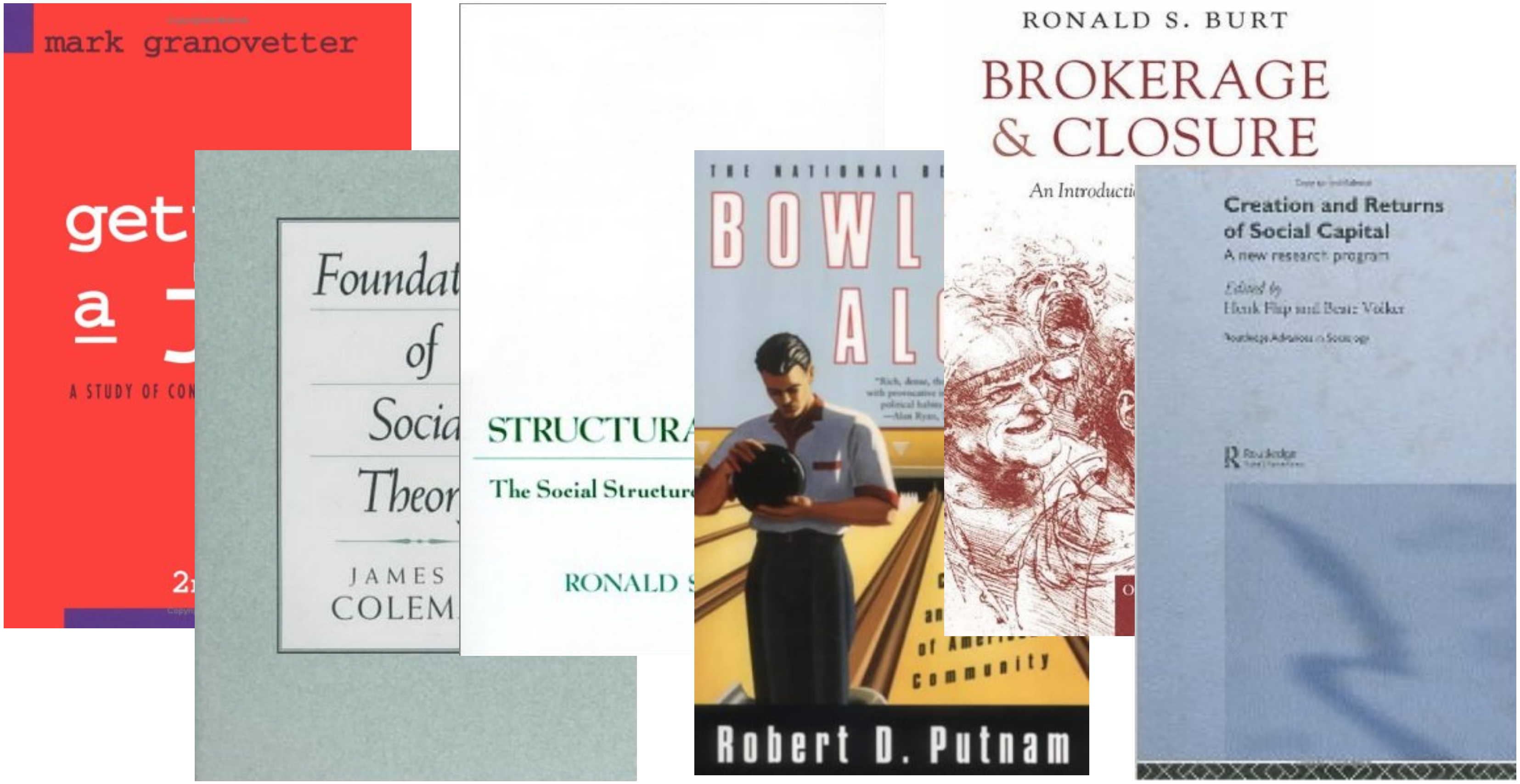

Much more evidence on the returns of social capital than the investments in social capital!

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Importance of Dynamic Networks

- If networks can cause benefits or damage for actors and actors are aware of those, they should attempt to optimize their networks? Examples:
  - Introducing one's friends to one another (*balance / trust*)
  - Avoiding contamination with diseases
  - Buying from dependent suppliers and vice versa (*network exchange theory*)
  - Maintaining many weak ties when searching for a job (*access to information*)
  - 'Networking' in the management world (*brokerage / control*)
  - Facilitating trust

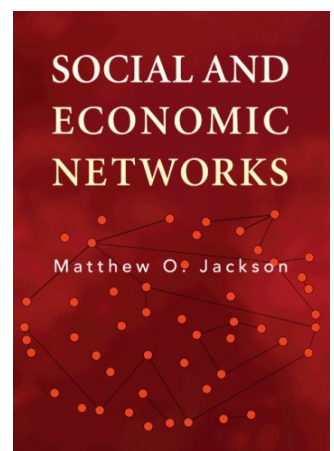
Research Questions

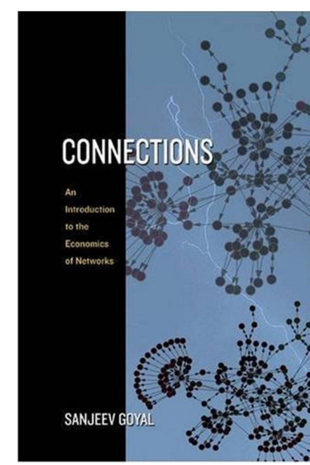
- If all actors build networks following such principles, which networks will emerge?
  - Which networks are stable?
- And, as a result, which benefits do actors receive?
  - Are stable networks also efficient?
- Might such networks have other unintended consequences?

Not the sole playground of sociologists anymore!

- E.g., economists have taken up the question: If networks are so valuable, can we predict which networks will emerge?
  - Jackson and Wolinsky (1996) in Journal of Economic Theory
  - Bala and Goyal (2000) in Econometrica

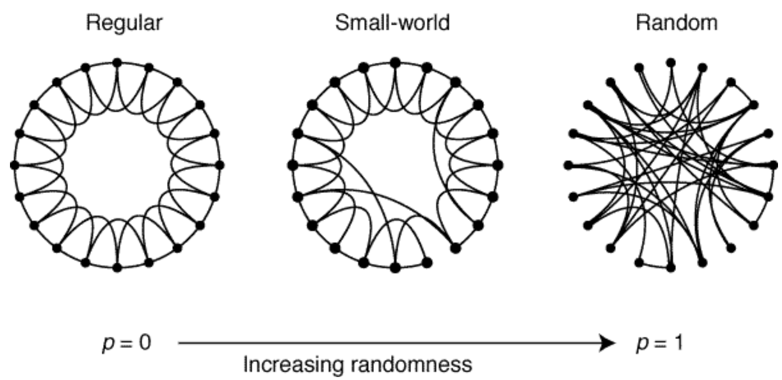
- See Goyal / Jackson for overviews

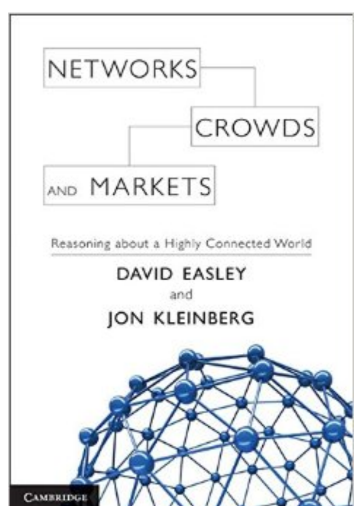


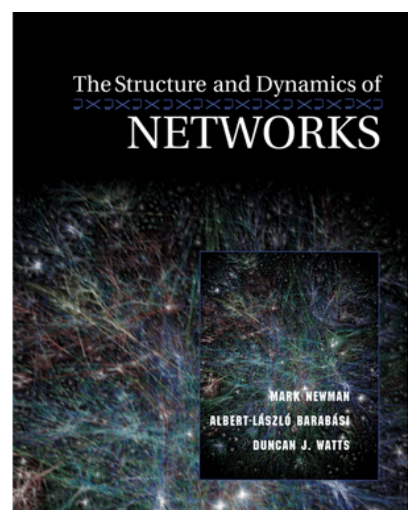


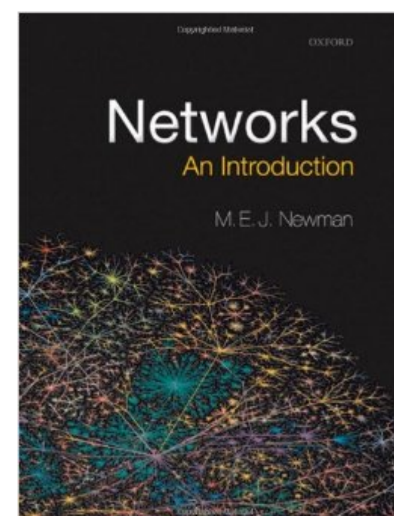
Also physicists have appeared in the arena!

- Random (Erdös-Renyi) networks
- Preferential attachment
- Small-world networks









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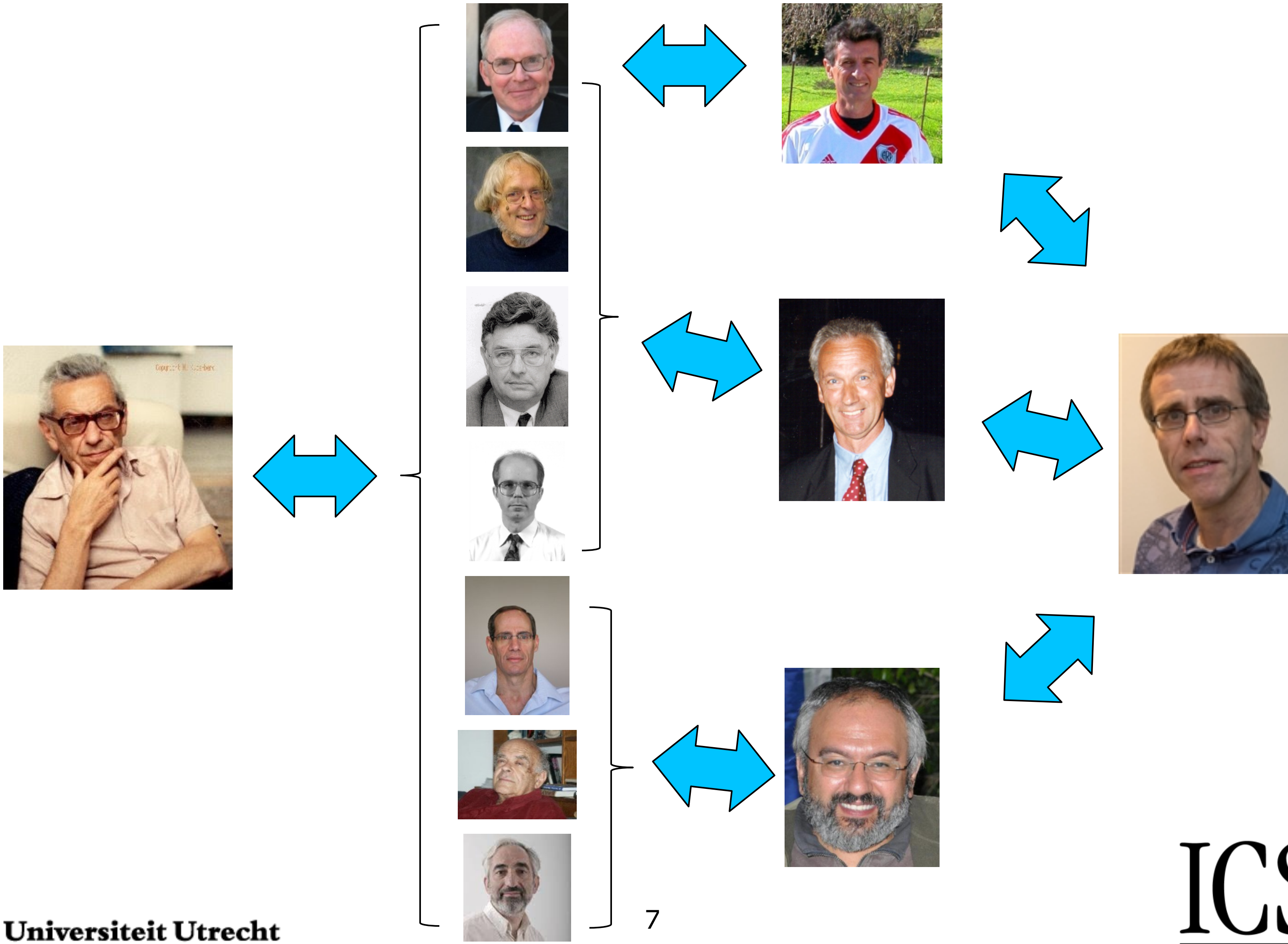
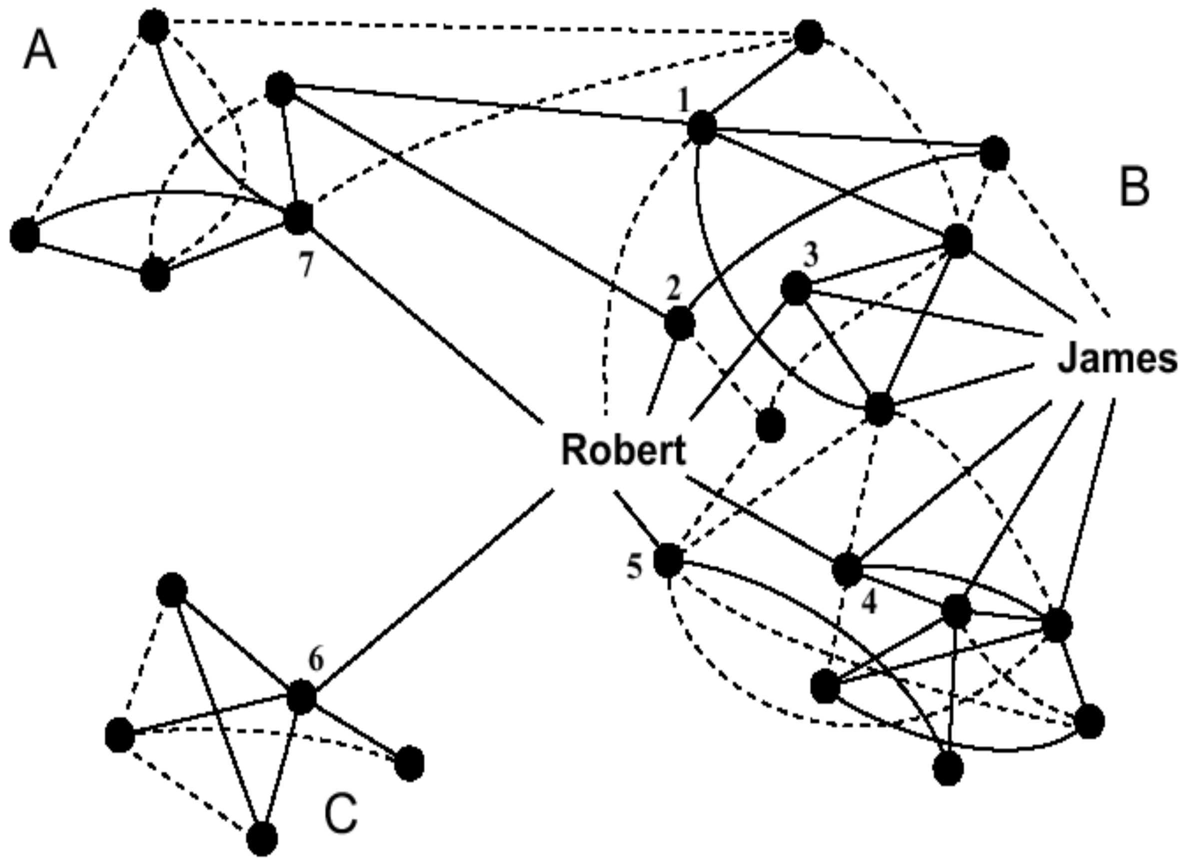
Erdös number 3



Using Game Theory to Predict Stable Networks

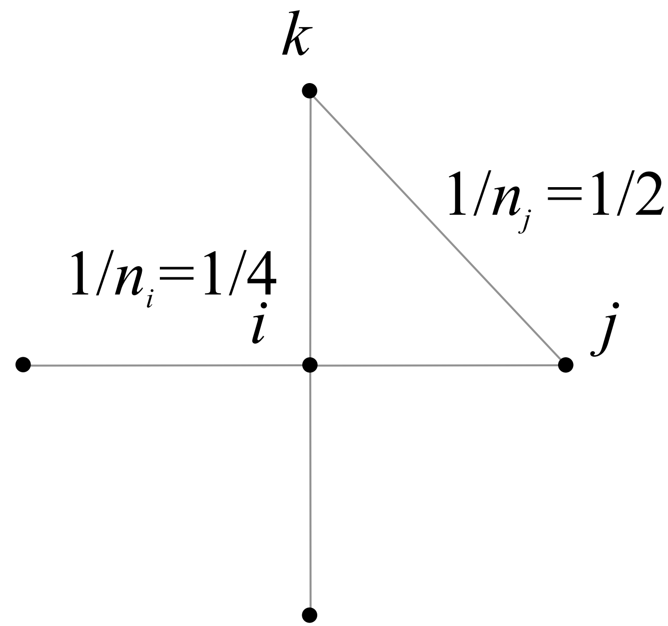
- Choosing ties is a conscious decision in which actors take their own and others incentives into account
- Networks are stable if everyone chooses a best reply against what others choose
- Best reply can be interpreted in a myopic sense, but also in a more general sense including forward-looking considerations
- As an example we consider structural holes

"Structural holes" Illustrated



“Structural Holes” Formalized

- Use the ‘network constraint’ measure (Burt 1992: 54):



$$c_i = \sum_j \left[\frac{1}{n_i} + \sum_k \left(\frac{1}{n_i} \frac{1}{n_j} \right) \right]^2 :$$

with n_i the number of ties of actor i

$$c_i = \left[\frac{1}{4} + \left(\frac{1}{4} \frac{1}{2} \right) \right]^2 + \left[\frac{1}{4} + \left(\frac{1}{4} \frac{1}{2} \right) \right]^2 + \left[\frac{1}{4} \right]^2 + \left[\frac{1}{4} \right]^2 = \frac{13}{32}$$

Properties Constraint Measure

- Closed triads are costly (increase constraint considerably)
- Closed triads are especially costly if they involve actors with few relations
- The more ties, the lower the constraint, namely 1/(number of ties), as long as no closed triads are made
- Only “redundancy” with respect to direct relations is taken into account (in contrast with how Goyal and Vega-Redondo formalize structural holes)

"Structural holes" in Practice

Evidence:

Bian 1994; Talmud 1994; Burt 1995, 1997, 1998, 2000, 2001, 2004; Granovetter 1995 [1974]; Yasuda 1996; Gabbay 1997; Jang 1997; Podolny and Baron 1997; Leenders & Gabbay 1999; Lin 1999; Burt, Hogarth, and Michaud 2000; Mehra, Kilduff, and Brass 2000; Lin, Cook, & Burt 2001; Mizruchi and Sterns 2001; Burt et al. 2002

- Jobs are found faster through ties that connect otherwise disconnected groups.
- Jobs found are more desirable
- Salaries are higher for managers occupying more structural holes
- Structural holes are positively correlated with income, positive performance evaluations, peer reputations, promotions, and good ideas.

What if EVERYONE Pursued Holes?

A Specific Type of Network Dynamics

- Actors minimize their "network constraint" as defined by Burt (1992)
- Utility is a strictly decreasing function of network constraint (no separate costs of ties, assumed to be part of the utility idea of Burt)
  - Assumption might become problematic if networks become really large
- In other words, actors optimize structural holes in their network

Our Approach

- In accordance with recently emerging economics literature on dynamic networks
- “Two-sided link formation” model
  - Ties are added if both actors agree on adding a tie
  - Ties can be removed without permission
- Specify stable and efficient networks

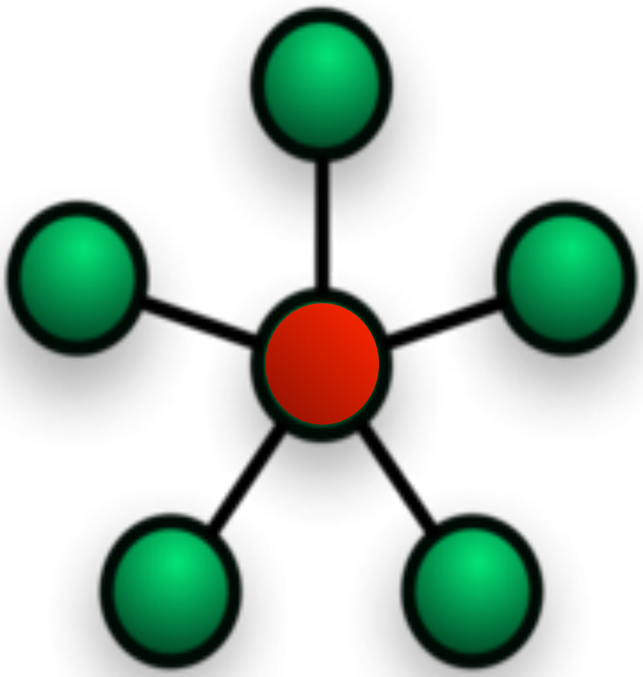
Stability and Efficiency Concepts

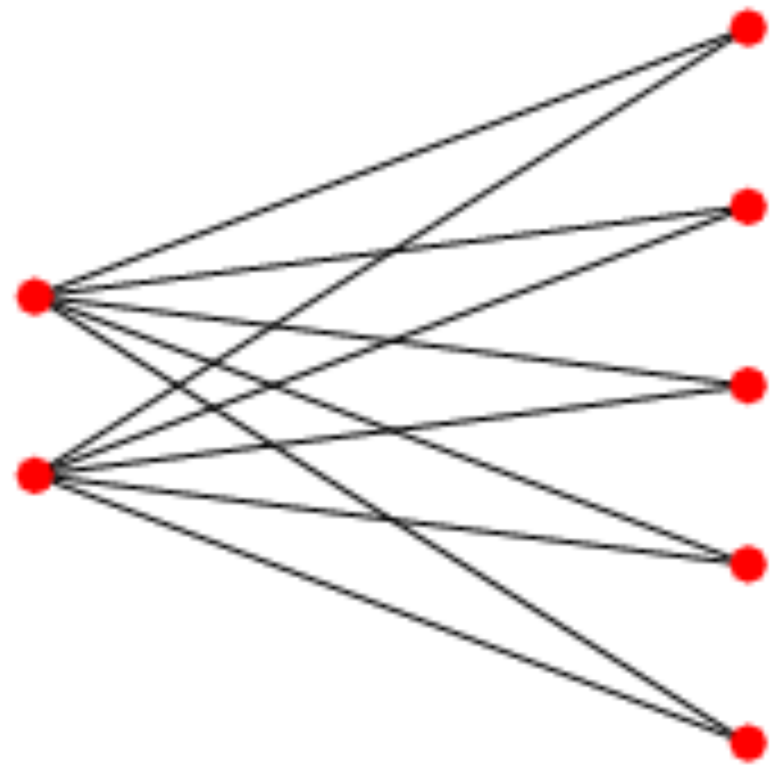
- *Pairwise stability*: No one wants to delete a link and no pair wants to add a link
- *Strong pairwise stability*: No one wants to delete a set of links and no pair wants to add a link (also *pairwise Nash equilibrium*)
- *Unilateral stability*: No one can profitably and with consent reconfigure his links

Definitions Bipartite Networks

- **Bipartite** networks: actors can be divided into two (non-empty) groups and there are no ties within the two groups
- **Complete bipartite** networks: bipartite networks in which all ties between the two groups exist

Examples Complete Bipartite Network





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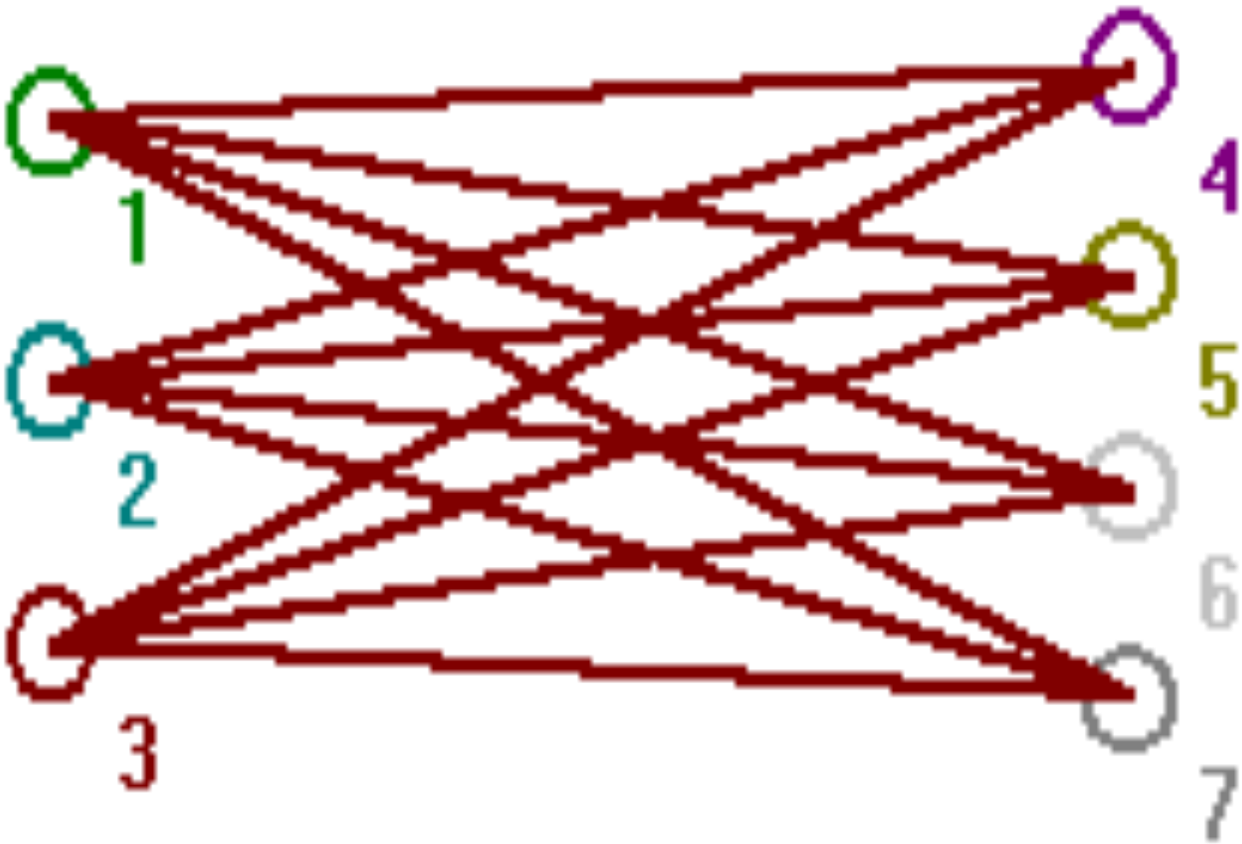
Definitions Bipartite Networks

- **Bipartite** networks: actors can be divided into two (non-empty) groups and there are no ties within the two groups
- **Complete bipartite** networks: bipartite networks in which all ties between the two groups exist (special case: star)
- **Balanced complete bipartite** networks: complete bipartite networks for which the number of actors in the two groups are as equal as possible

Example Balanced Complete Bipartite Network



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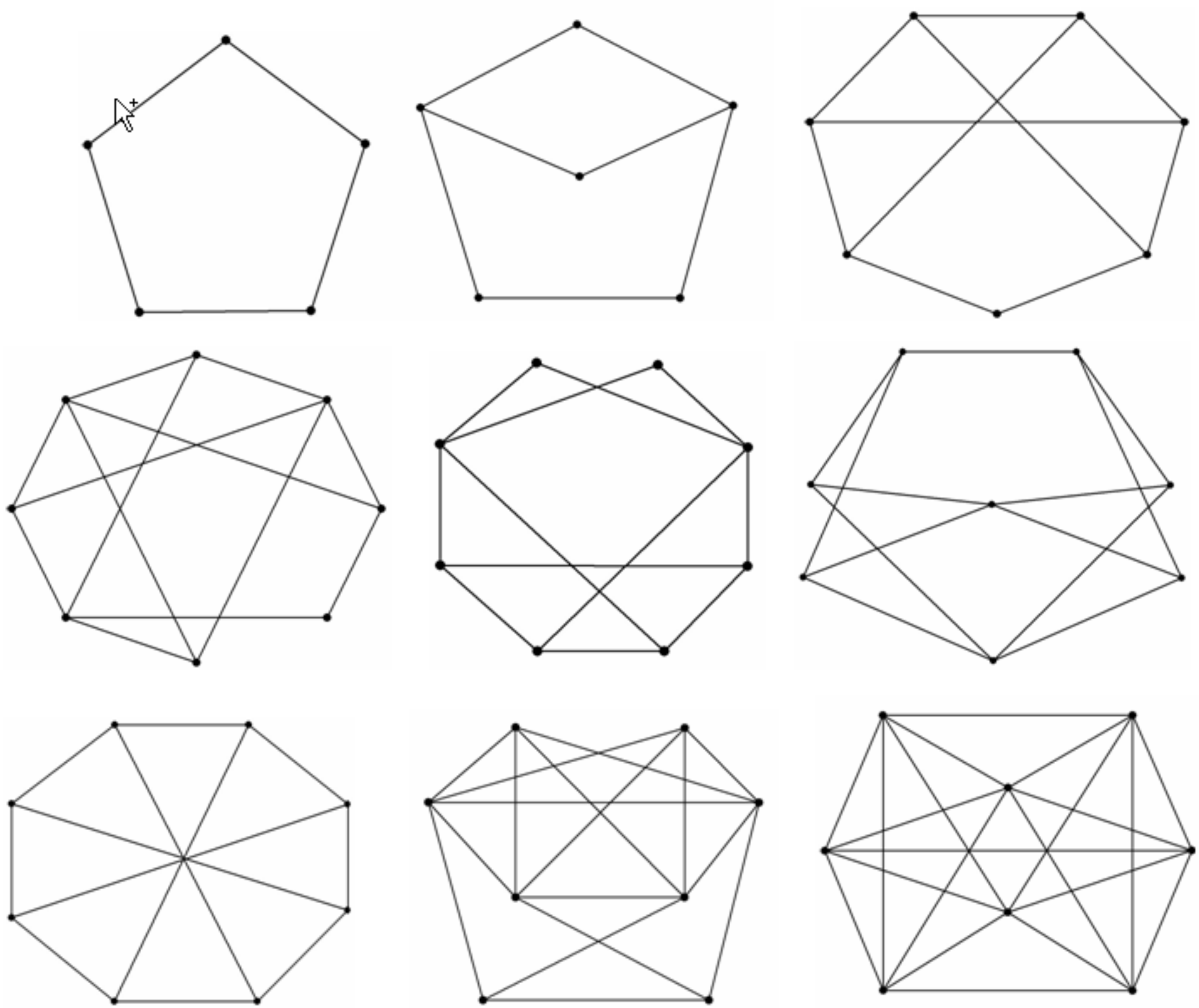
Three Theoretical Methods to Study Stability

- Analytic results on stability criteria
- Enumeration: check whether networks are stable for as many networks as possible
- Simulation: starting from a set of networks determine the likelihood that a myopic updating process ends in a specific network structure

Analytic Results Pairwise Stability

- Adding a tie without creating closed triads is always beneficial
  - Shortest path length in pairwise stable networks is smaller than or equal to 2
  - Pairwise stable networks are connected
  - Incomplete bipartite networks are not pairwise stable
- Complete bipartite networks are pairwise stable except for “stars” with more than 4 actors

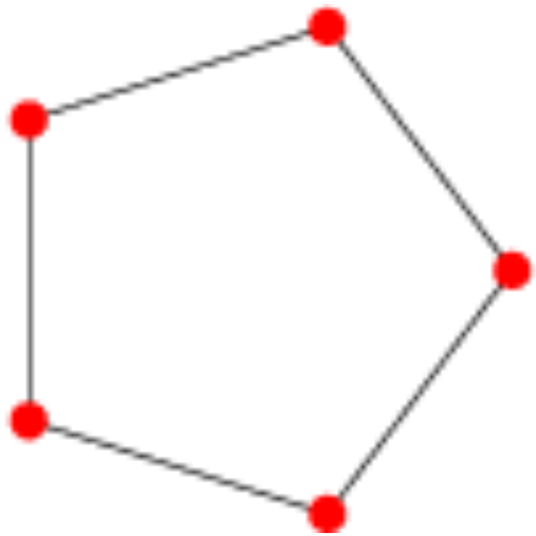
Pairwise Stable Networks



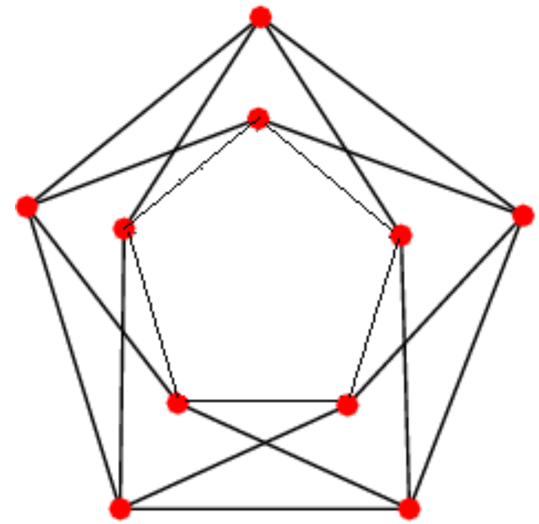
Analytic Results Unilateral Stability

- A complete bipartite network is unilaterally stable if and only if it is balanced
- Networks with a number of actors that is a multiple of 5 and that are generalizations of the Pentagon are unilaterally stable.
- Networks with a number of actors that is a multiple of 8 and that are generalizations of the Wheel are unilaterally stable.

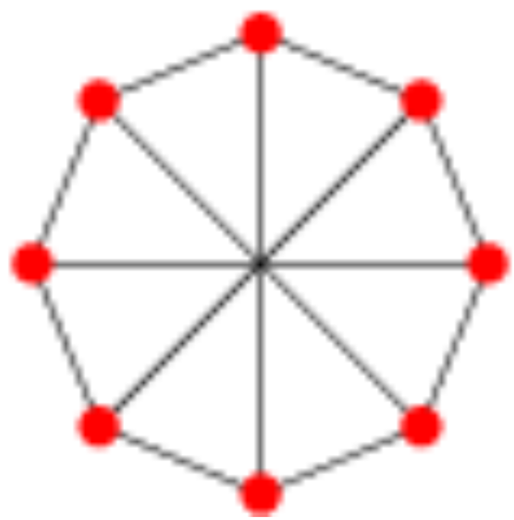
Other Unilaterally Stable Networks



Pentagon



Generalized pentagon



Wheel

Enumeration of Stable Networks

# actors	# networks	Pairwise	Strongly pairwise	Unilaterally
2	2	1	1	1
3	4	1	1	1
4	11	2	2	1
5	34	2	2	2
6	156	4	3	1
7	1044	3	3	1
8	12346	10	7	2
9	274668	9	7	1
10	$> 12 \cdot 10^6$	14	9	2

The Simulation

- Start from a “random” network (size 2-25)
- At each point in time, one randomly chosen actor may propose or delete one link (pairwise stability)
- For addition, consent is needed
- Actors are myopic, i.e., they will change the link that provides them with the largest utility gain given all other existing links
- No “noise” (also including random tie changes)
- Run until convergence (pairwise stable) for a set of networks

Emergence of Bipartite Networks

- Number of actors ≥ 8
- Convergences to a complete bipartite network in 87-97% of the cases (except for size = 8)
- Number of actors is odd
  - 80-91% to the balanced complete bipartite network
- Number of actors is even
  - 50-68% to the balanced complete bipartite network
  - Around 30% to the least unbalanced complete bipartite network

Convergence to Bipartite Networks

n	Number of starting networks	Balanced	Just unbalanced
8	12346	.61	.12
9	9292	.86	.01
10	10070	.68	.24
11	10898	.91	.03
12	10930	.61	.33
13	5078	.88	.07
14	5700	.57	.35
15	6358	.86	.07
16	7062	.58	.35
17	2346	.86	.09
18	2666	.55	.39
19	3006	.85	.10
20	3366	.53	.42
21	3746	.84	.13
22	4146	.52	.41
23	4566	.82	.14
24	5006	.50	.43
25	5466	.80	.16

Robustness Analyses

- Similar results if utility of structural holes is interpreted as relative utility compared to others

- If utility is simplified to two components
  - Benefits of direct relations
  - Relatively high costs of closed triads

  Still balanced complete bipartite networks are the dominant emerging structure

- Balanced complete bipartite networks are even stronger attractors if noise is added

Factors That Do Affect Results

- Substantive predictions ARE affected by
  - Redundancy over longer distances is relevant (Goyal and Vega Redondo)
  - Actors strive for other things
  - Actors have mixed motives
  - Different actors strive for different things (heterogeneity)

The Main Claims

- If everybody would strive for structural holes, we would obtain most likely many balanced complete bipartite networks

- We set-up a machinery that provides sharp predictions for emerging networks given that we know where actors strive for in social networks

- The machinery will even work for mixed motives and heterogeneous preferences as long as we have all actors' utility functions based on their network positions

Experiment

- Of course determining every actor’s utility based on the network is difficult, but we can manipulate this utility in an experiment

- Predictions are tested using such a computerized laboratory experiment

- Equipment:
  - z-Tree (Fischbacher, forthcoming)
  - ORSEE recruitment system (Greiner, 2004)
  - ELSE laboratory

Experimental Test of Stability Results

- Actors have benefits of ties
- Actors have increasing marginal costs of ties (implying a capacity constraint)

- Actors might have costs or benefits of closed triads
  - **Burtian** network formation context: Closed triads are costly
  - **Colemanian** network formation context: Closed triads are beneficial
  - **Neutral** network formation context: Closed triads do not matter

Utility Functions

- Burtian Network Formation Context

$$u_i(t_i, z_i) = b_1 t_i - c_1 t_i - c_2 t_i^2 - c_3 z_i$$

- Colemanian Network Formation Context

$$u_i(t_i, z_i) = b_1 t_i - c_1 t_i - c_2 t_i^2 + b_2 z_i$$

- Neutral Network Formation Context

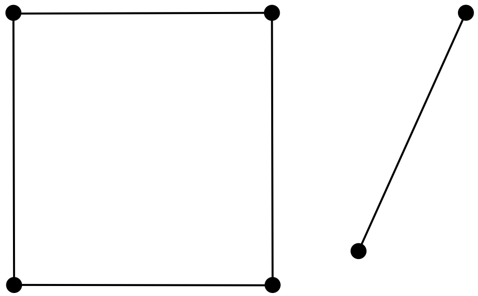
$$u_i(t_i) = b_1 t_i - c_1 t_i - c_2 t_i^2$$

- where t_i is the number of ties of *i* and z_i the number of closed triads in which *i* is involved

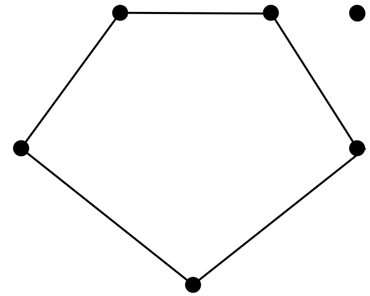
Experimental Design

Condition	Values
Starting network	Empty network
Size of the network	6 (156 different structures)
Network formation context	**Burtian, Colemanian, Neutral**
Linear Costs	0.20
Quadratic Costs (max. number of ties actors want in neutral context)	**0.10 (4), 0.20 (2)**
Costs or benefits of closed triads	0.20

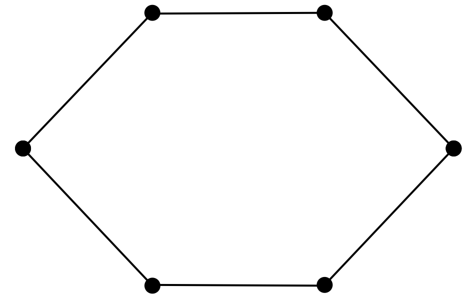
Stable Networks under High Quadratic Costs



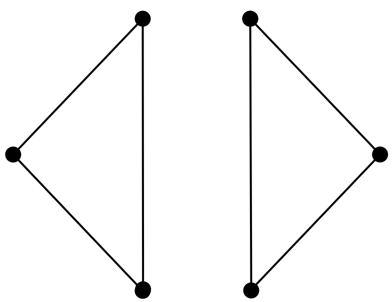
Square and Dyad (Burtian, Neutral)



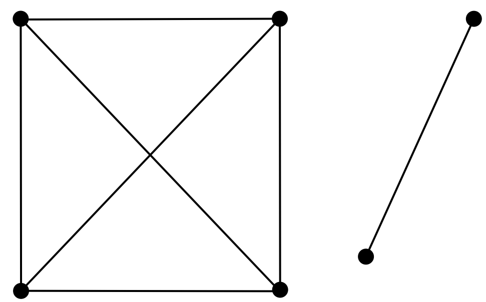
Pentagon and Isolate (Burtian, Neutral, Colemanian)



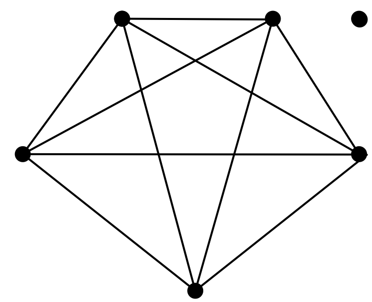
Hexagon (Burtian, Neutral, Colemanian)



Two triangles (Colemanian, Neutral)

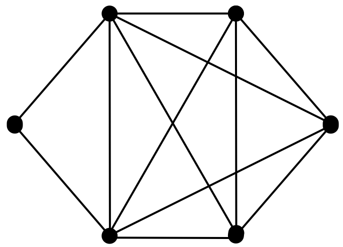


Full square and dyad (Colemanian)

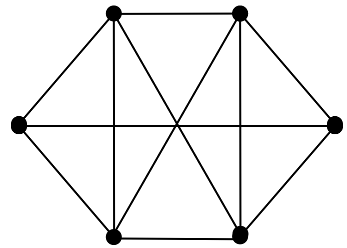


Full pentagon and isolate (Colemanian)

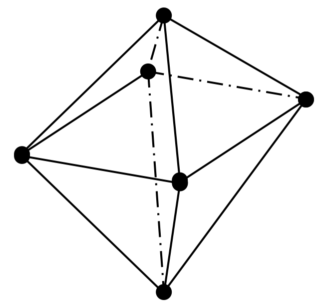
Stable Networks under Low Quadratic Costs



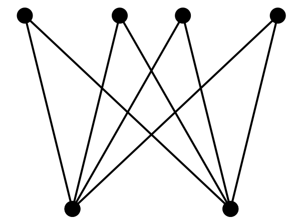
Tailed full pentagon (Neutral)



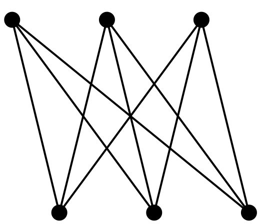
Single-crossed 3-prism (Neutral)



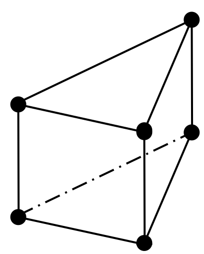
Octahedron (Neutral)



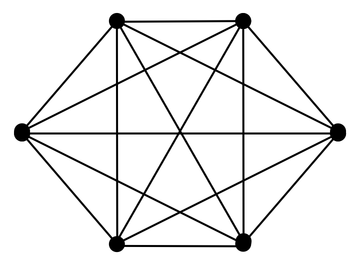
2,4-complete bipartite (Burtian)



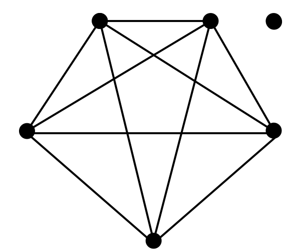
3,3-complete bipartite (Burtian)



3-prism (Burtian)



Full hexagon (Colemanian)



Full pentagon and isolate (Colemanian, Neutral)

Experiment: General Set-Up

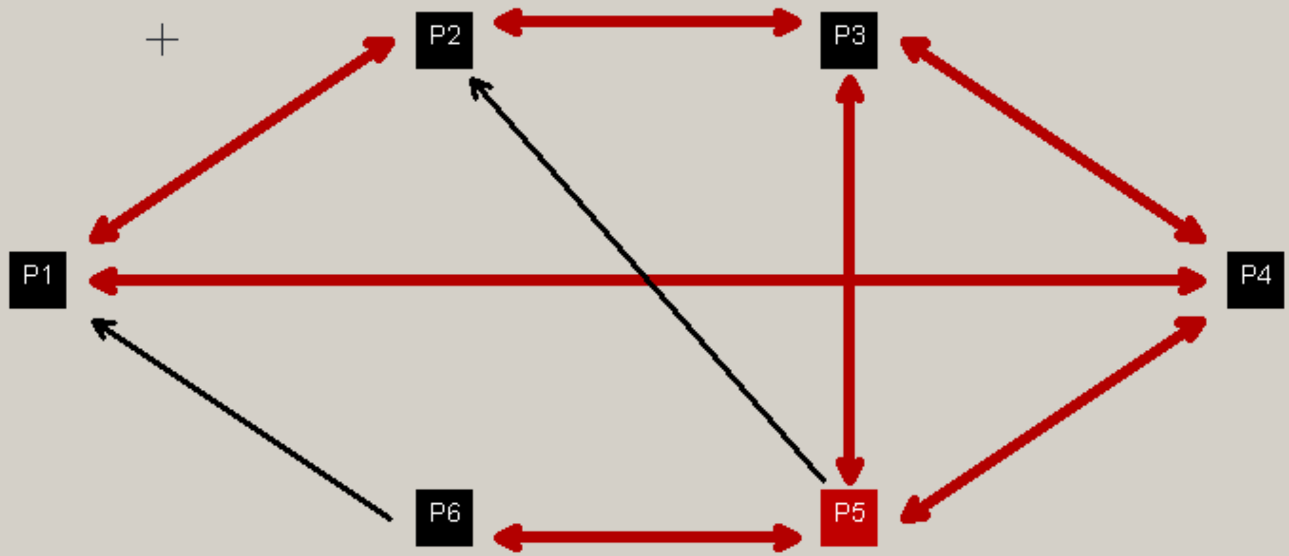
- 18 participants in each session, total 108 subjects in 6 session
- Participants had to interact in all three network formation contexts under one of the two costs functions
- Two costs functions and order of network formation contexts varied across sessions
- Every participant was matched anonymously with five other participants three times for each condition
- Every condition is repeated nine times within sessions and three times between sessions.

Experiment: "The Game"

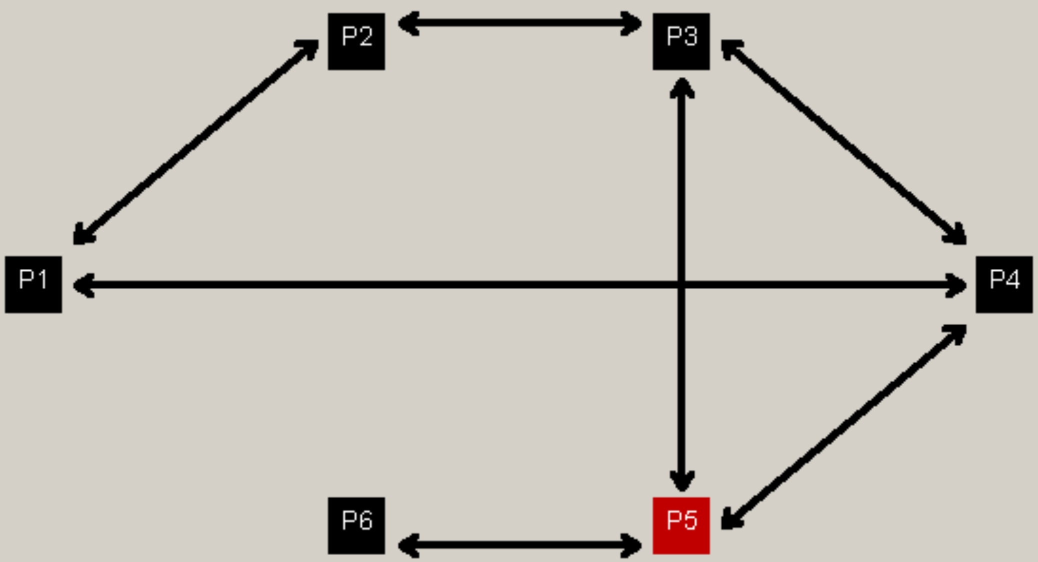
- 10 periods of 30 seconds each
- Everybody could click on others in the group to indicate that they want a link
- If the other also clicked, a tie was formed
- All clicks were shown instantly to all others in the group
- After every 30 second period, subjects obtained a number of points corresponding to their network position
- Maximum possible payoff: €16.80, maximum earned: €15.80, minimum earned: €10.80, average earned: €14.20

Please choose which relations you want to remove or which you would like to initiate with other participants.

(You can click on the other participants to remove or initiate a relation. If you click twice on another participant, your previous click will be undone)



Network in the previous round:



Results previous round

Benefits relations:	300
- Costs relations:	150
- Costs closed triads:	20
= Benefits previous round:	**130**
Total benfits within this scenario:	250

Data Analysis

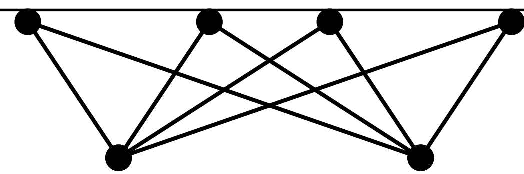
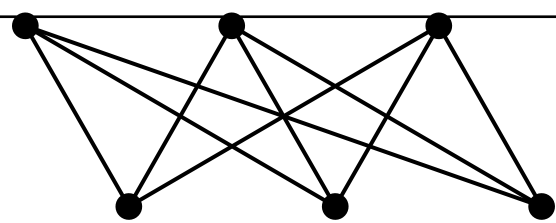
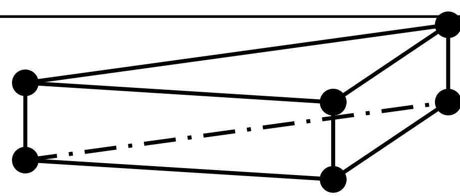
- Network dynamics for 27 networks in each of the 6 conditions
- We consider a network converged to a stable structure if the same configuration was chosen in three consecutive periods

- Results:
  - Comparison of converged networks with pairwise-stable networks
  - Comparison rank orders network measures

General Results

	Proportion 'Stable' Networks	Proportion 'Stable' Networks that are also Pairwise Stable
Low Costs		
Neutral	.815 (22 of 27)	1.000 (22 of 22)
Burtian	.519 (14 of 27)	1.000 (14 of 14)
Colemanian	.926 (25 of 27)	.600 (15 of 25)
High Costs		
Neutral	.963 (26 of 27)	1.000 (26 of 26)
Burtian	.815 (22 of 27)	.864 (19 of 22)
Colemanian	.778 (21 of 27)	.857 (18 of 21)
Overall	.802 (130 of 162)	.877 (114 of 130)

Probability of Each Network after Convergence

	Simulation	Experiment (15 of 28 times converged)
	0.070	0
	0.620	1
	0.310	0

Conclusion and discussion

- Adaptive model in combination with the stability criterion seems to predict behavior reasonably well
  - Empirically stable networks are very often the theoretically stable networks
  - Most likely stable network to emerge in simulation is also most likely to emerge in the experiment
  - Main structural differences in network characteristics emerge as predicted

Further research

- Precise likelihood of different stable networks more difficult to predict. Possible additions:
  - Stricter stability concepts
  - Additional selection arguments: inequality aversion

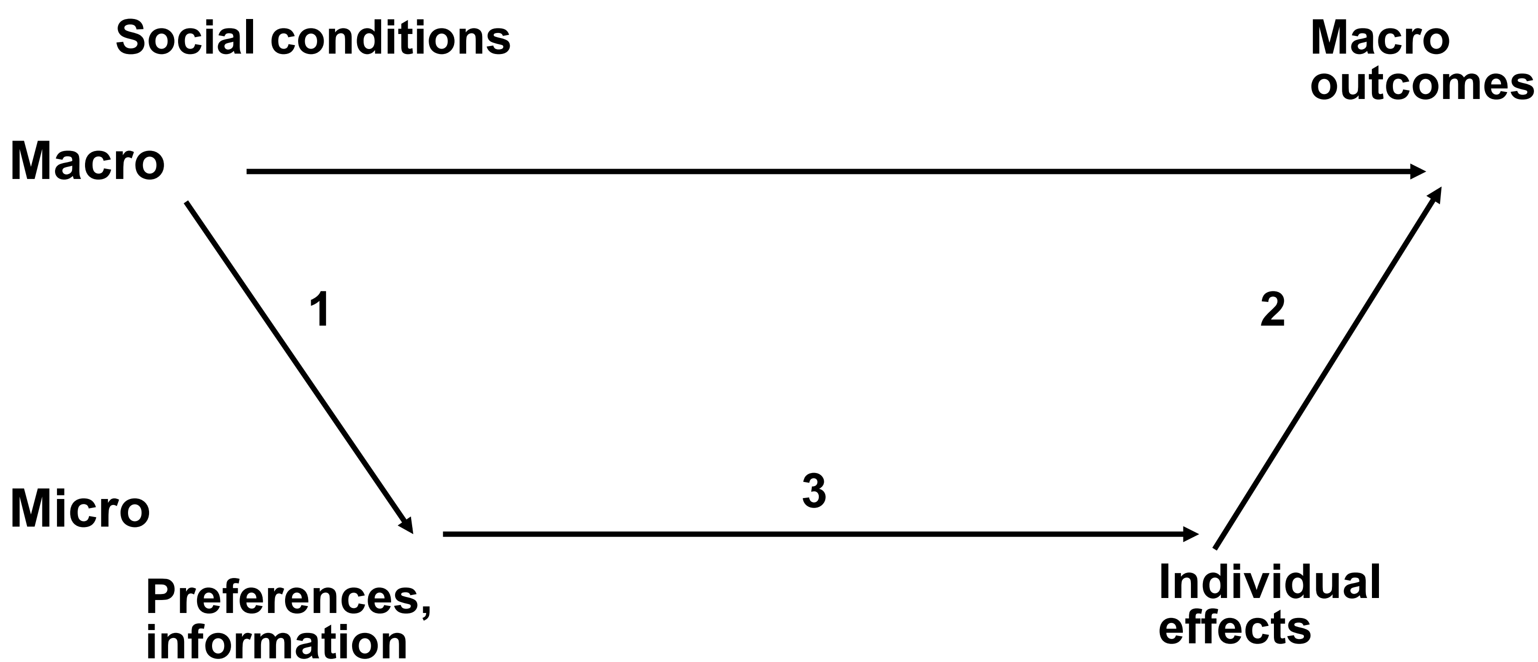
- Some limitations
  - All actors are the same
  - No hybrid utility functions

- These studies no not incorporate other types of behavior
  - Trust in dynamic networks
  - Cooperation or coordination in dynamic networks

General conclusion

- Nice example of analytical / micro-macro sociology
  - Mechanism driven
  - Social outcomes as result of interaction of individual choices

Coleman’s scheme for macro-micro-macro transitions



1: Bridge assumptions

3. Behavioral theory

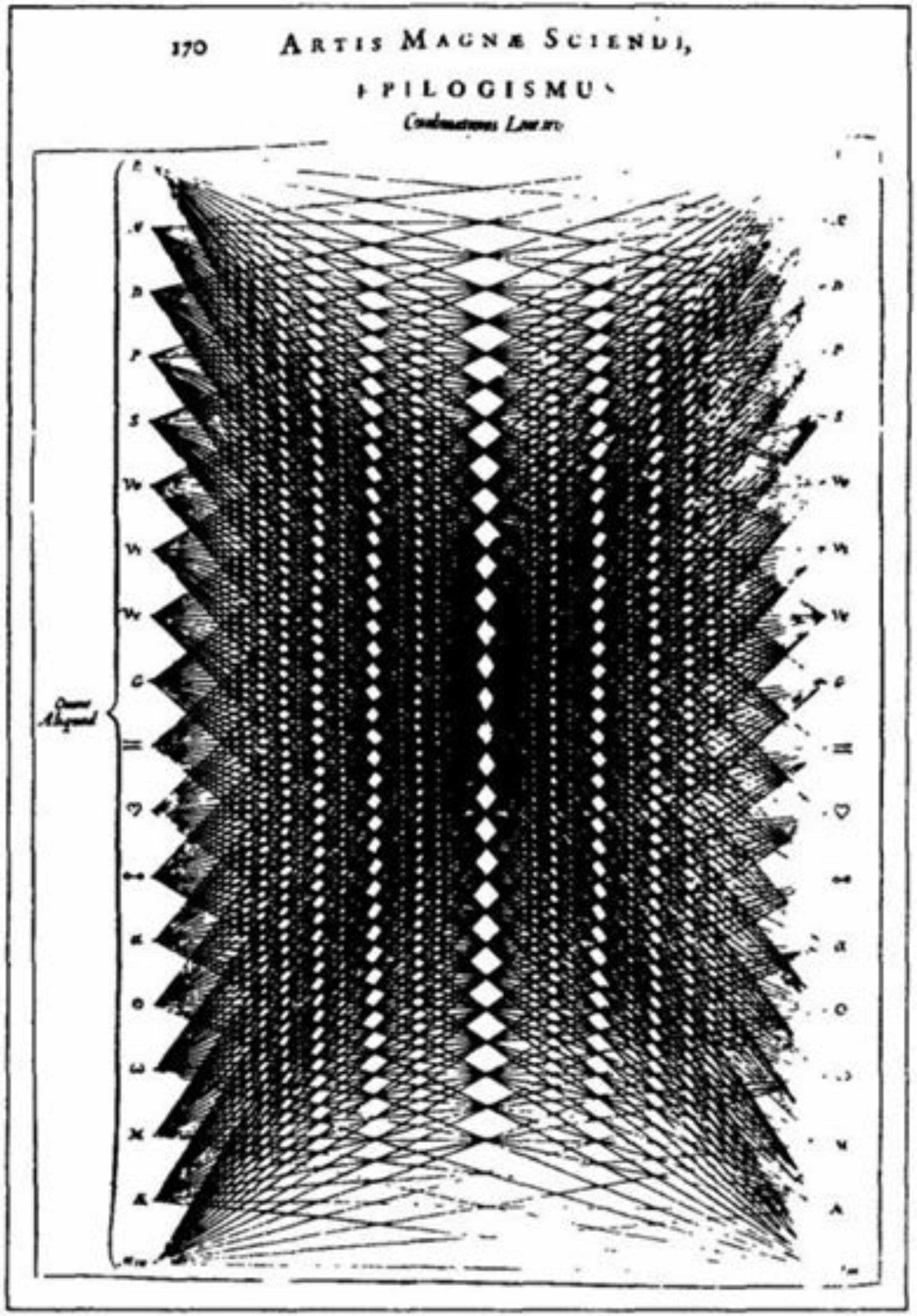
2. Transformation rules

General conclusion

- Nice example of analytical / micro-macro sociology
  - Mechanism driven
  - Social outcomes as result of interaction of individual choices
- Especially the micro-macro link can have complexity features
  - Macro-outcomes can dramatically change due to relatively small adaptations in micro-assumptions (adding some noise in the simulations, changes the likelihood of emergence to some stable states quite a bit)
  - Unintended consequences can emerge due to the interaction between individuals that were not intended by any of the individuals involved (the very equal network structure predicted was not intended)
- Simplifying assumptions crucial to obtain theoretical implications

What have we learnt?

- The theoretical framework based on pairwise stability or maybe a somewhat stronger equilibrium concept provides adequate predictions:
  - If network utility is known
  - All actors have the same utility
  - Stable networks are symmetric
- Theoretical framework also provides possibilities to design optimal network structures if an authority could impose the relations
- More research is still needed in empirical settings to understand where actors strive for in developing network relations



Trivium:

This balanced complete bipartite network with 36 nodes plays a role in Umberto Eco's ***Foucault's Pendulum*** **(1989, p. 473)**

Thank you for your attention!

Do you have any questions?

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Probability of Convergence by Noise Level for High Costs

Neutral context	Noise=.1	Noise=.4	Noise=.7
Two triangles	0.165	0.140	0.125
Square and dyad	0.190	0.110	0.130
Pentagon and isolate	0.215	0.190	0.205
Hexagon	0.430	0.560	0.540
Burt context			
Square and dyad	0.190	0.160	0.205
Pentagon and isolate	0.225	0.205	0.235
Hexagon	0.585	0.635	0.560
Coleman Context			
Full pentagon and isolate	0.000	0.000	0.005
Full square and dyad	0.035	0.105	0.190
Two triangles	0.645	0.595	0.465
Hexagon	0.170	0.195	0.260
Pentagon and Isolate	0.150	0.105	0.080

Probability of Convergence by Noise Level for Low Costs

	Noise=.1	Noise=.4	Noise=.7
Neutral Context			
Tailed full pentagon	0.225	0.220	0.345
Single-crossed 3-prism	0.425	0.400	0.295
Octahedron	0.215	0.340	0.345
Full pentagon and isolate	0.135	0.040	0.015
Burt Context			
2,4-complete bipartite	0.140	0.070	0.070
3,3-complete bipartite	0.735	0.620	0.495
3-prism	0.125	0.310	0.435
Coleman Context			
Full hexagon	0.720	0.860	0.875
Full pentagon and isolate	0.280	0.140	0.125

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Network measures

Indicator	Description
Density	The proportion of in the network
Full triads	The proportion of full triads
Centralization	The standard deviation of the proportion of ties each actor has. The measure is standardized, such that all values are between 0 (min.) and 1 (max.) for networks with six actors
Segmentation	The proportion of dyads with at least distance 3 of all dyads that have at least distance 2. We chose the maximal value 1 for disconnected networks and -1 for complete networks.

Testing Point-Predictions

	Proportion of full triads			Segmentation		
Low Costs	**EM (SD)**	**OM (SD)**	**z-test**	**EM (SD)**	**OM (SD)**	**z-test**
Neutral	.362 (.047)	.395 (.034)	3.29*	.040 (.196)	.045 (.213)	0.12
Burt	.031 (.046)	.000 (.000)	-2.52*	.000 (.000)	.000 (.000)	0.00
Coleman	.930 (.174)	.906 (.126)	-0.69	-.720 (.696)	-.600 (.500)	0.86
High Costs						
Neutral	.014 (.035)	.012 (.033)	-0.29	.627 (.332)	.428 (.230)	-3.06*
Burt	.000 (.000)	.000 (.000)	0.00	.577 (.322)	.328 (.042)	-3.63*
Coleman	.081 (.061)	.114 (.036)	2.48*	.870 (.265)	.972 (.086)	1.76

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Unilateral Stability Formalized

- A network $g' \subseteq g^N$ is *unilaterally obtainable* from g by i through $S \subseteq N \setminus \{i\}$ if
  - all ties that are in g' but were not in g involve actor i and an actor in S;
  - all ties that are not in g' but were in g involve actor i.

- A network $g \subseteq g^N$ is *unilaterally stable* if for all i, $S \subseteq N \setminus \{i\}$, and $g' \subseteq g^N$ unilaterally obtainable from g by i through S, $u_i(g') > u_i(g) \Rightarrow u_j(g') < u_j(g)$ for some $j \in S$.