Universiteit Utrecht

Cellular Automata

Jason Frank
Mathematical Institute

WISM484 Introduction to Complex Systems, Utrecht University, 2015

Cellular Automata

- Game of Life:
  Simulator: http://www.bitstorm.org/gameoflife/
  Hawking: https://www.youtube.com/v/CgOcEZinQ2I&feature=share&list=FLwikA_t8e6TSJW-L-IAHkKw
- Definition, concepts
- 1D, binary, nearest neighbor CA
- Game of Life, again: https://www.youtube.com/v/My8AsV7bA94
- Traffic models

Cellular Automata

- Defined on a structured lattice, e.g. $i \in \mathbb{Z}^d$
- Deterministic “evolution rule” (one-dimensional, radius r):

$$s(i,t) \in Q, \quad t \in \mathbb{N},$$
$$s(i,t+1) = f\left(s(i-r,t), s(i-r+1,t), \ldots, s(i+r,t)\right).$$

- Synchronous update
- Any discrete process on a finite space is eventually periodic. (Any bounded program that doesn’t terminate must eventually repeat).

Elementary cellular automata (1D, binary, nearest neighbor rule)

- $Q = \{0, 1\}, \quad r = 1$
- Defined via a look-up table. Example:

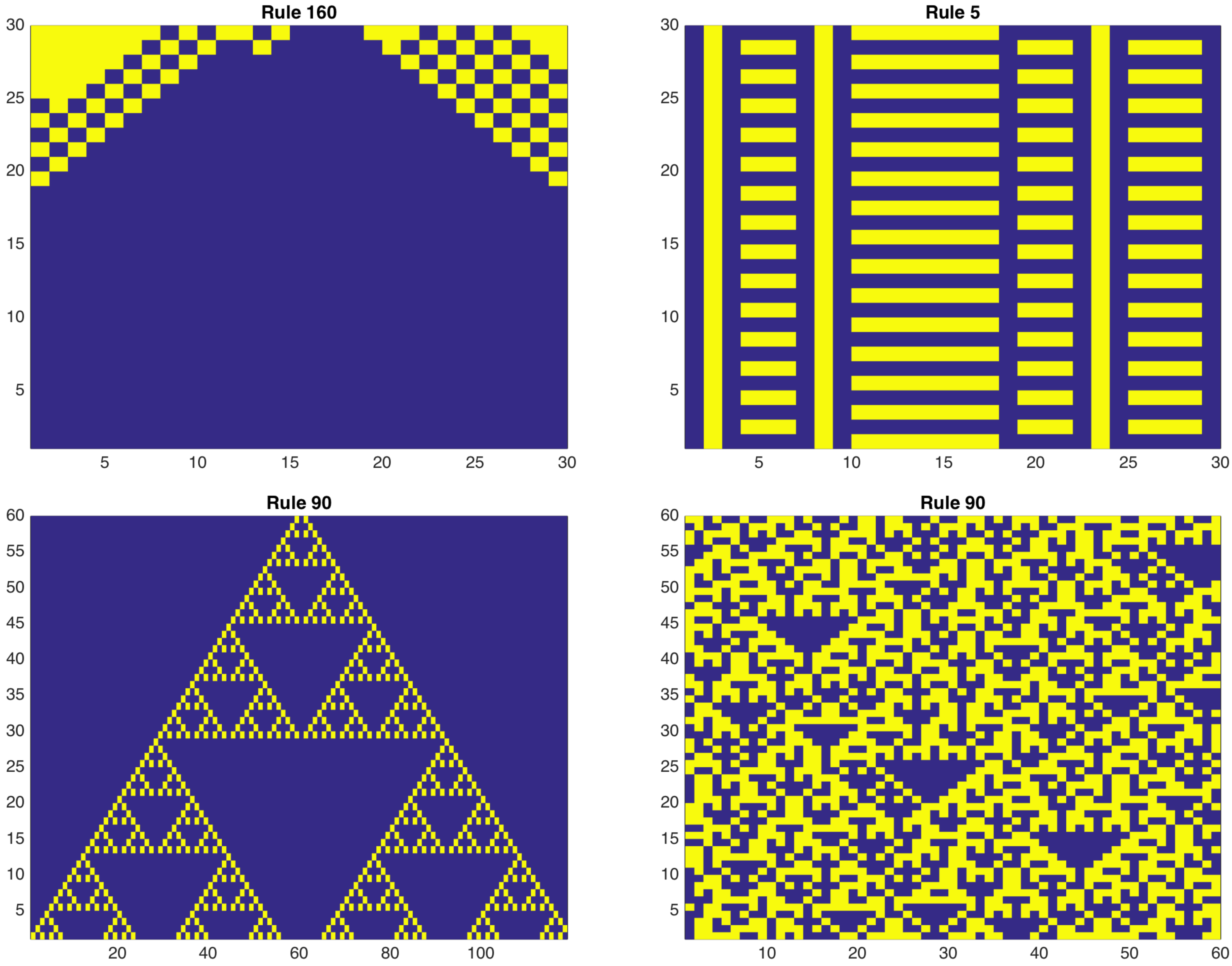
s(i-1,t)	s(i,t)	s(i+1,t)	s(i,t+1)
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

- Wolfram rule $10111000_2 = 184$, "Rule 184"

Elementary cellular automata (1D, binary, nearest neighbor rule)

- We must specify 2^r values to define the rule, 2^{2^r} rules.
- For r=1: 2^3=8 values and 2^8=256 different possible rules.
- Simplifying assumptions:
  - Observational symmetry (no bias towards left or right neighbor), means rows 2~5, 4~7 are equivalent ⟹ 2^6=64 distinct rules.
  - Outcome symmetry (looks only at neighbors, not at self), Assuming my neighbors will not change their strategies at the next time, there is is a unique best move for me ⟹ 2^3=8 distinct rules.
  - 0-1 symmetry 2^2=4 distinct rules:
    e.g. 0, 5, 90, 160
    Rule 0: trivial rule
    Rule 160: "0 unless both neighbors 1"
    Rule 5: "1 only if both neighbors 0"
    Rule 90: "Exactly two 1s in the 'hood"

s(i-1,t)	s(i,t)	s(i+1,t)	Rule 160	Rule 5	Rule 90
1	1	1	1	0	0
1	1	0	0	0	1
1	0	1	1	0	0
1	0	0	0	0	1
0	1	1	0	0	1
0	1	0	0	1	0
0	0	1	0	0	1
0	0	0	0	1	0

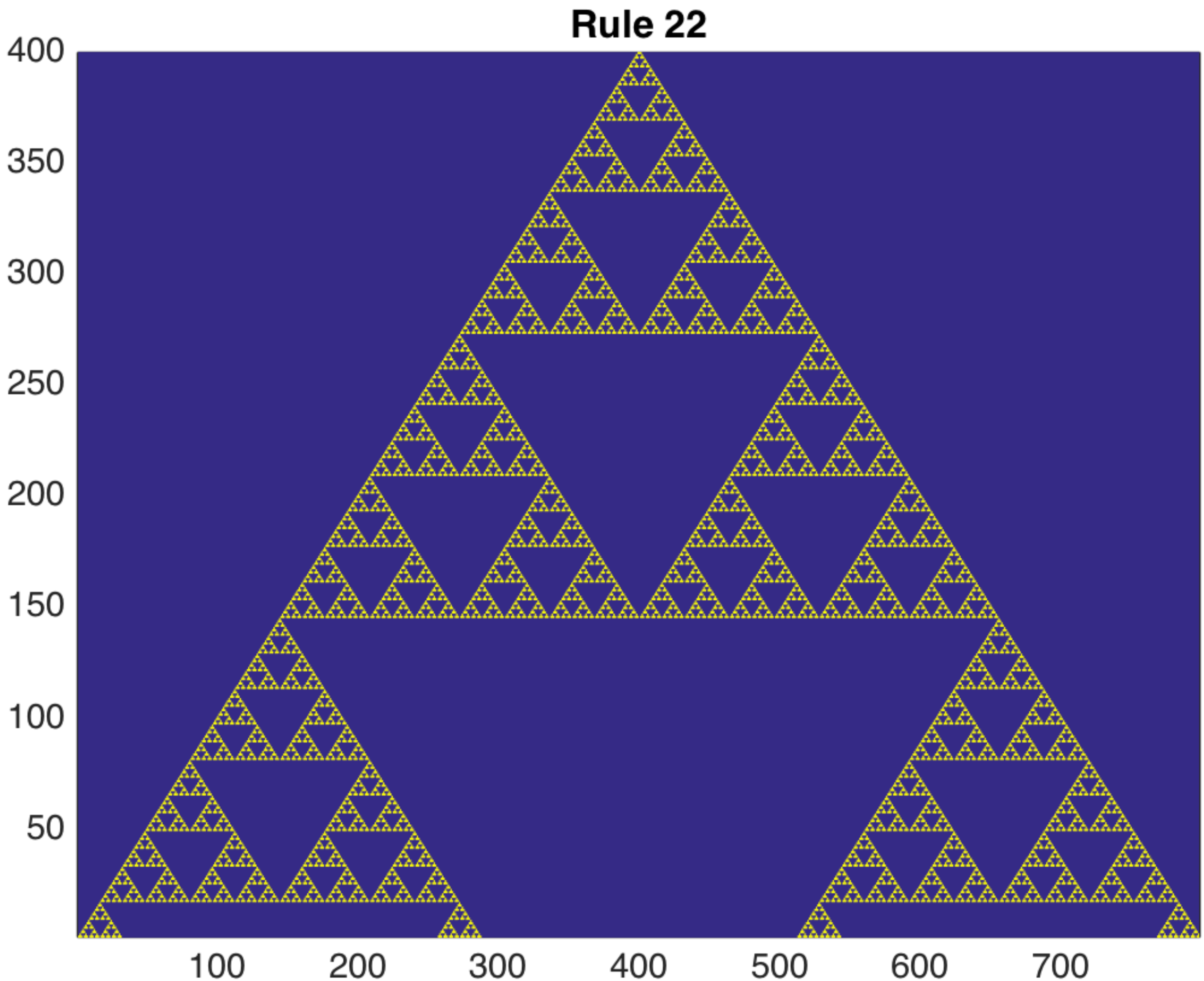


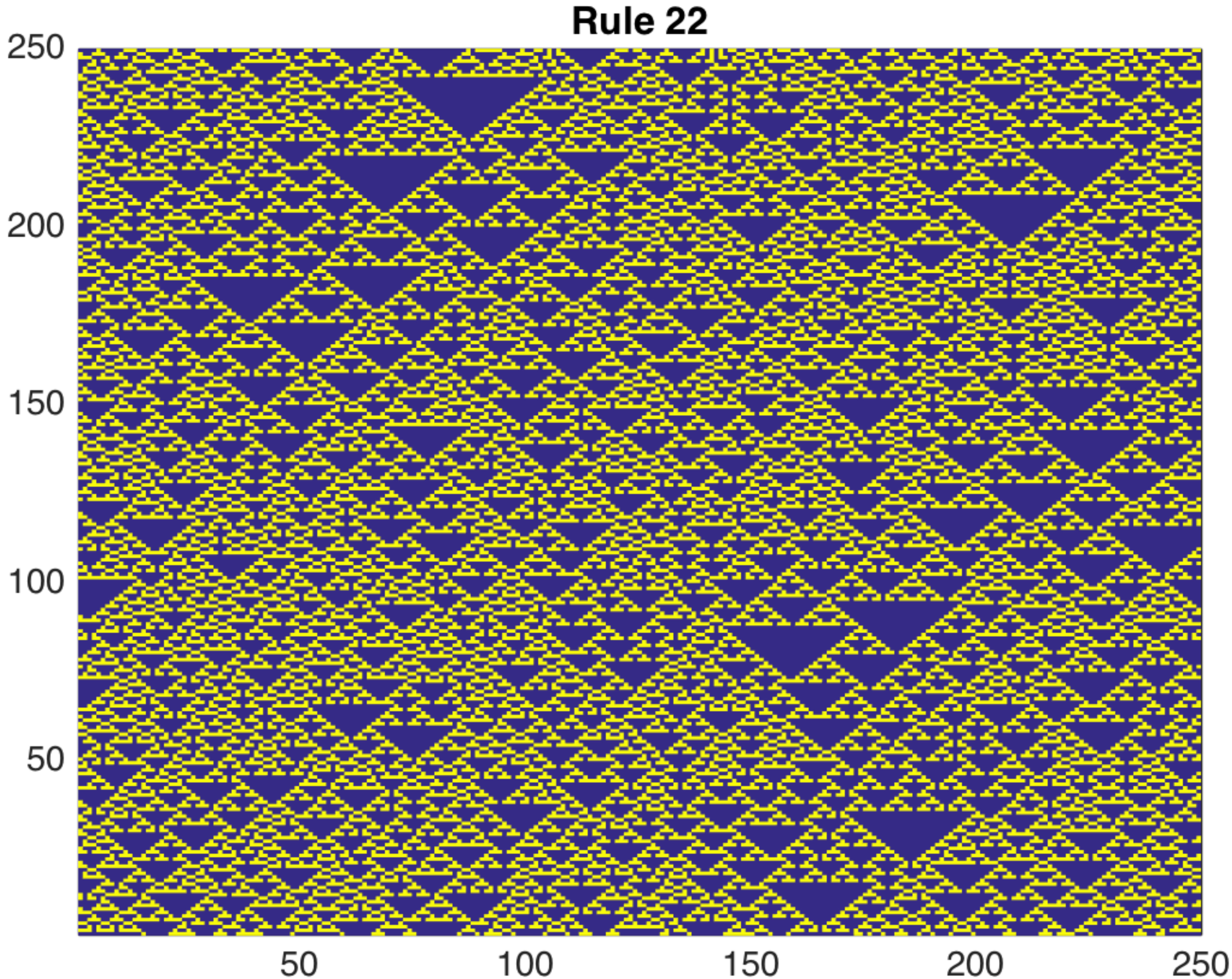
Rule 160
Rule 5
Rule 90
Rule 90

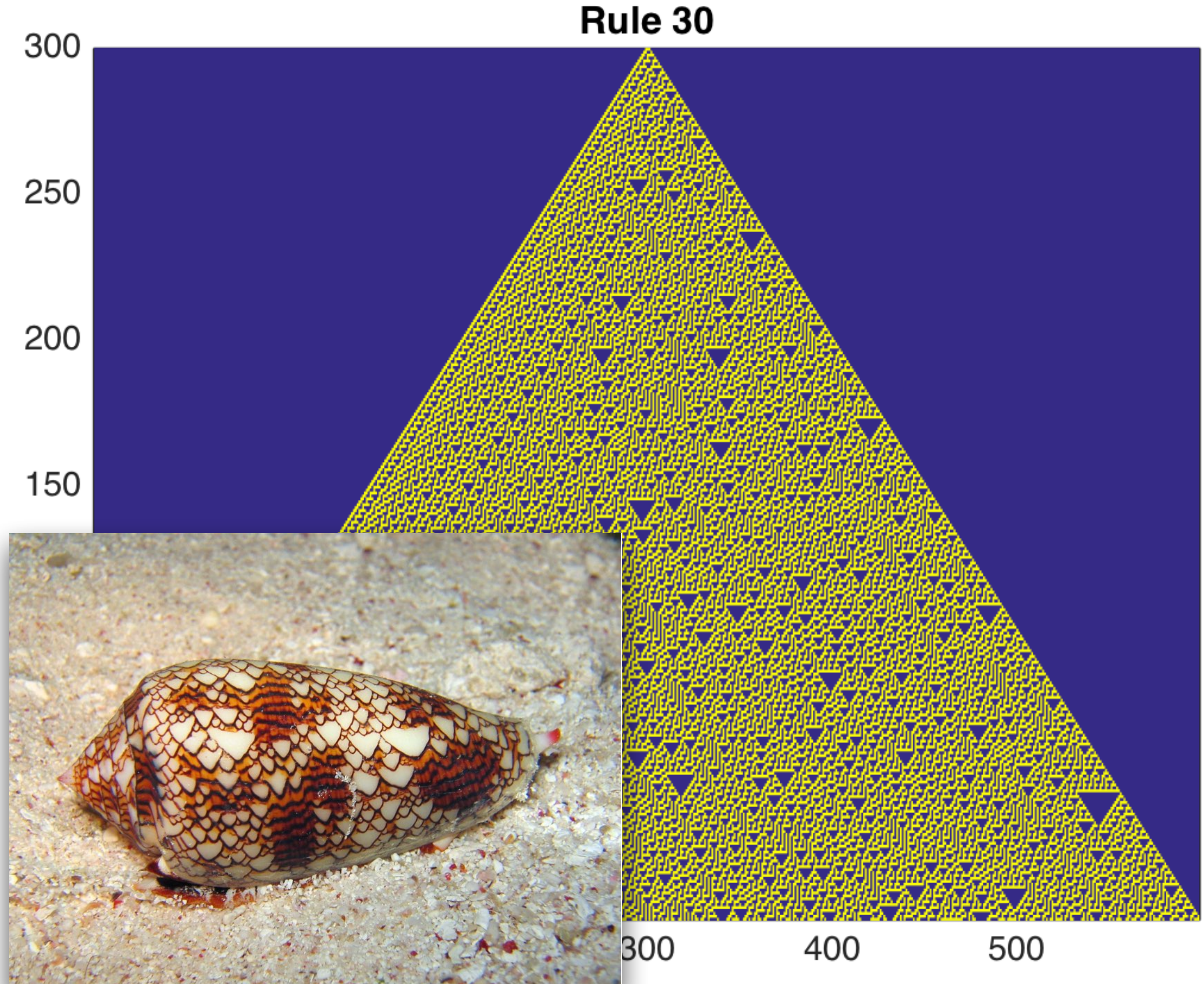
Wolfram’s classification

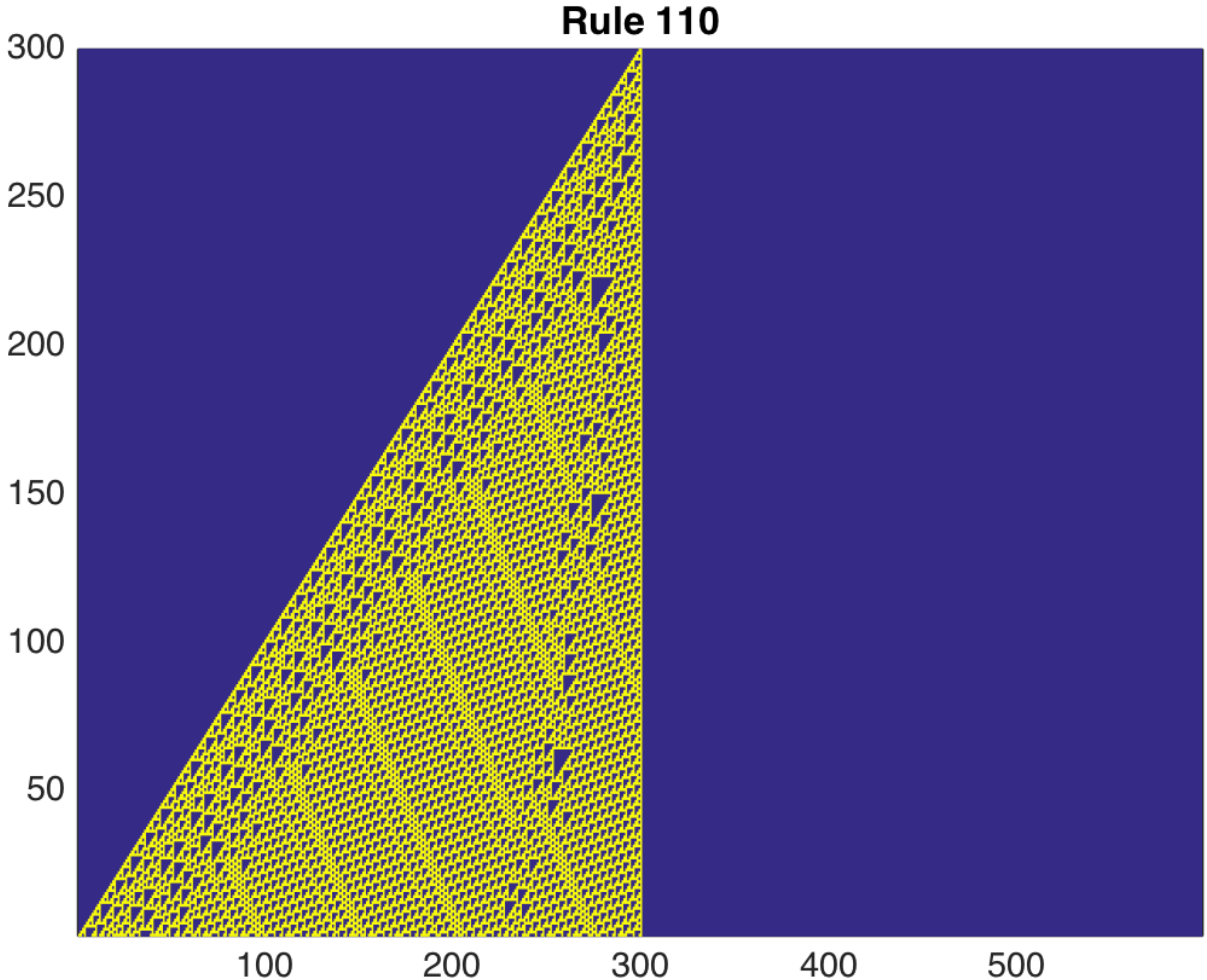
- Wolfram identifies 4 classes of elementary CAs:

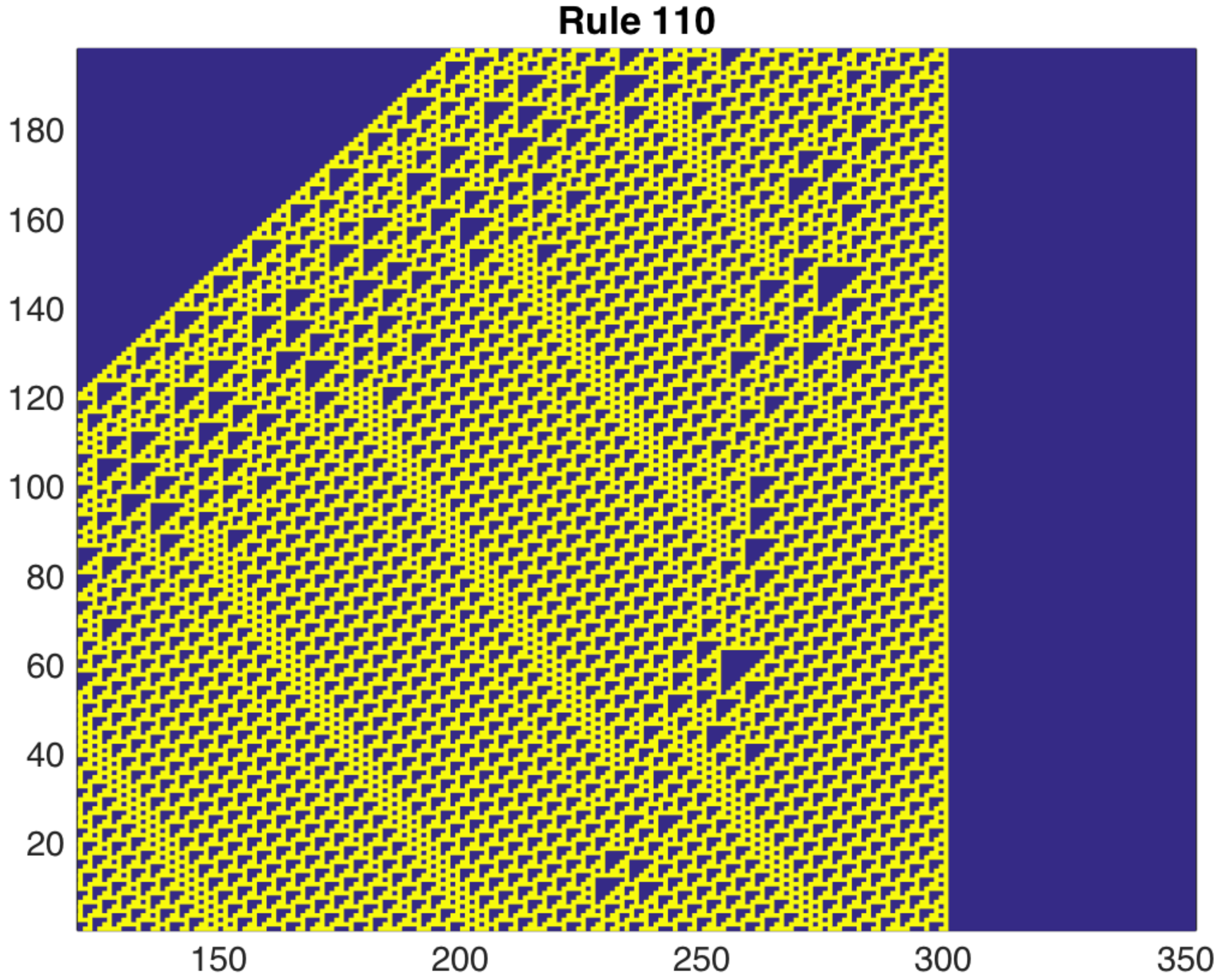
  1. Quick evolution to stable, homogeneous state. (Rule 0, 160)
  2. Stable or oscillating structures. Local changes to initial pattern remain local. (Rule 5)
  3. Chaotic: spread of initial perturbations, stable structures give was to noise. (Rules 90, 30)
  4. Complex: local structures persist for long times, capable of universal computation. (Rule 110)


Rule 22
400
350
300
250
200
150
100
50
100
200
300
400
500
600
700

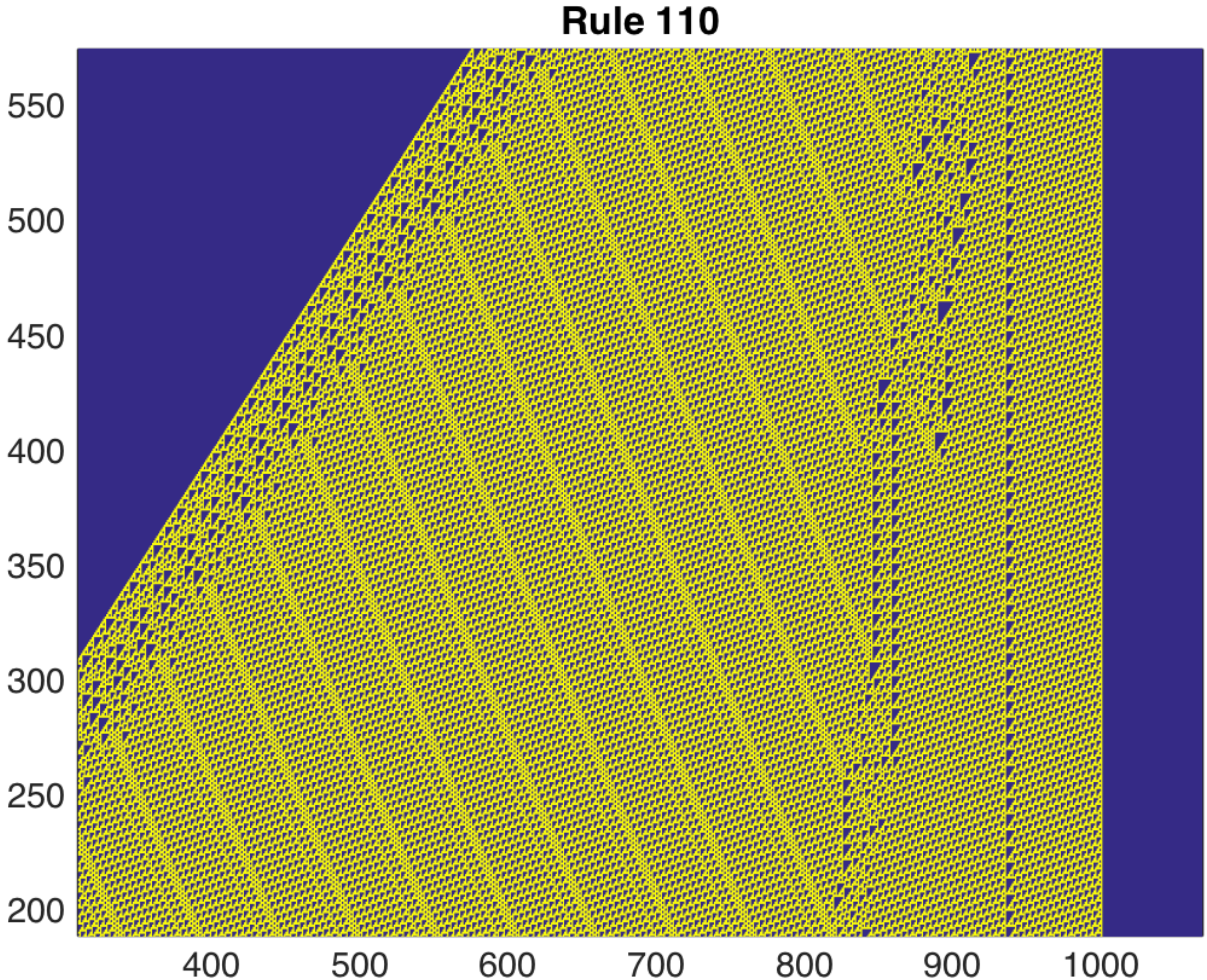

Rule 22
250
200
150
100
50
50
100
150
200
250


Rule 30
300
250
200
150
300
400
500


Rule 110
300
250
200
150
100
50
100
200
300
400
500

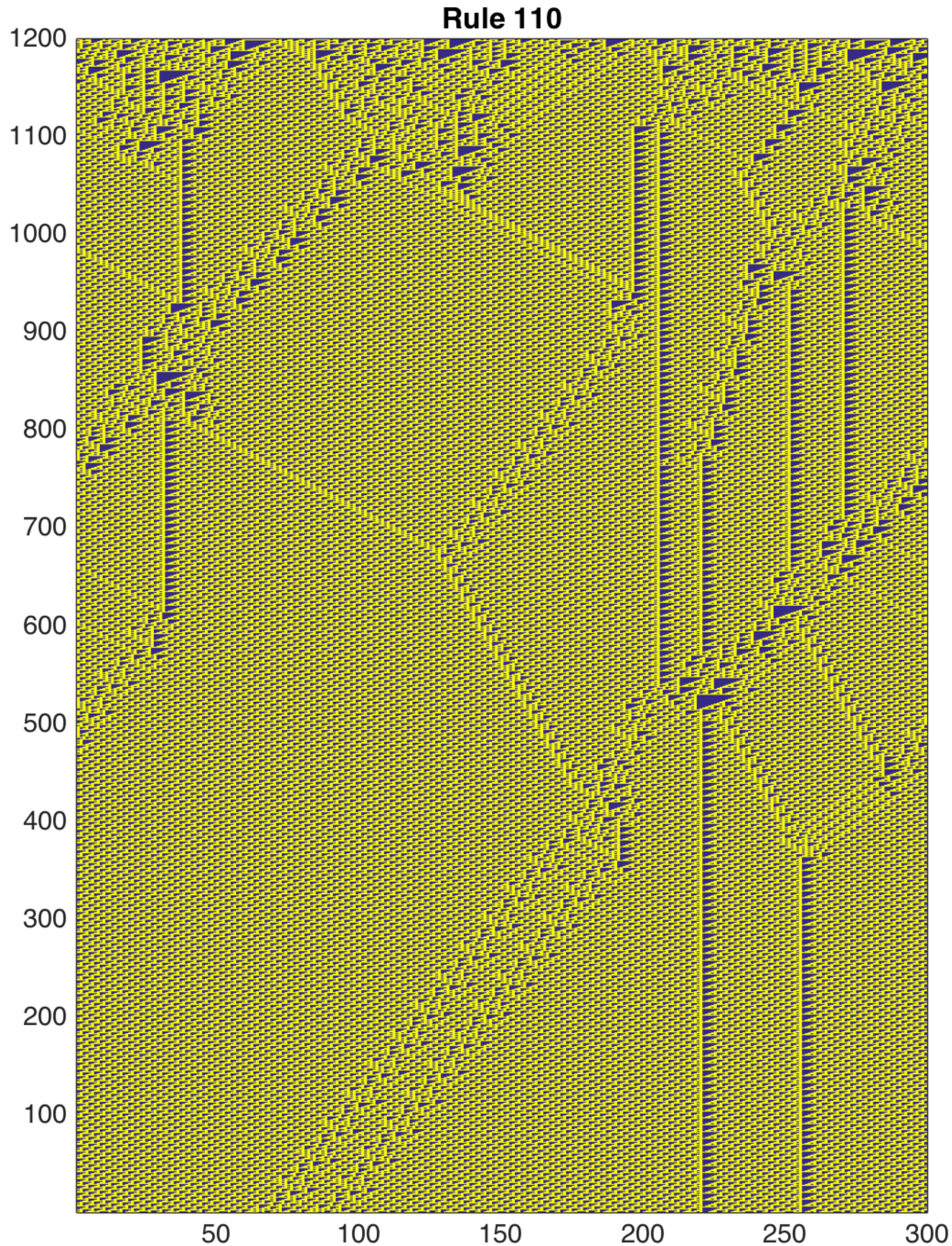


Rule 110
180
160
140
120
100
80
60
40
20
150
200
250
300
350


Rule 110
550
500
450
400
350
300
250
200
400
500
600
700
800
900
1000

Universal computation

- A **Turing machine** is an abstract “computer” that can perform a finite set of instructions on a data stream. This simple construction can emulate the logic of any computer algorithm (a **universal computer**)
- Cook and Wolfram proved that **Rule 110** is Turing complete, i.e. a universal computer.
- This means that any computer program can be simulated using Rule 110.
- Of course, the devil is in the details: one has to find the right initial condition, and be able to interpret the output.
- How a simple system can still generate complexity.


Rule 110
1200
1100
1000
900
800
700
600
500
400
300
200
100
50
100
150
200
250
300

Cellular Automata

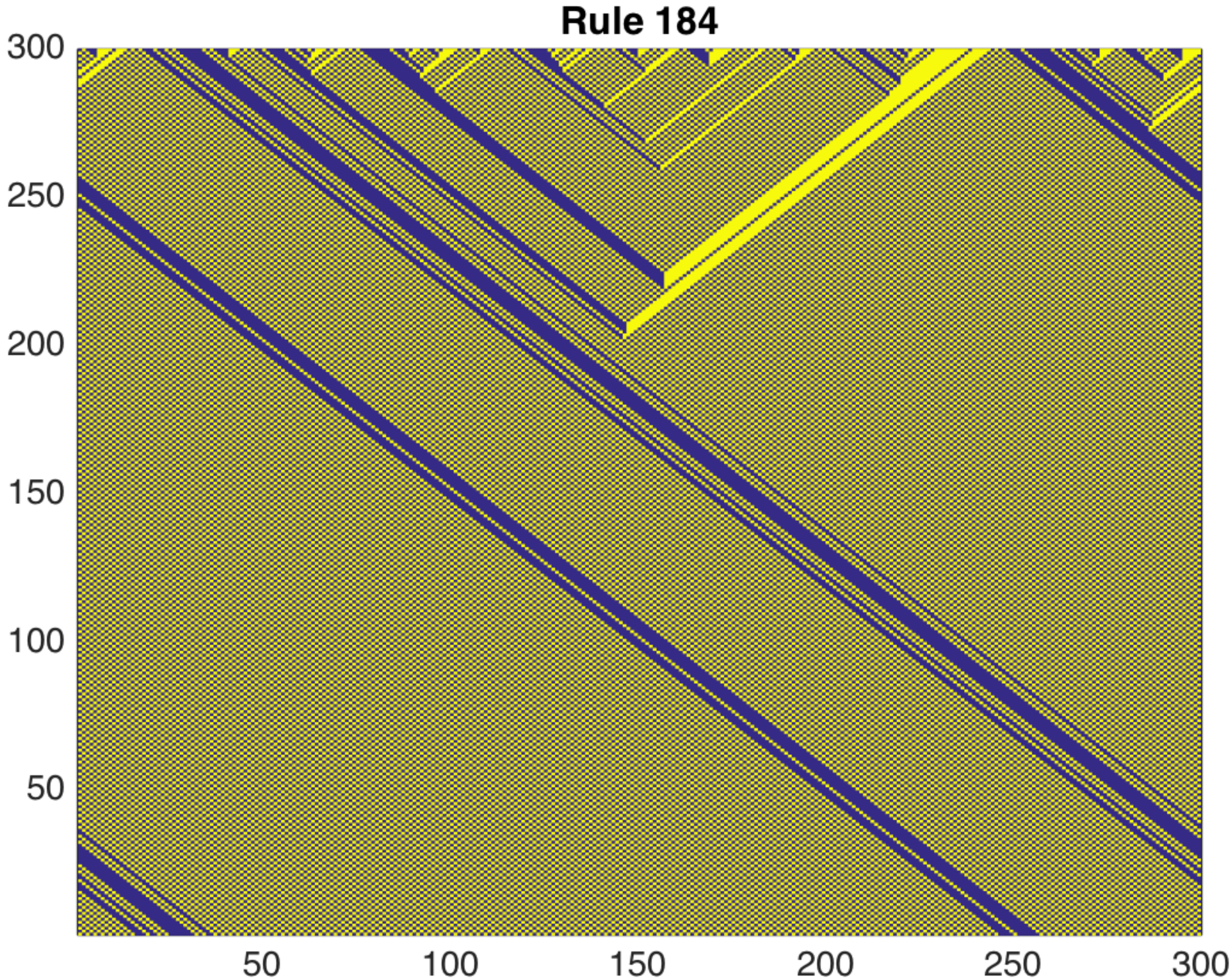
- Game of Life:
  Simulator: http://www.bitstorm.org/gameoflife/
  Hawking: https://www.youtube.com/v/CgOcEZinQ2I&feature=share&list=FLwikA_t8e6TSJW-L-lAHkKw
- Definition, concepts
- 1D, binary, nearest neighbor CA
- Game of Life, again: https://www.youtube.com/v/My8AsV7bA94
- Traffic models

Elementary cellular automata (1D, binary, nearest neighbor rule)

- $Q = \{0, 1\}, \quad r = 1$
- Defined via a look-up table. Example:

s(i-1,t)	s(i,t)	s(i+1,t)	s(i,t+1)
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

- Wolfram rule $10111000_2 = 184$, "Rule 184"


Rule 184
300
250
200
150
100
50
50
100
150
200
250
300

Rule 184

s(i-1,t)	s(i,t)	s(i+1,t)	Rule 160	Rule 5	Rule 90
1	1	1	1	0	0
1	1	0	0	0	1
1	0	1	1	0	0
1	0	0	0	0	1
0	1	1	0	0	1
0	1	0	0	1	0
0	0	1	0	0	1
0	0	0	0	1	0

- "Whenever there exists in the current state a 1 followed by a 0 (to its right), these swap places."
- Numbers of 1s and 0s is fixed: "number conserving"
- Symmetry: (left,right,0,1)
- Solves the "majority problem" in the sense that ultimately a repeated value indicates the majority.
- "Traffic flow"

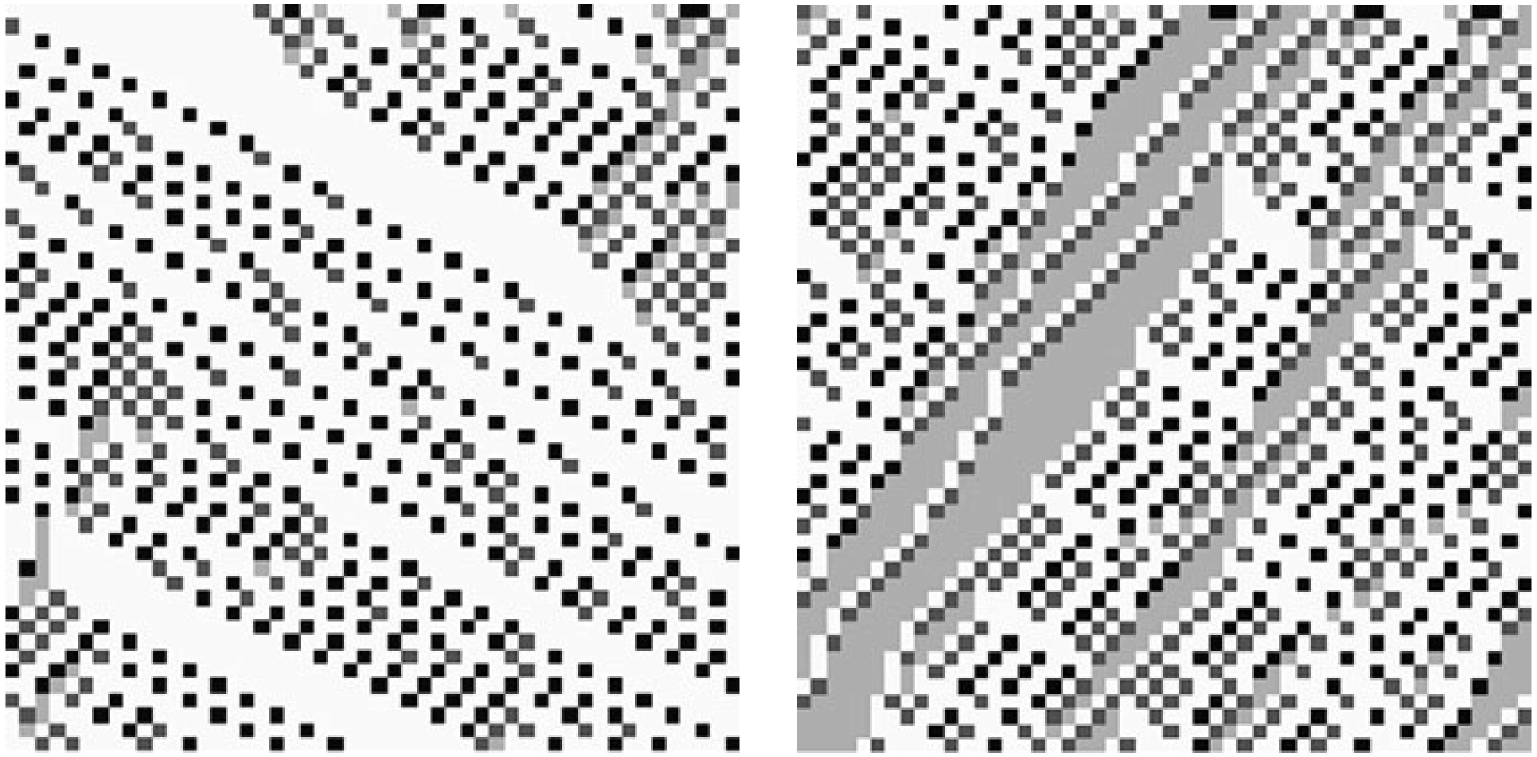


Fig. 6.3. First 50 iterations of the Nagel–Schreckenberg probabilistic cellular automaton traffic flow model. The initial configuration is random with a density equal to 0.24 in the left figure and 0.48 in the right one. In both cases $v_{\max} = 2$ and $p = 0.2$. The number of lattice sites is equal to 50. Empty cells are very light gray while cells occupied by a car with velocity v equal to either 0, 1, or 2 have darker shades of gray. Time increases downwards