



BRILL

NUNCIUS (2020) 1–18



Al-Khazini's Balance of Wisdom

A Masterpiece of Medieval Engineering

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Abstract

Abu al-Fath Khazini's "Balance of Wisdom" was one of the most sophisticated and advanced balances to be designed and manufactured in the medieval Islamic world. Hydrostatic balances were used to assess the value of the precious metals and gemstones entering the royal treasury. Al-Khazini presents a detailed description of his balance in *Al-Kitab Mizan Al-Hikma*. Among the advantages of this instrument were its higher degree of precision, its complex triple action, and its ability to identify and determine the percentage by weight of the constituent elements in a binary alloy, provided one of the elements in the alloy is known. It was also capable of measuring the weight and density of pure metals and alloys with a high degree of accuracy. This paper describes the construction of a prototype of Al-Khazini's balance and the testing of its accuracy in weighing, titration, and the identification of elements following the instructions contained in *Al-Kitab Mizan Al-Hikma*. The instrument was found to function as accurately as claimed by its inventor.

Keywords

Balance of Wisdom – Al-Khazini – titration of alloys

1 Introduction

The simple design of the earliest scales supports the thesis that mechanical balances may have been the first instruments invented for measuring the weight (mass) of objects. They usually consisted of two scale pans suspended at the extremities of a pivoting horizontal beam, with a pointer positioned at the centre of the beam that settled over a vertical line drawn on the fixed main body of the balance when the pans were in equilibrium. To measure the mass of an object, it was placed in one pan and a perfect balance was sought against a standard known mass placed in the pan opposite it. The design of these traditional balances was based on the principle of leverage.

However, the sultan's treasury required more accurate and sophisticated instruments, capable not only of weighing but also of titration and the determination of the purity of precious metals such as gold, and in this era the hydrostatic balance was perfected. The history of such balances goes back to Archimedes and culminated in the medieval period in the instrument known as the "balance of wisdom" designed and constructed by Al-Khazini.¹ This essay will present a summary of the life and achievements of the Persian scientist, and then describe the construction of a prototype of his balance and the results of tests of its functioning and accuracy.

2 The Life of Al-Khazini

Al-Khazini was a scientist, mechanical engineer, mathematician, and astronomer who lived in the 11th century A.C. His full name was Abu al-Fath Abd al-Rahman Mansour al-Khazini (usually simplified to Abu al-Fath Khazini).² It is believed that Al-Khazini was born a slave in the city of Marv (also known as Merv or Marwa), located at the southern extremity of the Qaraqum Sahara (Karakum Desert), which originally formed part of the territory of ancient Iran (Persia) and today takes up 70% of the total area of Turkmenistan.³ In medieval times Marv was a major Persian city in Khorasan, the north easternmost province of Persia, and its ruins can still be visited today.

1 Abol Fath Abdul Rahman Al-Khazini, *Mizan Al-Hikma*, translated by Moddaress Razavi (Tehran: Bonyad Farhang Publications, 1967).

2 Ibid.

3 Richard L. Scheffel, Suzanne J. Wernet, *Natural Wonders of the World* (Pleasantville: Reader's Digest Association, 1980), p. 199, and Lewis Owen, "Karakum Desert," in *Encyclopædia Britannica*, s.v. (online version).

Marv (or Merv) was renowned for its literary and scientific achievements⁴ and Al-Khazini was one of its greatest scholars.⁵ Little is known about his life, but he was apparently a slave of Byzantine origin who was bought by a treasurer (*khazin*) of the Seljuk court in Marv named Abu al-Husayn (or Abu al-Hasan) Ali ibn Muhammad Al-Khazin Al-Marwazi.⁶ The treasurer recognized the talents of his young slave, allowed him to be educated in the fields of science, mathematics and philosophy, and eventually freed him.⁷ Al-Khazini became a mathematician at the Seljuk court and pursued further studies under the famous mathematician and astronomer Umar Khayyam.⁸ Although Al-Khazini's name is quite well known in the Islamic world, especially amongst scholars, the international scientific community remains insufficiently aware of his extraordinary achievements.

3 Scientific Achievements

Apparently Al-Khazini rose to a high official rank in the government of the sultan of the Seljuk Empire, Ahmad Sanjar (Sinjar) ibn Malekshah (known as Sultan Sanjar).⁹ He benefitted from the rich scientific heritage stored in the libraries of Marv, which was where he did most of his scientific work.¹⁰ His best-known writings are *The Book of the Balance of Wisdom, Treatise on Astronomical Wisdom*, and his main astronomical book titled *Al-Zij- al mutabar al sanjari al-sultani* which is an astronomical book with tables dedicated to Sultan Sanjar.¹¹

4 Scheffel, Wernet, *Natural Wonders of the World* (cit. note 3).

5 Robert E. Hall, "Al-Khazini," in *Complete Dictionary of Scientific Biography*, 16 vols., Vol. 7, edited by Charles Coulston Gillispie, Frederic Lawrence Holmes, Noretta Koertge, Thomson Gale (New York: Charles Scribner's Sons, 1973), pp. 335–351, see <https://www.encyclopedia.com/science/dictionaries-thesauruses-pictures-and-press-releases/al-khazini-abul-fath-abd-al-raman-sometimes-abu-manur-abd-al-raman-or-abd-al-rahman-manur>. See also <https://muslimheritage.com/merv-history-science-and-learning/> (accessed 22 Oct. 2020).

6 Ibid.

7 Ibid. See also Boris Rosenfeld, Review of *Abu'l-Fath Abd al-Rahman al-Khazini (XII Century)*, by Mariam Mikhailovna Rozhanskaya, *Isis*, 1994, 85/4:686–688.

8 Rosenfeld, "Review" (cit. note 7).

9 Ahmad Sanjar was born in ca. 1086 in Sinjar, a town situated in the borderland between Syria and the Al-Jazira. He was a son of Malik Shah I the great king of Seljuq dynasty. In 1096, he was given the province of Khorasan to govern under his brother Muhammad I. Over the next several years Ahmed Sanjar became the ruler of most of Iran (Persia) with his capital at Nishapur.

10 Rosenfeld, "Review" (cit. note 7).

11 Ibid. Cf. also Al-Khazini, *Mizan Al-Hikma* (cit. note 1); Thomas A. Hockey (ed.), *Biographical Encyclopaedia of Astronomers* (New York: Springer, 2007), Vol. 1 (A–L), p. 630.

In the area of astronomy one of his great achievements was the invention of a 24-hour water clock for astronomical calculations. In his book, *Al-Zij Al-Mu'tabar Al-Sanjari*, he presented the position of the stars for the year 1115–1116 at the latitude of Marv.¹² In addition, he conducted an accurate determination of the obliquity of the Zodiac band (the apparent yearly path of the sun).¹³

Centuries before Isaac Newton (1643–1727), Al Khazini proposed a version of the law of gravitation,¹⁴ stating that: “For each heavy body of a known weight positioned at a certain distance from the centre of the universe, its gravity depends on the remoteness from the centre of the universe. For that reason, the gravities of bodies relate as their distances from the centre of the universe.”¹⁵ In addition, he was the first to surmise that the gravitational force between bodies varies according to their distance from the centre of the earth.¹⁶ Al-Khazini hypothesized that the density of water is inversely related to this distance, i.e. the nearer a body of water is to the centre of the earth, the greater its density. Roger Bacon (1220–1294) posited and verified the same hypothesis a century later.¹⁷

4 Written Works

There are nine surviving treaties, horoscopes (*zij*) and books by Al-Khazini which have been studied by many researchers and scientists.¹⁸ One of these is the succinct *Treatise on Astronomical Wisdom*,¹⁹ where in seven chapters the author explains the workings of seven different astronomical instruments – the triquetrum, the dioptra, a “triangular instrument,” a quadrant, devices involving reflection, and an astrolabe – and presented tips for viewing objects in the night sky with the naked eye.²⁰ Other works by the astronomer includes

12 Julio Samsö, “Al Khazini,” in *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, edited by Helain Selin (Dordrecht: Kluwer Academic Publishers, 1997), p. 135.

13 Ibid.

14 David E. Tivel, *Evolution: The Universe, Life, Cultures, Ethnicity, Religion, Science, and Technology* (Pittsburgh: Dorrance Publishing, 2012), p. 149.

15 Rosenfeld, “Review” (cit. note 7).

16 Mariam M. Rozhanskaya, I.S. Levinova (ed. and transl.), “Al-Khazini. Kniga vesov midrosti” [The Book of the Balance of Wisdom], in *Nauchnoye nasledstvo* [Scientific Heritage] (Moscow, 1983), Vol. 6, pp. 15–140.

17 Ibid.

18 Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

19 Hall, “Al-Khazini” (cit. note 5), and Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

20 Hall, “Al-Khazini” (cit. note 5).

the book *Tarkib al-Aflak va al-akr al-samavieh*.²¹ However, the most outstanding contribution of Al-Khazini to the world of science was *Al-Kitab Mizan Al-Hikma*.

5 *Al-Kitab Mizan al-Hikma*

Al-Kitab Mizan Al-Hikma was completed by Al-Khazini in 1121 at the court of Sultan Ahmad Sanjar in Marv.²² While serving in the royal treasury he invented a hydrostatic balance that he called the Mizan al-Hikma or the “balance of wisdom.” In his book he explains the laws of statics and hydrostatics underlying the design of the balance, describes its construction, and outlines procedures to measure the weight and density of materials and the proportions of the components in alloys.²³ In the mid-19th century a partial translation of his book into English was edited by N. Khanikoff.²⁴

Kitab Mizan Al-Hikma is composed of eight books divided into fifty chapters.²⁵ The first book is devoted to theories of centre of gravity and various aspects of specific gravity. He also discusses the achievements of eminent predecessors including Abu Rayhan al-Biruni, Al-Razi, ‘Umar al-Khayam, Al-Isfizari, and the ancient Greek savants Archimedes and Euclid.²⁶ He explains the differences between force, mass and weight, and shows he was aware that air was not weightless and that its density decreased with increasing altitude.²⁷ Before designing his own balance he carried out painstaking research, in particular on the instruments invented by al-Biruni,²⁸ measuring the specific gravity of fifty different substances ranging from liquids to precious stones and met-

21 Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

22 Ibid.

23 Hall, “Al-Khazini” (cit. note 5); Samsö, “Al Khazini” (cit. note 12), p. 135; Donald R. Hill, “Physics and Mechanics. Civil and Hydraulic Engineering, Industrial Processes and Manufacturing, and Craft Activities,” in *History of Civilization of Central Asia*, Vol. IV (France: UNESCO publishing, 2000), pp. 249–275, and Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

24 Hill, “Physics and Mechanics” (cit. note 23); Hall, “Al-Khazini” (cit. note 5).

25 Hall, “Al-Khazini” (cit. note 5); and Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

26 Al-Khazini, *Kitab Mizan Al-Hikma*, partial English translation by Nicholas Khanikoff (Hyderabad, 1860); Nicholas Khanikoff, “Analysis and Extracts from Kitab Mizan al-Hikma (Book of the Balance of Wisdom), an Arabic Work on Water Balances, Written by Al-Khazini in the Twelfth Century,” *Journal of the American Oriental Society*, 1860, 6:1–128; and Rozhanskay, Levinova, “Al-Khazini” (cit. note 16); and Hall, “Al-Khazini” (cit. note 5).

27 Al-Khazini, *Kitab Mizan Al-Hikma* (cit. note 26).

28 Amelia C. Sparavigna, “The Science of al-Biruni,” *International Journal of Sciences*, 2013, 2/2:52–60.

als. His admirably systematic approach is analysed by D.R. Hill, who presents modern values for the measurements made by the medieval Persian scientist.²⁹

In his book Al-Khazini explains in detail the procedures to make specific types of measurements. Of particular interest are the methods he devised to determine the specific gravity of pure substances (elements) and alloys, using his balance to measure the weight of objects in air and water, and the volume of air or water which they displaced.

6 Versions and Translations of *Kitab Mizan al-Hikma*

Al-Khazini's book has survived in four manuscripts, three of which are independent³⁰ and two of them are nearly identical versions of the same text.³¹ There have also been many studies published on the *Kitab al-Mizan al-Hikma*, as well as complete and partial translations. These include:

1. N. Khanikoff, who was Russian consul general in the city of Tabriz in Iran, translated an Arabic version of the book of the balance of wisdom into English. This work was presented to the American Oriental Society on October 29, 1857.³²
2. Later Khanikoff published an article entitled "Analysis and Extracts of the Book of the Balance of Wisdom" in the *Journal of the American Oriental Society*, 1860, 6:1–128, pp. 8–14.
3. Seyyed Hossein Nasr devotes an entire chapter in *Science and Civilization in Islam* (ABC International Group Inc., 2007, with a preface by Giorgio de Santillana) to Al-Khazini and his balance of wisdom, drawing on the exhaustive studies conducted by E. Wiedermann. The chapter includes a table compiled by A. Mieli that compares the values obtained by Al-Biruni and Al-Khazini with modern measurements. He also presents the formula developed by Al-Khazini to calculate the absolute weight and

29 Donald R. Hill, *Islamic Science and Engineering* (Edinburgh: Edinburgh University Press, 1993), p. 70.

30 Khanikoff, "Analysis and Extracts" (cit. note 26).

31 Sairah Yassir-Deane, *Oxford Museum of the History of Science Online Al-Mizan Exhibition*, <https://muslimheritage.com/article/al-mizan-online-exhibition> (accessed 22 Oct. 2020); Al-Khazini, *Kitab Mizan Al-Hikma* (cit. note 26); Khanikoff, "Analysis and Extracts" (cit. note 26); Rozhanskaya, Levinova, "Al-Khazini" (cit. note 16), and Hall, "Al-Khazini" (cit. note 5).

32 Khanikoff, "Analysis and Extracts" (cit. note 26).

consequently the specific weight of bodies composed of two components.³³

4. Faiza Larihi Bancel³⁴ published an admirable translation of Al-Khazini's text from Arabic into French. The book begins with five introductory chapters that contain information based on the study of important original sources. She discusses in turn:
 - i. Statics (the science of weighing) as a branch of mathematics
 - ii. The history of statics in relation to Al-Khazini's book.
 - iii. Mechanics in ancient Greece.
 - iv. The Greek and Arabic approaches to the law of leverage equilibrium.
 - v. The theory of gravity and centres of mass.
 Mrs Bancel's book is a valuable reference work for scholars studying the science of mechanics in the medieval Islamic world.³⁵
5. In 1995 the Max Planck Institute for the History of Science in Berlin launched a project to study Al-Khazini and his works. In 2017 it embarked on a project devoted to produce a translation and a study on Al-Khazini and on the history of balances in the medieval Islamic sciences.³⁶
6. In 1968 a translation from Arabic to Farsi of *Kitab al-Mizan al-Hikma* was published by the Foundation for Iranian Culture in Iran. This translation served as the primary source of data on the balance of wisdom for the current research project (see footnote 1).

7 Balances

The traditional balance or 'pair of scales' consists of a pivoted horizontal lever with a pointer at its fulcrum and two arms of equal length (the beam) from which two weighing pans are suspended. The weight of an object can be determined by placing it in one of the pans and balancing it against the weight of

33 The formula is as follows: $X = A \frac{\frac{1}{d_1} - \frac{1}{S}}{\frac{1}{d_1} - \frac{1}{d_2}}$ where A = the absolute weight of the sample being measured; d_1 and d_2 = densities of the two components; S = the specific gravity.

34 Al-Khazini, *Kitāb mīzān al-ḥikmat 'Abd al-Rahman al-Khāzinī*, edited, commented and translated by Faīza Laridhi Bancel, with a preface by Roshdi Rashed (Carthage: Tunisian Academy of Sciences, Letters and Arts [Bayt al-Hikma], 2008).

35 Hanif Ghalandari, "Kitab Al-Mizan Al-Hikma" (in Farsi), *Journal of Islamic and Iranian Heritage*, 2013, 1/1:83–85. See <http://www.mirasmaktoob.ir/fa/system/files/nashriat/ME1-Qalandari-Mizan.pdf> (accessed 22 Oct. 2020).

36 See: <https://www.mpiwg-berlin.mpg.de/research/projects/book-al-khazinis-book-balance-wisdom> (accessed 22 Oct. 2020).

a known mass. Over time various types of balances were developed for different uses. These balances include Roman scales, simple mechanical scales and hydrostatic balances. The latter were quite complicated instruments, some of them employing more than two weighing pans for titrations and to measure the density of materials.

The original design for Al-Khazini's balance of wisdom was conceived by one of his teachers, Abū Hātim al-Muẓaffar Al-Isfizari (or Al-Asfizari), a mathematician from the province of Khurasan in Persia.³⁷ The balance designed by Al-Isfizari could detect whether the precious metals and gemstones in the treasury were genuine or counterfeit. Since the treasurer was officially responsible for any discrepancies, naturally he did not favour the idea of the use of such a device in his jurisdiction. The surest way to avoid this was to destroy the instrument.³⁸ Al-Isfizari was deeply upset when he discovered the fate of his balance and died a disappointed man.³⁹ Undeterred, Al-Khazini continued his teacher's work, perfecting a version of the balance which he called *Al-Mizan al-Jaamea* or *Al-mizan Al-Hikmat* (the balance of wisdom).⁴⁰ This instrument had five weighing pans rather than the usual two, and could be used for various purposes from measuring the weight and specific gravity of samples to determining the composition of alloys, changing dirhams to dinars, and more.⁴¹ The fundamental principle of the balance can be traced back to Archimedes.⁴²

8 The Reconstruction of the Balance of Wisdom

Between 1908 and 1911 H. Bauereiss and F. Keller produced a replica of the balance of wisdom that can be seen today at the Deutsches Museum in Munich

37 Al-Khazini, *Mizan Al-Hikma* (cit. note 1), and Hall, "Al-Khazini" (cit. note 5).

38 Hall, "Al-Khazini" (cit. note 5).

39 Al-Khazini, *Mizan Al-Hikma* (cit. note 1); Mohammad Shahrzoori, *Nezhat Al-Arvah Va Rozat Al-Afrah: Tarikh al hokama*, translated from Arabic to Farsi by Ali Tabrizi, 2nd ed. (Tehran: Elmi & Farhangi Publications, 2012), pp. 394–398.

40 Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

41 Hall, "Al-Khazini" (cit. note 5); Al-Khazini, *Mizan Al-Hikma* (cit. note 1); Shahrzoori, *Nezhat Al-Arvah va Rozat Al-Afrah* (cit. note 40); Ali Asghar Halabi, *The History of Iranian Philosophers since Islam* (in Farsi) (Tehran: Zavar Publications, 2012).

42 Archimedes' law states that a body completely or partially submerged in a fluid at rest is acted upon by an upward or buoyant force, the magnitude of which is equal to the weight of the fluid displaced by the body. The volume of displaced fluid is equivalent to the volume of a fully immersed object or to that fraction of the volume below the surface for an object partially submerged in a liquid.

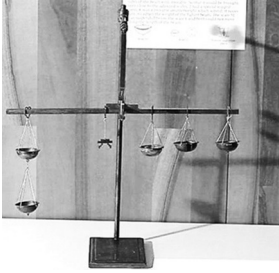


FIGURE 1
Reconstruction by Bauereiss and Keller of the balance of
wisdom
DEUTSCHES MUSEUM, MUNICH, GERMANY

(Fig. 1). However, they did not publish an account of their work and no other studies can be found in the literature on Al-Khazini's balance, nor are there any traces of other reconstructions of the instrument.

The project described here consisted of the reconstruction of a balance of wisdom based on the instructions laid out by Al-Khazini. Its mode of operation was tested and modifications were introduced to improve its precision. Mathematical relationships were developed to aid in the calibration of the balance and broaden its scope of operation.

8.1 *The Structure of the Balance*

Al-Khazini's balance (Fig. 1) consists of a base on which a vertical support is mounted, a square frame fixed at the top of the support, a horizontal beam which Al-Khazini called the *Ammood* from which up to five weighing pans can be suspended, and an open trapezoidal pendant.

In accordance with Archimedes' principle, the beam is divided into two arms of equal length. A fulcrum indicating when the balance is in equilibrium is mounted on a transverse bar fixed to the centre of the beam and is surrounded by an open trapezoidal pendant.⁴³ The bar and the fulcrum are mounted on the top of the horizontal beam. The beam is suspended from the trapezoidal pendant via transverse bar by eight pieces of parallel vertical strings. The trapezoidal pendant itself is also suspended from the square frame, so a double or dual suspension is achieved for higher precision.⁴⁴ A small solid triangular equilibrium pointer; with its vertex pointing downwards is fixed at the middle of the top horizontal side of the trapezoidal pendant showing the state of equilibrium when meeting the vertex of the fulcrum. Suspended from one arm of the beam is the gauge pan or 'first air pan', as Al-Khazini denominated it (in some manuscripts it is referred to as the 'stone pan' because it held the

43 Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

44 Ibid.

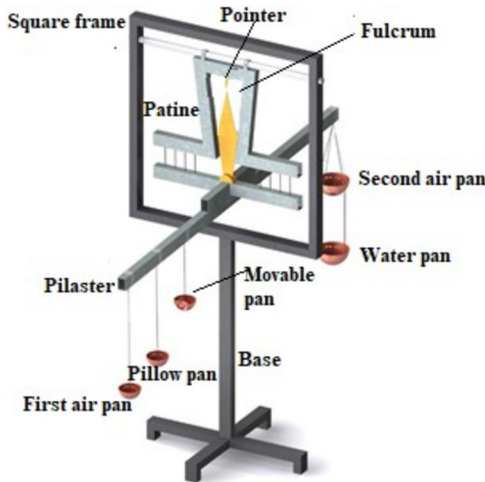


FIGURE 2
A digital image of our reconstruction
of Al-Khazini's balance of wisdom

counterweights), for the standard weighing of objects. Along the same arm two other pans hang, one called the *Al-Mojjanaheh* or 'pillow pan' and the other the 'movable pan'. On the opposite arm of the beam is 'the second air pan', beneath which hangs the water pan or 'the determining pan', which can be filled with water and used to measure the relative proportions of the metals in a binary alloy (Figs. 2 and 3).

8.2 *The Operation of the Balance*

8.2.1 Measuring Mass

Under conditions of equilibrium the beam stands perfectly horizontal, with the fulcrum and the equilibrium pointer vertices meeting. To measure the mass of an object, it is placed in 'the first air pan' and scale weights are gradually added to the gauge pan until a perfect balance is restored; the total mass of the scale weights is equivalent to that of the object.

8.2.2 Measuring Density⁴⁵

After measuring the mass in the air (according to step 1), the object is transferred into the water pan. According to the Archimedes principle and due to the fact that the pan is submerged in water and the density of water is one gram per cubic centimeter, the difference between the mass of the object in the water and in the air, divided by one, indicates the volume of water. Considering

45 All of the density measurements were made following the method described by Al-Khazini in *Al-Mizan Al-Hikma*.

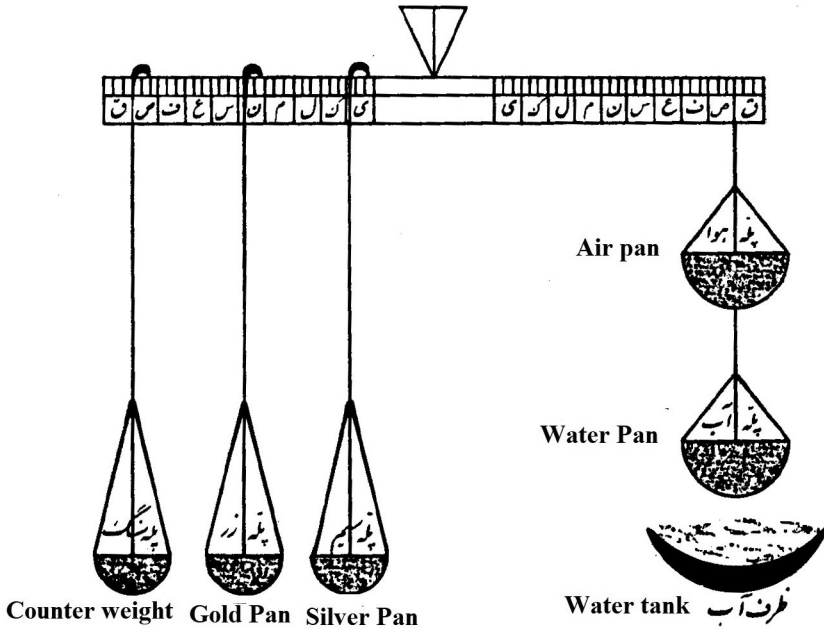


FIGURE 3 Hand drawing of the engraved main beam
 FROM AL-KHAZINI, *MIZAN AL-HIKMA* (CIT. NOTE 1), P. 124

the fact that the value for the volume of the displaced water and the object are the same, dividing the mass of the object by this value gives the density of the object.

$$\rho = \frac{m'}{v'}$$

8.2.3 Determining the Purity of the Elements in an Alloy by Titration⁴⁶
 To measure the relative proportions of gold and silver in a binary alloy, the balance must first be carefully calibrated. Al-Khazini's instructions explain that the five pans should be hung along the two arms of the beam in a certain order, and a precision pendant called a *rammaneh* is placed on the lighter side to create a perfect balance.

A known amount of pure gold is placed in the air pan and its mass is measured by adding scale stones to the scale pan. The gold is then removed from the air pan and placed in the water pan, where according to Archimedes' law

46 The titrations were carried out following the procedure described in *Al-Mizan Al-Hikma*.

the weight of the submerged gold decreases by an amount equivalent to the weight of the displaced water. Therefore, the weight on the side with the two movable pans becomes lighter. To restore the equilibrium, the scale stones in the scale pan are removed and placed in the gold pan, which is then moved along the beam to the point where balance is restored; this marks the index for gold.

To determine the indices for other elements one simply needs to repeat this process. To ensure their accuracy, Al-Khazini recommends that one repeat the calibration procedure more than once and record the average values. These indices were then written or engraved in alphabetical order on the beam of the balance (Figs. 2 and 3).

After the scale has been calibrated, the two movable pans are placed at the points along the beam that correspond to the indices of the two elements contained in the alloy.⁴⁷ To assess their relative proportions, after weighing the object in air it is placed in the water pan. The scale stones in the air pan are then distributed between the two movable pans until a perfect balance is achieved.

The mass of the stones in each of the two movable pans indicate the respective masses of the constituent elements in the alloy. In the case of an object composed of a single element, equilibrium is achieved only if all the scale stones are placed in one of the sub-pans.

8.3 *Improving the Performance of the Balance*

Al-Khazini's balance was not only capable of making different kinds of measurements; it was also vastly superior to all the other scales of the period in terms of its precision. According to contemporary sources the accuracy of the balance of wisdom was about 168 Mithqals per 1000.⁴⁸ This ratio will be referred to as the balance's "primary precision" hereafter.

Actually, the sensitivity of the balance of wisdom could be attributed to the length of its beam and its dual suspension mechanism. Therefore, the beam length and joining yarns should be considered as high as possible.

As a well-known experimental fact, repetition of any test process and taking the average of the results could increase the precision of the final result. So it is suggested to repeat each process several times to obtain better results (as suggested by Al-Khazini).

Also unsuspected factors such as air cavities, however small, in a sample could lead to measurement errors, especially if it were being used to calibrate

47 Al-Khazini, *Mizan Al-Hikma* (cit. note 1).

48 Mithqal is a unit of mass used in Muslim countries. One Mithqal is equivalent to 4.6875 grams.

the instrument. Therefore, although we followed Al-Khazini's procedure for measuring density, including his advice to repeat the measurement several times, we also checked the values obtained using a mathematical procedure devised by us.⁴⁹

- 49 The author has devised a method for calibrating Al-Khazini's balance in which the position of the index on the beam for any element can be calculated based on its density. Using this method, the precision of Al-Khazini's balance could be evaluated. Comparison of the values obtained using the mathematical procedure and the measurements made using Al-Khazini's instrument showed acceptable precision for the latter and therefore they were used for the calibrations and to determine the percentage of the elements in binary alloys. Al-Khazini's method requires the following data:

w : mass of the object in the air pan

w' : mass of the object in the water pan

d' : distance of the element's index from the midpoint of the beam

l : the beam's midpoint

$$\frac{w'}{w} = \frac{d'}{l} \quad (1)$$

$$w - v = w' \quad (2)$$

$$\text{Specific mass} = \frac{m'}{v'}$$

The index for a given element can be calculated by taking its mass and applying the specific mass ratio to determine its volume. The mass of the object in water can then be obtained using equation (2). Using equation 1 the location of the index on the beam can be determined. Example: The calculation for 200gr of the element gold is presented here:

$$V = \frac{200}{19.3} = 10.37$$

$$w' = 200 - 10.37 = 189.63$$

$$\frac{189.63}{200} = \frac{d'}{100}$$

$$d' = 95cm$$

By repeating this process for other elements, the balance can be calibrated and used for titration measurements. The specific gravity and index for various elements are presented in Table 3. Rewriting equation (1) using the mass of the object in air and the actual length of the beam:

$$\frac{w - w'}{w} = \frac{l - d'}{l} \quad (3)$$

$$\rho = \frac{w}{v} = \frac{w}{w - w'} \quad (4)$$

9 Building the Balance

A square frame was welded to a stable base and a horizontal rod was attached close to the top of this frame. The fulcrum and the beam assembly, which are connected to each other, are hanged in the form of a double suspension to the rod. (Fig. 3). The air and water pans hang in dual suspension from the right arm, while the two movable pans and the scale pan are suspended individually from the left arm.

To prevent the moveable pans from interfering with the movement of the scale pan during titration measurements, they hang from cords of different length and their joints are connected one above another (Fig. 4).

10 Testing the Instrument

Mass and density measurements were carried out as described above and the results are presented in Table 1. To assess the precision of the balance, the percentage discrepancy between the values obtained using Al-Khazini's balance and the actual volume and density of the elements are given in terms of (E_2) and (E_1) respectively.

Here the volume of the target object (v') is obtained by means of the following equation:

$$v' (cm^3) = (m' - m''(gr)) / 1(gr/cm^3) [\text{density of water}]$$

Using equation (5); obtained from equation (1); d' can be calculated for various elements without requiring any measurements.

$$\rho = \frac{l}{l - d'} \quad (5)$$

Comparing two different elements, the following relationship (6) holds.

$$\frac{\rho_1}{\rho_2} = \frac{l - d'_2}{l - d'_1} \quad (6)$$

Equation (6) represents the relationship between the densities of two different elements and their distances from the midpoint of the beam.

TABLE 1 Mass and density measurements made using the balance of wisdom

Variable	d (mm)	m'' (gr)	m' (gr)	m (gr)	E_2 (%)	v (cm ³)	v' (cm ³)	E_2 (%)	ρ (gr/cm ³)	ρ' (gr/cm ³)
Alloy										
Iron	18	108.8	125.1	125.6	0.40	16.02	16.30	1.30	7.80	7.70
Aluminium	55.1	127.9	199.20	199.1	0.05	71.49	71.30	0.00	2.70	2.70
Copper	11.1	34.7	39.300	36.6	1.800	4.565	4.600	1.100	8.400	8.540
Brass	50.2	217.4	246.1	248	0	29.77	28.70	2.00	8.33	8.50

TABLE 2 Measurements for samples of iron, aluminium and a binary alloy using the balance of wisdom

Quantity Ingredient	E (%)	T (gr)	m (gr)
Iron	4.0	131.1	125.6
Aluminium	2.8	193.6	199.1
Alloy	0.0	324.7	324.7

The next stage is the titration process, which begins with the calibration of the left arm of the beam following Al-Khazini's procedure. The accuracy of the calibration was checked using the mathematical method developed by us (footnote 49). Pure iron, pure aluminium and an iron-aluminium alloy were measured as described in Section 3 and the results are recorded in Table 2. As can be seen, the measurement error never exceeded 4%.

In addition, density measurements were performed on different substances. Table 3 presents the results of these tests, with the values obtained using the balance of wisdom alongside the actual values and the discrepancies between the two expressed as percentages. Here E is the percentage of discrepancy between the actual and measured values for the density of the object.

Figure 4 shows a digital image and a photograph of the balance of wisdom that was constructed by us at IROST.

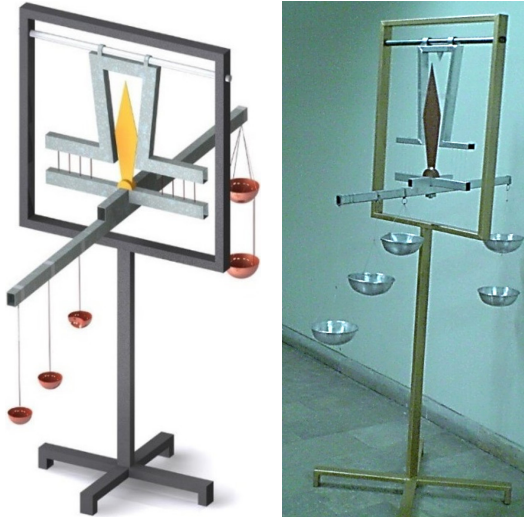


FIGURE 4
A digital image of the balance of wisdom (left) and the actual model manufactured at IROST (right)

TABLE 3 Results of further measurements using the balance of wisdom

Element Ingredient	Lead	Aluminium	Copper	Iron
γ (kg/m^3)	11400	2700	8920	7860
γ' (kg/m^3)	11370	2794	8543	7675
E (%)	0.263	3.481	4.226	2.354
d' (cm)	91	62	88	87

11 Results

Here we present the results of tests using a prototype of Al-Khazini's balance of wisdom.

11.1 Measurement Results

In this study, based on the information contained in Al-Khazini's book and other sources, a working model of the balance of wisdom was constructed. Tests carried out by us indicate that the balance works in a precise manner

and in accordance with the designer's claims. Repeated weighing and titrations showed an accuracy equal to 0.7% for weight measurements, 1.1% for density measurements, and 3.4% for the determination of the purity of elements and alloys.

11.2 *The Introduction of an Innovation to the Calibration Procedure*

The devising of a mathematical method to determine the location of the index for various elements on the balance arm was one of the most important achievements of our project, because it makes it possible for this medieval device to be calibrated for different elements in a simple and precise manner that represents an improvement on the procedure described by its inventor. Our method is based on the mathematical relationship between the mass and specific mass of the object being measured [(1) and (2) in footnote 49].

To determine the calibration index for an element Al-Khazini recommended that one measure the sample several times and calculate the average value in order to minimize the error in setting the position of the index on the balance arm. Using equation 5 [footnote 49] can facilitate the process and increase the precision of the determination. With Al-Khazini's method, the slightest error in measuring the mass or volume of the calibration sample will result in an inaccurate index. Our method eliminates this source of error as it utilizes the standard values for the density of elements and mathematical calculations to determine the position of the indices.

Al-Khazini's balance was designed to determine the degree of purity of the gold in a sample by measuring the proportion of silver present, but using our method the density of any pure substance can be determined. Using this value and its associated index on the balance arm, the element of which a sample is composed can be identified.

In addition, with this device all binary alloys whose indices have been marked on the balance can be evaluated for the type and percentage of their constituent elements, if one knows the identity of at least one of the two elements. In this case the identity and percentage of the other element can be determined.

12 The Superiority of the Balance of Wisdom

Another important result of this research was that it allowed us to demonstrate the superiority of the balance of wisdom to other medieval scales. Al-Khazini's balance can be calibrated not only for gold and silver, but also for any other element, and the index of each element can be positioned on the balance arm without interfering with other indices.

13 Other Investigations

In addition, experiments were conducted to investigate whether there was a direct relationship between the density of an element and the distance of its index on the balance arm from the fulcrum. Contrary to our expectations, no direct relationship was observed.

Symbols

d	cylindrical diameter of the object (mm)
ρ	real density (kg/cm ³)
d'	distance of the element being analyzed from the midpoint of the beam (cm)
ρ'	density measured by the balance (kg/cm ³)
T	titration of the degree of purity of the binary alloy (gr)
v	real volume of the object (cm ³)
v'	the difference in mass of an object measured first in the air pan and then in the water pan. The volume of water displaced is equivalent to the volume of the object (cm ³)
γ	real specific mass of the element (kg/m ³)
γ'	calculated specific mass of the element (kg/m ³)
E	error (%)
E'	average error (%)
E_1 and E_2	percentage of discrepancy between the actual and measured values for the density and volume of the object, respectively
m	real mass of the object (gr)
m'	mass of the object measured using the air pan of Al-Khazini's balance (gr)
m''	mass of the object measured using the water pan of Al-Khazini's balance (gr)