The Risk-Adjusted Carbon Price

Mathematics of the Economy and Climate, Soesterberg, July 2019

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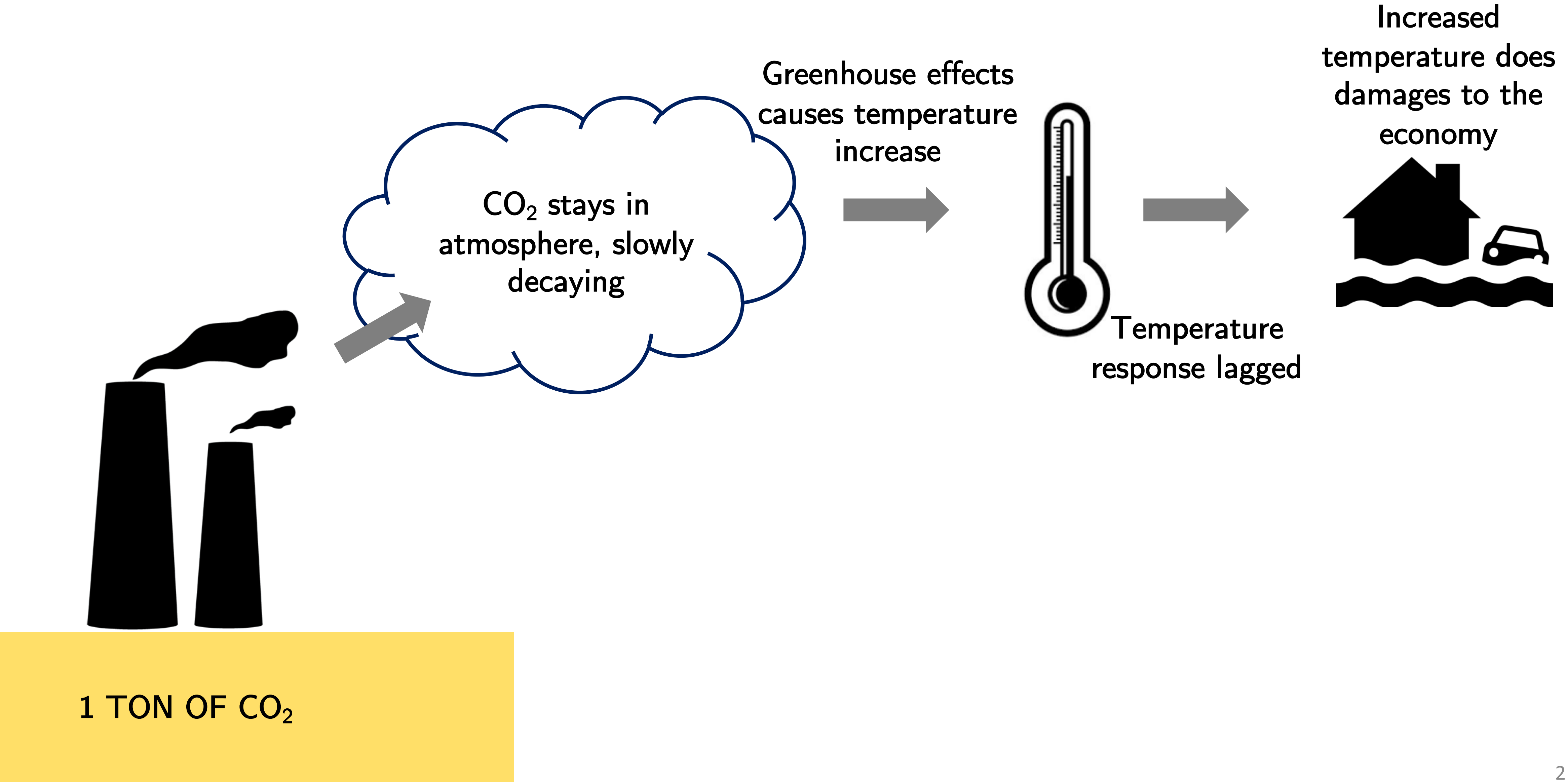
Optimal CO_2 tax = social cost of CO_2



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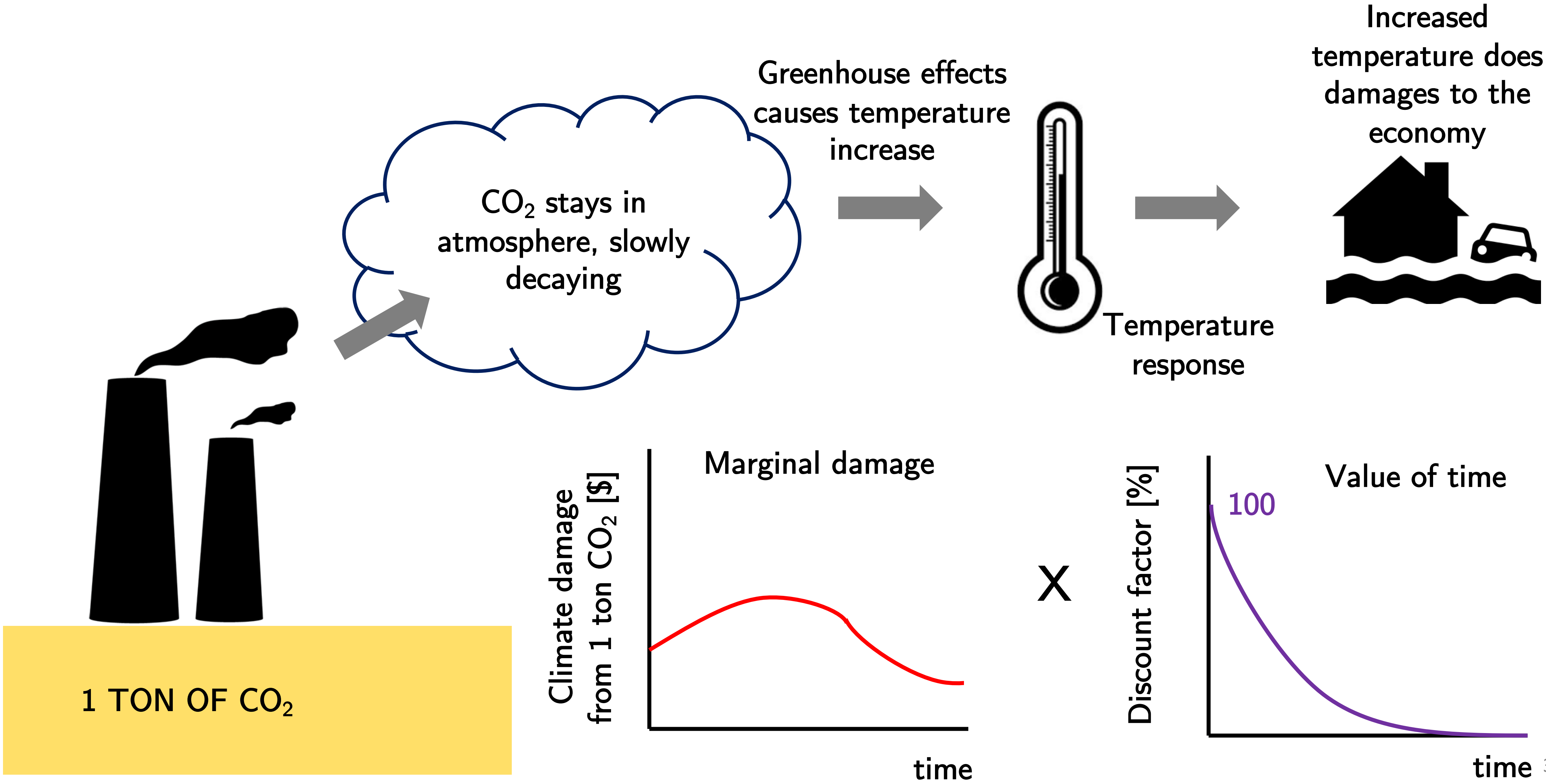

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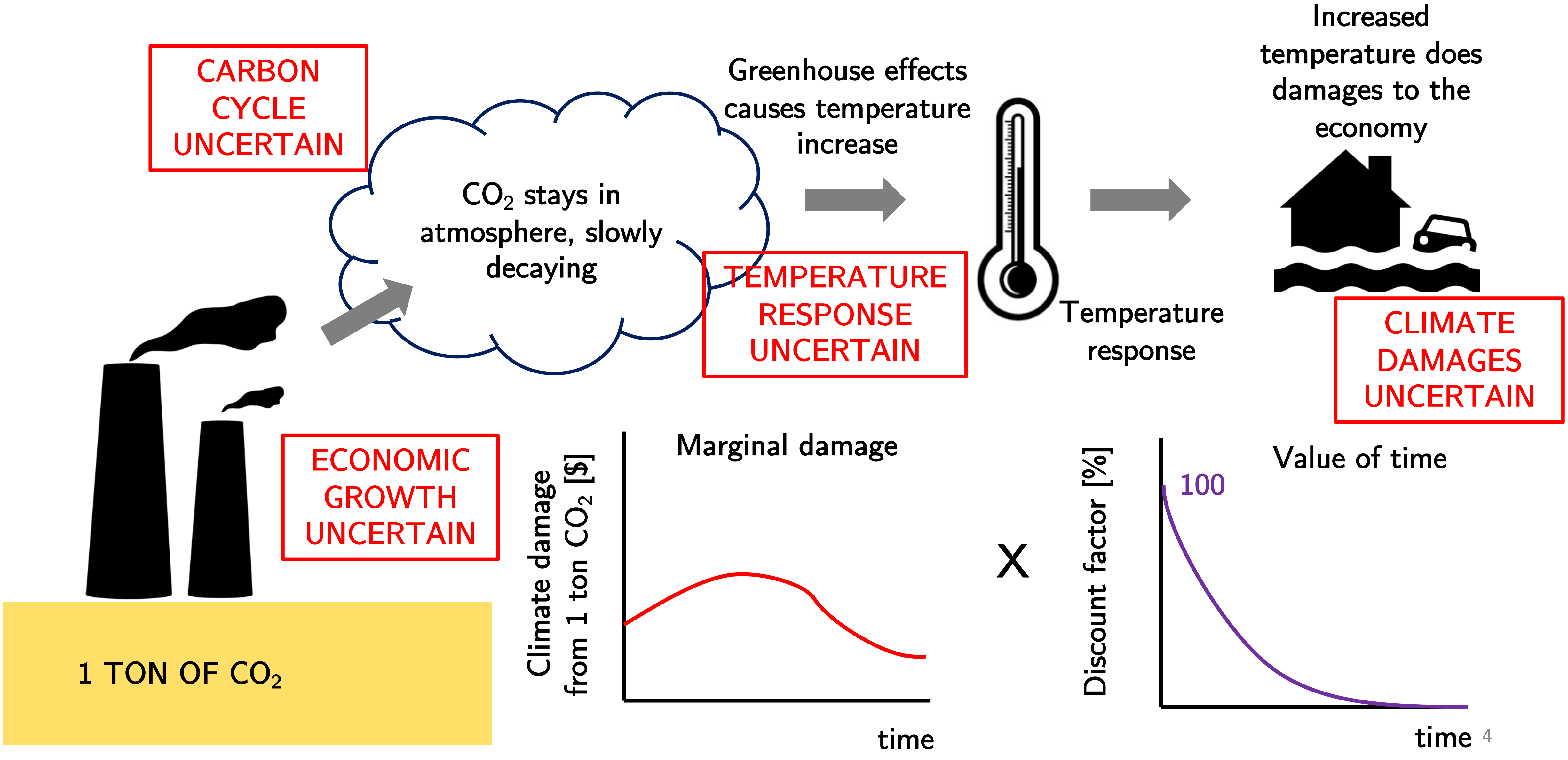
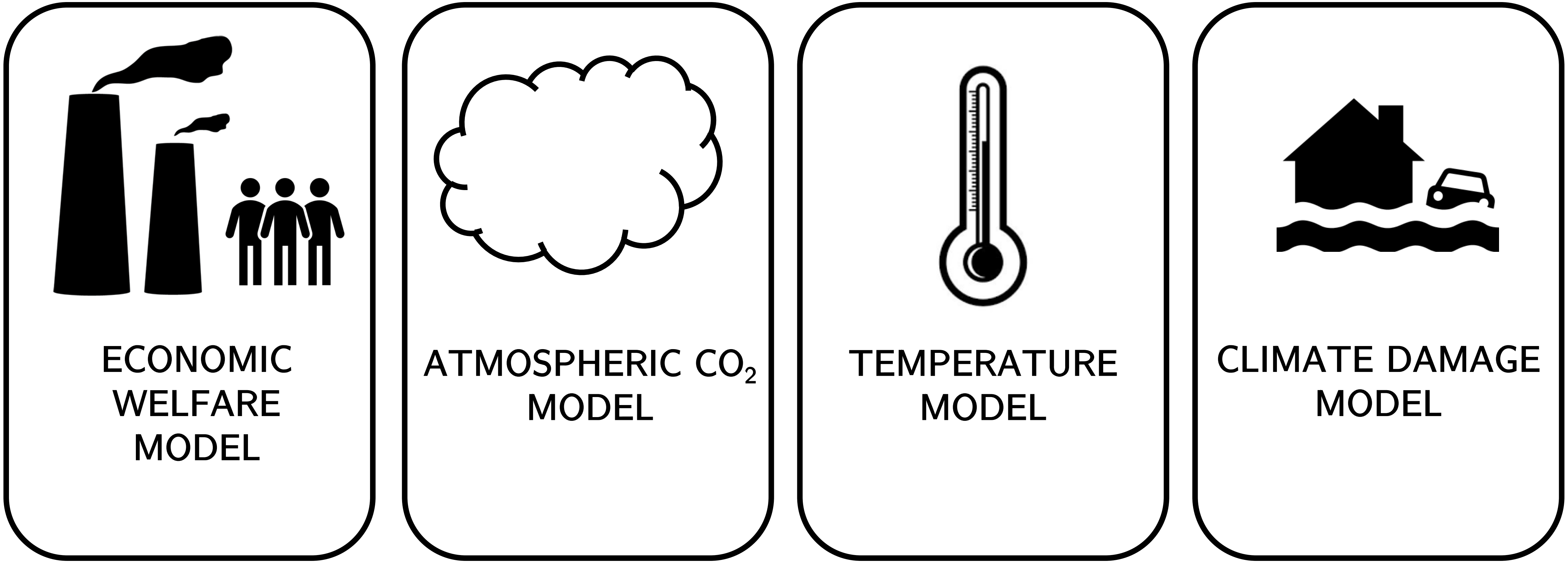


Dynamic Stochastic General Equilibrium Model



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1. Introduction and review of literature

For time-separable utility with exponential discounting, the social cost of carbon (SCC) is a Pigouvian tax:

$$P(t) \equiv \mathrm{E}_t\left[\int_t^{\infty}\left(U'\big(C(s)\big)\frac{\partial\Pi(s)}{\partial T(s)}\frac{\partial T(s)}{\partial S(s)}\frac{\partial S(s)}{\partial F(t)}\right)e^{-\rho(s-t)}ds\right]\frac{1}{U'\big(C(t)\big)},$$

1. Introductory example

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Marginal utility at time s

the effect of temperature T on damages to aggregate output Π

the effect of the atmospheric carbon stock S on temperature T

how much of one ton of carbon emitted at time t is left in the atmosphere at each future time s

Pure time preference discounting

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$$P(t) = \frac{\Theta C(t)}{r^* + \varphi}$$

with $r^* = \rho + (\gamma - 1)(g - \frac{1}{2}\gamma\sigma^2).$

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Pure time preference discounting

Iso-elastic utility function:

$$U = \frac{C^{1-\gamma}}{1-\gamma}$$

Marginal effect carbon on damages proportional to consumption:

$$\frac{\partial\Pi(s)}{\partial T(s)}\frac{\partial T(s)}{\partial S(s)} = \Theta C(s)$$

Consumption follows GBM:

$$dC = gdt + \sigma CdW$$

Constant decay rate of atmospheric carbon:

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The discount rate $r^* =$

$+$ safe rate $r_{\mathrm{rf}} = \rho + \gamma g - \gamma(1+\gamma)\dfrac{\sigma^2}{2}$

(a Keynes-Ramsey rule with a prudent correction for growth volatility)

$+$ risk premium for damages proportional to GDP $\gamma\sigma^2$

$-$ correction for growing damages g

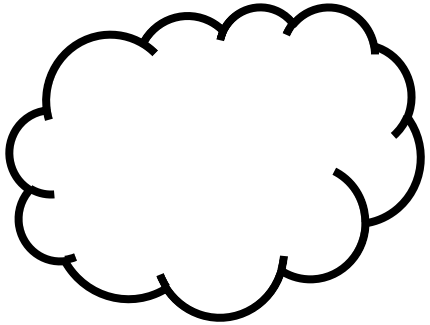
1. Review of literature on the optimal risk-adjusted SCC

Two strands:

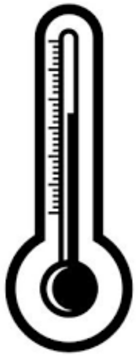
- Numerical studies using:
    - Monte-Carlo simulations (e.g., Ackerman and Stanton, 2012; Dietz and Stern, 2015).
    - Dynamic programming with advanced numerical methods (e.g., Crost and Traeger, 2013; Traeger, 2014a; Jensen and Traeger, 2014; Hambel et al., 2017).
- Analytical literature on discounting under uncertainty (typically deals with one uncertainty at a time, e.g., Gollier, 2012; Traeger, 2014b):
    - Golosov et al. (2014): simple rule using logarithmic utility, Cobb-Douglas production, 100% depreciation of capital each period, damages exponential function of the atmospheric carbon stock.
    - More simple rules: Gerlagh and Liski (2016), Van den Bijgaart et al. (2016), Bretschger and Vinogradova (2018).
    - Jensen and Traeger (2016), Lemoine (2017), Dietz et al. (2018): exogenous consumption, prudence and climate betas.
    - Traeger (2017): many uncertainties but strong restrictions on functional form.
- Complementary literature on tipping.

2. Dynamic Stochastic General Equilibrium Model

ECONOMIC WELFARE MODEL



ATMOSPHERIC CO_2 MODEL



TEMPERATURE MODEL

CLIMATE DAMAGE MODEL

2. Model: welfare function

Continuous-time recursive preferences (Duffie and Epstein, 1992) with value function:

$$J = E_t\left[\int_t^{\infty} f\big(C(s), J(s)\big)\,ds\right] \qquad \text{with} \qquad f\big(C,J\big) = \frac{1}{1-\gamma}\,\frac{C^{1-\gamma} - \rho\big((1-\eta)J\big)^{\frac{1-\gamma}{1-\eta}}}{\big((1-\eta)J\big)^{\frac{1-\gamma}{1-\eta}-1}}.$$

γ = IIA = 1/EIS — intergenerational inequality aversion or inverse of elasticity of intertemporal substitution

η = CRRA — coefficient of relative risk aversion

2. Model: capital accumulation and GDP uncertainty

AK growth with adjustment costs based on Pindyck and Wang (2013). Aggregate capital K as a GBM:

$$dK = \Phi(I,K)dt + \sigma_K K dW_1 \quad \text{with} \quad \Phi(I,K) = I - \frac{1}{2}\omega\frac{I^2}{K} - \delta K.$$

Add fossil fuel use F as a production factor: $Y = AK^{\alpha}F^{1-\alpha}$ with $0 < \alpha < 1$.

Production cost of fossil fuel b constant: $I = Y - C - bF$.

Total factor productivity A function of climate damage ratio (i.e. damages ∝ to GDP): $A \equiv A^*(1-D)$.

2. Model: carbon stock and uncertainty

A 1-box model for the atmospheric carbon stock associated with man-made emissions: $E \equiv S - S_{\mathrm{PI}}$.

Uncertainty of carbon stock projections for a given emission scenario modelled by a BM:

$$d\tilde{E} = (\mu F e^{-gt} - \varphi \tilde{E})dt + \sigma_E dW_2 \quad \text{and} \quad E = \max(0, \tilde{E}),$$

New emissions with emission intensity declining at the growth rate

Decay towards pre-industrial

Arithmetic Brownian Motion

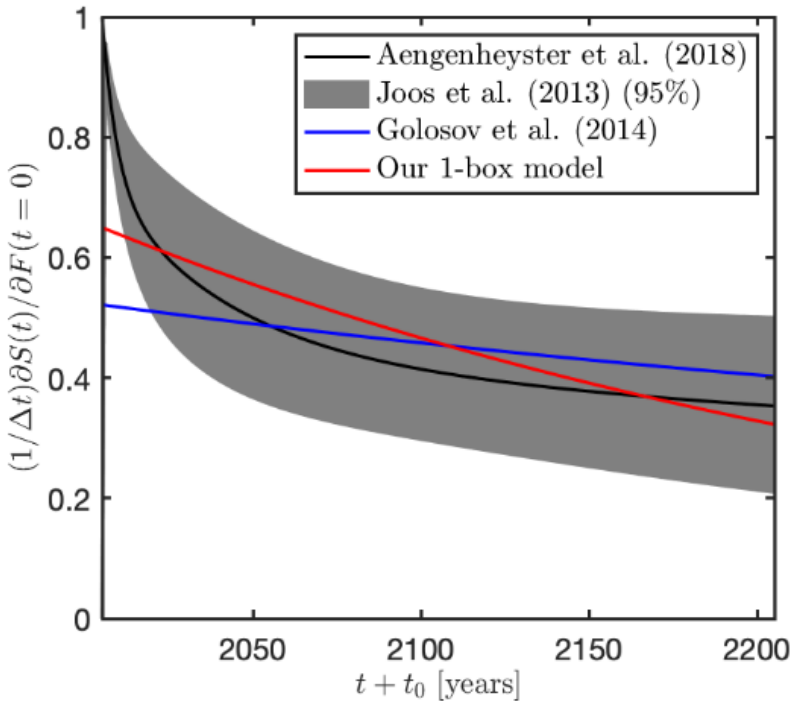
(technicality: negative E not allowed).

2. Model: carbon stock and uncertainty

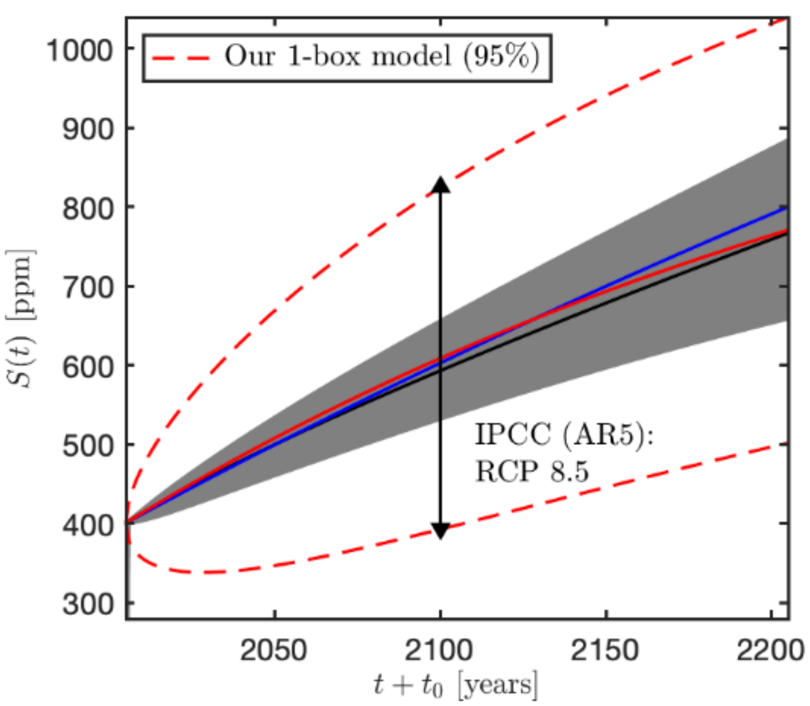
To calibrate: $d\tilde{E} = (\mu F e^{-gt} - \varphi \tilde{E})dt + \sigma_E dW_2$ and $E = \max(0, \tilde{E})$.

- Use 17 impulse response functions from survey in Joos et al. (2013): $\mu = 0.65$, $\varphi = 0.35\% / \text{year}$.
- Use 90% confidence range 794-1149 ppmv in 2100 predicted for RCP 8.5 (IPCC, AR5): $\sigma_E = 13$ ppmv/year$^{1/2}$.

Impulse response function



Atmospheric carbon stock



2. Model: temperature and uncertainty

Power-law temperature model: $T(E,\chi)=\chi^{1+\theta_\chi}(E/S_{\mathrm{PI}})^{1+\theta_E}$ with $\theta_E \geq -1$ and $\theta_\chi \geq -1$.

Climate sensitivity: $T_2 \equiv T(E=S_{PI},\chi)=\chi^{1+\theta_\chi}$, with skewness (to leading-order): $\mathrm{skew}\left[T_2\right]=3\theta_\chi(1+\theta_\chi)^3\mu_\chi^{3(1+\theta_\chi)}\dfrac{\Sigma_\chi^4}{\mu_\chi^4}$.

Ornstein-Uhlenbeck process: $d\tilde{\chi}=\nu_\chi(\bar{\chi}-\tilde{\chi})dt+\sigma_\chi dW_3$ with $\chi=\max(0,\tilde{\chi})$.

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Properties of the Ornstein-Uhlenbeck process

(mean-reverting arithmetic Brownian motion)

Normally distributed: $\tilde{\chi}(t)\sim \mathrm{N}(\mu_\chi,\Sigma_\chi^2)$

Time-varying mean: $\mu_\chi=\chi_0 e^{-\nu_\chi t}+\bar{\chi}(1-e^{-\nu_\chi t})$ Long time: $\mu_\chi \to \bar{\chi}.$

Time-varying variance: $\Sigma_\chi^2=\sigma_\chi^2\left(1-\exp(-2\nu_\chi t)\right)\big/2\nu_\chi$ Long time: $\Sigma_\chi^2 \to \sigma_\chi^2\big/2\nu_\chi.$

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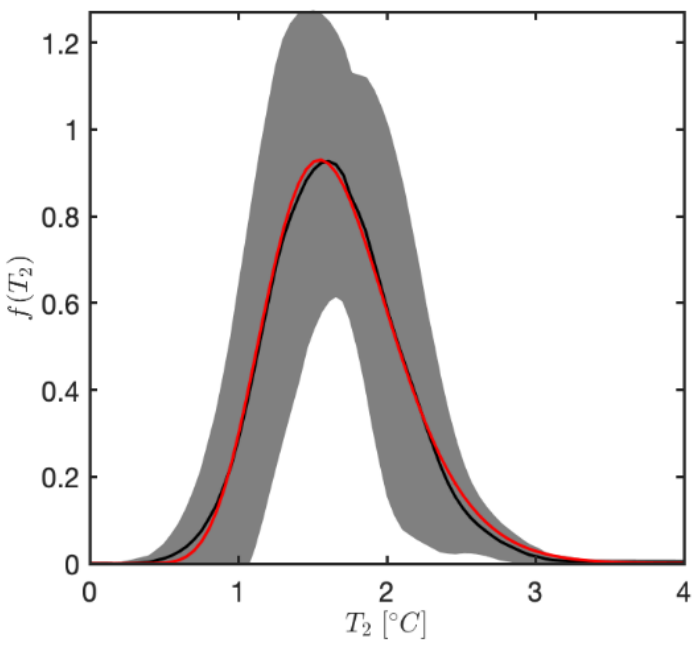
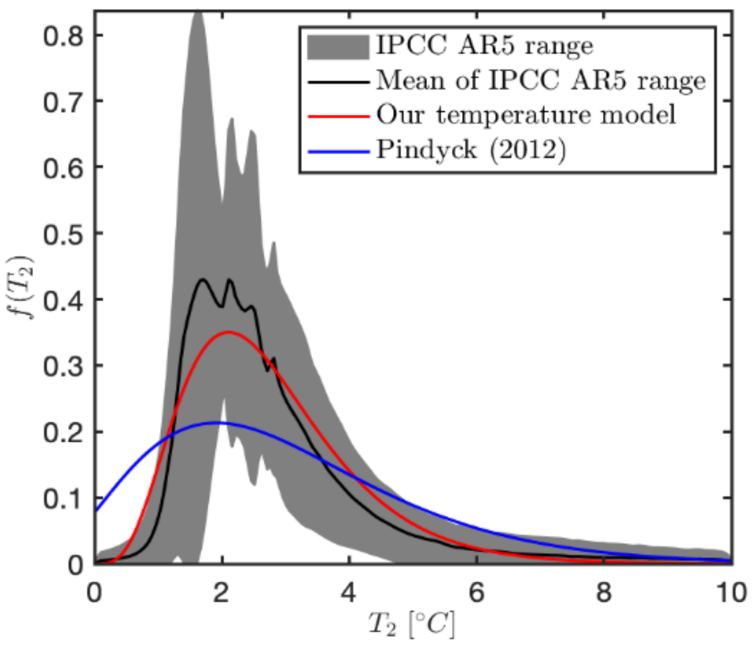
Transient Climate Response



Equilibrium Climate Sensitivity



2. Model: damage function and uncertainty

Power-law damage ratio function: $D(T,\lambda) = T^{1+\theta_T}\lambda^{1+\theta_\lambda}$ with $\theta_T \geq -1$ and $\theta_\lambda \geq -1$.

Ornstein-Uhlenbeck process: $d\tilde{\lambda} = \nu_\lambda(\bar{\lambda} - \tilde{\lambda})dt + \sigma_\lambda dW_4$ with $\lambda = \max(0,\tilde{\lambda})$.

Combined: $D(E,\chi,\lambda) = \chi^{1+\theta_{\chi T}}\lambda^{1+\theta_\lambda}\left(\dfrac{E}{S_{PI}}\right)^{1+\theta_{ET}}$ with

$\theta_{\chi T} \equiv \theta_\chi + \theta_T + \theta_\chi\theta_T$ curvatures of D(T) (+) + T(χ) (+)

$\theta_{ET} \equiv \theta_E + \theta_T + \theta_E\theta_T$ curvatures of D(T) (+) + T(E) (−)

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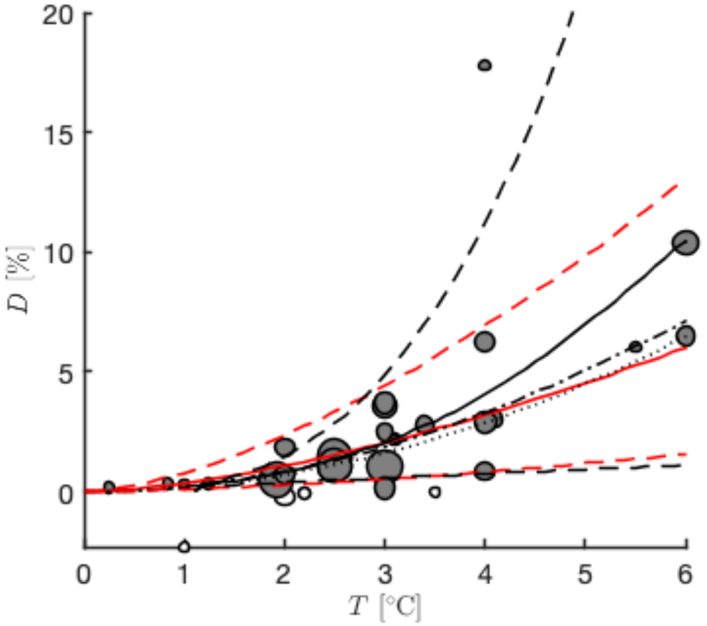
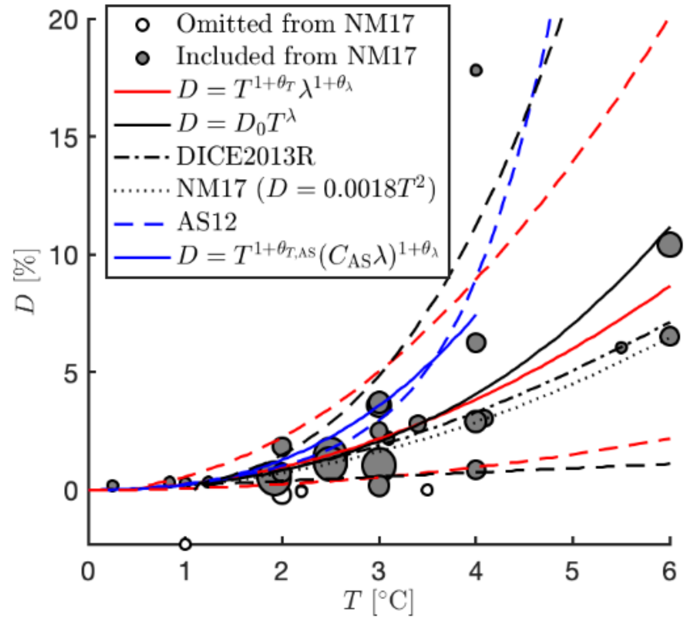
STEADY-STATE DISTRIBUTIONS

Proportional damages $\left(\theta_T = 0.56,\ \theta_{ET} = 0\right)$



Convex damages $\left(\theta_T = 1,\ \theta_{ET} = 0.28\right)$



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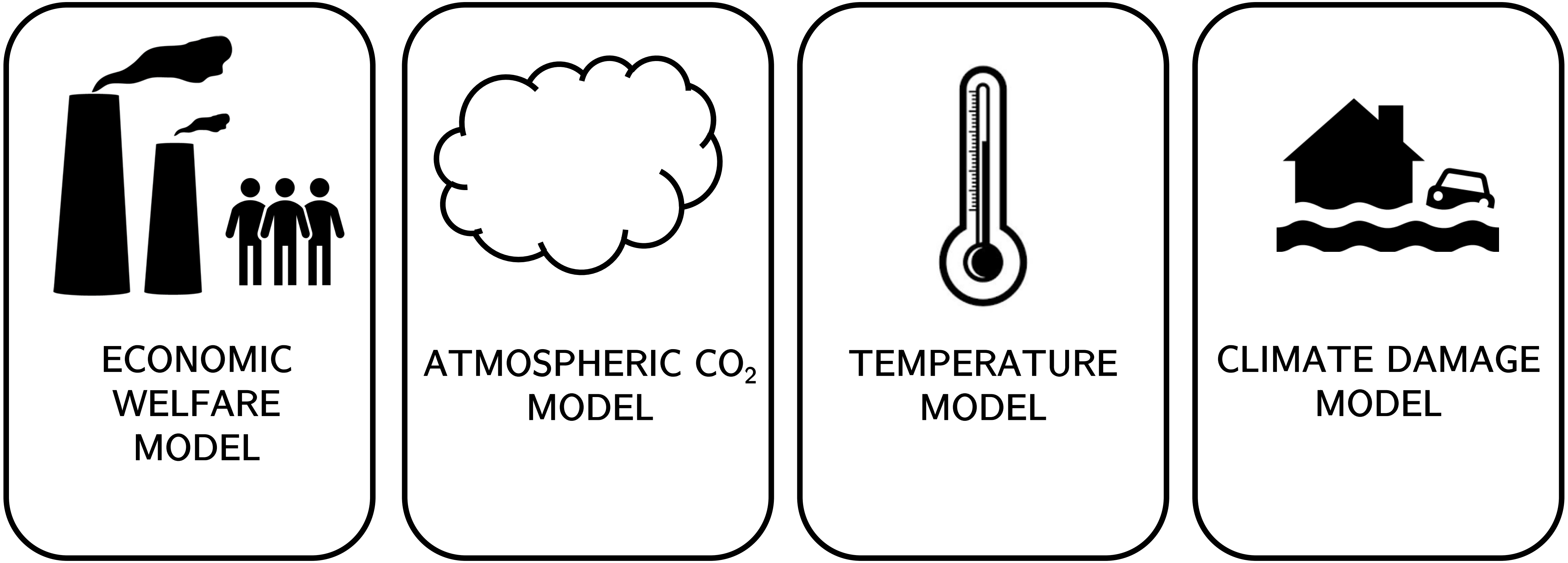

3. Asymptotic solutions

Small parameter is share of damages in GDP: $\epsilon \equiv D(E_0, \bar{\chi}, \bar{\lambda}) = \bar{\lambda}^{1+\theta_\lambda} \bar{\chi}^{1+\theta_{\chi T}} \left(\dfrac{E_0}{S_{PI}} \right)^{1+\theta_{ET}}.$

Truncated series solution: $J(K,\tilde{E},\tilde{\chi},\tilde{\lambda},t) = J^{(0)}(K,\epsilon D(\tilde{E},\tilde{\chi},\tilde{\lambda})) + \epsilon J^{(1)}(K,\tilde{E},\tilde{\chi},\tilde{\lambda},t,\epsilon D(\tilde{E},\tilde{\chi},\tilde{\lambda})) + O(\epsilon^2),$

'slow' dependence on climate states

Taylor-series expansion

Social costs of carbon or 'risk-adjusted carbon price': $P = -\dfrac{\mu\left(J_{\tilde{E}}^{(0)} + \epsilon J_{\tilde{E}}^{(1)}\right)}{\phi'(i^{(0)}) J_K^{(0)}}.$

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Three results:

Result 1: no additional assumptions. Solution in the form of multi-dimensional integral.

Result 2: proportional damages ($\theta_{ET} = 0, D \propto E$), leading-order effects of uncertainty and temperature and damage ratios χ and λ are at their steady-states.

Result 3: leading-order effect of uncertainty, but with other two assumptions relaxed.

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3. Result 2

The optimal risk-adjusted carbon price:

$$P = \frac{\mu\,\Theta Y\big|_{P=0}}{r^* + \varphi}(1 + \Delta_\chi + \Delta_\lambda + \Delta_{\mathrm{CK}}) \text{ with } \Theta = \frac{D_E}{1 - D} \text{ and } r^* = \rho + (\gamma - 1)(g^{(0)} - \frac{1}{2}\eta\sigma_K^2),$$

Uncertainty correction factors:

- Climate sensitivity (or temperature) uncertainty: $\Delta_\chi = \frac{1}{2}\theta_{\chi T}(1 + \theta_{\chi T})\frac{(\sigma_\chi / \bar{\chi})^2}{r^* + 2\nu_\chi + \varphi}.$

- Climate damage uncertainty: $\Delta_\lambda = \frac{1}{2}\theta_\lambda(1 + \theta_\lambda)\frac{(\sigma_\lambda / \bar{\lambda})^2}{r^* + 2\nu_\lambda + \varphi}.$

- Climate 'beta' terms: $\Delta_{\mathrm{CK}} = -(\eta - 1)\sigma_K\left((1 + \theta_{\chi T})\frac{\rho_{K\chi}\sigma_\chi / \bar{\chi}}{r^* + \nu_\chi + \varphi} + \frac{\rho_{K\lambda}\sigma_\lambda / \bar{\lambda}}{r^* + \nu_\lambda + \varphi}\right).$ (I will skip today)

3. Result 2: economic growth uncertainty and the climate beta

Including economic but not climatic uncertainty, Result 2 gives: $P = \dfrac{\mu\, \Theta Y\big|_{P=0}}{r^* + \varphi}$ with $\Theta = \dfrac{D_E}{1-D}$.

The optimal risk-adjusted discount rate:

$$r^* = \underbrace{\rho}_{\text{impatience}} + \underbrace{\gamma g^{(0)}}_{\text{rising affluence}} - \underbrace{g^{(0)}}_{\text{growing damages}} \underbrace{-\frac{1}{2}(1+\gamma)\eta\sigma_K^2}_{\text{prudence}} + \underbrace{\eta\sigma_K^2}_{\text{insurance}},$$

‘Built-in’ climate beta of 1 due to damages being proportional to GDP

γ = IIA = 1/EIS intergenerational inequality aversion or inverse of elasticity of intertemporal substitution,

η = CRRA coefficient of relative risk aversion,

(see also Dietz et al. (2018) for damages that are not proportional to GDP).

3. Result 2: climate and damage uncertainties

Climate sensitivity risk correction: $P = \frac{\mu\,\Theta Y|_{P=0}}{r^* + \varphi}\left(1 + \underline{\Delta_\chi}\right)$ with $\Theta = \frac{D_E}{1-D}$.

$$\Delta_\chi = \frac{1}{2}\underbrace{\theta_{\chi T}(1+\theta_{\chi T})}\frac{(\Sigma_\chi^\infty/\bar{\chi})^2}{1+\frac{r^*+\varphi}{2\nu_\chi}}.$$

Equilibrium climate sensitivity uncertainty

Ratio of discount rate and 'arrival rate' of equilibrium climate sensitivity uncertainty

$$\theta_{\chi T} \equiv \theta_\chi + \theta_T + \theta_\chi \theta_T$$

Combines positive skewness of the (equilibrium) climate sensitivity and the convex dependence of damages on temperature.

Climate damage risk correction: $P = \frac{\mu\,\Theta Y|_{P=0}}{r^* + \varphi}\left(1 + \Delta_\chi + \underline{\Delta_\lambda}\right)$ with $\Theta = \frac{D_E}{1-D}$.

$$\Delta_\lambda = \frac{1}{2}\underbrace{\theta_\lambda}(1+\theta_\lambda)\frac{(\Sigma_\lambda^\infty/\bar{\lambda})^2}{1+\frac{r^*+\varphi}{2\nu_\lambda}}$$

Parameter that measures skewness of climate damage ration

4. Calibration

TABLE 1 – SUMMARY OF BASE CASE CALIBRATION

Impatience and aversion to intergenerational inequality and risk	$\rho = 5.8\%$/year, IIA = 1/EIS = $\gamma = 1.5$, RRA = $\eta = 4.3$
World economy	$A^* = 0.113$ /year, GDP PPP= 116\$T/year, $g^{(0)} = 2.0$ %/year
Investment, depreciation and adjustment cost	$i^{(0)} = 2.8\%$/year, $\delta = 0.33\%$/year, $\omega = 12.5$ year
Asset volatility and returns	$\sigma_K = 12\%/\text{year}^{1/2}$, $r^{(0)} = 7.2\%$/year, $r_{\text{rf}}^{(0)} = 0.80\%$/year, $r^{(0)} - r_{\text{rf}}^{(0)} = \eta\sigma_K^2 = 6.4\%$/year
Share of fossil fuel and production cost	$1 - \alpha = 4.3\%$, $b = \$5.4\times10^2$ /tC
Preindustrial and 2015 ($t = 0$) carbon stocks Concavity of Arrhenius'law & stochastic carbon stock dynamics	$S_{PI} = 596$ GtC, $S_0 = 854$ GtC, $E_0 = 258$ GtC, $\theta_E = -0.36$, $\mu = 0.65$, $\varphi = 0.35\%$/year, $\sigma_E = 13$ ppmv/year$^{1/2}$
Distribution of the climate sensitivity	$\chi_0 = 1.1109$, $\bar{\chi} = 1.2619$, $\sigma_\chi = 0.020\%/\text{year}^{1/2}$ $\nu_\chi = 0.0086\%$/year, $\theta_\chi = 3.0$
Distribution of the damage ratio	$\theta_T = 0.56$ ($\theta_{ET} = 0$), $\bar{\lambda} = 0.21$, $\sigma_\lambda = 2.3\%/\text{year}^{1/2}$, $\theta_\lambda = 2.7$, $\nu_\lambda = 0.20$/year
Flow impact of global warming damages	$\Theta_0 = 2.07\%$ GDP/TtC
Conversion factors	1 ppmv CO_2 = 2.13 GtC, 1 tC = 3.664 tCO_2

4. Estimates of the optimal risk-adjusted carbon price

TABLE 4. ESTIMATES OF THE SCC: MARKET- VS. ETHICS-BASED

	Market-based calibration			Ethics-based calibration		
	base case	$\chi_0 = \bar{\chi}$	$\nu_\chi = \infty$	base case	$\chi_0 = \bar{\chi}$	$\nu_\chi = \infty$
Deterministic SCC ($/tCO₂)	4.1	8.4	8.4	11.5	20.8	20.8
due to economic uncertainty ($/tCO₂)	1.3	2.4	2.4	18.7	26.2	26.2
due to carbon stock uncertainty	0	0	0	0	0	0
due to climate sensitivity uncertainty	0.4	0.6	2.6	4.7	6.4	11.2
due to damage ratio uncertainty	0.8	1.4	1.7	4.9	7.5	8.1
Risk-adjusted SCC ($/tCO₂)	**6.6**	**12.8**	**15.0**	**39.8**	**61.0**	**66.3**
Economic risk mark-up	32%	29%	29%	163%	126%	126%
Climate sensitivity risk mark-up	9%	7%	31%	41%	31%	54%
Damage ratio risk mark-up	18%	17%	20%	43%	36%	39%
Total risk mark-up	59%	53%	80%	247%	193%	219%
Discount rate $r^{(0)}$ (per year)	7.2%	7.2%	7.2%	2.9%	2.9%	2.9%

Estimates in this table are for *proportional* damages ($\theta_{ET} = 0$), asset return volatility ($\sigma_K = 12\%/\text{year}^{1/2}$), and $\rho =$ 5.8%/year (*market-based* calibration) or $\rho = 1.5\%$/year (*ethics-based* calibration).

4. Estimates of the optimal risk-adjusted carbon price

TABLE 6. ESTIMATES OF THE SCC: CONVEXITY OF THE DAMAGE FUNCTION

	Proportional damages ($\theta_{ET} = 0$)	Convex damages ($\theta_{ET} = 0.28$)	Highly convex damages (AS12, $\theta_{ET} = 0.63$)
Deterministic SCC ($/t$CO_2$)	25.5	26.8	77.2
Risk-adjusted SCC ($/t$CO_2$)	**34.1**	**41.9**	**140.8**
Economic risk mark-up	2%	1%	-1%
Carbon stock risk mark-up	0%	-1%	-1%
Climate sensitivity risk mark-up	15%	30%	61%
Damage ratio risk mark-up	16%	26%	15%
Total risk mark-up	34%	56%	82%
Discount rate $r^{(0)}$ (per year)	3.1%	3.1%	3.1%
Estimates in this table are for $\rho = 0.1\%$/year (*ethics-based* calibration) and GDP growth volatility ($\sigma_K = 1.5\%/\text{ year}^{1/2}$).			

4. Estimates of the optimal risk-adjusted carbon price

TABLE 8. ESTIMATES OF THE SCC: COMPARISON WITH OTHER CALIBRATIONS

Model	Base	Golosov et al. (2014)	Gollier (2012)		Stern (2007) +AS12
Volatility based on	asset returns	-	asset returns	GDP	GDP
Deterministic SCC ($/t$CO_2$)	11.5	19.0	14.4	14.4	86.9
Risk-adjusted SCC ($/t$CO_2$)	**39.8**	**24.6**	**62.6**	**18.5**	**165.2**
Economic risk mark-up	163%	0%	225%	1%	0%
Carbon stock risk mark-up	0%	0%	0%	0%	-1%
Climate sensitivity risk mark-up	41%	13%	57%	12%	65%
Damage ratio mark-up	43%	16%	54%	16%	26%
Total risk mark-up	247%	29%	336%	29%	90%
Discount rate $r^{(0)}$ (per year)	2.9%	3.5%	2.5%	4.0%	2.5%

Estimates in this table are for *proportional* damages ($\theta_{ET} = 0$), except for the final column, which assumes highly convex AS12 damages. The base case is for $\rho = 1.5\%$/year (*ethics-based* calibration).

5. Conclusions and context

- We have derived analytical expressions for the risk-adjusted social cost of carbon using perturbation methods:
  - Economic uncertainty: huge if based on market volatility and CRRA = EIS $\neq 1$.
  - Carbon stock uncertainty: small and zero for proportional damages.
  - Temperature uncertainty: only for equilibrium climate sensitivity and depends on its timescale.
  - Damage uncertainty: only if skew, for which there is some evidence.

- Although we have Epstein-Zin preferences, the non-climatic part of the model remains primitive (AK model + Geometric Brownian Motion).

- Future work should:
  - Distinguish volatility of equity returns and GDP growth;
  - Include long-run risk in economic growth and a downward-sloping term structure (Bansal and Yaron, 2004; Gollier and Mahul, 2017);
  - Allow for compound Poisson shocks to temperature and damages (cf. Hambel et al., 2018; Bretschger and Vinogradova, 2018; Bansal et al., 2016).

Thank you for your attention!