Discounting the Future: on Climate Change, Ambiguity Aversion and Epstein-Zin preferences

S. Olijslagers S. van Wijnbergen

University of Amsterdam

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Outline

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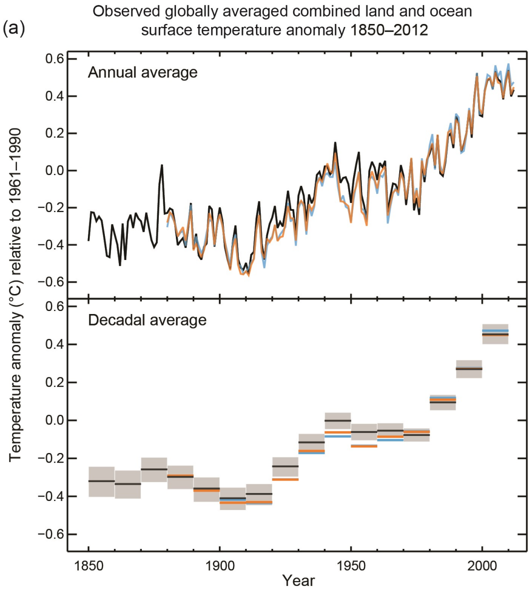
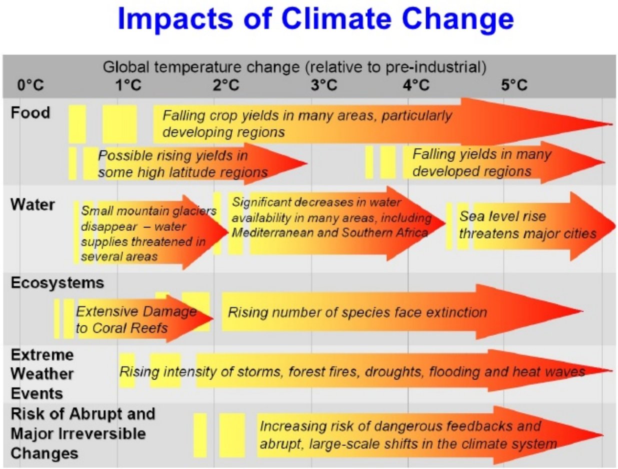
Temperature anomaly



Source: IPCC, 2013: Climate Change 2013: The Physical Science Basis.

Climate Change Impacts



Source: Stern Review, adapted from IPCC

Related literature and contribution

- Numerical models
  - Deterministic models: DICE (W. Nordhaus, 2014), PAGE (Hope, 2006) and FUND (Tol, 2002).
  - Stochastic models: LRR-T model (Bansal, Kiku & Ochoa, 2016) and 4-stated DICE (Traeger, 2014).
- Analytic models: Golosov, Hassler, Krusell and Tsyvinski (2014), Bretscher and Vinogradova (2018), Bremer and van den Ploeg (2018) and Traeger (2018).
- Contribution: More general preference structure (Epstein-Zin preferences and Ambiguity aversion) and modeling climate risk explicitly as disaster risk. The focus is on valuation of climate risk (i.e. Social Cost of Carbon), not (yet) on optimal policy.

Outline

Lucas tree economy with climate disasters

- Representative agent
- Exogenous endowment follows a geometric Brownian motion with a jump process:

$$dC_t = (\mu - \lambda_t m) C_t dt + \sigma C_t dZ_t + Y_t C_{t-} dN_t$$

where $m = E[Y_t] = e^{\mu_J + \frac{1}{2}\sigma_J^2} - 1$.

- μ = growth rate, σ = volatility, λ_t = arrival rate of disasters and m = expected disaster size.
- Jump size Y_t has a lognormal distribution.
- The disasters are compensated, the *expected* effect of a disaster is zero. Focus on risk.
- We do calculate expected loss component at the end.

Epstein-Zin preferences

- Continuous time version of Epstein-Zin preferences:

$$V_t = E_t\Big[\int_t^\infty f(C_s, V_s)ds\Big] \quad \text{where}$$

$$f(C,V) = \frac{\beta}{1-1/\epsilon}\frac{C^{1-1/\epsilon} - ((1-\gamma)V)^{\frac{1}{\zeta}}}{((1-\gamma)V)^{\frac{1}{\zeta}-1}} \quad \text{for } \epsilon \neq 1$$

$$\text{with} \quad \zeta = \frac{1-\gamma}{1-1/\epsilon}.$$

- β equals the pure rate of time preference.
- Separate risk aversion (γ) and elasticity of intertemporal substitution (ϵ).
- EIS very important in climate change setting due to nature of the problem: damages will take place in the (far) future.

Ambiguity aversion

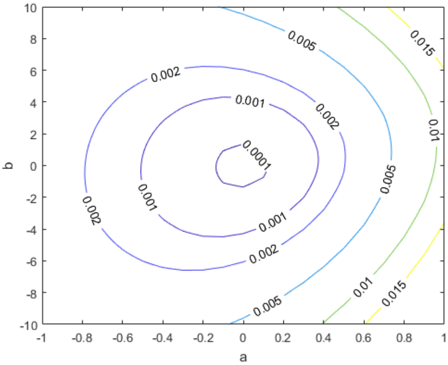
- Intertemporal version of Gilboa-Schmeidler maxmin utility.
- Under the reference measure $\mathbb{P}$, the arrival rate equals λ_t and the expected jump size m.
- Consider alternative measure $\mathbb{Q}_t^{a,b}$ with arrival rate $\lambda_t^{\mathbb{Q}} = e^{a_t}\lambda_t$ and expected jump size $m_t^{\mathbb{Q}} = e^{\mu_J + \frac{1}{2}\sigma_J^2 + b_t\sigma_J^2} - 1$.
- Preferences become (Chen & Epstein, 2002):

$$V_t = \min_{\{a_s, b_s\}_{s \geq t}} E_t^{\mathbb{Q}}\left[\int_t^{\infty} f(C_s, V_s) ds\right]$$
$$\text{s.t. } d(a_t, b_t) \leq \theta_t \quad \forall t$$

- θ_t = ambiguity aversion parameter.
- Optimization yields $\lambda_t^* > \lambda_t$ and $m_t^* < m_t$.

Ambiguity aversion

- Distance measure: (instantaneous) relative entropy RE.
  - RE concept is closely related to the Maximum-Likelihood ratio
- $RE(a_t, b_t) = \lim_{\Delta \to 0} \frac{1}{\Delta} E_t^{\mathbb{Q}}\left[\log\left(\frac{\xi^{a,b}_{t+\Delta}}{\xi^{a,b}_t}\right)\right]$ where $\xi^{a,b}_t = \frac{d\mathbb{Q}^{a,b}}{d\mathbb{P}}|_{\mathcal{F}_t}$ is the Radon-Nikodym derivative.



Admissible region for different values of θ.

Outline

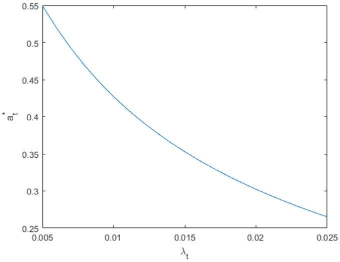
Temperature-dependent arrival rate of disasters

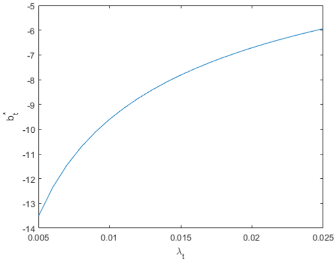
- The arrival rate is temperature-dependent: $\lambda_t = \lambda_T T_t$.
- T_t is temperature at time t with respect to the temperature in pre-industrial age (Temperature anomaly).

- Climate model: Continuous time version of DICE.
- Emissions scenario: Business as usual. We do not (yet) look at optimal abatement policies.

Optimal a_t and b_t with time-varying arrival rate λ_t.

- a_t and b_t are a function of both $\lambda_t = \lambda_T T_t$ and θ.
- Assumption: Budget θ is constant over time.





Climate Model (1/2)

- Emissions are modeled exogenously to match the base scenario of the DICE model. E_t is first increasing and peaks around 2100.

$$dg_t^E = \delta_{g^E}(g_\infty^E - g_t^E)dt$$
$$dE_t = g_t^E E_t dt$$

- Carbon cycle: three box model. $\delta_{a\to b}$ is the rate at which carbon moves between the boxes.

$$dM_t^{at} = \Big(-\delta_{at\to up}M_t^{at} + \delta_{up\to at}M_t^{up} + E_t\Big)dt$$
$$dM_t^{up} = \Big(\delta_{at\to up}M_t^{at} - (\delta_{up\to at} + \delta_{up\to lo})M_t^{up} + \delta_{lo\to up}M_t^{lo}\Big)dt$$
$$dM_t^{lo} = \Big(\delta_{up\to lo}M_t^{up} - \delta_{lo\to up}M_t^{lo}\Big)dt$$

Climate Model (2/2)

- The atmospheric carbon concentration leads to positive radiative forcing (F_t). Furthermore, non-carbon forcing (EF_t) is modeled as an exogenous process.

$$F_t = \kappa_1 \frac{\log(M_t^{at}/M_{pre}^{at})}{\log(2)}$$

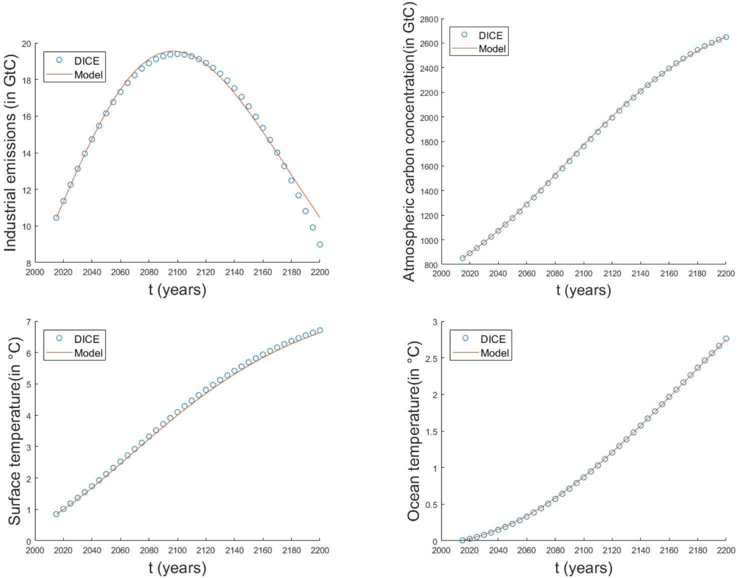
$$dEF_t = \kappa_2(EF_\infty - EF_t)dt$$

- Temperature responds slowly to radiative forcing due to thermal inertia. Oceans slow down this process.

$$dT_t = \frac{1}{\tau_1}\left(F_t^{tot} - \frac{\kappa_1}{\tau_2}T_t - \frac{\tau_4}{\tau_3}(T_t - T_t^{oc})\right)dt$$

$$dT_t^{oc} = \frac{1}{\tau_4}\frac{\tau_4}{\tau_3}(T_t - T_t^{oc})dt$$

Evolution of climate state variables



Outline

The Social Cost of Carbon (1/8)

Preliminaries

- Define X_t as the vector of state variables.
- A consumption strip $H_t(C_t, X_t, s-t) = E_t\left[\frac{\pi_s}{\pi_t} C_s\right] = C_t D(X_t, u)$, pays out C_s in $u = s - t$ years. Here π_t is the state-price density at time t and $D(X_t, u)$ is the consumption discount factor with maturity u.

- $\frac{\partial}{\partial \epsilon} D(X_t, u) > 0$.
- $\frac{\partial}{\partial \theta} D(X_t, u) > 0$ if $\epsilon < 1$.
- $\frac{\partial}{\partial \theta} D(X_t, u) < 0$ if $\epsilon > 1$.

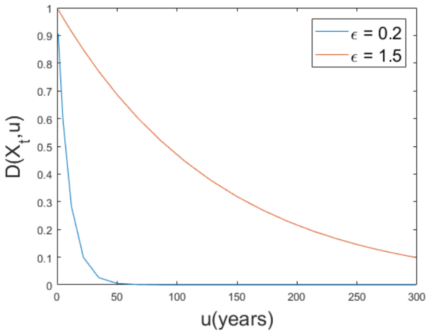
The Social Cost of Carbon (2/8)

- SCC = welfare loss of one additional ton of carbon emissions today in 2015 \$ terms.
- $$SCC_t = -\frac{\partial V_t / \partial M_t^{at}}{f_C(C_t, V_t)} = -C_t \int_0^\infty \int_t^{t+u} \underbrace{\frac{\partial}{\partial M_t^{at}} \lambda_s^*}_{A}$$
$$\underbrace{\left(\frac{e^{(1-\gamma)(\mu_J + b_s^* \sigma_J^2 + \frac{1}{2}(1-\gamma)\sigma_J^2)} - 1}{1-\gamma} - m_s^* \right)}_{B} ds \, \underbrace{D(X_t, u)}_{C} du$$
- A = marginal effect of emissions on arrival rate
- B = difference between certainty equivalent and expected loss
- C = consumption discount factor

The Social Cost of Carbon (3/8)

Effect of elasticity of intertemporal substitution on the SCC.

Figure: Consumption discount factor



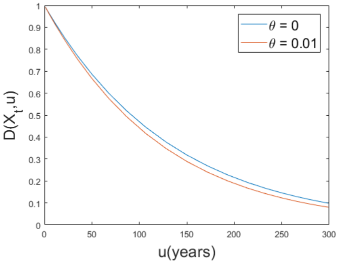
- Larger EIS $\Longrightarrow$ Lower discount rate.
- Yearly discount rate: $\approx$ 10% ($\epsilon = 0.2$) vs. $\approx$ 0.8% ($\epsilon = 1.5$).

The Social Cost of Carbon (4/8)

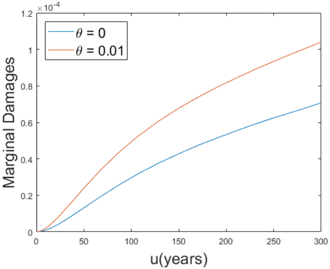
Effect of ambiguity aversion on the SCC

- Assume $\epsilon > 1$ (for $\epsilon < 1$ the result is the opposite).

(a) Consumption discount factor



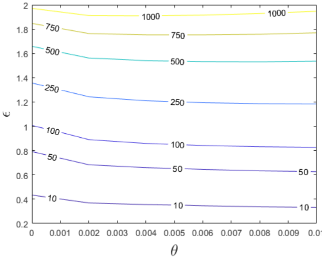
(b) Marginal damages

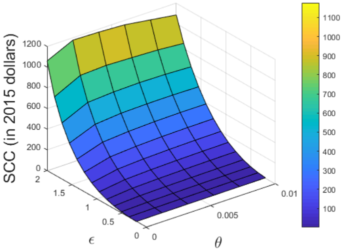


- Larger θ has two offsetting effects: 1) Higher discount rate, 2) Larger marginal damages.

The Social Cost of Carbon (5/8)

Par.	Description	Value
C_t	Initial consumption level (PPP, in trillion 2015 $)	83
μ	Growth rate of consumption (per year)	0.025
σ	Volatility of consumption (per year)	0.03
γ	Risk aversion	5
β	Pure rate of time preference	0.015
λ_T	Arrival rate parameter (per year)	0.0029
m	Expected disaster size	-0.09

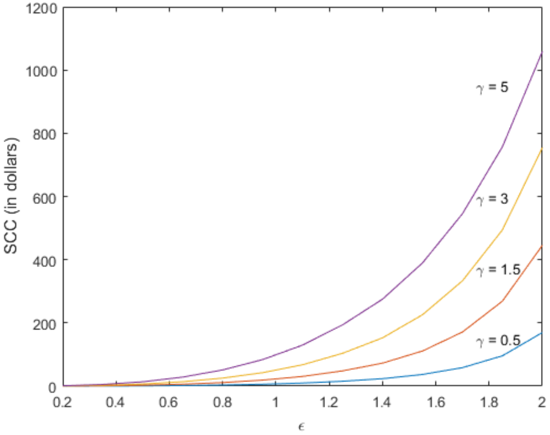




Baseline calibration ($\epsilon = 1.5$, $\theta = 0.005$): SCC = 465 in 2015$.

The Social Cost of Carbon (6/8)

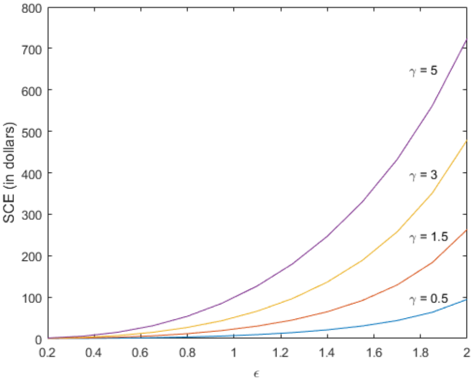
Effect of risk aversion on the SCC



The Social Cost of Carbon (7/8)

- Alternative measure welfare loss: Social Cost of Emissions.
- Marginal loss of shifting the path of emissions up scaled by marginal welfare of shifting the path of consumption up.

$$SCE_t = -\frac{\partial V_t / \partial E_t}{\partial V_t / \partial C_t}$$



The Social Cost of Carbon (8/8)

Parameters: $\epsilon = 1.5$, $\theta = 0$.

	Compensation	No Compensation
SCC	347	763
SCE	300	655

Outline

Summing Up

- Mitigation policies have to be implemented today while damages will occur far into the future. Discounting is therefore one of the main issues in the valuation of climate damages.
- We develop a stochastic IAM with a realistic climate model that does not suffer from the curse of dimensionality.
- Analytic solutions provide intuition about how parameters affect the SCC.
- The SCC is very sensitive to preference assumptions, especially to risk aversion and the EIS.

Directions for future research

- Look at stochastic processes for state variables. Model must be solved numerically.
  - Implications for SCC.
- Optimal policy: Production based economy, directed technological change...
- Prices vs. quantities?

Ambiguity aversion

- Hansen and Sargent (2001,2008) constraint approach:

$$V_t = \min_{\{a_s, b_s\}_{s \geq t}} E_t^{\mathbb{Q}} \Big[\int_t^{\infty} f(C_s, V_s) ds \Big]$$
$$\text{s.t. } \int_t^{\infty} e^{-\beta(s-t)} d(a_s, b_s) ds \leq \eta \tag{1}$$

- Use lifetime relative entropy budget η instead of instantaneous relative entropy θ_t like in Chen-Epstein (1992).
- Disadvantage: Preferences are not homothetic.

Relative entropy and maximum likelihood.

- Assume that we have a random sample of n observations of X, namely $x_1, ..., x_n$. Consider the following hypothesis:

$$H_0 : \mathbb{P} \text{ is true} \quad vs. \quad H_1 : \mathbb{Q} \text{ is true}$$

- Most powerful test (Neyman-Pearson Lemma) is to reject H_0 if $\log\left(\frac{\prod_1^n f^{\mathbb{Q}}(x_i)}{\prod_1^n f(x_i)}\right) > \log(k)$ where k is chosen to minimize the type I error.
- Relative entropy: instead of random sample, take expectation of log-likelihood ratio assuming that the alternative measure is true. Measure of 'distance' between two probability distributions.

Consumption strip

$$H_t = C_t \exp\Big\{ -\underbrace{ku}_{A} - \underbrace{(1/\epsilon - 1)}_{B}$$
$$\underbrace{\int_t^{t+u} \lambda_s^* \left(\frac{e^{(1-\gamma)(\mu_J + b_s^* \sigma_J^2 + \frac{1}{2}(1-\gamma)\sigma_J^2)} - 1}{1-\gamma} - m_s^* \right) ds}_{C} \Big\}$$

where $\lambda_t^* = \lambda_T e^{a_t^*} T_t$, $m_t^* = e^{\mu_J + b_t^* \sigma_J^2 + \frac{1}{2}\sigma_J^2}$ and
$k = \beta + (1/\epsilon - 1)\left(\mu - \frac{\gamma}{2}\sigma^2\right)$.