Increasing the sample size for climate extremes by several orders of magnitude cost

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With J. WOUTERS and F. RAGONE (climate heat waves)

Mathematics of the Economy and Climate– July 2019.

ENS DE LYON

European Research Council
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Outline

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1. Rare events in complex dynamical systems
   - Rare events with a huge impact: extreme heat waves
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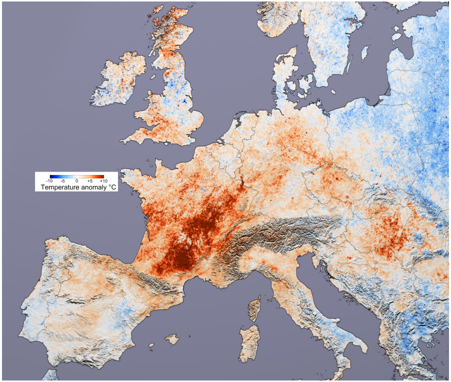
Extreme Heat Waves

Example: the 2003 heat wave over western Europe



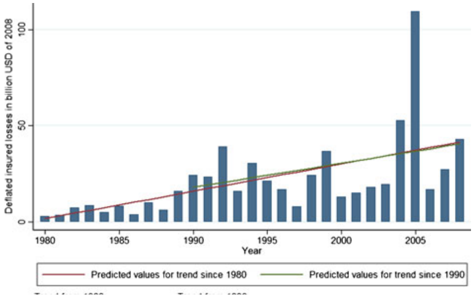
July 20 2003-August 20 2003 land surface temperature minus the average for the same period for years 2001, 2002 and 2004 (TERRA MODIS).

The Few More Extreme Events Have More Impact than All the Others?

(b) WORLD	DEATHS	LOCATION	DATE
DROUGHT/FAMINE	9,000,000	China	1876-79
FLOOD	2,000,000	Yellow River, China	1931
TROPICAL STORM	500,000	Coastal Bangladesh	11/12-13/1970
HEAT WAVE	35-52,000	Western Europe	8/2003
BLIZZARD	4,000	Iran	2/1972
TORNADO	1,300	Manikganj District, Bangladesh	4/26/1989
HAILSTORM	246	Moradabad, India	4/30/1888
SMOG	4,000	London, England	12/5-10/1952

(Ahrens)

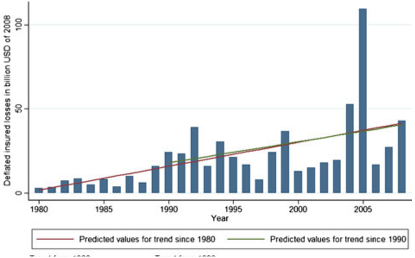
The Few More Extreme Events Have More Impact than All the Others?



Global deflated insured losses from natural disasters
(F. Barthel and E. Neumayer, 2012)

- Katrina (2005): 110 billion \$, Harvey (2017): between 80 and 190 billion \$, about 0.5% to 1% of USA GDP (Est. 21 trillion in 2019).

Natural Disaster Losses Increase

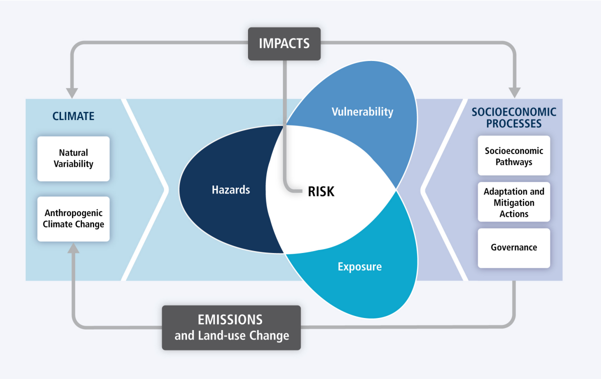


Global deflated insured losses from natural disasters

(F. Barthel and E. Neumayer, 2012)

- FFA (Fédération Française des Assurances) projections for France in 2015: 90% increase of natural disaster costs (92 *billion* €), for the next 25 years, compared to the previous 25 years, among which 30% due to climate change.

Hazard, Exposure and Vulnerability



(from IPCC, AR5, WG2)

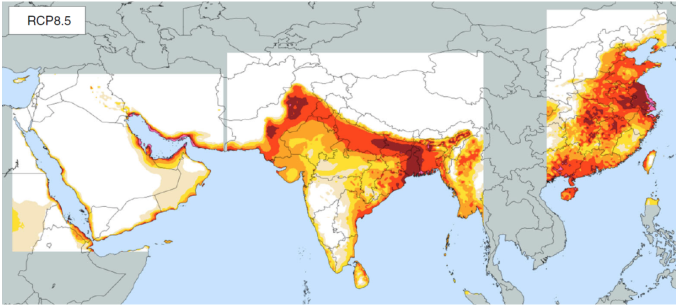
For Physical Damages, Climate Change is Not the Main Driver

- *"Climate change neither is nor should be the main concern for the insurance industry. The accumulation of wealth in disaster-prone areas is and will always remain by far the most important driver of future economic disaster damages. Nevertheless, insurance companies are concerned about climate change as the predicted increase in the frequency and/or intensity of natural hazards is likely to lead to higher economic damages, unless defensive mitigating measures make exposed wealth less vulnerable to the impacts of hazards."*

  (F. Barthel and E. Neumayer, Climatic Change, 2012)

- This points even more to anthropogenic responsibility.

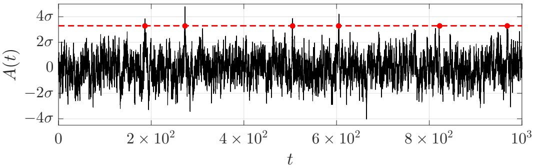
Heat Waves in East Asia by the End of the Century



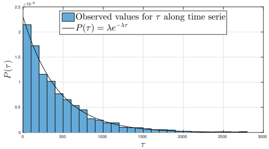
TWmax (red =31-32°C) in 2070 with RCP8.5 scenario.
(Russo et al, 2017)

- Hundreds of million of people live in world area that will be uninhabitable because of heat, before the end of the century, if climate change is not mitigated.

Extreme Events, Poisson Statistics, and Return Times



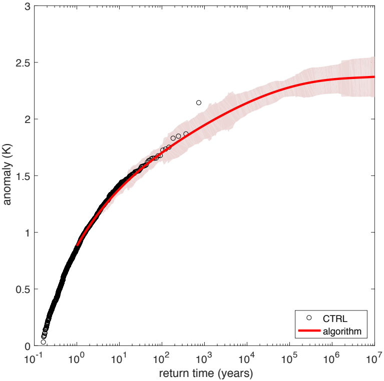
Ornstein–Uhlenbeck extremes



Waiting time statistics

For systems with a single state, rare enough events are uncorrelated and have a Poisson statistics

The Return Time of Extreme Heat Waves



Return time of 90 day European heat waves

F. Ragone, J. Wouters, and F. Bouchet, PNAS, 2018

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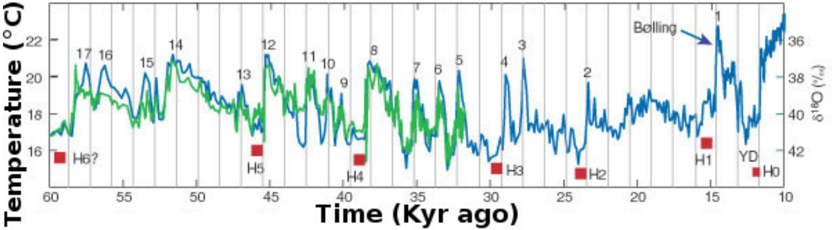
Abrupt Climate Changes (Last Glacial Period)

Long times matter

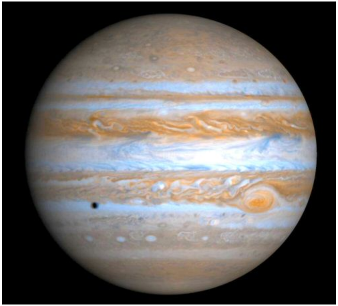


Temperature versus time: Dansgaard–Oeschger events (S. Rahmstorf)

- What is the dynamics and probability of abrupt climate changes?

Jupiter's Zonal Jets

An example of a geophysical turbulent flow (Coriolis force, huge Reynolds number, ...)

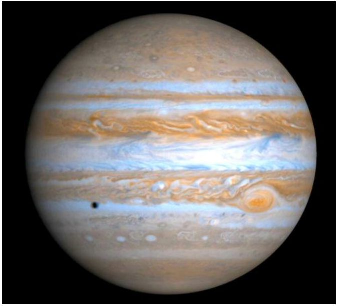


Jupiter's troposphere

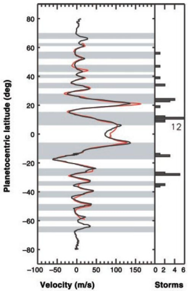
Jupiter's motions (Voyager)

Jupiter's Zonal Jets

We look for a theoretical description of zonal jets



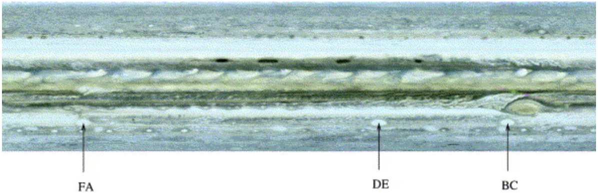
Jupiter's troposphere

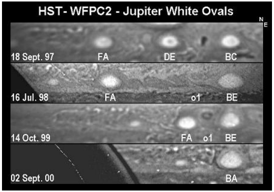


Jupiter's zonal winds (Voyager and Cassini, from Porco et al 2003)

Jupiter's Abrupt Climate Change

Have we lost one of Jupiter's jets ?



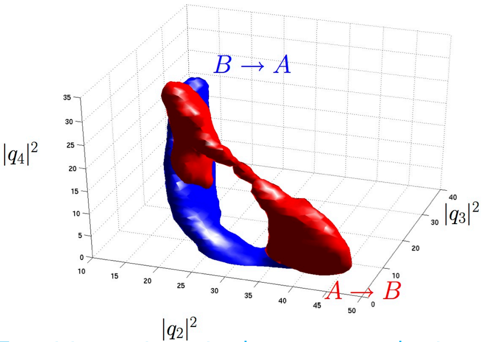


Jupiter's white ovals (see Youssef and Marcus 2005)

The white ovals appeared in 1939-1940 (Rogers 1995). Following an instability of one of the zonal jets?

Atmosphere Jet "Instantons" Computed using the AMS

AMS: an algorithm to compute rare events, for instance rare reactive trajectories



Transition trajectories between 2 and 3 jet states

- The dynamics of turbulent transitions is predictable.

F. Bouchet, J. Rolland, and E. Simonnet, PRL, 2019

Rare Events in Complex Dynamics

The scientific questions:

- What is the probability and the dynamics of those rare events?
- Is the dynamics leading to such rare events predictable?
- How to sample rare events, their probability, and their dynamics?
- Are direct numerical simulations a reasonable approach?
- Can we devise new theoretical and numerical tools to tackle these issues?

Large Deviation Theory

- Large deviation theory is a general framework to describe probability distribution in asymptotic limits

$$P\left[X_\varepsilon = x\right] \underset{\varepsilon \ll 1}{\asymp} \mathrm{e}^{-\frac{\mathscr{F}[x]}{\varepsilon}}.$$

For equilibrium statistical mechanics, $\mathscr{F}$ is the free energy, and $\varepsilon = k_B T/N$.

Maths: Cramer 30', Sanov 50', Lanford 70', Freidlin–Wentzell 70' and 80', Varadhan, ... In parallel with theoretical physicist ideas.

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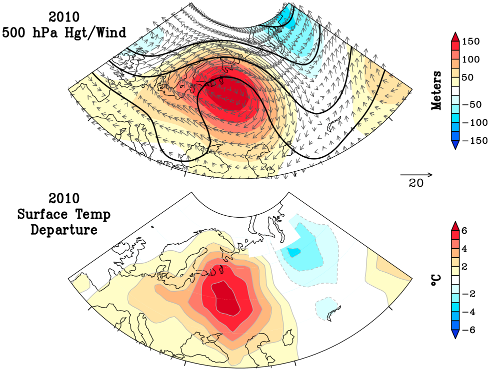
Extreme Heat Waves and Anticyclonic Anomalies

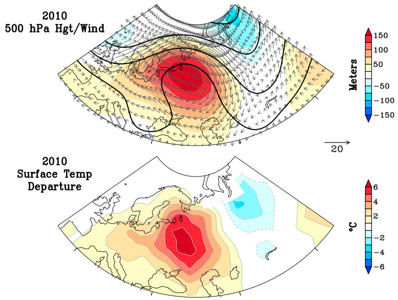


2010 Heat Wave over Eastern Europe (Dole and col., 2011)

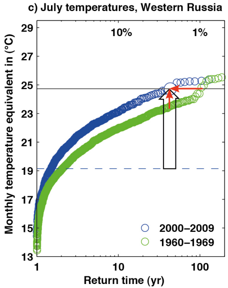
The Jet Stream, Rossby Waves, and Blocking Events

roposphere winds (NASA)

Anthropogenic Causes of the 2010 Heat Wave



(Dole et al., 2011)



Return time of monthly temperature

(Otto et al., 2012)

- A clear anthropogenic impact.
- What are the dynamical mechanisms for such extreme events?

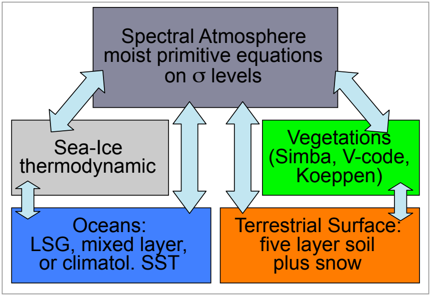
How to Study 10 000 Year Heat Waves with a 200 Year Computation?

Sampling rare events in dynamical systems

The scientific questions:

- What is the probability and the dynamics of those rare events?
- How to sample rare events?
- Are direct numerical simulations a reasonable approach?

Model: the Planetary Simulator (Plasim) - Hamburg



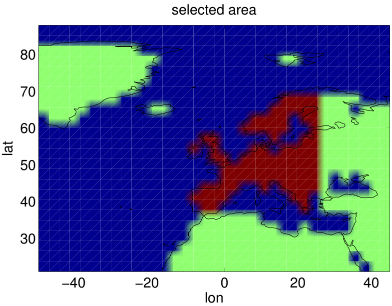
Plasim main properties

- We use the Planet Simulator (PlaSim) model (an Earth system model of intermediate complexity) developed at Hamburg.
- T42 horizontal resolution (64x128 grid points), 10 vertical layers, about $10^5 - 10^6$ degrees of freedom.

Plasim Northern Hemisphere Dynamics

opotential height and temperature anomalies

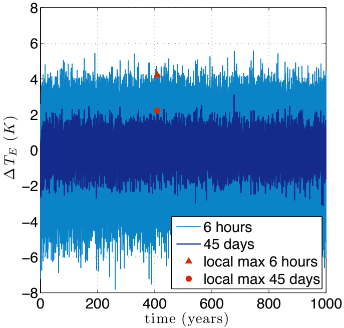
Observable: Averaged Surface Temperature



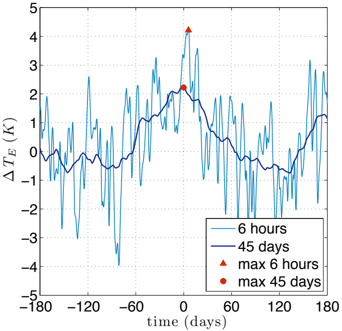
The observable will be Europe averaged surface temperature

$$a = \frac{1}{T} \int_0^T \mathrm{d}t \, \langle Temp \rangle_{\mathrm{Europe}} (t)$$

45-Day Averaged Temperature over Europe (Plasim Model)



Full timeseries



Close to a heat wave

$$a = \frac{1}{T}\int_0^T \mathrm{d}t\ \langle Temp \rangle_{\text{Europe}}(t) \text{ with } T{=}45 \text{ days}$$

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Large Variances for Estimators of Rare Event Probabilities

- Monte Carlo sampling of small probabilities (sampling the probability from iid random variables)

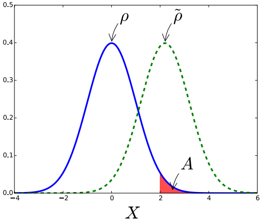
$$\gamma_A = \int \mathrm{d}x\, \rho(x) 1_A(x) = \mathbb{E}\left(1_A\right). \text{ Estimator: } \hat{\gamma}_A = \frac{1}{N}\sum_{n=1}^{N} 1_A(X_n).$$

- The variance of $\hat{\gamma}_A$ is $\mathrm{Var}\left(1_A\right)/N = \left(\gamma_A - \gamma_A^2\right)/N$. The relative error is

$$\mathrm{Er} \simeq \frac{1}{\sqrt{\gamma_A N}}.$$

- The number of observations has to grow at least as fast as the probability decreases to keep the relative error constant.

Importance Sampling



Tilted probability: $\rho(x) = L(x)\tilde{\rho}(x)$

- We sample a tilted probability with PDF $\tilde{\rho}(x)$.

$$\gamma_A = \int_A \rho(x)\,\mathrm{d}x = \int_A L(x)\tilde{\rho}(x)\,\mathrm{d}x. \text{ Estimator } \hat{\gamma}_A = \frac{1}{N}\sum_{n=1}^{N} L(X_n)1_A(X_n).$$

- If L is well chosen, rare events for ρ are common for $\tilde{\rho}$, and the variance is much lower.
- How to perform importance sampling for a climate model?

Donsker–Varadhan Large deviations for time averaged observables

- Donsker–Varadhan: large time asymptotics for time averaged observables.
- Time averaged observables

$$P\left[\frac{1}{T}\int_0^T \langle Temp\rangle_{\text{Europe}}\,\mathrm{d}t = a\right] \underset{T\to\infty}{\asymp} C\mathrm{e}^{-TI[a]}.$$

- $I(a)$ is the large deviation rate function. It has a minimum for the most probable value a_*, its second derivative at a_* describes the Gaussian fluctuations, but it describes also much rarer fluctuations.

Numerical Computation of Donsker–Varadhan Large Deviations

- Importance sampling: how to sample efficiently the tilted distribution

$$\tilde{P}_k\left(\{X(t)\}_{0\leq t\leq T}\right)=\frac{1}{\exp(T\lambda(k))}P_0\left(\{X(t)\}_{0\leq t\leq T}\right)\exp\left[k\int_0^T A(X(t))\,\mathrm{d}t\right]?$$

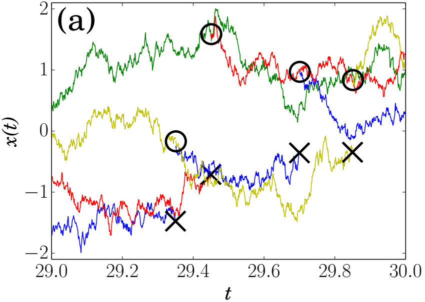
- We use the Giardina–Kurchan algorithm (Giardina et al 2006).
- We consider an ensemble of N trajectories $\{x_n(t)\}$. At each time $t_i = i\tau$, each trajectory may be killed or cloned according to the weights

$$\frac{1}{W_i(k)}\exp\left(k\int_{t_{i-1}}^{t_i} A(x_n(t))\,\mathrm{d}t\right)\text{ with }W_i(k)=\sum_{n=1}^{N}\exp\left(k\int_{t_{i-1}}^{t_i} A(x_n(t))\,\mathrm{d}t\right).$$

- For the mathematical aspects, see Del Moral's book (2004).

Genealogical Algorithm: Selecting and Cloning Trajectories

The trajectory statistics is tilted towards the events of interest.

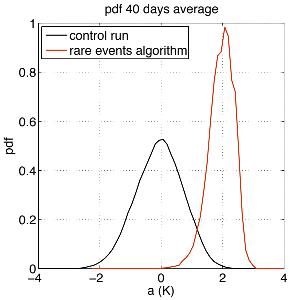


Sample paths for the GKLT algorithm

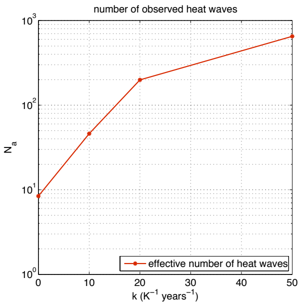
(from Bouchet, Jack, Lecomte, Nemoto, 2016)

- Computing numerically Donsker–Varadhan large deviations through a genealogical algorithm.

Importance Sampling of Extreme Heat Waves in a Climate Model



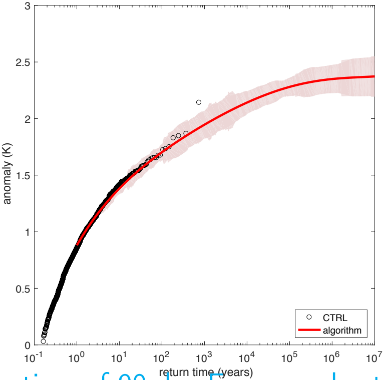
PDF of time averaged temperature



Heat wave number

- At a fixed numerical cost, we get hundreds more heat waves with the large deviation algorithm than with the control run.
- We can consider interesting dynamical studies.

The Return Times of Extreme Heat Waves



Return time of 90 day European heat waves

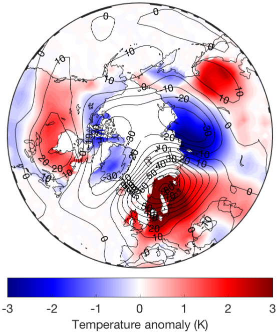
- At a fixed numerical cost, with the large deviation algorithm, we can study events which are several orders of magnitude rarer than the ones we could study with the control run.

F. Ragone, J. Wouters, and F. Bouchet, PNAS, 2017

A Typical Heat Wave

opotential height and temperature anomalies

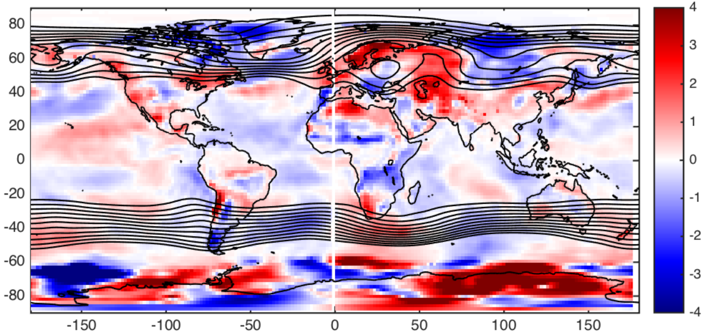
Heat Wave Conditional Statistics and Teleconnection Patterns



500 HPa geopotential height anomalies and temperature anomalies

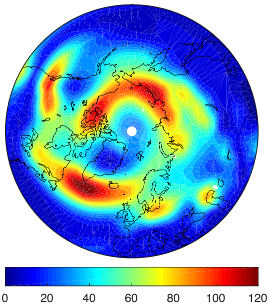
Heat wave statistics defined as statistics conditioned with
$\frac{1}{T}\int_0^T \langle Temp \rangle_{\text{Europe}}(t)\,\mathrm{d}t > 2^{\circ}\mathrm{C}$, with $T = 40\,\text{days}$.

The July 2018 Heat Wave(s)

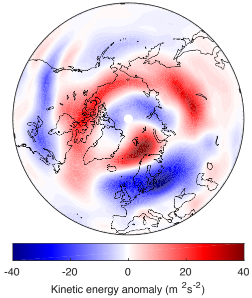


July 2018 observed (reanalysis) 500 HPa geopotential height and temperature anomalies (with respect to the last ten years).

Heat Waves and Shift of the Jet Stream



Northern hemisphere mean kinetic energy



Kinetic energy anomaly during the heat waves

- The European heat waves are associated with a northward shift of the jet stream over Europe and a southward shift over Asia.

Extreme Heat Waves: Conclusions

Conclusions:

- Large deviation algorithms provide a wonderful tool to sample rare events, for instance heat waves.
- It should open a new range of dynamical studies in GCM, even the more complex ones.

Work in progress:

- A dynamical study of heat waves based on hundreds of sampled heat waves.
- Relation with blocking events? Are they different types of dynamics leading to heats waves? Which physical processes? Relation between heat waves precursors and instantons?

F. Ragone, J. Wouters, and F. Bouchet, PNAS, 2018