Tipping Point Dynamics in Climate-Economy Systems

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Mathematics of the Economy an Climate - Soesterberg

Outline of My Presentation

- General remarks
- Modeling tipping points and their interaction
- More complex formulations

General Remarks

- Simulation vs. Optimization
- Deterministic vs. Stochastic Optimisation Models
- Uncertainty Quantification

Global warming might trigger climate system tipping points. For example:.

- Meltdown of the Greenland ice sheet
- Dieback of the Amazon rain forest

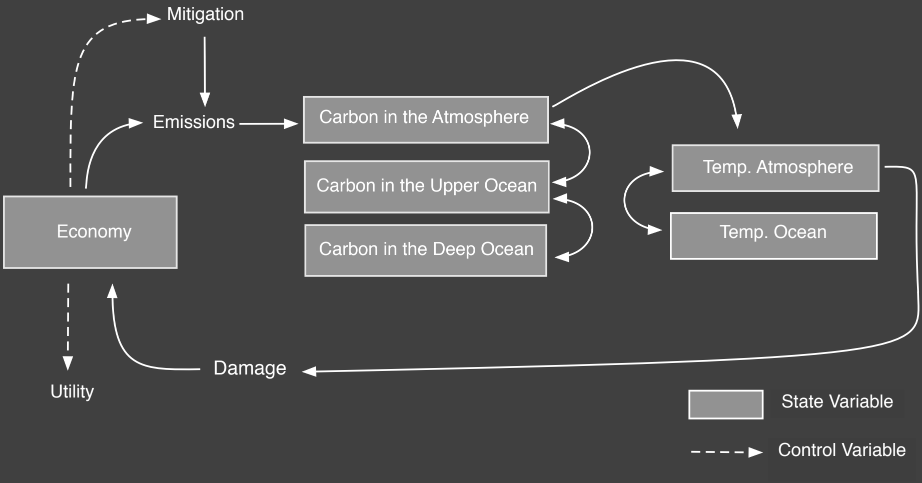
Assessments of the social costs of carbon (SCC) for regulatory policy point out that those possible externalities are not appropriately dealt with.

- IPCC (2014), US Government Interagency study (2013)
- US council of Economic Advisors (2014)
- EU Commission Joint Research Centre (2015)

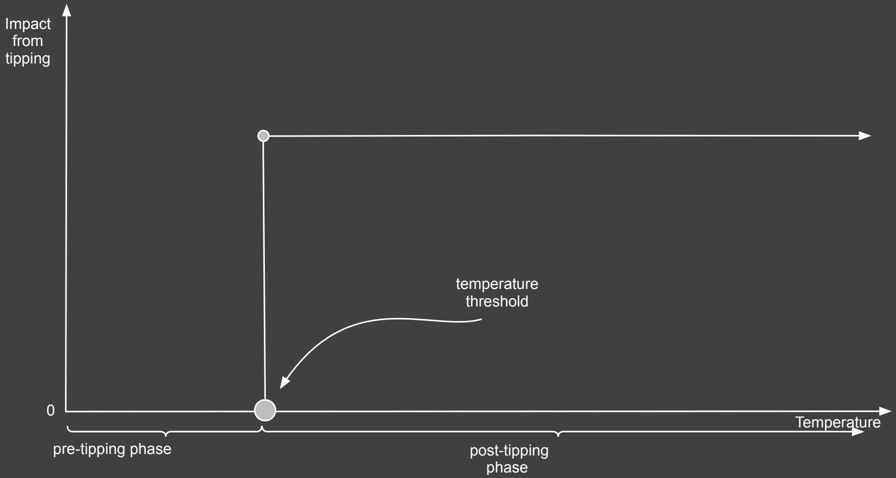
- Adding Tipping points to an economic integrated assessment model
- Calibration
- Implications for the social cost of carbon

Based on: Cai et al. Nature Climate Change 2016

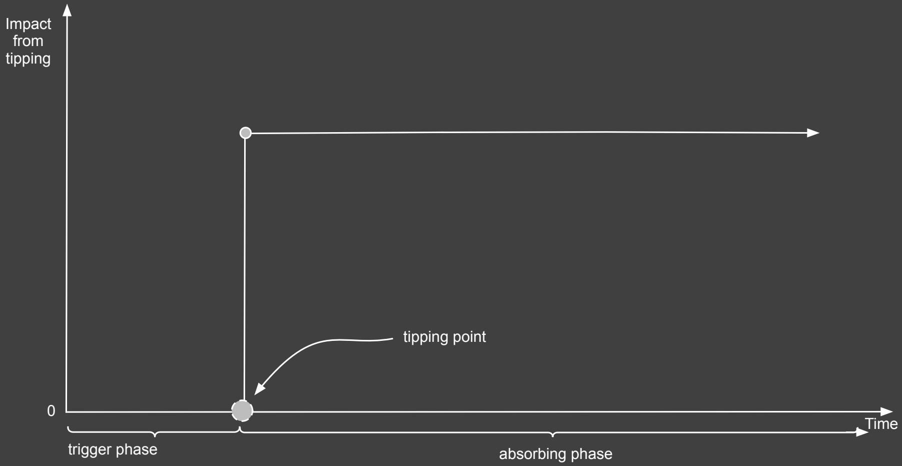
The DICE Model (Nordhaus, 1992, 2002, 2008, 2013, 2016)



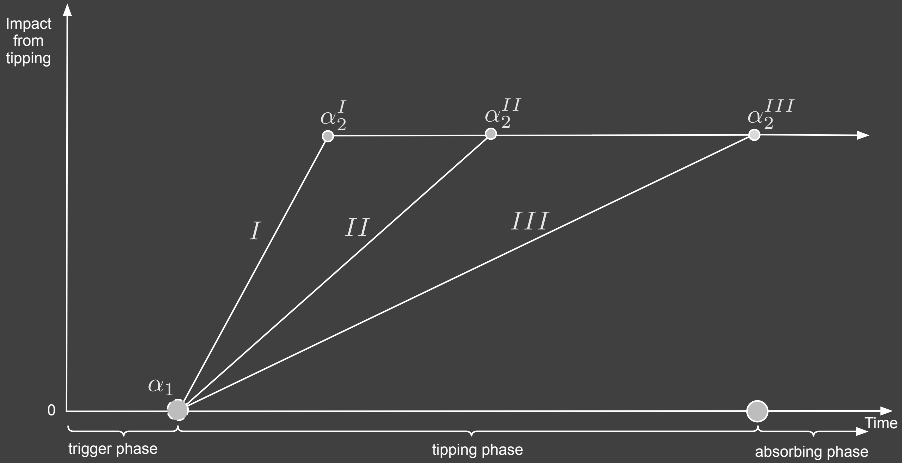
Deterministic Tipping Process: Known Temperature Threshold



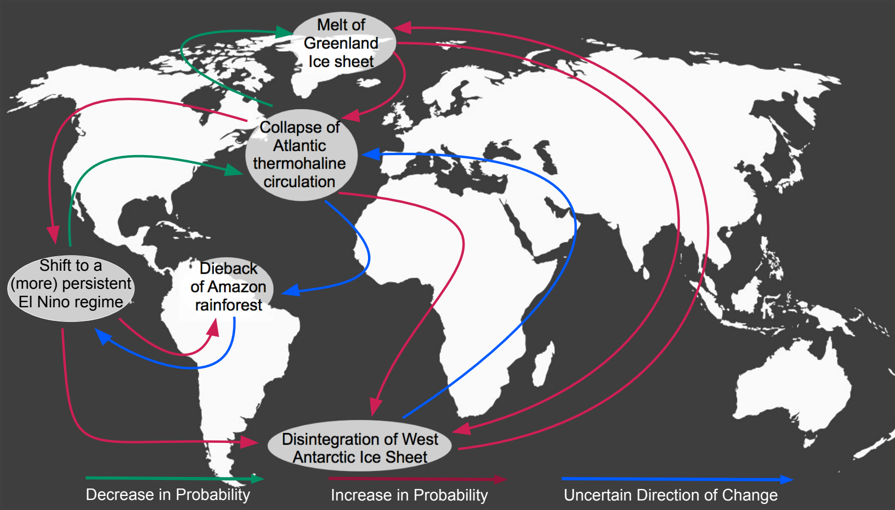
Stochastic Tipping Process



Stochastic Tipping Process with Impact Transition Time



Interaction Between Climate Tipping Elements (adapted from Kriegler et al., 2009)



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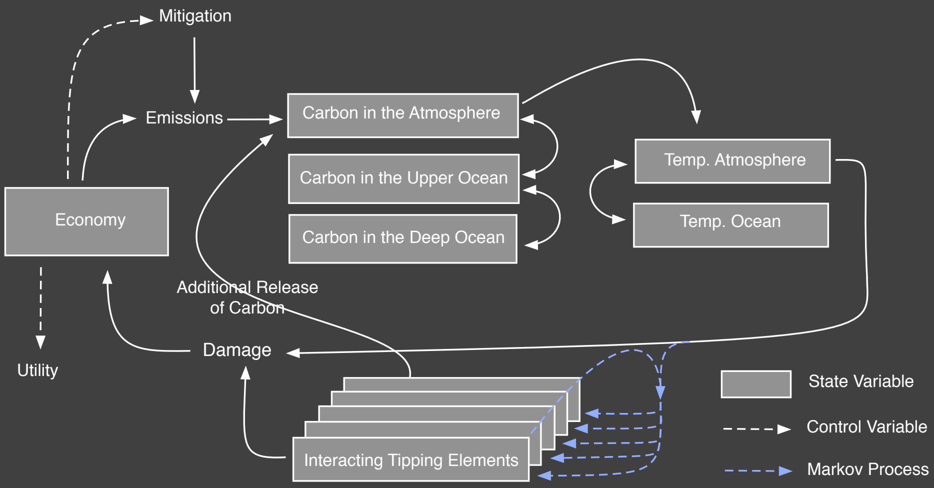

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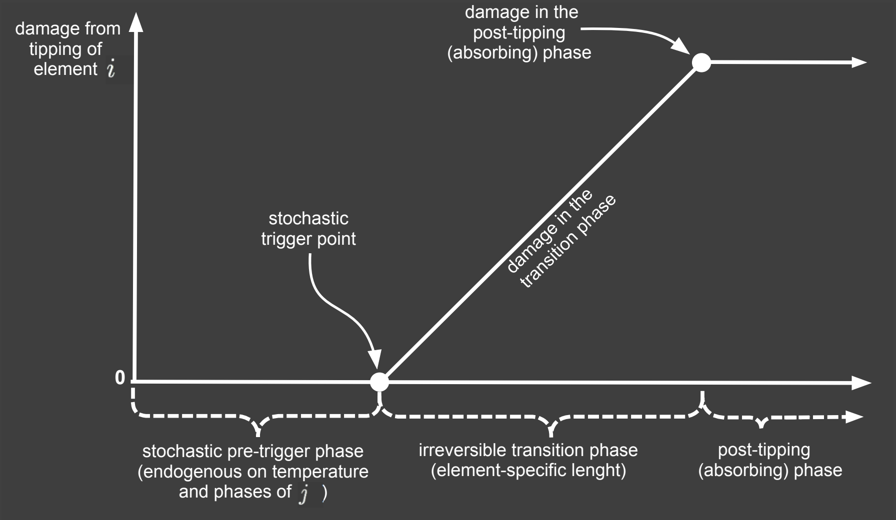


Parameters to Calibrate for Each Tipping Element

- The hazard rate of tipping: depends on temperature and on the tipping state of other tipping elements
- The impact transition time: depends on the internal dynamical timescale of the system
- The final-stage impact of tipping: is realised when the tipping process is completed

- We infer hazard factors from Kriegler et al., (2009) - Method developed in Lontzek et al. (2015)
- The calibrated hazard rate factor b_i for the elements is:
  - 0.00063 for AMOC (Atlantic Meridional Overturning Circulation)
  - 0.00188 for GIS (Greenland ice sheet)
  - 0.00104 for WAIS (West Antarctic ice sheet)
  - 0.00163 for AMAZ (Amazon rainforest)
  - 0.00053 for ENSO (El Nino Southern Oscilation)

Calibration of Impact Transition Time and Final-Stage Impacts

Tipping element	Transition time (years)	Final damage (in % of GDP)
AMOC	10 - **50** - 250	10 - **15** - 20
GIS	300 - **1500** - 7500	5 - **10** - 15
WAIS	100 - **500** - 2500	2.5 - **5** - 7.5
AMAZ	10 - **50** - 250	2.5 - **5** - 7.5
ENSO	10 - **50** - 250	5 - **10** - 15

- **Transition time**: Default based on current model and paleo-data based understanding - lower limits based on limits of physical plausibility / past experience - upper limits based on conservative assessment

- **Final damage**: Some assessment of the relative final damages of different tipping events based on physical climate knowledge, e.g. GIS melt will contribute twice the final sea-level rise as WAIS disintegration.

Calibration of Hazard Interaction Factors

- We implement the interactions as direct, conditional alterations to the hazard rate of individual tipping events.
- We consider the effect of causal interactions between tipping events based on Kriegler et al. (2009).
- "By how much is the hazard factor for tipping element *j* is affected if tipping element *i* has tipped?"

	AMOC	GIS	WAIS	AMAZ	ENSO
AMOC		-0.235	0.12	0.55	0.121
GIS	1.62		0.378	0.108	0
WAIS	0.107	0.246		0	0
AMAZ	0	0	0		0
ENSO	-0.083	0	0.5	2.059	

The System of Interacting Tipping Elements

$$\Omega_t\left(T_t^{\mathrm{AT}}, \mathbf{J}_t, \mathbf{I}_t\right) = \frac{\prod_i (1 - I_{i,t} J_{i,t})}{1 + \pi_2 (T_t^{\mathrm{AT}})^2}$$ (Damage factor to GDP)

$$J_{i,t+1} = \min\left\{J_{i,t} + \triangle_{i,t},\, \overline{J_i}\right\} I_{i,t}$$ (Additional impact from TE i)

$$I_{i,t+1} = g_i(\mathbf{I_t},\, T_t^{\mathrm{AT}}, \omega_{i,t})$$ (Indicator function for TE i)

$$\begin{bmatrix} 1 - p_{i,t} & p_{i,t} \\ 0 & 1 \end{bmatrix}$$ (Markov transition matrix for $I_{i,t+1}$)

$$B_{i,t}(\mathbf{I_t}) = b_i(1 + \sum_j (I_{j,t} f_{ji}))$$ (Hazard rate function for TE i)

$$p_{i,t} = 1 - \exp\left\{-B_{i,t}(\mathbf{I}_t) \max\left\{0,\, (T_t^{\mathrm{AT}} - T_{2010}^{\mathrm{AT}})\right\}\right\}$$

$\triangle_t$: Incremental damage level from t to $t+1$ for TE i
T_t^{AT}: Atmospheric temperature
$\omega_{i,t}$: Random process.
$p_{i,t}$: Conditional "trigger" probability of the tipping element i at time t
b_i: TE-specific hazard rate parameter
f_{ji}: Interaction factor

The Dynamic Programing Problem

$$
\begin{aligned}
V_t(\mathbf{S}) \quad &= \quad \max_{C_t,\mu_t} u(C_t, L_t) + \beta \left[\mathbb{E}_t \left\{ \left(V_{t+1}\left(\mathbf{S}^+\right)\right)^{\frac{1-\gamma}{1-1/\psi}} \right\} \right]^{\frac{1-1/\psi}{1-\gamma}} && \text{(Welfare)} \\
\text{s.t.} \quad & K^+ = (1-\delta)K + \mathcal{Y}_t(K, T_{\mathrm{AT}}, \mu, \mathbf{I}, \mathbf{J}) - C_t - \Psi_t && \text{(Production)} \\
& \mathbf{M}^+ = \Phi_M \mathbf{M} + \left(\mathcal{E}_t\left(K, \mu\right), 0, 0\right)^\top && \text{(Carbon cycle)} \\
& \mathbf{T}^+ = \Phi_T \mathbf{T} + \left(\xi_1 \mathcal{F}_t\left(M_{\mathrm{AT}}\right), 0\right)^\top && \text{(Climate)} \\
& I_i^+ = g_i(\mathbf{I}, T_{\mathrm{AT}}, \omega_i) && \text{(Tipping Indicators)} \\
& J_i^+ = min\{J_i + \Delta_i, \overline{J_i}\} I_i && \text{(Tipping Impacts)}
\end{aligned}
$$

for $t = 0, 1, \ldots, 299$, and any $\psi > 1$

ψ: Inter-temporal elasticity of substitution

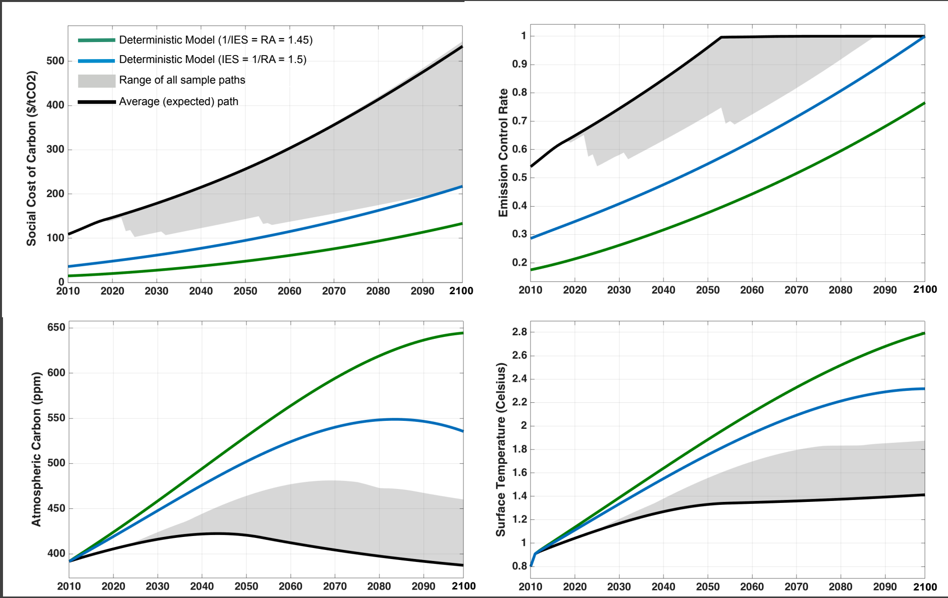
γ: Relative risk aversion parameter

$\mathbf{S} \equiv (K, \mathbf{M}, \mathbf{T}, \mathbf{I}, \mathbf{J})$: Sixteen-dimensional state variable vector

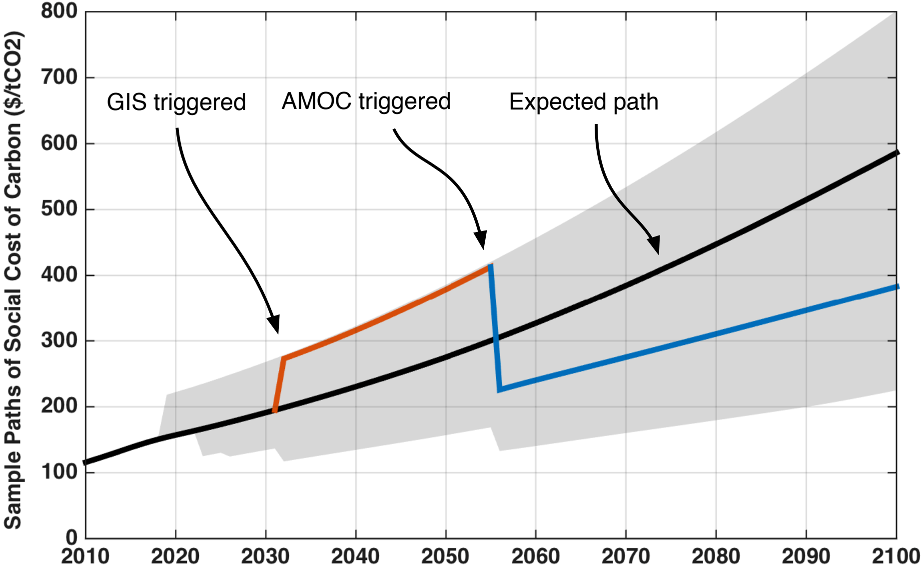
$\mathbf{S}^+$: Next period's state vector

$V_{300}(\mathbf{S})$: Terminal value function

Results: Model Version without Interaction Effects



Results: Sample Path with Interaction Effects



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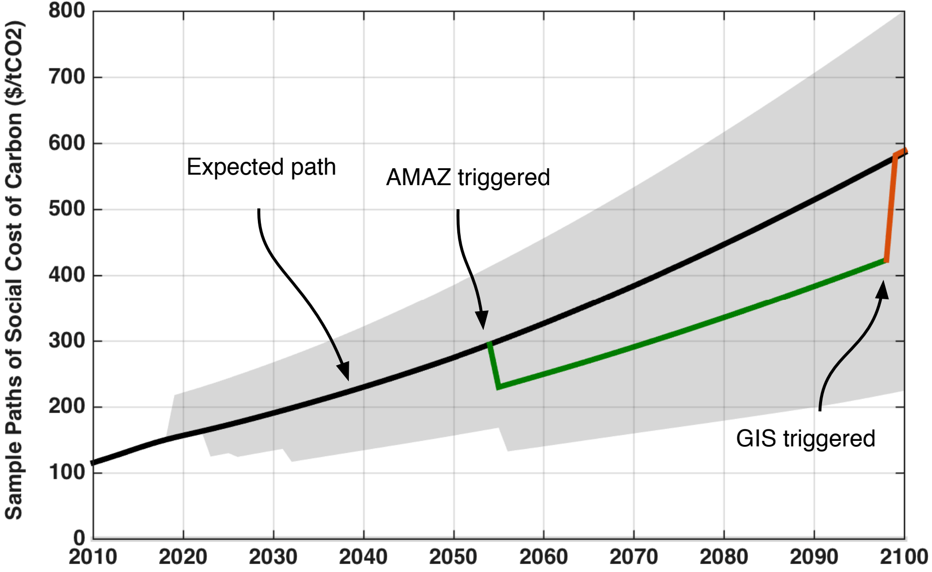
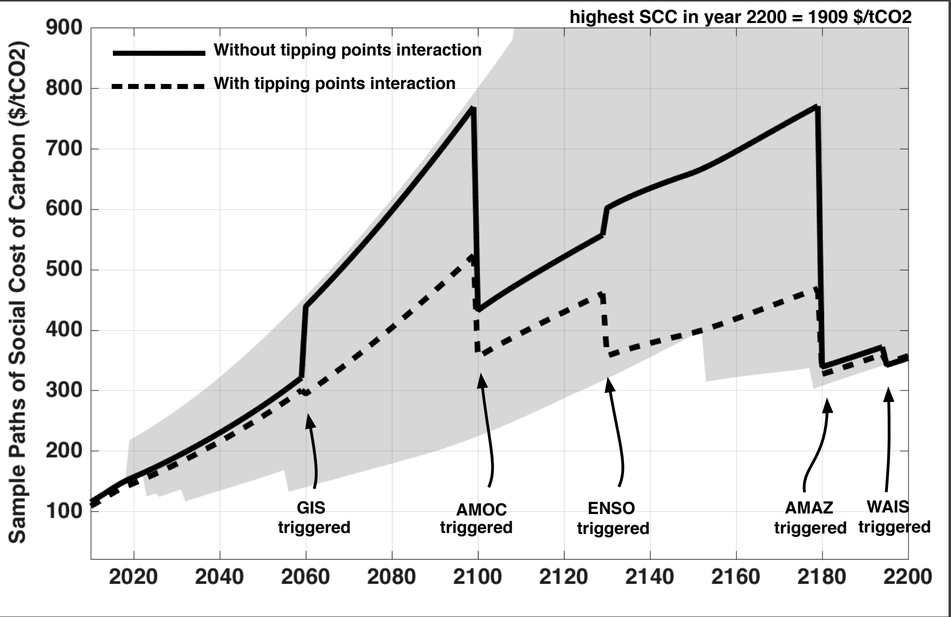
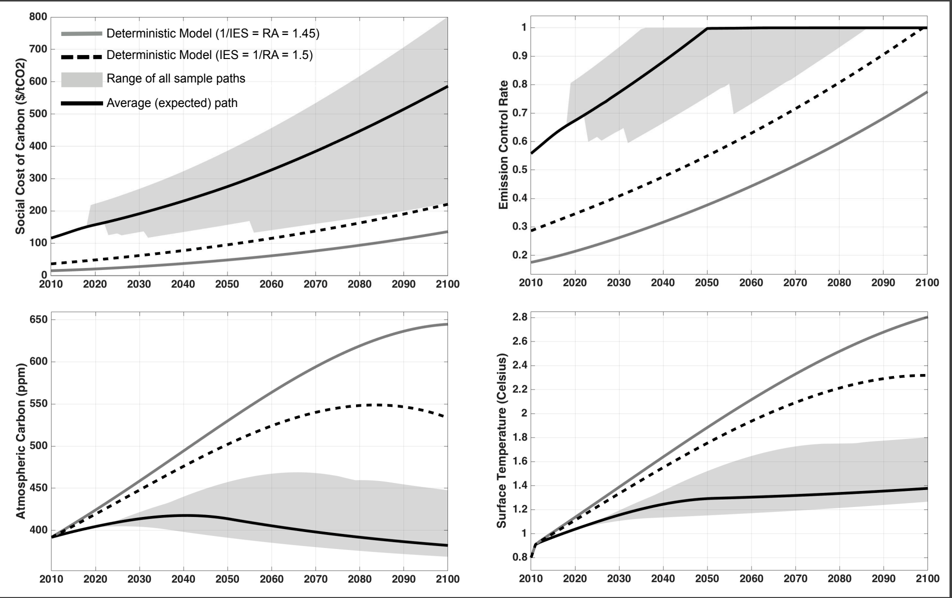


Results: Tipping Point Cascade with and without Interaction Effects



Interaction Effects



Results: Tipping Point Likelihoods

Number of tipping events	Stochastic tipping points (interacting) 2100	Stochastic tipping points (interacting) 2200	Stochastic tipping points (no interaction) 2100	Stochastic tipping points (no interaction) 2200
1	10.8	24.38	12.04	26.88
2	0.65	4.14	0.72	4.08
3	0.04	0.42	0.05	0.41
4	0	0.02	0	0.02
5	0	0.01	0	0
Cumulative probability	11.49	28.97	12.81	31.39

Number of tipping events	Baseline model temperature path* 2100	Baseline model temperature path* 2200	RCP8.5 temperature path† 2100	RCP8.5 temperature path† 2200
1	34.28	23.03	29.69	0
2	10.03	31.31	30.73	0
3	1.81	24.7	19.08	0.33
4	0.18	10.1	6.76	16.87
5	0	2.29	0.85	82.80
Cumulative probability	46.30	91.43	87.11	100

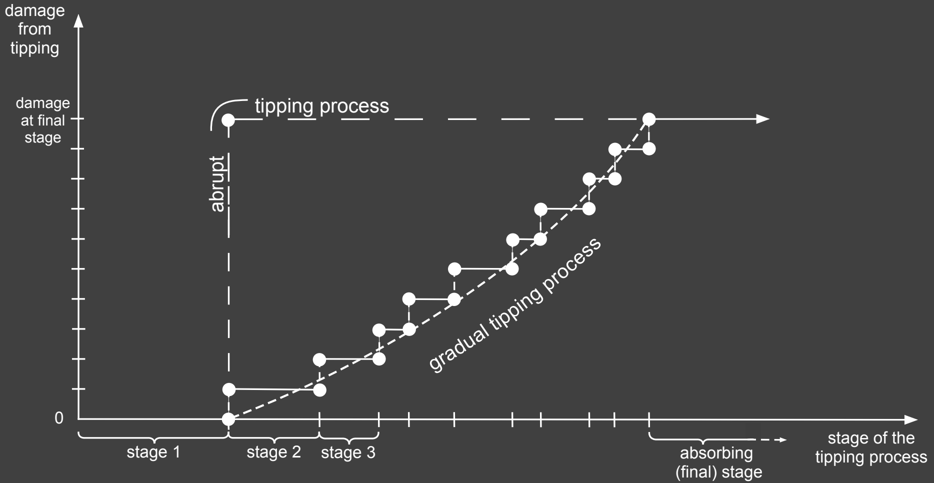
*2.8 °C warming in 2100, 2.76 °C in 2200. †4.7 °C warming in 2100, 7.5 °C in 2200.

More Complex Formulations

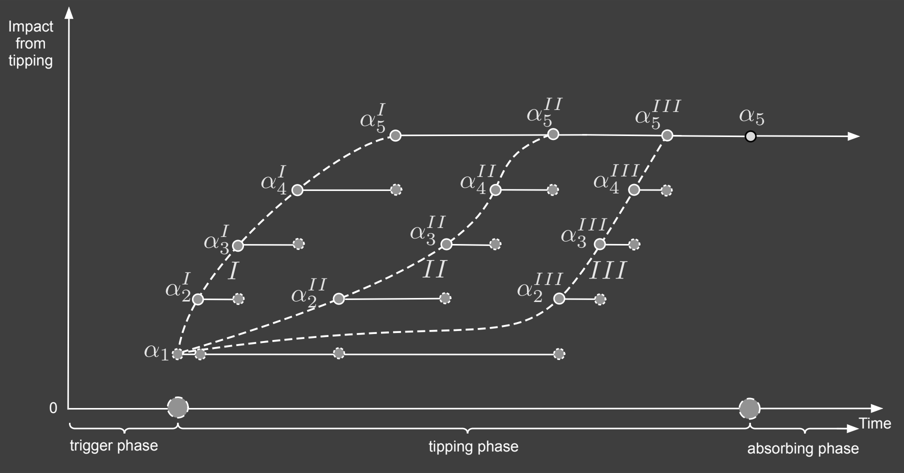
Based on Cai and Lontzek, Journal of Political Economy, 2019

- Stochastic Economic Growth with Long run risk
- Representative Tipping Point with a Stochastic trigger, Stochastic Tipping Duration and Stochastic Impact
- Epstein Zin Preferences

Multi-Stage Tipping Process



Stochastic Impact Transition Time



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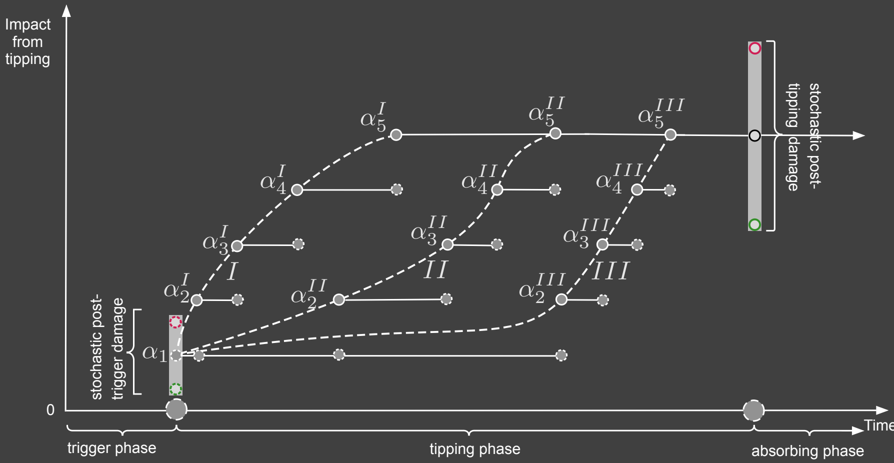


Multi-Stage Tipping System

$$
\begin{bmatrix}
* & * & * & * & & & \\
 & * & & & * & & \\
 & & \ddots & & & \ddots & \\
 & & & * & & & * \\
 & & & & 1 & & \\
 & & & & & 1 & \\
 & & & & & & 1
\end{bmatrix}
$$

$p_{1,1,t} = \exp\left\{-\lambda \max\left\{0,\ T_{\mathrm{AT},t} - \underline{T_{\mathrm{AT}}}\right\}\right\}$ (No-trigger probability)

$p_{1,j,t} = \left(1 - p_{1,1,t}\right)/3$ (first tipping stage probability for $j = 2, 3, 4$)

$\mathcal{J}_{3i+j-2} = \frac{i}{5}\left(1 + (j-2)\sqrt{1.5q}\right)\overline{\mathcal{J}}_\infty$ (Damage for $i = 1, ..., 5$ and $j = 1, 2, 3$)

$1 - \exp\left(-4/\overline{\mathcal{D}}\right)$ (Transition probability in the tipping process)

λ : hazard rate parameter

$\underline{T_{\mathrm{AT}}}$: temperature for which $p_{1,1,t} = 1$.

$q\overline{\mathcal{J}}_\infty^2$: variance of the long-run tipping damage level $\overline{\mathcal{J}}_\infty$

$\overline{\mathcal{D}}$: Expected duration of the tipping process.

Multi-Stage Tipping System: Calibration

- Hazard rate parameter: $\lambda \in [0.0025, \mathbf{0.0035}, 0.0045]$

  Calibrated from expert elicitation studies (e.g., Kriegler et al., 2009)

- Mean duration of the tipping process (in years): $\overline{\mathcal{D}} \in [5, \mathbf{50}, 200]$

  Calibrated from Lenton (2008)

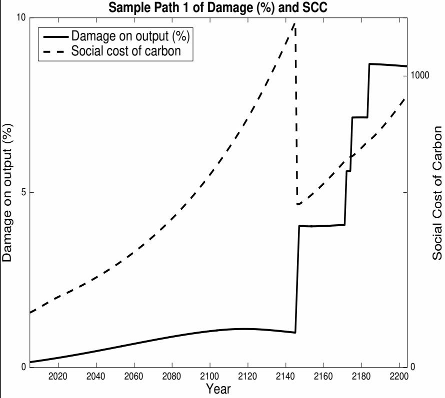
- Expected long-run damage (in % of world GDP): $\overline{\mathcal{J}}_\infty \in [2.5, \mathbf{5}, 10]$
- Mean-squared to variance ratio of damage: $q \in [0, \mathbf{0.2}, 0.4]$

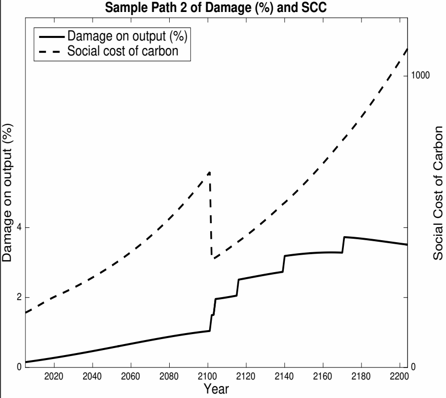
  Calibrated from Stern (2007), Nordhaus (2008) and Hope (2009)

  $\rightarrow$ This results in possible long-run damage levels of

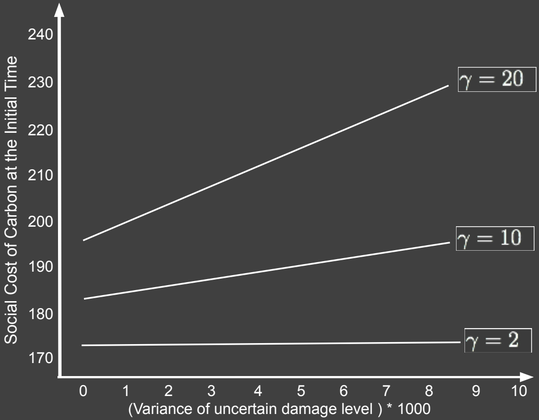
  0.56%, 2.5% or 4.44% for $q = 0$ and $\overline{\mathcal{J}}_\infty = 2.5$
  2.26%, 5% or 7.74% for $q = 0.2$ and $\overline{\mathcal{J}}_\infty = 5$
  2.25%, 10% or 17.75% for $q = 0.4$ and $\overline{\mathcal{J}}_\infty = 10$

Results: Sample Multi-Stage Tipping Paths





Value of Knowledge about Tipping Point Impacts



More Complex Formulations

Augmented formulation: $p_{i,t}^{k \to l} = f(\nu, \mathcal{S}, \mathcal{C}, \dot{\mathcal{S}})$

- hysteresis or reversibility
- fluctuating hazard rate
- interaction

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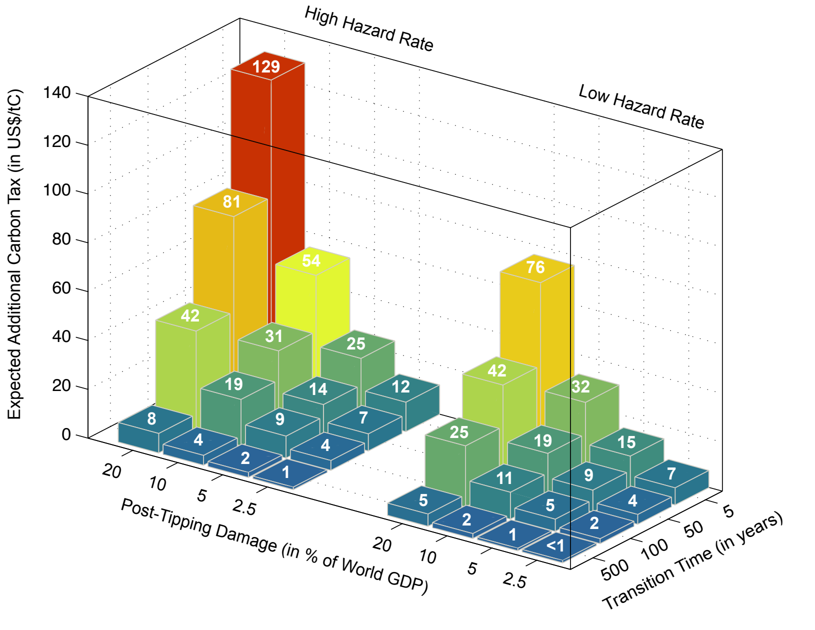
p_t: Conditional probability of the tipping point to occur at time t
ν: Hazard rate parameter
T_t^{AT}: Atmospheric temperature
i: Type of tipping point element
k: curent stage of the tipping transition process
l: Subsequent stage of the tipping transition process
$\mathcal{S}$: Set of states (including discrete states, i.e. tipping elements and stages)
$\mathcal{C}$: Set of controls


High Hazard Rate
Low Hazard Rate
Expected Additional Carbon Tax (in US$/tC)
0
20
40
60
80
100
120
140
129
81
54
42
31
25
19
14
12
8
9
7
4
4
2
1
76
42
32
25
19
15
11
9
7
5
5
4
2
2
1
<1
20
10
5
2.5
Post-Tipping Damage (in % of World GDP)
20
10
5
2.5
5
50
100
500
Transition Time (in years)

We use expert elicitation studies (Kriegler et al. 2009 and Lenton, 2010)

$T^A_{2100} - T^A_{2000}$	$1°C$	$2°C$	$3°C$	$4°C$	$5°C$	$6°C$
Probability of climate tipping triggered until 2100	12.5%	25%	37.5%	50%	62.5%	75%
Inferred hazard rate parameter λ	0.00267	0.00288	0.00313	0.00347	0.00392	0.00462

We choose $\lambda = 0.0035$ as our default.

Range for the sensitivity analysis is $\lambda \in [0.0025, 0.0045]$

Conclusion

- Interacting tipping points increase today's social cost of carbon by 700%
- Warming limited to less than $1.5C^{\circ}$ and full decarbonisation by 2050
- There is a good chance that future SCC in will be so large that optimal policy would use technologies that remove carbon from the atmosphere.
- Our model has implications for the ranking of alternative mitigation options
- R&D decisions today will determine future portfolios of mitigation options
- The development of such technologies could take decades to complete.
- Recommendation: Focus on the present value of having such technologies in those states of the world where the SCC justifies their deployment.

Conclusion

Growth Uncertainty:

- Economic productivity is a stochastic process with significant variability
- Factor productivity growth displays long-run risk (e.g., Bansal and Yaron, 2004; Beeler and Campbell, 2011)
- Stochastic productivity growth process calibrated to match observed moments of the growth rates for per capita consumption.

Damage Uncertainty:

- A representative *tipping point* in the climate system
- Scientifically plausible specification of abrupt climate impacts

Preferences about Risks:

- Epstein-Zin preferences, consistent with observations about how much people are willing to pay to reduce consumption risk.

DSICE: Preferences

DICE assumes that per capita consumption, $c_t = C_t/L_t$, is constant across the population and that each individual has the same power utility function, $c_t^{1-1/\psi}/(1-1/\psi)$, where ψ is the inter-temporal elasticity of substitution (IES). At each time t, the world social welfare function is

$$u(C_t, L_t) = \frac{(C_t/L_t)^{1-1/\psi}}{1-1/\psi} L_t,$$

and is discounted at rate β.
If the stochastic consumption process and the population process are denoted by (C_t, L_t) and $\psi > 1$, the social welfare at time t in DSICE is defined recursively by

$$U_t = \left\{ (1-\beta)\, u(C_t, L_t) + \beta \left[\mathbb{E}_t \left\{ U_{t+1}^{1-\gamma} \mid C_t, L_t \right\} \right]^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}$$

where $\mathbb{E}_t\{\cdot\}$ is the expectation conditional on the states at time t, β is the discount factor, and γ is the risk aversion parameter.
When $0 < \psi < 1$, the utility function $u(C_t, L_t)$ is negative, the formula becomes:

$$U_t = -\left\{ -(1-\beta)\, u(C_t, L_t) + \beta \left[\mathbb{E}_t \left\{ (-U_{t+1})^{1-\gamma} \mid C_t, L_t \right\} \right]^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}$$

If $\psi\gamma = 1$, we have the separable utility case used in DICE.

Sensitivity Around the Benchmark Case

Social cost of carbon in 2010 ($\$/tCO_2$)	High damage	Default damage	Low damage
Short transition time	166	145	94
Default transition time	145	116	77
Long transition time	75	62	50

Endogenous transition time for GIS

- Starts at 7500 years but can reduce to 300 years if warming reaches $6^\circ C$
- SCC in 2010 is $114/tC (-$\$2/tCO_2$)

Pessimistic assessment from experts on the interaction effects

- SCC in 2010 is $121/tC ($+\$5/tCO_2$)

Sensitivity Around the Benchmark Case