A Survey of European Astronomical Tables in the Late Middle Ages

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A Survey of European Astronomical Tables in the Late Middle Ages

By

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PREFACE

The compilation of astronomical tables throughout the Middle Ages was a major and dynamic intellectual enterprise. These tables respond to a wide variety of astronomical problems and computational needs, and contain a large number of ingenious solutions proposed by astronomers over the centuries. In the absence of algebraic notation and mathematical graphing techniques, a table was often the best way to transmit precise information to the reader. Indeed, an astronomical table is not a just a list of data, but a structured way to present numerical information of astronomical interest. An astronomical table is characterized by at least three features. Underlying a table is usually what we now call an algorithm, whether a function (i.e., assigning exactly one numerical value to each argument in a certain numerical range) or a combination of functions, although it is not always easy to determine their modern algebraic form. Often a table replaces a long sequence of computations based on a geometrical model, together with one or several parameters. The third characteristic of an astronomical table is its layout, and it is often the case that tables display the same basic data in different formats, probably because different computational approaches were used for the same purpose. For example, this is the case for the tables in the Alfonsine corpus for the computation of mean motions: some follow the layout used in the Toledan Tables (see Table 5.1A for computing a mean motion by groups of years, months, days, etc.) whereas others follow a purely sexagesimal layout (see Table 5.1C for computing a mean motion for a given sexagesimal number of days since the radix). These three basic features (purpose, underlying parameters, and format) gave rise to hundreds of different tables during the Middle Ages. Cracking a table, that is, bringing to light the parameters on which it is based and discovering the algorithm which enables one to recompute its entries, is usually a difficult task. Sometimes it is also a rewarding undertaking, and it is always a way to gain insight into the methods and cleverness of the authors. But, above all, the analysis of tables has proved to be an important tool for understanding the transmission of ideas and computational techniques during that period.

This survey is a first attempt to classify and illustrate the numerous astronomical tables compiled from about the 10th century to the early 16th century in the Latin West. The boundaries in time and space are left somewhat vague, but we emphasize that the Iberian Peninsula was generally the locus from which astronomical knowledge flowed to other parts of Europe, particularly in the first centuries considered here, due to the dependence on Muslim astronomy. Thus we discuss tables in many Western languages, including Hebrew, but not those in Arabic or Greek, to avoid overlapping with similar surveys of tables, e.g., Kennedy 1956a, and Samsó, King, and Goldstein 2001. As will be indicated in the appropriate places, a set of tables composed in one language was sometimes translated into other languages in the Middle Ages; indeed, the original version is not always extant. Many of the tables in this survey have been studied by the authors of this monograph, sometimes in joint publications: Tables of Abraham Bar Ḥiyya (Hebrew); Alfonsine Tables; Tables of John of Murs; *Tabule permanentes* of John of Murs and Firmin of Beauval; Tables of John Vimond; Tables of Levi ben Gerson (Hebrew and Provençal); Tables of Jacob ben David Bonjorn (Hebrew, Latin, Catalan, and Greek); Tables of Barcelona (Catalan, Hebrew, and Latin); Tables of Isaac Ibn al-Ḥadib (Hebrew); Tables of Prosdocimo de' Beldomandi; Tables of Giovanni Bianchini; Tables of Flavius Mithridates; Abraham Zacut's *ha-Ḥibbur ha-gadol* (Hebrew and Latin); *Almanach Perpetuum*; Tables of Judah ben Verga (Hebrew); *Tabulae Resolutae*; Tables of Queen Isabella; etc. We have also taken advantage of some previous studies carried out by other authors on various sets of tables, e.g., the Toledan Tables, the Oxford Tables, and *The Six Wings* of Immanuel ben Jacob Bonfils of Tarascon (originally composed in Hebrew, with translations into Latin and Greek), as well as on specific tables within such sets. The examples reproduced here come to about 160 tables reproduced in full or in part, and are taken from complete sets of tables or from isolated tables found mostly in manuscripts, but occasionally in printed editions. The tables reviewed in this survey are those widely used by astronomers and practitioners in the period under consideration and tables closely related to them. We have omitted a few isolated tables, known to us in only a single manuscript, which do not seem to be characteristic of mainstream usages, or are trivial by medieval standards (for discussion of this matter, see ch. 19). We have also excluded other genres, which present material in tabular form and contain astronomical information, such as calendars and ephemerides. In sum, the aim of this survey is neither to be exhaustive nor to edit tables, but to give a framework, with respect both to the approach and classification, for future studies of medieval astronomical tables, a subject that certainly merits further attention. We emphasize that in this study we present a survey of these astronomical tables, not a history of them. Although our main goal is to summarize and organize the results of previous research, we have addressed many items not considered in earlier publications; in some cases we have included new findings: tables analyzed for the first time and new explanations of "old" tables. Specifically, we would like to highlight Zacut's table for the cusps of the astrological houses (Table 18.1B), which has impressed us for its user-friendliness, and a peculiar table for daily lunar progress (Table 18.5B).

We thank Fritz S. Pedersen and an anonymous referee whose insightful comments have been very helpful; indeed, they served as a guide for making final revisions of the draft of this work. We are most grateful to the libraries where we have consulted the manuscripts and printed books cited in this work as well as the libraries that have provided us with copies of books, articles, and manuscripts; the list is too long to thank all these libraries individually. We also thank our respective universities for the use of interlibrary loan.

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INTRODUCTION

What we mean by mathematical astronomy is fixed by the contents of the *Almagest*, a Greek treatise by Ptolemy (Alexandria, 2nd century), in which he presented his methods in a way that allowed his results to be modified based on new observational data. He also established models and parameters for the motions of the luminaries and the planets, and constructed tables describing these motions (for a modern translation of the *Almagest*, see Toomer 1984; see also O. Pedersen and Jones 2010). In this comprehensive treatise Ptolemy discussed plane and spherical trigonometry and described astronomical instruments, models for the Sun, Moon, and the planets (both in longitude and latitude), lunar parallax, criteria for solar and lunar eclipses, and a star catalogue. He included several tables for trigonometric functions, mean motion tables for the Sun, the Moon, and the planets, tables for correcting mean positions of celestial bodies to their true positions, tables for planetary latitude, as well as tables for determining the circumstances of solar and lunar eclipses. Ptolemy also explained how the entries in these tables could be computed using trigonometry. These tables allow one to treat all problems concerning the daily rotation of the heavens (e.g., finding the time of day from the altitude of the Sun or a fixed star), and to compute a planetary position at any given time. It is noteworthy that the tables for finding true planetary positions require only addition, subtraction, and multiplication, without appealing to trigonometric functions that would be required for determining them directly from the models. One of the keys to Ptolemy's success was to include columns for interpolation with instructions for using them to find values between extremal values in other columns, thus reducing the number of entries in the table and simplifying the rules for computing a position from them. In other words, Ptolemy sought to make his tables "user-friendly."

In his later work, the *Handy Tables*, Ptolemy presented a full set of astronomical tables to be used for solving the major problems of mathematical astronomy, built on his previous work: for the *Handy Tables*, see Stahlman 1959, Tihon 2011, and Mercier 2011. Some time after the *Almagest* and the *Handy Tables*, Ptolemy wrote the *Planetary Hypotheses* where he introduced several changes in his previous models and parameters, and added a physical approach to the geometrical models presented in the *Almagest* (see Neugebauer 1975, pp. 900–913). In the 4th century, Theon of Alexandria wrote important commentaries on Ptolemy's works in Greek, remaining strictly within the framework of Ptolemy's astronomy: see, e.g., Mogenet and Tihon 1985, Tihon 1991, and Tihon 1999.

In order to clarify the terminology used here, it should be stressed that the expression "set of tables," frequently appearing in this text, refers to a consistent collection of astronomical tables, as well as some lists (e.g., geographical places), embracing all or some aspects of mathematical astronomy and usually accompanied by a text, here called "canons," explaining their use. Most sets of tables compiled in Western Europe in the Middle Ages followed the structure of Arabic sets of tables, included in handbooks called zijes (from the Arabic *zīj*, plural *zījāt*): see Samsó, King, and Goldstein 2001, where a zij, as Kennedy put it, is "a document which contains the numerical tables sufficient to enable an astrologer, or astronomer, to solve the standard problems of his profession" (Kennedy *et al.* 2009–2010, p. 81).

Ptolemy's works had an enormous influence on scholars during the period considered here, especially his masterpiece, the *Almagest*. It was translated from Greek into Arabic several times in the 9th century in what is now called Iraq, and two versions survive: (1) the version by al-Ḥajjāj, *ca*. 827, at the behest of the Caliph, al-Ma'mūn (d. 833), and (2) the version by Isḥāq b. Ḥunayn as revised by Thābit Ibn Qurra, *ca*. 892 (Morelon 1996, p. 22). Ptolemy's treatise was widely known in the Islamic world, both in the East and in the West (i.e., al-Andalus [Muslim Spain] and the Maghreb [North Africa]), and was the point of departure for subsequent developments in mathematical astronomy. Theon's commentaries were also available in Arabic translation in the 9th century, but the Arabic versions have not survived (Morelon 1996, p. 23). For our purposes the most important example of this dissemination in the East is the zij of al-Battānī (d. 929), produced in Raqqa (a town on the Euphrates now in northern Syria), because it was also very influential in al-Andalus and the Maghreb (Nallino 1903–1907; for a recent assessment of Nallino's edition, see van Dalen and F. S. Pedersen 2008). While maintaining a strict adherence to Ptolemy's works, al-Battānī introduced new values for a few parameters based on new observations in the 9th century. Another eastern zij, associated with Baghdad and the Caliph, al-Ma'mūn, is the Mumtaḥan zij by Yaḥyā ibn Abī Manṣūr (d. 830), a version of which is extant in Escorial, MS Ar. 927 (Kennedy 1956a, pp. 132, 145–147; Vernet 1956;

Sayılı 1960, pp. 56–61; Kennedy and Faris 1970; Kennedy 1977a). A second version of the Mumtaḥan zij has recently been described (van Dalen 2004). The Mumtaḥan zij (meaning "verified tables") was the result of the observations of al-Ma'mūn's astronomers and remains globally within the Ptolemaic framework with some Indian influence. It too was available in al-Andalus, and in Latin it was called *Tabulae probatae*.

In the late 8th century, before the translations of Greek astronomical works into Arabic, some Arabic astronomical texts were produced in Iraq in the Indian (or Hindu) tradition, mainly by Yaʿqūb ibn Ṭāriq and al-Fazārī, but these texts only survive in fragmentary form (Pingree 1968 and 1970; Kennedy 1968). The earliest complete set of tables in this tradition is the *Zīj al-Sindhind* by al-Khwārizmī (*fl.* 830), but the original version does not survive. Rather, we have a Latin version produced in Christian Spain by Adelard of Bath (*fl.* 1116–1149) who in turn depended on a recension (no longer extant) made in Muslim Spain by Maslama b. Aḥmad al-Majrīṭī (d. 1007): for the tables, see Suter 1914, and for an analysis of them, see Neugebauer 1962a; see also van Dalen 1996. In this survey citations of the zij of al-Khwārizmī refer to this extant Latin version, unless otherwise indicated. The Indian tradition was stronger in the West than it was in the East, where it was largely displaced by Greek astronomical procedures (Morelon 1996, p. 21; see also Pingree 1976; and Samsó 1992, pp. 81–93). In al-Andalus the Indian tradition had a significant influence on Ibn Muʿādh of Jaén (d. 1093), the author of a zij which has not survived (but see Hermelink 1964, and Samsó 1992, pp. 152–166), and on Ibn al-Kammād (*ca.* 1100), active in Córdoba and the author of two zijes as well as a compilation of them, called *al-Muqtabis* (Chabás and Goldstein 1994). Only a single chapter of the canons *al-Muqtabis* is extant in the original Arabic (Alger, MS 1454,2, ff. 62–63), although the tables are preserved in two versions: one in Latin by John of Dumpno (1260, Sicily) in Madrid, Biblioteca Nacional, MS 10023; and the other in Hebrew by Solomon Franco (14th century, Spain) in Vatican, MS Heb. 498 (Goldstein 2005, p. 16; Goldstein 2011).

In al-Andalus the Greek tradition flourished and co-existed with the Indian tradition. Shortly before the Christian conquest of Toledo in 1085, Ṣāʿid al-Andalusī and his group of scholars, which included the outstanding astronomer, Ibn al-Zarqālluh (Azarquiel), compiled a zij, mainly based on the Ptolemaic framework as well as on some new observations, known as the Toledan Tables (Toomer 1968, Richter-Bernburg 1987, and F. S. Pedersen 2002). The Toledan Tables reflect both of these traditions: the Greek, through the zij of al-Battānī; and the Indian, through the zij of al-Khwārizmī. Azarquiel was also the author of an extant *Almanac*, possibly based on an ancient tradition going back to Ammonius (Millás 1943–1950). The Toledan Tables were arranged for the Hijra calendar and originally written in Arabic; this version is not extant, whereas there are numerous copies in Latin. They were widely used both to the north and to the south of the frontier between Christian and Muslim kingdoms and, in the 12th century, they were adapted to the Christian calendar, giving rise to sets of tables for various cities, such as Novara, Toulouse, and Marseilles. Through these translations and adaptations, the Toledan Tables became the standard computational tool for mathematical astronomy in Europe for more than two centuries.

Abraham Bar Ḥiyya of Barcelona (d. after 1145) was the first to compose a zij in Hebrew, and it is largely based on the zij of al-Battānī. A list of the tables displayed in it appears in Millás 1959 which also includes transcriptions of a few of them. For the remaining tables, it is necessary to consult manuscript copies of which there are many (e.g., Paris, Bibliothèque nationale de France, MS Heb. 1046). Another member of the Jewish intellectual elite in Spain was Abraham Ibn Ezra (d. 1167); he wrote extensively in Hebrew on astronomy and astrology (see Goldstein 1996, Sela and Freudenthal 2006). Ibn Ezra refers to tables he compiled for Pisa in a Latin text edited by Millás (1947, pp. 87–88) and, according to Mercier (1987, pp. 108–112), they are extant in a few Latin manuscripts (see also Sela 2003, pp. 22–27). Some copies of Bar Ḥiyya's tables indicate that Ibn Ezra redacted a version of them. Ibn Ezra also translated into Hebrew the commentary on the zij of al-Khwārizmī by Ibn al-Muthannā (10th century) of which the original Arabic is not extant (Goldstein 1967; for an edition of the Latin version of this commentary, see Millás Vendrell 1963).

In al-Andalus and the Maghreb zijes in Arabic were compiled by many astronomers in the tradition of Azarquiel: Ibn al-Kammād (mentioned above), Ibn al-Hā'im of Seville (*ca.* 1205), Ibn Isḥāq al-Tūnisī (beginning of the 13th century), Ibn al-Raqqām of Granada (d. 1315), Ibn al-Bannā' of Marrākesh (d. 1321), and Ibn ʿAzzūz al-Qusanṭīnī (d. 1354): on Ibn al-Hā'im, see Samsó 1992, pp. 320–326, and Calvo 1998; on Ibn Isḥāq, see Comes 1996, and Mestres 1996 and 1999; on Ibn al-Raqqām, see Kennedy 1997; on Ibn al-Bannā', see Vernet 1952, and Samsó and Millás 1994 and 1998; on Ibn ʿAzzūz, see Samsó 1997.

The Castilian Alfonsine tables were compiled in Toledo in about 1272, under the patronage of Alfonso X, king of Castile and León (reigned: 1252–1284), by Isaac ben Sid and Judah ben Moses ha-Cohen. This set of tables, of which only the original canons in Castilian have survived, was heir to a long tradition of table-making that had flourished in al-Andalus in the preceding centuries and superseded the Toledan Tables as well as their Latin variants. The Alfonsine Tables of Toledo began to circulate throughout Europe, mainly in the version produced in Paris, beginning about 1320 (Chabás and Goldstein 2003).

At the turn of the 14th century, Paris hosted a group of remarkable astronomers who produced theoretical treatises as well as tables and almanacs. This group included John of Sicily, Peter of Dacia, William of Saint-Cloud, and Peter of St. Omer, all of whom based their work on Latin versions of the Toledan Tables (for John and the two Peters, see F. S. Pedersen 1983–84 and 1986). However, in the 1320s several astronomers began to use the tables compiled in Toledo under the patronage of Alfonso X, and recast the Castilian Alfonsine Tables in the form known as the Parisian Alfonsine Tables. Outstanding among these astronomers were John Vimond, John of Murs, John of Lignères, and John of Saxony (see Poulle 1973a, 1973b, 1973c, and 2005; Saby 1987; Chabás and Goldstein 2003, 2004, and 2009a). From then on, the center stage in the field of European astronomical tables was occupied by the Alfonsine corpus, that is, a series of adaptations, modifications, and variants of the Parisian Alfonsine Tables which are found throughout most of Europe in a vast number of manuscripts, as well as in several printed editions, from the early 14th century to the middle of the 16th century. Unfortunately, no systematic survey of this material has yet been carried out. Just to cite the names of a few astronomers who produced tables in this tradition before 1440, we may mention William Batecombe in England (see North 1977), Nicholaus de Heybech in Germany (see Chabás and Goldstein 1992), John of Gmunden in Austria (see Porres 2003), and Prosdocimo de' Beldomandi in Italy (see Chabás 2007). It is worth noting that the *Tabule Anglicane* (often called the Oxford Tables), associated with Batecombe (*fl.* 1348), introduced rather elaborate tables including double argument planetary tables. Moreover, Heybech addressed and solved in a very user-friendly way the intricate problem of determining the time from mean to true syzygies (i.e., conjunctions and oppositions of the Sun and the Moon). Two Hebrew versions of Batecombe's tables are also extant, one of which is by Mordecai Finzi of Mantua (*fl.* 1440–1475): Goldstein 1987, pp. 120, 138 n. 18; Langermann 1988, p. 26; Chabás and Goldstein 2000, p. 22. We have identified a Latin version of Heybech's table produced in Salamanca in the 15th century as well as a subsequent Hebrew version by Abraham Zacut, which he included in his tables of 1513 (Goldstein and Chabás 2008).

There was another tradition, independent of the Alfonsine Tables, which flourished among Jewish scholars, especially in southern France. Levi ben Gerson of Orange, France (d. 1344), was the most original astronomer to write in Hebrew in the Middle Ages and his *Astronomy* was translated into Latin in his lifetime. He invented new instruments and produced new planetary models, and made more observations by far than any of his contemporaries (Jewish, Christian, or Muslim): see Goldstein 1979a, 1985, and 1988; and Mancha 1997. His astronomical tables have been published and, in particular, his lunar tables are based on an entirely new model intended to reproduce Levi's own observations rather than those of Ptolemy (Goldstein 1974). For the Provençal version of Levi's eclipse tables, see Mancha 1998. Levi is the first medieval astronomer known to have produced lunar tables that did not exaggerate the variation in lunar distance from the Earth which is a consequence of Ptolemy's lunar model (Goldstein 1997). Immanuel Bonfils of Tarascon, France (*fl.* 1350) is best known for his set of tables in Hebrew, *Six Wings*, of which many copies are preserved in the original as well as in Greek translation (one copy in Latin is also extant). A description of the tables appears in Solon 1970; the Hebrew version was printed in Zhitomir in 1872, and the Greek version was edited (but not published) by Solon in 1968. Although there are no planetary tables in *Six Wings*, Bonfils produced another set of tables which included them, based on the zij of al-Battānī. This set has not been published but is preserved in at least three manuscripts: Munich, Staatsbibliothek, MSS Heb. 343 and 386; and New York, Jewish Theological Seminary of America, MS Heb. 2597 (see Goldstein 1999, p. 226 n. 4). Jacob ben David Bonjorn (also called ha-Poʿel, i.e., the [table-]maker), a follower of Levi, composed tables for syzygies and eclipses for Perpignan beginning in 1361,which are extant in Hebrew, Latin, Catalan, and Greek versions (Chabás 1992, pp. 152–165; Tihon and Mercier 1998, p. 11). These tables were much appreciated in the 14th and 15th centuries, for we have identified 60 copies of them, of which 43 are in Hebrew (Chabás 1992, pp. 151–172). They are based on a cycle introduced by Bonjorn, comprising a non-integer number of synodic months (383.5), corresponding to 767 consecutive syzygies, lasting about 11,325 days and thus almost equal to 31 Julian years and 2 days, with years beginning March 1 (Chabás 1991).

In the 14th century the Indian tradition in the Iberian Peninsula, partially transmitted by Ibn al-Kammād, was still alive, and its echoes are present in various tables in Hebrew script, namely, those by Abraham Ibn Waqār, Solomon Franco, and Juan Gil, none of which has yet been edited. The Tables of Barcelona, compiled by Jacob Corsuno of Seville at the behest of the king of Aragon, Pere el Cerimoniós (1319–1387), were completed in Barcelona around 1381 and are extant in Catalan, Hebrew, and Latin; they depend heavily on Ibn al-Kammād as well as on Azarquiel (Millás 1962; Chabás 1996a). Judah ben Asher II of Burgos (d. 1391), the great grandson of the famous chief rabbi of Toledo, Asher ben Yeḥiel (d. *ca*. 1328), was the author of a set of tables in Hebrew, dated 1364, which are original in their presentation. They have not yet been edited and are poorly preserved in a unique copy (Vatican, MS Heb. 384). A student of his, Isaac Ibn al-Ḥadib (d. *ca*. 1426), was active in Castile in the 1370s, but it was not until after his arrival in Sicily, no later than 1396, that he composed a set of tables called *Oraḥ selulah* ("the paved way") for conjunctions of the Sun and the Moon, and the times and circumstances of eclipses (see Goldstein 1987 and 1999). This set of tables was originally composed in Hebrew based on Ibn al-Kammād's works, and was later translated into Greek: see Tihon and Mercier 1998, p. 258. It had a certain impact in the Italian Peninsula; in fact, a Latin version of these tables was compiled by Flavius Mithridates (*fl*. 1460–1485), a Jewish convert to Christianity who served as an advisor to the renowned humanist, Pico della Mirandola (d. 1494): see Goldstein and Chabás 2006.

Standard sets of tables, as most of those cited so far, are usually composed of auxiliary tables used to compute positions of the celestial bodies for any arbitrary date. In addition to these sets, another way of facilitating astronomical computations was developed, namely, almanacs in which these positions were given directly for specific dates, not requiring further computation. The almanac tradition is well represented in the Iberian Peninsula by the Almanac of Azarquiel, cited above, and in southern France by the *Almanach Perpetuum* of Jacob ben Makhir Ibn Tibbon (d. 1304), also called Profatius Judaeus (Boffito and Melzi d'Eril 1908). In Paris some authors also made almanacs: William of Saint-Cloud, John of Lignères, and John of Saxony (Chabás and Goldstein 2003, pp. 282–289). At the very end of the

14th century Ferrand Martines of Seville compiled an almanac preceded by a text in Castilian that consisted of tables adapted for 1391 from those in the Almanac of 1307, a perpetual almanac of Arabic origin, traditionally called the "Almanac of Tortosa" and consisting of a text and tables beginning in 1307 (Chabás 1996b). The distinction between an almanac and a perpetual almanac is that the former includes positional data for a specific set of cycles of years for each planet and the Sun and the Moon, whereas the latter also provides numerical rules to use the data in a given cycle for other cycles, whether prior or subsequent.

During the 15th century there was a noticeable increase in the production of astronomical tables. Many more copies survive than from previous centuries, due in part to the fact that the number of university students had increased considerably. Moreover, new, more user-friendly, versions of earlier sets of tables were compiled, which were then widely diffused. At the same time, almanacs and ephemerides proliferated. An ephemeris contains much the same information as an almanac, but they differ in presentation. Broadly speaking, in an ephemeris we are given, on a daily basis and for a certain number of years, the positions of the planets and the Sun and the Moon, and this information is presented in a single row for a given day. On the other hand, in an almanac we find tables for each of the planets and the Sun and the Moon, and their positions are given at intervals of one day or a few days; hence, the information concerning a specific day is scattered among various tables.

However, for the most part this material was compiled within the framework of the Parisian Alfonsine Tables, which was not challenged until the middle of the 16th century. Among the authors of complete sets of tables is Abraham Zacut of Salamanca (1452–1515), the most celebrated astronomer in the Iberian Peninsula of his time. He is the author of a set of astronomical tables in Hebrew called *ha-Ḥibbur ha-gadol* (*The Great Composition*), completed in 1478. Zacut's fame rests mainly on the *Almanach Perpetuum* consisting of a set of canons by Joseph Vizinus and a great number of astronomical tables, almost all drawn from the *Ḥibbur*. The *Almanach Perpetuum* was first printed in Latin and Castilian versions in Leiria, Portugal, in 1496, probably without the intervention of Zacut himself, and was reprinted several times, not always mentioning the author's name. For the *Ḥibbur* Zacut depended on the Jewish astronomical tradition as well as on the Parisian Alfonsine Tables, brought to Spain from Poland by Nicholaus

Polonius, the first incumbent of the chair of astronomy/astrology at the University of Salamanca (Chabás and Goldstein 2000).

Various forms of presenting the Parisian Alfonsine Tables were developed in central Europe (especially Poland), the most successful being the *Tabulae Resolutae* (Dobryzcki 1987, Chabás 1998 and 2002). The most visible difference is that the *Tabulae Resolutae* maintain the system of cyclical radices with intervals of 20 years (as explained in the canons to the Castilian Alfonsine Tables), rather than the organization in days to be counted sexagesimally, as in the *editio princeps* of the Parisian Alfonsine Tables. In Italy, Giovanni Bianchini (died after 1469) constructed another set of tables, quite different from, and larger than, all previous sets, entirely based on the Parisian Alfonsine Tables; there are many manuscript copies as well as three editions of Bianchini's tables (dated 1495, 1526, and 1553). Despite the large number of copies of these tables, we have found few citations in astronomical texts of the relevant time period (Chabás and Goldstein 2009b). However, his tables were used by the leading European astronomers of his time, Georg Peurbach (1423–1461), and his disciple Johannes Müller, called Regiomontanus (1436–1476), who worked together in Vienna. These two astronomers fully understood Ptolemy's work and compiled new astronomical tables with substantial improvements, notably in trigonometry and the computation of eclipses (Zinner 1990).

During the 15th century and the early 16th century several other astronomers compiled sets of tables. Among them we may cite a set in Hebrew by Judah ben Verga (*fl.* 1470) of Lisbon (Goldstein 2001 and 2004) and another by Alfonso de Córdoba, the *Tabule Astronomice Elisabeth Regine*, which have as epoch 1474, the year of the accession of Isabella to the throne of Castile, and were printed repeatedly (Chabás 2004). Mordecai Finzi of Mantua compiled a set of tables in Hebrew, preserved in his own hand, that are based in part on the Parisian Alfonsine Tables, and in part on tables included in earlier Hebrew sets of tables (Oxford, Bodleian Library, MS Mich. 350: Goldstein 1987, p. 138; Langermann 1988, pp. 20–23). This set of tables is different from Finzi's translation of Batecombe's tables, mentioned above. In 1460, while in Avignon, Moses ben Abraham of Nîmes produced a Hebrew version of the Parisian Alfonsine Tables with the canons by John of Saxony (Steinschneider 1964, pp. 196–197; Goldstein 1999, p. 232).

The invention of movable type in the mid-15th century greatly contributed to the dissemination of astronomical tables because printers quickly recognized that this material had a ready market. Therefore, it comes as no surprise that we find tables ascribed to four authors in incunabula editions: Alfonso (King of Castile), Regiomontanus, Bianchini, and Zacut. In fact, between 1483 and 1504 there were nine editions of sets of astronomical tables of which four were adaptations of sets that had already been printed. This short list also reveals the prominence of Venetian printing houses in this domain (seven out of the nine items) and indicates that only one language (Castilian) other than Latin was used for the canons accompanying these sets of tables.

1483 *Tabule astronomice illustrissimi Alfontij regis castelle*. Venice: Erhard Ratdolt. (Parisian Alfonsine Tables)
1490 *Tabule directionum profectionumque*. Augsburg: Erhard Ratdolt. (Regiomontanus)
1492 *Tabule Astronomice Alfonsi Regis*. Venice: Johannes Lucilius Santritter. (Parisian Alfonsine Tables)
1495 *Tabulae coelestium motuum earumque canones*. Venice: Simon Bevilaqua. (Giovanni Bianchini)
1496 *Almanach Perpetuum*. Leiria: Samuel d'Ortas. (Abraham Zacut, one version with canons in Latin and another with canons in Castilian)
1498 *Almanach Perpetuum*. Venice: Johannes Lucilius Santritter. (Abraham Zacut in Latin)
1502 *Almanach Perpetuum*. Venice: Petrus Liechtenstein. (Abraham Zacut in Latin)
1503 *Tabule astronomice Elisabeth Regine*. Venice: Petrus Liechtenstein. (Alfonso de Córdoba)
1504 *Tabule directionum profectionumque*. Venice: Petrus Liechtenstein. (Regiomontanus)

A Classification of Astronomical Tables

As mentioned above, a huge number of astronomical tables were compiled in the Middle Ages and they were presented in a great many different ways. Modern authors who have dealt with specific sets (see, e.g., Kennedy 1956a, Toomer 1968, and F. S. Pedersen 2002) group them by their contents, but there is no definite consensus on the categories proposed.

We offer the following classification for such tables and we give examples of some tables, fully aware that they represent only a tiny fraction of those produced in the Middle Ages.

1. Chronology
2. Trigonometry and spherical astronomy
3. Equation of time
4. Precession and apogees
5. Mean motion and radices
6. Equations
7. True positions
8. Velocity
9. Latitudes
10. Stations and retrogradations
11. Visibility of planets and fixed stars
12. Parallax
13. Syzygies
14. Planetary conjunctions
15. Eclipses
16. Star lists
17. Geographical lists
18. Astrology
19. Miscellaneous

To illustrate tables in various categories, we selected a few of them that are reproduced here (in some instances the entire table, while in others an excerpt only), in each case taking from the table as much as is necessary to establish its character and to identify the extremal values of the entries or the parameters on which the table is based, for our aim here is not to edit tables but to give the reader the tools to identify and understand them. We follow the standard convention for sexagesimal notation, that is, an expression such as a,b;c,d,… is to be understood as $a \cdot 60 + b + c/60 + d/60^2 + \ldots$. For zodiacal signs of 30° (i.e., dividing the zodiacal circle into 12 equal arcs), we use the symbol "s"; for example, 2s = 60°. It should be noted that these zodiacal signs are not identical with the zodiacal constellations (which bear the same names), for the latter refer to groups of stars rather than to equal arcs on the ecliptic. We also distinguish zodiacal signs of 30° from physical signs of 60° (= 1,0°), which were commonly used in the Alfonsine corpus: see, e.g., Table 7.1A.

We distinguish arc-degrees (°), where 360° = 1 revolution, from time-degrees, also labeled (°), where 360° = 1 day = 24 hours. Note also that a minute of an hour, 0;1h, is labeled (min), whereas (′) stands for an arc-minute, 0;1°. Coefficients of interpolation appear in some tables in a column headed "Min. prop." (i.e., minutes of proportion), where the entries range from 0 to 60 minutes (see, e.g., Table 6.3B).

In modern terms the tables have an independent variable (here called the argument), usually in the first column (or the first two columns) and the entries in a subsequent column (or columns) are functions of that argument. In some cases the arguments for different columns in the same table are different (see, e.g., Table 6.2A). There are also double argument tables, where the table has two arguments: one argument is at the head of each column and the other argument is at the head of each row; the entry is a function of both arguments (see, e.g., Table 6.3E).

CHAPTER ONE

CHRONOLOGY

The astronomical interest in chronology is, in the first instance, to enable one to establish the number of days between two observations so that they may be used to determine the parameters of a model. Reports of observations are given with a date in some calendar, and the study of chronology is intended to allow one to find the corresponding date in the calendar preferred by the astronomer who wishes to use them. In the *Almagest* Ptolemy includes observational reports in various calendars used by observers and then converts those dates to the dates in his calendar: Egyptian years each of which contains 365 days and an epoch, "Era Nabonassar," that corresponds to Feb. 26, 747 BC. For example, in *Almagest* X.9 Ptolemy cites an observation of Mars "in the 13th year of the calendar of Dionysius, Aigon 25, at dawn…i.e., in the 476th year from Nabonassar, Athyr [month III] 20/21 in the Egyptian calendar," corresponding to Jan. 17/18, 272 BC (Toomer 1984, p. 502). However, Ptolemy does not describe any calendar and does not explain how he made these conversions. Moreover, Ptolemy decided to begin the day (daytime and nighttime together, i.e., our 24-hour day) at noon, in contrast to the Egyptians who began the day at sunrise and the Babylonians who began the day at sunset. This convention of starting the astronomical day at noon continued in the Middle Ages despite the usages in the civil calendars on which they were based. To explain the dating of observations and the conversion of dates from one calendar to another, astronomers from the 9th century on included a section on chronology in sets of astronomical tables. Each calendar was defined, giving the names of the months, their lengths, the length of the year, cycles for leap years, and the epoch (the initial day of year 1 of the calendar). In this context a calendar was intended to serve as a device for counting days so that one could easily determine the number of days between two dates. This meant that observation did not play a role in defining the calendar and that a civil calendar based on phases of the Moon (e.g., New Moon) had to be recast as a "mean" calendar which allowed for a simple counting of days from the epoch. This applies specifically to the Hijra calendar used by Muslims. In the civil Hijra calendar the day began at sunset and some months were determined by observation of the New Moon. But in the astronomical Hijra calendar the day began at noon, and the pattern of month lengths and leap years was fixed. The most extensive study of calendars in the Middle Ages is al-Bīrūnī's *Chronology of Ancient Nations* (tr. Sachau 1879).

The next issue to be considered is the procedure for converting dates from one calendar to another. The modern solution is to convert a date in one calendar into the corresponding Julian Day Number (where the day begins at noon), a simple day count, with no division into months or years, from an arbitrary epoch, and then to convert the Julian Day Number into the date in some other calendar. The medieval solution, however, was to display different tables for converting the date in one calendar to the corresponding date in another calendar, i.e., each pair of calendars was treated separately. In the course of the Middle Ages the original astronomical rationale for including a section on chronology in sets of astronomical tables was largely forgotten, but tables for chronology persisted (in some cases for astrological purposes or for establishing a fixed chronology for Biblical events). There are, however, instances where a medieval astronomer sought to revise a parameter by comparing his own observation with one taken from the *Almagest* (e.g., to determine the length of the mean synodic month, Levi ben Gerson compared a lunar eclipse that he observed in 1335 with one observed by Ptolemy in 134: see Goldstein 2003, pp. 70–71).

1. *Epochs and Intervals*

The compilers of medieval sets of tables referred to many calendars which use different epochs and specified the intervals in days between the various epochs. These differences in days, sometimes expressed in sexagesimal form, were presented in the form of a list, of which there are a great many variations (see, e.g., Suter 1914, p. 109; Ratdolt 1483, f. c8r). In Table 1.1A below, we display the names of the most frequently used epochs, together with their corresponding dates in the Julian calendar and weekdays. We indicate their Julian Day Number (JDN), based on a modern counting of days, beginning arbitrarily in year −4712; note that the Julian Day begins at noon and it has nothing to do with the Julian calendar. We also accept the convention of negative years in the Julian calendar: for instance, −4712 corresponds to 4713 BC.

Table 1.1A: Epochs

	Date	Weekday	JDN
Flood	February 17, –3101	Thursday	588465
Nabonassar	February 26, –746	Wednesday	1448638
Philippus	November 12, –323	Sunday	1603398
Seleucid or Alexander	October 1, –311	Monday	1607739
Caesar or Spanish	January 1, –37	Sunday	1707544
Christian or Incarnation	January 1, 1	Saturday	1721424
Diocletian	August 29, 284	Friday	1825030
Hijra or Arabic*	July 15, 622	Thursday	1948439
Yazdegird or Persian	June 16, 632	Tuesday	1952063
King Alfonso**	January 1, 1252	Sunday	2178351
Queen Isabella	December 23, 1474	Saturday	2259794

* For a discussion of the epoch of Hijra calendar, see Neugebauer 1962a, pp. 10–11.
** The epoch of King Alfonso used in the Parisian Alfonsine Tables is June 1, 1252 (JDN 2178503).

Table 1.1B: Weekdays of epochs

	Weekday numbers
Radix diluvij	5
Radix Nabuchodonosor	4
Radix mortis philippi	1
Radix Alexandri magni	2
Radix Caesaris	1
Radix Incarnationis iesu xpi	7
Radix Diocletiani	6
Radix Arabum	5
Radix Persarum	3
Radix Regis Alfonsi	7

Most often chronological tables displayed only the number of days between two epochs but did not specify the weekdays of the different epochs, which were sometimes addressed in separate tables. In Table 1.1B we reproduce such a table, entitled *Tabule radicum notarum anni*, taken from Florence, Biblioteca Laurenziana, MS San Marco 184, f. 18r, a 15th-century manuscript containing, among many other items, a set of Parisian Alfonsine Tables for Florence. The numerical values for the weekdays follow the standard rule according to which Sunday = 1, Monday = 2, and so on.

With the passage of time many of these epochs became obsolete and were generally abandoned, but it is interesting to note that those listed here survived as late as 1492, when the Parisian Alfonsine Tables were published for the second time.

2. *Number of Days from the Beginning of an Era*

To compute the number of days elapsed from a certain epoch, tables were constructed indicating the number of days in groups of years (collected years), single years (expanded years), and months, for each of the calendars involved. The number of years in a group of years varied according to the calendar used. Table 1.2A displays an excerpt of the table for counting days in the Hijra (or Arabic) calendar found in the *editio princeps* of the Parisian Alfonsine Tables (Ratdolt 1483, f. c3v), with sub-tables for collected years, expanded years, and months. Note that it is the same table as in the Toledan Tables (Toomer 1968, p. 15; F. S. Pedersen 2002, p. 886). One slight difference is that here, in the subtable for expanded years, there is an added column, labeled "m". In the medieval astronomical literature similar tables are found for many different calendars.

The Hijra calendar consists of 12 lunar months of 29 or 30 days. In an ordinary year there are 354 days, and in a leap year there are 355 days. The cycle of 30 Hijra years has 19 ordinary years and 11 leap years: in the central sub-table for expanded years in the Hijra calendar a leap year is preceded by the letter "b". In the headings for the sub-tables in Table 1.2A, the symbols "4," "3," "2," and "p" (meaning "1") refer, in modern terms, to 1 less than the power of 60; hence,

Table 1.2A: Number of days in the Hijra calendar (excerpt)

Collected Years	4	3	2	p
30	0	2	57	11
60	0	5	54	22
. . .				
870	1	25	38	19
900	1	28	35	30
1800	2	57	11	0
. . .				
8100	13	17	19	30

Expanded Years		3	2	p	m
1		0	5	54	11
2	b	0	11	49	22
. . .					
29	b	2	51	17	19
30		2	57	11	0

Month	2	p
Almuharaç	0	30
Saphar	0	59
. . .		
Dulchida	5	25
Dulcheya	5	54

Table 1.2B: Weekdays in the Hijra calendar (excerpt)

Collected Years	Weekday	Expanded Years		Weekday	Month	Weekday
30	5	1		4	1	1
60	3	2	b	1	2	3
. . .		. . .			. . .	
870	5	29	b	7	11	2
900	3	30		5	12	4

the first entry in the sub-table on the left is to be read as 0,2,57,11 days or $2{\cdot}60^2 + 57{\cdot}60 + 11$ (= 7200 + 3420 + 11 = 10631) days. If we divide 10631d by 30y we find a year length of 354;22 days. In the central sub-table it is easy to recognize the equivalence of 1 year, 5,54d 11m = $354^{11}/_{30}$ days, for "m" in the heading indicates here a thirtieth, not a sixtieth, of a day. The Arabic names of the months in the Hijra calendar shown in the sub-table on the right reflect one of many medieval transcriptions of these month names.

The tables for the number of days were often accompanied by other tables to determine the weekdays. Table 1.2B displays an excerpt of such a table, also associated with the Hijra calendar, this time taken from Florence, Biblioteca Laurenziana, MS San Marco 185, f. 59r, a 13th/14th-century manuscript containing a copy of the Toledan Tables (see Toomer 1968, p. 24; F. S. Pedersen 2002, p. 931).

3. *Conversion of Dates*

The cumbersome tasks of converting a date from one calendar to another and determining the weekday of the beginning of a year were also solved by means of tables. As an example, we display an excerpt of a table for conversion of Hijra dates to Julian dates, taken from the Toledan Tables (Toomer 1968, p. 20; F. S. Pedersen 2002, pp. 907–911).

The year length in the Arabic calendar is 354;22 days, and the year length in the Julian calendar is 365;15 days. Hence, in 30 Arabic years there are 10,631 days or 29 Julian years (of 365;15 days) and $38^3/_4$ days: see Table 1.3, Expanded Arabic years, where the entry for 30 Arabic years is 29y 1m 8d 3q, where "y" stands for a Julian year (of 365;15d), "m" stands for a month (of 30 days), "d" stands for a day,

Table 1.3: Conversion of Hijra dates to Julian dates (excerpt)

Collected Arabic years	Collected Julian years	m	d	q
Radix	621	6	15	0
30	650	7	23	3
60	679	9	2	2
. . .			. . .	
870	1465	7	13	0
900	1494	8	21	3

Expanded Arabic years	Expanded Julian years	m	d	q
1	0	11	24	0
2 b	1	11	13	3
. . .				
29 b	28	1	20	0
30	29	1	8	3

Arabic months	m	d
Almuharan	1	0
Saphar	1	29
. . .		
Dulcheyda	10	25
Dulcheya	11	24

"q" stands for a quarter of a day, and "b" indicates a leap year of 355 days. The entry for 30 collected Arabic years is the sum of the entry for the radix and the entry for 30 expanded Arabic years. The entry for the radix, 621 (completed) Julian years plus 6 (completed) months (of 30 days) and 15 days (or 621 years plus 195 days), corresponds to July 15, 622 AD which, in turn, takes us to to Muḥarram 1, A. H. 1 (see Table 1.1A).

CHAPTER TWO

TRIGONOMETRY AND SPHERICAL ASTRONOMY

1. *Sines and Chords*

In *Almagest* I.11 Ptolemy displayed a table of chords, the construction of which is explained in the previous chapter. The chord of a central angle α in a circle is the segment subtended by that angle α. If the radius of the circle (the "norm" of the table) is taken as 60 units, the chord can be expressed by means of the modern sine function:

$$\text{crd}\ \alpha = 120 \cdot \sin(\alpha/2).$$

In Ptolemy's table (Toomer 1984, pp. 57–60), the argument, α, is given at intervals of ½° from ½° to 180°. Columns II and III display the chord, in units, minutes, and seconds of a unit, and the increase, in the same units, in crd α corresponding to an increase of one minute in α, computed by taking $^1/_{30}$ of the difference between successive entries in column II (Aaboe 1964, pp. 101–125). For an insightful account on Ptolemy's chord table, see Van Brummelen 2009, where many issues underlying the tables for trigonometry and spherical astronomy are addressed.

By contrast, al-Khwārizmī's zij originally had a table of sines for integer degrees with a norm of 150 (Neugebauer 1962a, p. 104) that has uniquely survived in manuscripts associated with the Toledan Tables (Toomer 1968, p. 27; F. S. Pedersen 2002, pp. 946–952). The corresponding table in al-Battānī's zij displays the sine of the argument at intervals of ½° with a norm of 60 (Nallino 1903–1907, 2:55–56). This table is reproduced, essentially unchanged, in the Toledan Tables (Toomer 1968, p. 29; F. S. Pedersen 2002, pp. 957–959) and it was rarely modified during the Middle Ages. For instance, it is found in the Alfonsine corpus as the first of a series of tables ascribed to John of Lignères (e.g., Oxford, Bodleian Library, MS Can. Misc. 27, ff. 88v–89r, where this table is found under the title: *Incipiunt tabule sinuum et cordarum (...) composuit magister iohannes de lineris picardus diocesis ambianensis anno domini nostri ihesu christi 1322*), but close inspection reveals that it was just copied from the Toledan

Table 2.1A: Sines (excerpt)

Argument (°, ′)	(°, ′)	*Corde mediate* (°)
0 30	179 30	0;31,25
1 0	179 0	1; 2,50
. . .	. . .	. . .
30 0	150 0	30; 0, 0
. . .	. . .	. . .
60 0	120 0	51;57,42
. . .	. . .	. . .
89 30	90 30	59;59,52
90 0	90 0	60; 0, 0

Tables. Table 2.1A displays an excerpt of the table ascribed to John of Lignères.

The most extensive table of this type in a 14th-century manuscript that we have found is in Paris, Bibliothèque nationale de France, MS lat. 7316A, ff. 89r–111r, where the argument is given at intervals of 1′, thus displaying 2,700 entries for the sine of the argument. At the end of this table there is a trigonometric circle, similar to that displayed in Figure 1, and a small text which includes the name of John of Murs (f. 111r).

In some cases, columns for other trigonometric functions are displayed next to the column for the sine (see Figure 1 for the geometric definitions of these functions). For example, in the Almanac of Azarquiel columns for cosine and versine appear next to the column for sine. Table 2.1B displays such a table, as presented in Paris, Bibliothèque de l'Arsenal, MS 8322, f. 127r.

Towards the middle of the 15th century substantial progress was made in this field. In his *Tabulae primi mobilis* Bianchini compiled a table of sines with a norm (i.e., a value of Sin 90°) of 60,000. This table is later found in Regiomontanus's *Tabule directionum*, first printed in 1490 in Augsburg, although Regiomontanus composed tables with a norm of 6,000,000 in his *De triangulis* as well as one with a norm of 10,000,000, in effect displaying decimal values of the sine function to 7 places (Zinner 1990, pp. 55 and 95).

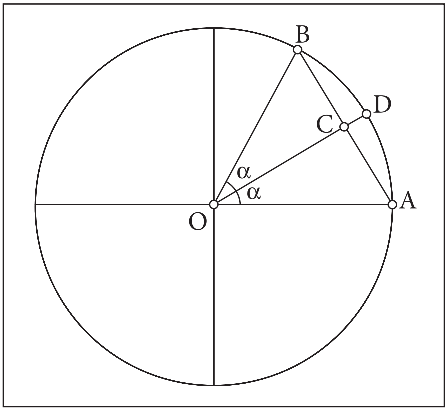

Figure 1: Trigonometric functions: when OA = 60, AB = Crd 2α, AC = Sin α (= 60 · sin α), OC = Cos α (= 60 · cos α), and CD = Vers α = 60 − Cos α

Table 2.1B: Trigonometric functions (excerpt)

(°)	Sine	Cosine	Versine
3	3; 8	59;45*	0;35 (*sic*)
6	6;16	59;40	0;24 (*sic*)
9	9;23	59;16	0;44
. . .			
30	30; 0	51;58	8; 2
. . .			
60	51;58	30; 0	30; 0
. . .			
87	59;55	3; 8	56;52
90	60; 0	0; 0	60; 0
93			63; 8
96			66;16
. . .			
177			119;45 (*sic*)
180			120; 0

* Instead of 59;55.

2. *Solar Declination*

The solar declination, δ, is the angular distance between the Sun and its projection on the equator along a great circle from the celestial poles. It can be computed by means of the modern formula

$$\sin \delta = \sin \lambda \cdot \sin \varepsilon,$$

where λ is the solar longitude and ε is the obliquity of the ecliptic, that is, the angle between the celestial equator and the ecliptic (see Figure 2).

According to modern theory, this angle is not constant, but oscillates slowly between the extremes of about 22;37° and 24;14° with a period of 39,120 years; its value for year 2000 was about 23;26,21°, and it is decreasing at a rate of about 0.47″/y. With the modern formula (Meeus 1991, p. 135), the value for 1300 is computed to be 23;31,50°.

In *Almagest* I.15 Ptolemy displayed a table for the solar declination to seconds of arc as a function of the solar longitude, given at intervals of 1° from 1° to 90°, and used an obliquity, ε = 23;51,20° (rounded to 23;51° in the *Handy Tables*, where all entries for the solar declination are only given to minutes). This type of table became standard although in some cases the basic parameter for the obliquity was

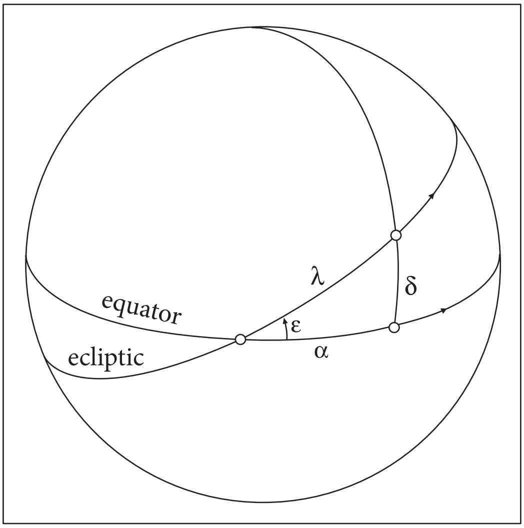

Figure 2: Solar declination

Table 2.2A: Some historical values of the obliquity

ε		Reference
24°	Indian value	Kennedy and Muruwwa 1958, p. 120
23;51,20°	Ptolemy	*Almagest* I.15
23;51°	Ptolemy	*Handy Tables*
	al-Khwārizmī	Neugebauer 1962a, pp. 96–97
23;35°	al-Battānī	Nallino 1903–1907, 1:12
23;33,30°	Toledan Tables	F. S. Pedersen 2002, pp. 961–966
23;33, 8°	Abraham Ibn Ezra	Goldstein 1974, p. 96
	Levi ben Gerson	Goldstein 1974, pp. 95–96
23;33°	Yaḥyā ibn Abī Manṣūr	Kennedy 1956a, p. 145; van Dalen 2004, 18–21
	Almanac of Azarquiel	Millás 1943–1950, p. 174
	Ibn al-Kammād	Chabás and Goldstein 1994, pp. 29–30
	Peter of St. Omer	F. S. Pedersen, 1983–84, p. 347
	Tables of Barcelona	Chabás 1996a, pp. 499–500
23;32,30°	Isaac ben Sid	Rico 1863–1867, 4:6
	Ibn Isḥāq	Comes 1996, p. 363; Mestres 1999, p. 275
23;31,15°	BnF, MS lat. 7316A	f. 114v; Evans 1998, p. 107
23;30,30°	Giovanni Bianchini	Chabás and Goldstein 2009b, p. 19
23;30°	*Tabule Anglicane*	Florence, MS San Marco 185, f. 120v
	Regiomontanus	*Kalendarium* 1483; *Tabule directionum*
23;28°	Regiomontanus	*Epitome* I.17
	Copernicus	Swerdlow and Neugebauer 1984, p. 105

modified. Some values for the obliquity that occur in medieval astronomy are listed in Table 2.2A.

A table for solar declination is found in almost all sets of astronomical tables, and there are numerous variants. We have chosen to reproduce excerpts of three such tables, with different basic parameters and levels of precision, that depend on different computational methods. Table 2.2B displays an excerpt of a table for solar declination associated with the Toledan Tables, found in Naples, Biblioteca Nazionale, MS VIII.C.49, f. 62v; Table 2.2C comes from the *Tabule Anglicane*, as found in Florence, Biblioteca Laurenziana, San Marco 185, f. 120v; and Table 2.2D displays an excerpt of the corresponding table in Regiomontanus's *Tabule directionum profectionumque*, ed. 1504, f. 25r.

For tables of variable obliquity, see § 4.1, below.

Table 2.2B: Solar declination in the Toledan Tables

Argument (°)	Declin. (°)
0	0; 0, 0
1	0;24, 0
2	0;48, 0
. . .	
30	11;31,36
. . .	
60	20;15, 0
. . .	
88	23;32, 2
89	23;33,13
90	23;33,30

Table 2.2C: Solar declination in the *Tabule Anglicane*

Argument (°)	Declin. (°)
0	0; 0
1	0;24
2	0;48
. . .	
30	11;30
. . .	
60	20;11
. . .	
88	23;28
89	23;29
90	23;30

Table 2.2D: Solar declination in the *Tabule directionum*

Argument (°)	Declin. (°)
0	0; 0
1	0;26
2	0;52
. . .	
30	12;16
. . .	
60	20;38
. . .	
88	23;29
89	23;30
90	23;30

3. *Shadow Table*

The purpose of this table is to determine the length of the shadow, s, in the horizontal plane, cast by a gnomon of 12 units, as a function of the solar altitude, h, given in degrees at intervals of 1° from 1° to 90° (see Figure 3). The entries, in units and minutes of a unit, can be computed with the modern formula

$$s = 12 \cdot \text{cotan } h.$$

This type of table is found in many medieval sets of tables with minor variants: the zij of al-Khwārizmī (Suter 1914, p. 174), the zij of al-Battānī (Nallino 1903–1907, 2:60), the Toledan Tables (F. S. Pedersen 2002, p. 993), among others. Table 2.3 displays an excerpt taken from the Tables of Barcelona (Ripoll, Biblioteca Lambert Mata, MS 21, f. 144v). The values shown here, especially for arguments of 3° and 4°, sometimes differ from those in other sets of tables or even from those transcribed from other copies of the Tables of Barcelona (see, e.g., Millás 1962, p. 225).

4. *Right Ascension*

For the solution of some trigonometric problems the equatorial system of coordinates (right ascension, α, and declination, δ) was preferred

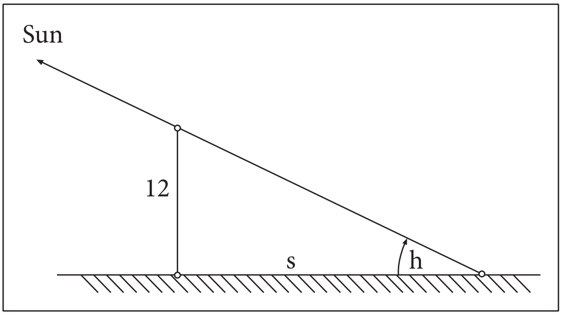

Figure 3: Shadows

Table 2.3: Shadows (excerpt)

Argument (°)	Shadow
1	687;26
2	343;39
3	293;28
4	171;42
5	137;10
. . .	
30	20;47
. . .	
60	6;56
. . .	
89	0;13
90	0; 0

over ecliptic coordinates (longitude, λ, and latitude, β). Let V be the first point of Aries, the intersection of the celestial equator and the ecliptic. To find the equatorial coordinates of a point R on the celestial sphere, we drop a perpendicular, RG, from R onto the equator at point G. Then the right ascension of R, α, is the angular distance VG along the equator measured eastwards from point V, and the declination of point R, δ, is the angular distance RG (see Figure 4). Right ascension is sometimes expressed in hours, minutes, and seconds from 0h to 24h, where 1h = 15°.

Explanations for computing right ascension are often found in medieval texts, as in the canons to the Toledan Tables (F. S. Pedersen 2002, pp. 612–13) and Ibn al-Muthannā (Goldstein 1967, pp. 69ff. and 202–203), which are very similar to each other.

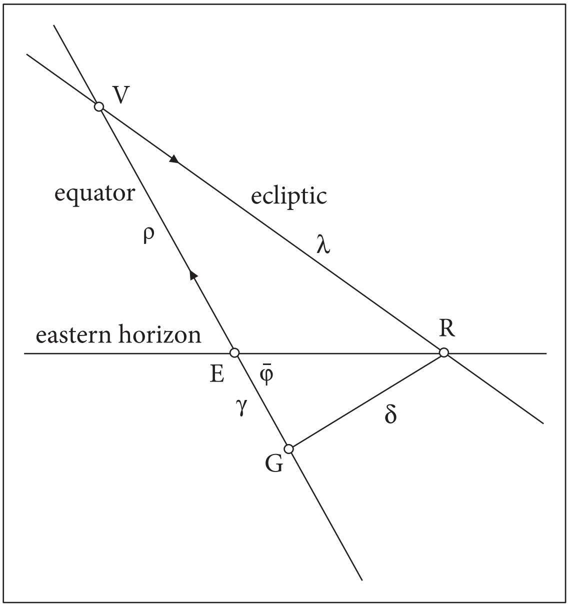

Figure 4: Right ascension, oblique ascension, and ascensional difference

In addition to tables of right ascension, α, there are also tables of normed right ascension, α', which represent the right ascension increased by 90°, in which case the table begins with Cap 0°, rather than with Ari 0°. In the Middle Ages right ascensions were sometimes called the rising times at *sphera recta*. In the usual table, the entries are given in degrees, to minutes or to seconds, as a function of each degree of λ. The right ascension, α, and thus the normed right ascension, α', depend on the obliquity of the ecliptic, ε. This relation can be expressed by means of the modern formula (see Fig. 2):

$$\sin \alpha = \tan \delta \,/\, \tan \varepsilon,$$

where

$$\delta = \arcsin (\sin \lambda \cdot \sin \varepsilon),$$

as in § 2.2, above. Right ascensions can also be found directly, using a modern formula whose equivalent was available in the Middle Ages (see Van Brummelen 2009, p. 191):

$$\tan \alpha = \cos \varepsilon \cdot \tan \lambda.$$

In many cases a column for the equation of time was included in the same table (for the equation of time, see ch. 3, below). There follow the normed right ascensions of the individual signs in some major medieval tables. Note that the three values in each column add up to 90°. As is the case with many other specific tables, two main traditions emerge: that of al-Khwārizmī and that of al-Battānī.

	Ptolemy	al-Khwārizmī	al-Battānī
Cap	32;16°	32;15,48°	32;13°
Aqr	29;54°	29;54, 3°	29;54°
Psc	27;50°	27;50, 9°	27;53°

The data under Ptolemy are found both in the *Almagest* II.8 (Toomer 1984, p. 100) and the *Handy Tables* (Stahlman 1959, pp. 206–209). For the corresponding table in the tradition of al-Khwārizmī, see Suter 1914, pp. 171–173; Neugebauer 1962a, pp. 104–105. In Table 2.4A we reproduce an excerpt of it, as found in Florence, Biblioteca Nazionale Centrale, MS Conv. Soppr. J.V.5 (San Marco 191), f. 75r, a manuscript from about 1300 containing a copy of the Toledan Tables with some values for Cremona.

Table 2.4A: Right ascension in the tradition of al-Khwārizmī (excerpt)

Arg. (°)	Capricorn Asc. (°)	Aquarius Asc. (°)	Pisces Asc. (°)	. . .	Sagittarius Asc. (°)
1	1; 5,33	33;18,13	63; 6,57		328;46,50
2	2;11, 7	34;20,29	64; 3,54		329;49,41
. . .					
10	10;54,34	42;32, 6	71;35,15		338;18, 4
. . .					
20	21;41,56	52;29,48	80;50,20		349; 5,26
. . .					
29	31;13,10	61;12,34	89; 5, 8		358;54,37
30	32;15,48	62; 9,51	90; 0, 0		360; 0, 0
	32;15,48	29;54, 3	27;50, 9		32;15,48*

* The last row contains the right ascensions of the individual signs.

Table 2.4B: Right ascension in the tradition of al-Battānī (excerpt)

Arg.	Capricorn		Aquarius		Pisces		. . .	Sagittarius	
	Asc.	Eq. t.	Asc.	Eq. t.	Asc.	Eq. t.		Asc.	Eq. t.
(°)	(°)	(°)	(°)	(°)	(°)	(°)		(°)	(°)
1	1; 6	3;41	33;15	0;37	63; 4	0;12		328;50	6;59
2	2;11	3;33	34;17	0;33	64; 1	0;15		329;53	6;55
. . .									
10	10;53	2;37	42;29	0;10	71;33	0;38		338;21	6; 6
. . .									
20	21;39	1;31	52;25	0; 1	80;49	1;21		349; 7	5; 4
. . .									
29	31;10	0;44	61; 9	0; 8	89; 5	2; 5		358;54	3;56
30	32;13	0;41	62; 7	0;10	90; 0	2;10		360; 0	3;49

The tradition based on the table in al-Battānī's zij (Nallino 1903–1907, 2:61–64) is found in the Almanac of Azarquiel, the Toledan Tables, Ibn al-Kammād's zij, the Tables of Barcelona, the *editio princeps* of the Alfonsine Tables, among many others. Note that entries in this table are given to minutes, whereas those in the tradition of al-Khwārizmī are given to seconds. Sometimes the tables for right ascension have an extra column for the equation of time (see ch. 3, below). This is the case in Table 2.4B, displaying an excerpt of the table for right ascension found in Vienna, Nationalbibliothek, MS 2288, ff. 46v–47r, a 14th-century manuscript of Italian origin containing, among other items, a set of the Parisian Alfonsine Tables.

5. *Oblique Ascension*

The oblique ascension, ρ, of a point on the ecliptic, R, rising on the horizon is the angular distance, VE, along the equator from the first point of Aries, V, to the East point, E (see Figure 4). From this definition it follows that the right ascension is a special case of oblique ascension, that is, when the observer is on the equator and therefore the geographical latitude is 0°. The term "ascension," or "rising time," refers to the arc on the equator that rises above the horizon in the same time as a given arc on the ecliptic.

The oblique ascension depends on the obliquity of the ecliptic as well as on the geographical latitude of the observer. Thus, medieval sets of tables included tables for various values of the latitude, as well as tables for specific localities. In the *Handy Tables*, Ptolemy displayed tables for the seven climates, that is, specific geographical latitudes associated with the longest daylight at that latitude. This system of climates became standard and was followed in all major sets of tables (see Table 2.7B, below). Despite the fact that in the *Handy Tables* the oblique ascensions begin in Cancer, most medieval sets of tables begin in Aries. To compute the oblique ascension, ρ, as a function of the right ascension, α, the declination, δ, and the geographical latitude of the observer, φ, one may use for following modern formula:

$$\rho = \alpha - \gamma$$

where

$$\gamma = \arcsin(\tan\delta \cdot \tan\varphi).$$

Note that in Figure 4, $\overline{\varphi} = 90 - \varphi$ (see also Figure 5), and $\alpha = \rho + \gamma$.

In many cases a column for the length of the diurnal seasonal hour (see § 2.7, below) is included, as Ibn al-Kammād did in his table for the oblique ascension for Córdoba (φ = 38;30°) in his zij, *al-Muqtabis* (Madrid, Biblioteca Nacional, MS 10023, ff. 49v–51r), an excerpt of which appears in Table 2.5.

Table 2.5: Ibn al-Kammād's table for the oblique ascension for Córdoba (excerpt)

Arg.	Aries		Taurus		Gemini	
	ascen.	s. hour	ascen.	s. hour	ascen.	s. hour
(°)	(°)	(°)	(°)	(°)	(°)	(°)
1	0;35	15; 3	19; 5	16;37	41; 3	17;56
2	1;11	15; 6	19;45	16;40	42; 8	17;58
. . .						
29	17;46	16;[b]	39;27	17;52	68;27	18;24
30	18;25	16;[b]	40;13	17;55	69;32	18;24

Arg.	Cancer		Leo		Virgo	
	ascen.	s. hour	ascen.	s. hour	ascen.	s. hour
(°)	(°)	(°)	(°)	(°)	(°)	(°)
1	70;38	18;24	106; 3	17;52	144; 0	16;31
2	71;44	18;24	107;22	17;50	145;15	16;28
. . .						
29	103;33	17;56	141;25	16;[b]	178;46	15; 3
30	104;44	17;55	142;45	16;[b]	180; 0	15; 0

[b] means "blank".

6. *Ascensional Difference*

The ascensional difference, γ, of a point R is the difference between its right ascension and its oblique ascension (EG = VG – VE, in Figure 4), and can be computed by means of the modern formula

$$\sin \gamma = \tan \delta \cdot \tan \varphi,$$

where δ is the declination of point R.

Tables for the ascensional difference are not common in the medieval astronomical literature, but several examples survive. In all of them the argument is given at intervals of 1° from 1° to 90°, and the tabulated entries are the sines of the ascensional difference. In two of these tables the entries reach maximum values of 5;31,34 and 2;12,38. Both are associated with al-Khwārizmī and were computed for ε = 23;51° and for a unit radii of 150 and 60, respectively (Neugebauer and Schmidt 1952; Lesley 1957, pp. 126–127; Goldstein 1967, pp. 204–206; Toomer 1968, p. 33; and F. S. Pedersen 2002, pp. 986–990). In Table 2.6A we present an excerpt of one such table in the tradition of al-Khwārizmī, where the unit radius is 150, taken from Florence, Biblioteca Laurenziana, MS San Marco 185, f. 59v.

In the West there is yet another variant of this table, where the maximum is 2;10,50 for a unit radius of 60. It was computed for

Table 2.6A: Ascensional difference in the tradition of al-Khwārizmī (excerpt)

Argument (°)	Sine of ascensional difference
1	0; 5,18
2	0;10,35
3	0;15,52
...	
30	2;34,49
...	
60	4;40,23
...	
88	5;31,19
89	5;31,30
90	5;31,34

Table 2.6B: Ascensional difference for ε = 23;33° (excerpt)

Argument (°)	Sine of ascensional difference
1	0; 2, 6
2	0; 4,11
3	0; 6,16
...	
30	0; 1, 9*
...	
60	1;50,41
...	
88	2;10,40
89	2;10,44
90	2;10,46

* Instead of 1;1,9°.

$\varepsilon \approx 23;33°$ and, according to F. S. Pedersen (2002, p. 991), it appears in a collection under the name of John of Lignères. In fact, it is almost the same as that found in the Almanac of Azarquiel, of which we reproduce an excerpt in Table 2.6B (Paris, Bibliothèque de l'Arsenal, MS 8322, f. 131r).

7. *Length of Daylight*

The length of daylight (from sunrise to sunset) in time degrees, $d(\lambda)$, is given by the formula

$$d(\lambda) = \rho(\lambda + 180) - \rho(\lambda),$$

where ρ is the oblique ascension when the Sun is at λ: see Neugebauer 1975, p. 40. To transform $d(\lambda)$ into the length of daylight in equinoctial hours and minutes, D, one has to divide it by 15:

$$D = d(\lambda)/15.$$

These are exactly the instructions given in *Almagest* II.9 (Toomer 1984, p. 99).

A diurnal seasonal hour is defined as $^1/_{12}$ of the time from sunrise to sunset, and a nocturnal seasonal hour is $^1/_{12}$ of the time from sunset to sunrise. To find the duration of a diurnal seasonal hour, h^*, in time-degrees, one has to divide the length of daylight by 12:

$$h^* = d(\lambda)/12.$$

Values for the length of daylight often appear in a column inserted in tables for oblique ascensions (see, e.g., Table 2.5 for Córdoba). Note that in Table 2.5 the entries are given in time-degrees, but it is also common to find tables for the same purpose where the entries are given in (equinoctial) hours and minutes. The coefficient relating corresponding entries is $12 \cdot 24\text{h} / 360° = 12/15$, where the coefficient, 12, only applies to conversion between the length of a seasonal hour and the length of daylight.

In Table 2.7A we reproduce an excerpt of the table for the length of daylight for Toledo (in hours and minutes) in Madrid, Biblioteca Nacional, MS 10053, f. 13r–v; see also Chabás and Goldstein 2003, pp. 212–213, and F. S. Pedersen 2002, pp. 1125–1127.

The maximum length of daylight (14;51h) occurs at Gem 30° (= Cnc 0°) and the minimum length (9;9h), at Sgr 30° (= Cap 0°).

Table 2.7A: Length of daylight in hours and minutes (excerpt)

Arg. (°)	Ari (h)	Tau (h)	Gem (h)	. . .	Aqr (h)	Psc (h)
1	12; 3	13;21	14;25	. . .	9;28*	10;40****
2	12; 5	13;23	14;27	. . .	9;40	10;47
. . .						
29	13;16	14;22	14;51	. . .	9;39 **	11;57
30	13;18	14;24	14;51	. . .	9;42 ***	12; 0

* Instead of 9;38.
** Instead of 10;39.
*** Instead of 10;42.
**** Instead of 10;44.

These data correspond to a geographical latitude of 39;54°, which is attested for Toledo. To determine the length of the longest daylight, M, from the geographical latitude, φ, one can appeal to the following modern expression:

$$M/2 = 90 - \gamma,$$

where

$$\sin \gamma = \tan \varphi \cdot \tan \varepsilon,$$

and ε is the obliquity of the ecliptic. See Fig. 4, where δ, the declination, is set equal to its maximum value, ε.

In this type of table, the critical parameter is the value for the longest daylight. Table 2.7B shows the correspondence between M and φ in Ptolemaic astronomy; see, e.g., Neugebauer 1975, p. 38 (we have also included a column for the associated climates).

Table 2.7B: Length of longest daylight and geographical latitude

Length of longest daylight (M)	Geographical latitude (φ)	Climate
13h	16;27°	I
13½h	23;51°	II
14h	30;22°	III
14½h	36; 0°	IV
15h	40;56°	V
15½h	45;34°	VI
16h	48;32°	VII

8. *Meridian Altitude of the Sun*

The meridian altitude of the Sun, h (= arc SC in Figure 5), is related to the geographical latitude of the observer, φ (= arc NP), and the solar declination, δ (= arc QC), by means of the expression:

$$h = 90° - \varphi + \delta,$$

where the declination may be either positive or negative. In Figure 5 arc SQCZPN is the meridian, O is the observer, S is the south point on the horizon, N is the north point on the horizon, Z is the zenith, P is the north celestial pole, Q is the intersection of the celestial equator and the meridian, and C is the culminating point where the Sun crosses the meridian. QO is perpendicular to PO; hence angle SOQ + angle NOP = $\bar{\varphi} + \varphi = 90°$.

Usually, the argument is the solar longitude, as in the examples given by F. S. Pedersen 2002, pp. 1122–1124; sometimes, however, when the table is associated with a calendar, the argument is the day of the year. For the second type, see Table 2.8, displaying a table for a latitude of 50;6° (Liège, Belgium) in a 15th-century astronomical notebook: Liège, Bibliothèque de l'Université, MS 354C, f. 194v (Chabás 1997, pp. 3–16).

Note that the maximum altitude is 64° (for June, 13–27) and the minimum, 16° (for December, 9–23). Thus, the latitude for which the table is valid is $\varphi = 90° - (h_{max} + h_{min})/2 = 50°$, in agreement with the latitude of Liège given in the manuscript (50;6°).

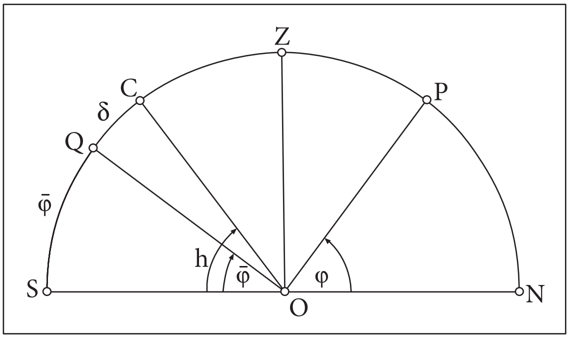

Figure 5: Meridian altitude of the Sun

Table 2.8: Meridian solar altitude for Liège

Days	Altitude (°)											
	J	F	M	A	M	J	J	A	S	O	N	D
1	18		36	48	57	63	62	56	45	33	23	17
2		26										
3					58			55	44		22	
4			37	49						32	21	
5		27										
6							62	54	43			
7	19		38	50	59					31		
8		28										17
9			39					53	42	30		16
10				51								
11		29			60		61				20	
12	20		40			63		52	41	29		
13				52		64						
14		30	41						40			
15								51		28		
16	21		42	53	61				39		19	
17		31					60					
18							59	50		27		
19		32	43	54					38			
20	22											
21								49	37	26	18	
22		33	44	55	62		58					
23	23											16
24			45					48	36	25		17
25		34		56								
26							57	47				
27	24	35	46						35	24	17	
28						63						
29				57				46	34			
30	25		47			62	56			23	17	
31												

9. *Time of Day from the Altitude of the Sun*

Once the altitude of the Sun at noon is known, it is not difficult to construct tables giving the altitude of the Sun at different times of day or, alternatively, the time of day as a function of the altitude. For examples of the first type, see Toomer 1968, p. 155, and F. S. Pedersen 2002, pp. 1136–1138; and for the second type, see F. S. Pedersen 2002, pp. 1134–1136. Table 2.9A displays an excerpt of the first type of table, taken from the Toledan Tables (F. S. Pedersen 2002, pp. 1137–1138), where the argument is the mer(idian) alt(itude).

Table 2.9B displays an excerpt of the second type of table, taken from Florence, Biblioteca Laurenziana, MS San Marco 185, f. 78r, a 13th/14th-century manuscript containing a copy of the Toledan Tables.

The few types of tables for trigonometric functions and spherical astronomy presented here are those most frequently found in medieval manuscripts, but certainly not the only ones.

Table 2.9A: Solar altitude from the time of day according to the Toledan Tables (excerpt)

	1st hour				. . .	6th hour			
Mer. Alt. (°)	15 (°)	30 (°)	45 (°)	60 (°)	. . .	15 (°)	30 (°)	45 (°)	60 (°)
10; 0	0;40	1;18	1;56	2;35	. . .	9;48	9;55	9;59	10; 0
10;30	0;42	1;21	2; 2	2;43	. . .	10;17	10;25	10;29	10;30
. . .									
43;30	2;34	5; 8	7;42	10;16	. . .	42;28	43; 2	43;24	43;30
44; 0	2;39	5;11	7;47	10;22	. . .	42;57	43;32	43;54	44; 0
45; 0	3;12	5;17	7;54	10;33	. . .	43;55	44;31	44;54	45; 0
. . .									
80; 0	3;41	7;23	11; 5	14;48	. . .	74;56	77;26	79;20	80; 0
90; 0	3;45	7;30	11;14	14;59	. . .	78;45	82;30	86;15	90; 0

Table 2.9B: Time of day from the solar altitude according to the Toledan Tables (excerpt)

	Meridian solar altitude										
	21°	22°	. . .	30°	. . .	45°	. . .	60°	. . .	79°	80°
Altitude (°)	(h)	(h)		(h)		(h)		(h)		(h)	(h)
1	0;11	0;11		0; 9		0; 6		0; 5		0; 4	0; 4
2	0;22	0;21		0;16		0;11		0; 9		0; 8	0; 8
3	0;34	0;22*		0;24		0;17		0;14		0;12	0;12
. . .											
21	6; 0	4;52		3; 4		2; 0		1;38		1;25	1;25
22		6; 0		3;14		2; 5		1;43		1;29	1;29
. . .											
29				5;30		2;53		2;15		1;59	1;58
30				6; 0		3; 0		2;21		2; 3	2; 2
. . .											
44						5;16**		3;33		3; 0	2;59
45						6; 0		3;38		3; 4	3; 4
. . .											
59								5;24		4; 3	4; 2
60								6; 0		4; 8	4; 9
. . .											
78										5;41	5;33
79										6; 0	5;47***
80											6; 0

* Instead of 0;32. ** Instead of 5;17. *** Instead of 5;42.

CHAPTER THREE

EQUATION OF TIME

The equation of time is the difference between apparent and mean time, where apparent time is counted from true noon, i.e., the moment that the true Sun crosses the meridian, and mean time is counted from mean noon. The time from true noon to the next true noon (called the true solar day) varies throughout the year, whereas the time from mean noon to the next mean noon (called the mean solar day) is always the same. This difference between a true solar day and the mean solar day, can be represented by the modern formula for E, the equation of time:

$$E = \alpha_1(\lambda_1) - \alpha_0(\lambda_0) - (\bar{\lambda}_1 - \bar{\lambda}_0)$$

where α_1 and α_0 are the right ascensions of the true solar longitudes, λ_1 and λ_0, corresponding to some day in the year and a day taken to be the epoch for this equation, respectively, and $\bar{\lambda}_1$ and $\bar{\lambda}_0$ are the mean solar longitudes on those days, respectively. The equation of time has two components: one is due to the variable daily solar velocity, $\bar{\lambda} - \lambda$, and the other is due to the variation in the correspondence of degrees on the ecliptic with degrees on the equator, $\alpha(\lambda) - \lambda$. In other words, the above equation can be recast as:

$$E = \alpha_1(\lambda_1) - \lambda_1 - [\alpha_0(\lambda_0) - \lambda_0] - (\bar{\lambda}_1 - \lambda_1 - [\bar{\lambda}_0 - \lambda_0]),$$

such that λ_1 and λ_0 are canceled out, for each of them occurs in this equation with opposite algebraic signs. Note that time is measured in equatorial degrees, whereas the Sun progresses on the ecliptic. Hence, even if the Sun moved uniformly on the ecliptic, its successive meridian crossings would not take place at equal time intervals. Figure 6 displays a graphic representation of the equation of time in degrees and minutes in the table described by Peter of Saint Omer (F. S. Pedersen 2002, p. 984) that is usually attributed to John of Lignères (Oxford, Bodleian Library, MS Can. Misc. 27, f. 79r).

In the *Almagest* Ptolemy defines the equation of time but does not tabulate it, whereas it is tabulated in his *Handy Tables* in minutes and seconds of an hour in such a way that this correction always has

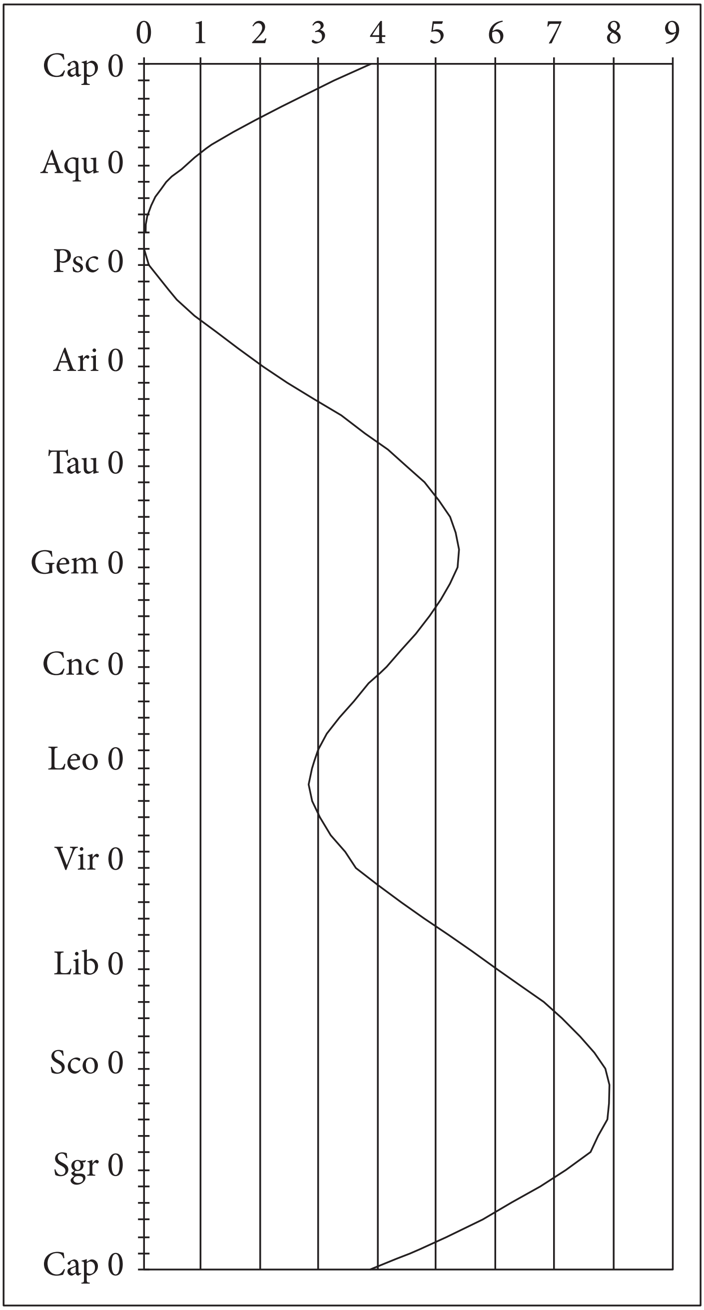

Figure 6: Equation of time

the same algebraic sign (Stahlman 1959, pp. 206–210; cf. Neugebauer 1975, pp. 61–68). For an analysis of Ptolemy's table, see van Dalen 1994; for an analysis of al-Khwārizmī's table, see van Dalen 1996. For discussion of the equation of time in modern astronomy, see Smart [1931] 1965, pp. 42, 146.

In the Middle Ages the equation of time was called "correction of the days with their nights" (*equatio dierum cum noctibus suis*) and was tabulated either together with the right ascensions (see Table 2.4) or separately. The argument is usually the longitude of the Sun (generally without specifying whether it is the mean longitude or the true longitude), and very rarely the argument is the day of the year. The entries are usually given in time-degrees and minutes, but other tables display them in minutes of an hour (min); since 1d is equal to 360° (here meaning time-degrees) or 24h, the conversion factor between them is 360°/24h = 15°/h. In Table 3A we display an excerpt of a table with entries expressed in time-degrees, taken from Cracow, Jagiellonian Library, MS 1865, ff. 84v–85r, a 15th-century manuscript containing a copy of the *Tabulae Resolutae*. Note that the maximum value, 7;57°, is the same as that in the tables of Peter of St. Omer.

Table 3A: Equation of time, in time-degrees and minutes (excerpt)

Arg. (°)	Cap (°)	Aqr (°)	. . .	Tau (°)	. . .	Leo (°)	. . .	Sco (°)	Sgr (°)
1	3;46	0;38	. . .	4;33	. . .	2;51	. . .	7;53	7; 5
2	3;37	0;34	. . .	4;36	. . .	2;50	. . .	7;54	7; 0
. . .									
5	3;14	0;23	. . .	4;46	. . .	2;49	. . .	7;55	6;45
. . .									
8	2;52	0;16	. . .	4;56	. . .	2;51	. . .	7;57	6;28
9	2;47	0;14	. . .	4;58	. . .	2;52	. . .	7;57	6;22
. . .									
18	1;45	0; 0	. . .	5;17	. . .	3; 5	. . .	7;48	5;23
. . .									
25	1; 5	0; 0	. . .	5;21	. . .	3;26	. . .	7;28	4;32
26	1; 1	0; 1	. . .	5;21	. . .	3;29	. . .	7;25	4;24
27	0;57	0; 2	. . .	5;21	. . .	3;32	. . .	7;21	4;17
. . .									
30	0;42	0; 6	. . .	5;20	. . .	3;37	. . .	7;10	3;54

Table 3B: Some historical examples of the equation of time

	Minimum in February	maximum in May	minimum in July	Maximum in October
Handy Tables	0;33,23h (Aqr 18°)	0;6,12h (Tau 30°)	0;16,21h (Leo 9°)	0;0,0h (Lib 30°–Sco 3°)
al-Khwārizmī	0;0,0h (Aqr 22°)	0;23,28h (Tau 30°–Gem 2°)	0;14,40h (Leo 1°–7°)	0;34,28h (Sco 9°–10°)
al-Battānī, Toledan Tables, Parisian Alf. Tables	0;0° (Aqr 18°–19°)	5;33° (Tau 28°–Gem 5°)	3;4° (Leo 1°–9°)	7;54° (Sco 8°–10°)
John of Murs	0;0h (Jan 24–Feb 12)	0;20,58h (May 6)	0;12,0h (July 18–24)	0;32,38h (Oct 24)
Peter of St. Omer, John of Lignères	0;0° (Aqr 18°–25°)	5;21° (Tau 25°–27°)	2;49° (Leo 5°)	7;57° (Sco 8°–9°)
Levi ben Gerson	0;0° (Aqr 20°–24°)	5;8° (Tau 24°–28°)	2;45° (Leo 1°–5°)	8;13° (Sco 9°–10°)
Abraham Zacut	0;0h (Jan 22–Feb 6)	0;22h (Apr 27–May 21)	0;12h (July 12–25)	0;32h (Oct 20–24)

Table 3B shows some of the main features of various characteristic tables of the equation of time.

Note that the definition of the equation of time—adopted in the zijes of al-Khwārizmī and al-Battānī, the Toledan Tables, and by Peter of Saint Omer—differs in sign from that in Ptolemy's *Handy Tables*.

As can be seen from Table 3B, there are three different presentations of the equation of time:

(i) the argument is the solar longitude expressed in degrees and the entries are given in time, whether to minutes or to seconds of an hour (this is the pattern followed by the *Handy Tables* and al-Khwārizmī);

(ii) the argument is the solar longitude expressed in degrees and the entries are given in time-degrees, to minutes (this is the pattern followed by al-Battānī and the Toledan Tables; see Table 3A);

(iii) the argument is the day of the year and the entries are expressed in time, in minutes and seconds of an hour.

We only know of three instances of the third type of presentation: John of Murs's Tables of 1321 (Chabás and Goldstein 2009a, pp. 311–312), Abraham Zacut's *Ḥibbur*, and the *Tabule Verificate* for Salamanca (Chabás and Goldstein 2000, pp. 29, 108–109). Note that, although expressed in minutes and seconds of an hour, the entries in Abraham Zacut's table are equivalent to those in the Toledan Tables and the Parisian Alfonsine Tables. In Table 3C we display John of Murs's equation of time as given in his Tables of 1321 (Lisbon, MS Ajuda 52-XII-35, f. 59r). As far as we know, these values are unprecedented in the astronomical literature.

Table 3C: Equation of time, in minutes and seconds of an hour

Day		(min)	Day		(min)	Day		(min)
Jan.	6	5; 2	May	6	20;58	Sept.	6	22; 2
	12	3; 1		12	20:57		12	24; 2
	18	1;27		18	20;43		18	25;59
	24	0;24		24	20; 7		24	27;50
	31	0; 0		30	19; 9		31	29;32
Feb.	6	0; 5	June	6	18; 5	Oct.	6	30;52
	12	0;37		12	16;55		12	31;58
	18	1;35		18	15;30		18	32;30
	24	2;55		24	14;30		24	32;38
	28	3;57		30	13;30		31	32;20
Mar.	6	5;49	July	6	12;42	Nov.	6	31;23
	12	7;41		12	12;10		12	29;55
	18	9;41		18	12; 0		18	28; 4
	24	11;44		24	12; 0		24	25;43
	31	14; 3		31	12;35		30	23; 4
Apr.	6	15;52	Aug.	6	13;32	Dec.	6	20; 0
	12	17;26		12	14;45		12	16;53
	18	18;47		18	16;11		18	13;39
	24	19;52		24	17;52		24	10;40
	30	20;37		31	20;20		31	7;29

CHAPTER FOUR

PRECESSION AND APOGEES

In *Almagest* VII.2 (Toomer 1984, pp. 327–329), Ptolemy noted that the return of the Sun to the vernal equinox did not have the same duration as the return of the Sun to the fixed stars. He considered the length of the year to be the tropical year, that is, the period of the Sun's return to the vernal equinox, and introduced a motion of the fixed stars with respect to the vernal equinox in the direction of increasing longitude—the difference between these two definitions of the year is due to what is now called the precession of the equinoxes. For Ptolemy, however, the equinoxes were fixed whereas the positions in longitude of the fixed stars increase uniformly at a rate of 1° in 100 Egyptian years (where an Egyptian year is exactly 365 days), and he saw no need to tabulate the accumulated motion of the fixed stars due to precession. There were also theories in antiquity in which the rate of precession varied over time (Ragep 1996), but no tables for such a motion survive (if there were any). In 9th-century Baghdad astronomers noticed that the length of the tropical year no longer had the value Ptolemy assigned to it in *Almages*t III.1 (365;14,48d; see Toomer 1984, p. 140), whereas the sidereal year (the period of return of the Sun with respect to the fixed stars) remained essentially unchanged. Since they had no reason to deny that Ptolemy's results were valid for his own time, they introduced models for variable precession, or trepidation (called in the Middle Ages "the motion in access and recess"), that could account for the data they collected as well as the data in the *Almagest*. The earliest treatise on trepidation accompanied by tables was ascribed in the Middle Ages to Thābit Ibn Qurra (erroneously in all probablity: see, e.g., Ragep 1996, pp. 267–268, n. 4), and it only survives in a Latin translation (see, e.g., Neugebauer 1962b). The tables in Pseudo-Thābit's *On the motion of the 8th sphere* also appear in the Toledan Tables that survive in many Latin copies.

In the Middle Ages all fixed stars were assumed to lie at the same distance from the center of the Earth on a sphere which was called the 8th sphere, that is, it was farther from the Earth than the spheres of the seven planets (Moon, Mercury, Venus, Sun, Mars, Jupiter, and

Saturn). Some medieval astronomers (e.g., al-Battānī and Levi ben Gerson: see Nallino 1903–1907, 1:124; Goldstein 1975) did not appeal to a theory of trepidation; rather, they continued to use precession, that is, a uniform motion of the fixed stars, although the parameter for this motion often differed from the value that Ptolemy had proposed. For instance, al-Battānī's value for precession was 1° in 66 years or about 0;0,0,9°/d.

1. *Trepidation (Access and Recess)*

Pseudo-Thābit's description of trepidation is difficult to reconcile with his tables and the figure in the manuscripts is ambiguous but, according to one interpretation, the sidereally fixed point, Aries 0°, moves on a small circle, the center of which lies at the intersection of the equator and the mean ecliptic, both of them great circles fixed on the celestial sphere. The point Y at Aries 0° moves, together with the sphere of the fixed stars and the spheres of the planets, at a constant velocity on the small circle, through which a movable ecliptic passes. The second point on the movable ecliptic, M, which is required to fix its position on the celestial sphere, is defined as the point on the mean ecliptic 90° from its intersection with the equator (see Figure 7). For discussion of this model and different interpretations of it, see Neugebauer 1962b, p. 298; Goldstein 1965; North 1967 and 1976, 3:155–158; Mercier 1976, 1977, and Mercier 1996, pp. 303–306.

Pseudo-Thābit's tables (Neugebauer 1962b, pp. 296–298) also appear in the Toledan Tables (F. S. Pedersen 2002, pp. 1542–1545): the mean motion on the small circle is tabulated under the title "table for the mean motion of access and recess of the 8th sphere," or variants of it; the table for equation of the motion in access and recess; and the table for the equation of radius of the small circle. The table for the mean motion of access and recess (i.e., trepidation) displayed in Table 4.1A is based on the Hijra as the epoch and Arabic years, and it gives the mean motion about O of Y, Aries 0°. The excerpt reproduced here is taken from Florence, Biblioteca Nazionale Centrale, MS Conv. Soppr. J.V.6 (San Marco 189), f. 48v.

The mean motion, i, derived from the preceding table serves as argument for the table of the equation of access and recess of the 8th sphere, an excerpt of which is displayed in Table 4.1B, as it

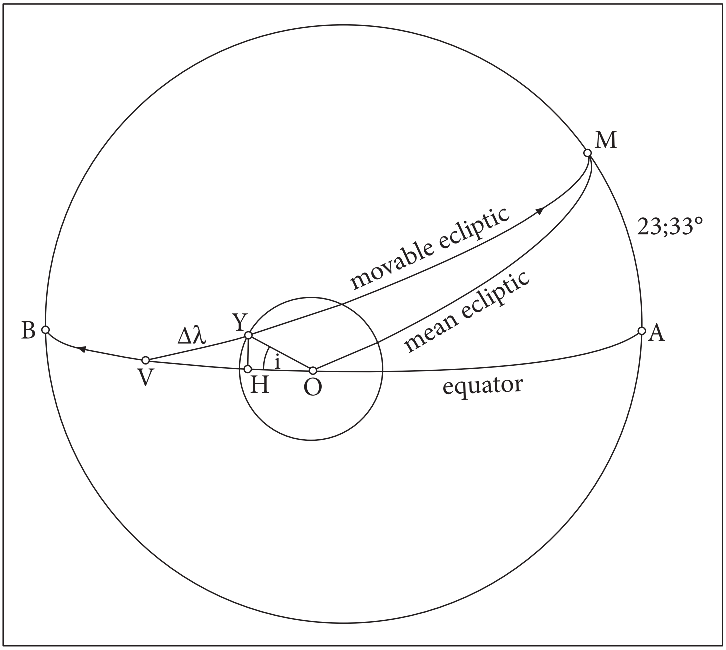

Figure 7: Pseudo-Thābit's model for trepidation: O is the center of the small circle; arcs MO and AO are 90°; angle MOA is 23;33°; Y is Aries 0° (sidereally fixed) and V is the vernal equinox; the amount of trepidation, Δλ, is arc YV; the variable obliquity is angle YVO; the mean motion, i, is angle VOY; and YH is called the equation of the radius of the small circle, OY.

appears in Vatican, Biblioteca Apostolica, MS Lat. 3118, f. 53r (see also F. S. Pedersen 2002, pp. 1558–1559). It has two columns, both representing sinusoidal functions, with different amplitudes. The first column gives the trepidation, that is, the distance Δλ between Y and the vernal point, V. The second column gives YH, the declination of Y with respect to the equator, and its maximum value is the radius of the small circle.

In a model for trepidation by Azarquiel, based on Thābit's model, the angle, ε, between the movable ecliptic and the equator varies regularly between a minimum of 23;33° and a maximum of 23;53°, and it is tabulated in Azarquiel's *Treatise on the motion of the fixed stars*, that only survives in a single Hebrew manuscript (Paris, MS Heb. 1036, reproduced in facsimile, with a Spanish translation, in Millás 1943–1950, p. 332): Table 4.1C. For discussion of this table, see Samsó 1987.

Table 4.1A: Mean motion of access and recess of the 8th sphere (excerpt)

Collected Arabic years	motion (s)	(°)
Radix	0	1;34, 2
30	0	4; 9, 0
60	0	6;43,58
90	0	9;18,56
120	0	11;53,54
. . .		
420	1	7;43,35
. . .		
780	2	8;43,12
810	2	11;18,10
840	2	13;53, 8

Single Arabic years	motion (s)	(°)
1	0	0; 5, 9
2	0	0;10,20
3	0	0;15,29
. . .		
30	0	2;34,58

Table 4.1B: Equation of access and recess of the 8th sphere

Arg. (°)	Equation of access and recess (°)	Equation of the radius of the small circle (°)
5	0;55,52	0;22,40
10	1;50,36	0;44,31
. . .		
30	5;22,30	2; 9,21
. . .		
60	9;17,44	3;44,46
. . .		
85	10;42,13	4;17,30
90	10;45, 0	4;18,43

Table 4.1C: Variable obliquity

Argument (°)	Obliquity (°)
10	23;33, 9
20	23;33,36
30	23;34,20
40	23;35,20
50	23;36,34
60	23;38, 0
70	23;39,35
80	23;41,16
90	23;43, 0
100	23;44,44
110	23;46,25
120	23;48, 0
130	23;49,26
140	23;50,40
150	23;51,40
160	23;52,24
170	23;52,51*
180	23;53, 0

* MS 23;53,51, emended to 23;52,51.

2. *Apogees*

In the *Almagest* Ptolemy argued that planetary apogees are sidereally fixed and subject to precession (*Alm.* IX.7), in contrast to the solar apogee which he took to be tropically fixed at Gem 5;30° (*Alm.* III.4). But, in the 9th century, astronomers in Baghdad fixed the solar apogee sidereally so that it too was subject to variable precession (or trepidation). Later, in the 11th century, Azarquiel realized that the solar apogee had a proper motion in addition to precession, and fixed its amount as 1° in 279 Julian years or about 0;0,0,2°/d (Samsó 1992, p. 212; Chabás and Goldstein 1994, p. 28), thus making the anomalistic year (the return of the Sun to its apogee) differ from the sidereal year. In one Andalusian tradition, this proper motion of the solar apogee was applied to the planetary apogees as well (Samsó and Millás 1998, p. 269; cf. Mestres 1996, pp. 394–395).

For those medieval astronomers who advocated a theory of trepidation, the solar apogee was subject to trepidation. In the Toledan Tables the solar apogee is fixed sidereally, whereas in the Parisian Alfonsine

Table 4.2: Apogees for 1320

	(s)	(°)
Solar apogee	2	21; 7,15,39
Apogee of Saturn	8	3; 5,34,42
Apogee of Jupiter	5	13;18,52,43
Apogee of Mars	4	4;54, 5,43
Apogee of Mercury	6	20;21,25,43
Equation for the motion of the 8th sphere	0	8;17, 6,48

Tables it is subject to trepidation. There is no proper motion of the solar apogee in either the Toledan Tables or in the Parisian Alfonsine Tables. By contrast, in the Tables of John Vimond (produced in Paris at about the same time as the Parisian Alfonsine Tables) there is a double motion of the solar apogee (precession and proper motion), and the planetary apogees are fixed with respect to the solar apogee (Chabás and Goldstein 2004, p. 243).

In Table 4.2 we reproduce the list of apogees of the Sun and the planets taken from a set of tables associated with John of Lignères, extant in Cambridge, Gonville and Caius College, MS 110 (179), p. 17.

3. *Parisian Alfonsine Tables*

The canons to the Castilian Alfonsine Tables fix the sidereal year and vary the tropical year (Chabás and Goldstein 2003, p. 255). In contrast to Pseudo-Thābit, the Toledan Tables, and the Castilian Alfonsine Tables, the Parisian Alfonsine model for precession/trepidation uses a fixed tropical year of 365d 5;49,15,59,34,3h [= 365;14,33,9,59,20,7,30d], a value that first appeared in the *Expositio* by John of Murs, who explicitly says that he took it from the Castilian Alfonsine Tables (Poulle 1980, pp. 251 ff.), and implies a variable sidereal year. Fixing the tropical year makes no sense astronomically, since the problem arose from Ptolemy's poor value for the tropical year, and his unwillingness to evaluate the sidereal year. But his poor value for the tropical year combined with his poor value for precession implies a relatively accurate value for the sidereal year. Hence, if one wished to accommodate Ptolemy's value for the tropical year with more recent values for it, the way to do so was to fix the sidereal year and vary the length of the tropical year over time (see Goldstein 1994).

The Parisian Alfonsine Tables consider two terms for precession/trepidation: (i) a linear term associated with the difference between the calendar year of 365;15 days and a fixed tropical year, and (ii) a periodic term associated with the difference between a variable sidereal year and the calendar year of 365;15 days. These two terms require the use of three tables. The linear term is found from a table usually called "table for the mean motion of the apogees and the fixed stars," or variants of it, and is based on a mean motion of 0;0,0,4,20,41,17,12°/d, corresponding to one revolution in exactly 49,000 years. In this context the planetary apogees are sidereally fixed. Thus, the linear term can be written as

$$p_1 = 360\ t\ /\ 49{,}000$$

where t is the number of years from the epoch (cf. Mercier 1977, pp. 58–60; for the epoch, see below). In Table 4.3A we reproduce some entries listed in a table together with other mean motions, in St. Gallen, Kantonsbibliothek Vadiana, MS 426 (f. 11v), containing the tables by the 15th-century Paduan astronomer, Prosdocimo de' Beldomandi. It should be noted that Prosdocimo uses a 28-year cycle for mean motions, and that the presentation of his tables for the mean motions follows the Toledan Tables and the Tables of Novara, rather than the sexagesimal presentation which is standard in the Parisian Alfonsine Tables (see e.g. Table 5.1C for the mean motion of the Sun). It would seem that Prosdocimo adapted the Table for Novara to the "new" parameters offered by the Alfonsine Tables and, in particular, to Alfonsine precession (Chabás 2007, pp. 271–272).

Table 4.3A: The linear term for precession/trepidation in the Parisian Alfonsine Tables (excerpt)

Years	m.m. access & recess (s)	m.m. access & recess (°)	Years	m.m. access & recess (s)	m.m. access & recess (°)	Months	m.m. access & recess (s)	m.m. access & recess (°)
Radix	0	0; 0, 0	1	0	0; 0,26	January	0	0; 0, 2
28	0	0;12,21	2	0	0; 0,53	February	0	0; 0, 4
56	0	0;24,41	3	0	0; 1,19	March	0	0; 0, 7
84	0	0;37, 2	4	0	0; 1,46	April	0	0; 0, 9
. . .			. . .			. . .		

To obtain the periodic term, two tables are needed. One is usually called "table for the mean motion of access and recess of the 8th sphere," or variants of it, and is based on a mean motion of 0;0,0,30,24,49,0°/d, corresponding to one revolution in exactly 7,000 years. Note that this value is precisely 7 times that of the linear term. The value provided by this table serves as the argument to enter another table, usually called "table for the equation of access and recess of the 8th sphere." We reproduce excerpts of the two tables as they appear in St. Gallen, Kantonsbibliothek Vadiana, MS 426 (ff. 11v and 13v): see Tables 4.3B and 4.3C.

Table 4.3B: The mean motion of access and recess in the Parisian Alfonsine Tables (excerpt)

Years	m.m. (s)	apogees (°)	Years	m.m. (s)	apogees (°)	Months	m.m. (s)	apogees (°)
Radix	5	59;12,34	1	0	0; 3, 5	January	0	0; 0,16
28	0	0;38,58	2	0	0; 6,10	February	0	0; 0,30
56	0	2; 5,22	3	0	0; 9,15	March	0	0; 0,46
84	0	3;31,46	4	0	0;12,21	April	0	0; 1, 1
. . .			. . .			. . .		

Table 4.3C: The Alfonsine equation of the 8th sphere (excerpt)*

	Equation (°)		Equation (°)		Equation (°)
1	0; 9,25	31	4;37,17	61	7;51,42
2	0;18,49	32	4;45,18	62	7;56, 8
. . .	. . .	. . .			
15	2;19,13	45	6;21, 2	75	8;40,50
. . .	. . .	. . .			
29	4;20,55	59	7;42,23	89	8;59,50
30	4;29,10	60	7;47,10	90	9; 0, 0

* The entries for arguments 61°–90° given by Prosdocimo de' Beldomandi differ slightly from those in the *editio princeps* of the Alfonsine Tables (Ratdolt 1483, f. d3v).

Note that the function represented in Table 4.3C is of the form

$$y = \arcsin(\sin 9° \cdot \sin x),$$

which is equivalent to

$$\sin y = \sin 9° \cdot \sin x,$$

and not a simple sinusoidal function of the form $y = 9° \cdot \sin x$. Thus, the tabulated periodic term agrees very closely with the function

$$p_2 = \arcsin(\sin 9° \cdot \sin(360\ t / 7{,}000)),$$

and the total precession/trepidation in Alfonsine astronomy can be written as

$$p = p_1 + p_2 = 360\ t / 49{,}000 + \arcsin(\sin 9° \cdot \sin(360\ t / 7{,}000))$$

or

$$p = p_1 + \arcsin(\sin 9° \cdot \sin 7\ p_1).$$

In the 15th century Giovanni Bianchini merged all this information into a single table for total precession. Figure 8 shows the two components of Alfonsine precession/trepidation and their sum, labeled "total precession," appears as a column in Bianchini's table (Chabás and Goldstein 2009b, pp. 28–33).

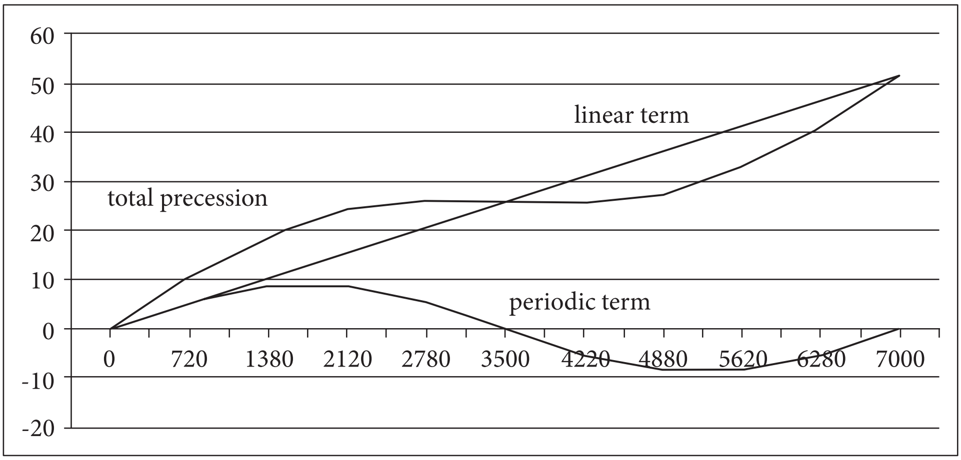

Figure 8: Alfonsine trepidation

In Bianchini's table the epoch for precession is May 17, 16 AD. He took this date to be the epoch of Ptolemy's star catalogue because it is the epoch of the Alfonsine trepidation model, as he explains in the canons to his tables. This is the earliest source we have found that gives this date explicitly for the epoch of the Alfonsine model. However, according to Ptolemy, the epoch for his star catalogue is 137 AD: Toomer 1984, p. 340.

It is not at all common to find lists of the positions of the apogees for various epochs, although the Alfonsine Tables display radices for the apogees of the Sun and the planets (except for Venus, which was assumed to have the same apogee as the Sun). Table 4.3D displays an excerpt of this presentation taken from Madrid, Biblioteca Nacional, MS 7856, ff. 4r and 5r. The apogees, given in the manuscript for only 8 epochs (the two epochs missing here with respect to the *editio princeps* are those for Philippus and Diocletianus), were computed for Toledo, as is the case in most manuscripts containing the Parisian Alfonsine Tables. (For epochs, see § 1.1.)

Table 4.3D: Alfonsine apogees (excerpt)

Sun and Venus (°)		Saturn (°)	
Diluvii	0,48;38, 0	*Diluvii*	3,33;36,20,41
Nabugodo.	1, 5;56, 1	*Nabugod.*	3,47;54,28,41
Alex. Magni	1, 9; 8, 4	*Alex. Magni*	3,51; 6,29,45
Cesaris	1,11; 8,16	*Cesaris*	3,53; 6,56,47
Incarnationis	1,11;25,23	*Incarnationis*	3,53;23,42, 4
Alhighera	1,15;59,18	*Arabum*	3,57;57,40,58
Iesdaiert	1,16; 3,43	*Persarum*	3,58; 2, 3,24
Alfonsii	1,20;36, 0	*Alfonsii*	4, 2;35,20,41

For the Sun and Venus most entries differ in the seconds from those in the *editio princeps* of the Alfonsine Tables. Saturn's apogee for the era of Caesar is given in the *editio princeps* of the Alfonsine Tables as 3,53;6,56,57° (differing in the sexagesimal thirds).

CHAPTER FIVE

MEAN MOTIONS AND RADICES

The Sun, the Moon, and the planets, as well as some special points treated as celestial bodies, were assigned constant mean velocities, called mean motions.These special points include the first point of Aries (Υ in Figure 7) and the ascending and descending lunar nodes, i.e., the intersections of the lunar orb with the ecliptic called, respectively, *caput draconis* and *cauda draconis* in the Latin literature (see Figure 9).

The longitude of a celestial body, λ, is a function of the time, t, after some epoch, t_0, and it was generally computed by means of the formula

$$\lambda(t) = \overline{\lambda}(t_0) + \mu \cdot (t - t_0) + c,$$

where $\overline{\lambda}(t_0)$ is the mean longitude at epoch (called the radix), μ is the mean motion, and c is the correction, i.e., the difference between the true longitude and the mean longitude of a celestial body at time t. In this section we focus our attention on tables for μ. In each case tables for the mean motions were computed based on a single parameter. These tables differ greatly in presentation, depending on such factors as the calendar used and the date taken as beginning of the year but, from Ptolemy's time on, the underlying parameters vary only slightly. For many centuries there was a single basic presentation, which we

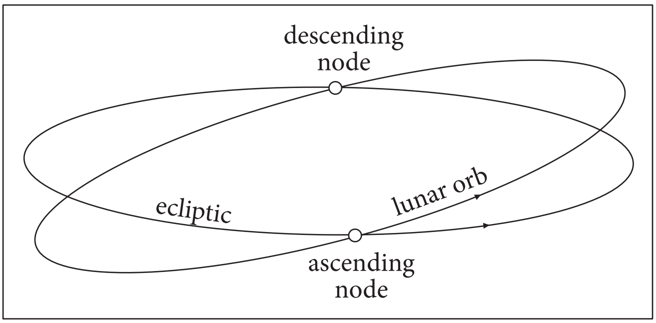

Figure 9: Lunar nodes

shall refer to as "traditional or Arabic style," for it is found in all Arabic zijes. It consists of a series of sub-tables for each celestial body where the mean motions are given for collected and expanded years, months, days, hours, and minutes. In the 14th century the Parisian Alfonsine Tables inaugurated a new presentation displaying 60 consecutive multiples of each parameter, expressed in sexagesimal form.

1. *Mean Motions*

In the traditional style, the tables gave the mean motions for expanded years (that is, for a succession of years from one year up to a certain number of years, called a calendaric cycle) and collected years (groups of calendaric cycles of years), as well as for the months in a year, the days in a month, the hours in a day, and the minutes in an hour. The duration of the cycle depended greatly on the calendar used, and in Arabic zijes it is usually 30 lunar years (= 10,631 days). This is indeed the case in the zij of al-Khwārizmī, the zij of al-Battānī, the Toledan Tables, and the zij *al-Muqtabis* of Ibn al-Kammād, to mention just a few. In adaptations of the Toledan Tables a variety of cycles were used. The Tables of Marseilles and those for Novara have 28 Julian years (= 10,227 days) as the duration of the cycle, whereas the Tables of Toulouse have 24 Julian years (= 8,766 days), and still others have 20 Julian years (= 7,305 days) as is the case for the Tables for London and those for Pisa (Mercier 1987, pp. 108–110). Table 5.1A displays excerpts of the different sub-tables for the mean motion of the Sun in the Tables of Toulouse, as found in Paris, Bibliothèque nationale de France, MS lat. 16658, f. 70r–v (see also Poulle 1994, pp. 68, 73; F. S. Pedersen 2002, p. 1200).

The radices also depend on the calendar used. Thus, for the calendar with Arabic months and years, the epoch is usually the Hijra, whereas the Incarnation applies for the Julian calendar. The beginning of the year also varies with the calendar chosen and, in the case of the Julian calendar, we find both January 1 and March 1. The precision of the entries may also differ: they are usually given to seconds, although in some cases, as in one version of the Tables of Novara, the precision climbs to six or seven sexagesimal places (see F. S. Pedersen 2002, p. 1206); a copy of this set of tables transliterated into Arabic written in Hebrew characters displays the mean motions to six places (see Goldstein 1979b, pp. 34–35).

Table 5.1A: Mean motion of the Sun in the Tables of Toulouse (excerpts)

Collected years	(s)	(°)
Radix	11	13;51, 7
24	11	13;41,52
48	11	13;32,36
…		
1392	11	4;53,54
1416	11	4;44,38

Expanded years	(s)	(°)
1	11	29;44,50
2	11	29;29,40
3	0	0;13,38
4	11	29;58,27
…		
24	11	29;50,44

Months	(s)	(°)
March	1	0;33,14
April	2	0; 7,20
…		
January	11	2; 9, 0
February	11	29;44,50

Days	(°)
1	0;59, 8
2	1;58,16
…	
29	28;34,57
30	29;34, 5

A different presentation is that found in the so-called *Tabule magne*, a set of tables ascribed to John of Lignères (Poulle 1973b, p. 123; Husson 2007, pp. 81–84), where we are given the mean motions of the luminaries, the planets (both in longitude and anomaly), and the lunar node for each day in a year which is not specified. Table 5.1B displays the mean motions in longitude of the Sun, Venus, Mercury, and the lunar node; it is excerpted from Erfurt, Biblioteca Amploniana, MS CA 2° 388, ff. 25r–26v. In the heading we are also told that, in the case of the Sun and the two planets, the radix for the Incarnation ("tempore Christi") is "98 18 27 43 53" (*sic*), which we interpret as 9s 8;18,27,43,53° = 278;18,27,43,53°, in contrast to 4,38;21,0,38 = 278;21,0,38°, the value given in the Parisian Alfonsine Tables (Ratdolt 1483, f. c8r). Note that John of Lignères is using signs of 30°, as opposed to signs of 60°, which has been claimed to be the norm of the Parisian Alfonsine astronomers. In the case of the lunar node, the radix for the Incarnation is given as "3 1 55 44 28 19," i.e., 91;55,44,28,19°, in contrast to 1,31;55,52,41°, the value in the Parisian Alfonsine Tables (Ratdolt 1483, f. c8v).

The Parisian Alfonsine Tables offer yet another presentation of the mean motions, for they only give lists of 60 consecutive multiples of the basic parameter in each case. The *editio princeps* (Ratdolt 1483) displays the following for the Sun, Venus, and Mercury (f. d5r), where

Table 5.1B: Mean motion of the Sun in the *Tabule magne* (excerpt)

	Sun, Ven., Merc.		Node			Sun, Ven., Merc.		Node	
	(s)	(°)	(s)	(°)		(s)	(°)	(s)	(°)
	January					December			
[1]	0	0;59, 8	0	0; 3	. . .	11	0;11,30	0	17;44
[2]	0	1;58,17	0	0; 6	. . .	11	1;10,38	0	17;47
[3]	0	2;57,25	0	0; 9	. . .	11	2; 9,46	0	17;50
. . .									
[29]	0	28;35, 1	0	1;32	. . .	11	27;47,23	0	19;13
[30]	0	29;34,10	0	1;35	. . .	11	28;46,31	0	19;16
[31]	0	0;33,18*	0	1;38	. . .	11	29;45,39	0	19;19

* Read: 1s 0;33,18°.

the basic parameter is 0;59,8,19,37,19,13,56°/d. The presentation is such that when the computer wishes to know the mean motion for, say, 120 days (= 2,0 days), he only has to shift to the left by one column all numbers in the second row in order to obtain 1,58;16,39,14,38,27,52°. Analogously, when the computer wishes to know the mean motion for, say, 2 sixtieths of a day (= 0;2 days) he only has to shift to the right by one column all numbers in the second row in order to obtain 0;1,58,16,39,14,38,27,52°. It is clear that the basic parameters were derived by computation (rather than by observation), and that their presumed precision is unwarranted, much more so when considering huge multiples of them.

In the following lists we display the mean motions of the Sun, the Moon, and the planets as used in various sets of tables (see Tables 5.1D and 5.1E). These lists of parameters are not intended to be exhaustive, but they are representative of the tables that circulated widely in the Middle Ages. They also illustrate the differences between them or, in some cases, the absence of any difference between them. The entries are given in degrees per day. In the case of the Sun the entries appear in increasing order and the list is divided into two parts, one for values yielding a solar year longer than 365;15 days, and the other for those yielding a solar year shorter than 365;15 days (corresponding to a sidereal and tropical year, respectively). Also, one often finds tables for the lunar elongation, $\mu_m - \mu_s$, and for the argument of lunar latitude, $\mu_m - \mu_n$, where μ_m is the daily lunar motion in longitude, μ_s is the daily solar motion in longitude, and μ_n is the daily mean motion of the lunar node.

Table 5.1C: Daily mean motion of the Sun in the *editio princeps* of the Alfonsine Tables (excerpt)

1	0	0	59	8	19	37	19	13	56
2	0	1	58	16	39	14	38	27	52
…									
30	0	29	34	9	48	39	36	58	0
…									
59	0	58	9	11	17	41	54	42	4
60	0	59	8	19	37	19	13	56	0

Table 5.1D: Some historical values of the mean motions of the Sun and the length of the year

	Daily mean motion of the Sun (°/d)	Length of the year (days)
Sidereal coordinates		
Ibn al-Kammād	0;59, 8, 9,21,15	365;15,36,35
al-Khwārizmī	0;59, 8,10,21	365;15,30,26
Toledan Tables	0;59, 8,11,28,27	365;15,23,29
Ibn al-Raqqām	0;59, 8;11,28,26,22, 5	365;15,23,29
Tropical coordinates		
Ptolemy	0;59, 8,17,13,12,31	365;14,48
Tables of Barcelona	0;59, 8,19,10,13	365;14,35,57
Vimond*	0;59, 8,19,37, 5	365;14,33,11
Parisian Alfonsine Tables	0;59, 8,19,37,19,13,56	365;14,33,10
Levi ben Gerson	0;59, 8,20, 8,44, 6, 3,14	365;14,29,56
al-Battānī	0;59, 8,20,46,56,14	365;14,26, 0

* In the case of Vimond, the mean motion in longitude is obtained by adding the mean motion in anomaly (0;59,8,8,23,30°/d) to the motion of the solar apogee (0;0,0,11,13,35°/d). Thus, Vimond's anomalistic year of 365;15,42,32d corresponds to a tropical year of 365;14,33,11d, which is very close to the value in the Parisian Alfonsine Tables.

Most sets of tables also list the mean motions in anomaly of the superior planets, in which case the following relation holds for a given interval of time: the sum of the mean motion in longitude and the mean motion in anomaly is equal to the mean motion of the Sun. For the inferior planets, it was usual to give only their mean motion in anomaly because their mean motions in longitude are simply those of the Sun (see Tables 5.1F and 5.1G).

Table 5.1E: Some historical values of the mean motions of the Moon and the lunar node

	Daily mean motion of the Moon in longitude (°/d)	Daily mean motion of the Moon in anomaly (°/d)
Ibn al-Kammād	13;10,34,52,46	13; 3,53,56,19
Ibn al-Raqqām	13;10,34,52,46,51,19,31	13; 3,53,56,17,51,25,27
al-Khwārizmī	13;10,34,52,46	13; 3,53,58,49
Toledan Tables	13;10,34,52,48,47	13; 3,53,56,17,57
Ptolemy	13;10,34,58,33,30,30	13; 3,53,56,17,51,59
		13; 3,53,56,29,38,38*
Vimond	13;10,35, 1,12, 0	13; 3,53,57,27,11
Parisian Alfonsine Tables	13;10,35, 1,15,11, 4,35	13; 3,53,57,30,21, 4,13
Tables of Barcelona	13;10,35, 1,23,16	13; 3,53,54,51,29
Levi ben Gerson	13;10,35, 1,39,35,43,49	13; 3,53,55,55,33,30
al-Battānī	13;10,35, 2, 7,17,10	13; 3,53,56,17,51,59

* For the two values of Ptolemy's mean motion in anomaly, see Neugebauer 1975, p. 70.

	Daily mean motion of the lunar node (°/d)
al-Khwārizmī	–0; 3,10,48,22
Toledan Tables	–0; 3,10,46,42,33
Ibn al-Kammād	–0; 3,10,46,41
Ibn al-Raqqām	–0; 3,10,46,40,59,49,30
Ptolemy	–0; 3,10,41,15,26, 7
Tables of Barcelona	–0; 3,10,38,55,53
Parisian Alfonsine Tables	–0; 3,10,38, 7,14,49,10
Levi ben Gerson	–0; 3,10,37,38,56, 2,10
al-Battānī	–0; 3,10,37,18,40,26
Vimond	–0; 3,10,29,20,23

Besides the quantities mentioned above, tables for mean motions for the access and recess of the 8th sphere and for the planetary apogees are found in some sets of tables, and they have already been treated in ch. 4 on precession and apogees. It is much more uncommon to find tables for the elongation between pairs of planets, such as Jupiter and Saturn, and Mars and Saturn, which are found in Budapest, National Museum, MS 62, ff. 13v and 33r, respectively.

Comparison of Ibn al-Raqqām's values for Venus and Mercury and those given in the Parisian Alfonsine Tables show very close agreement

Table 5.1F: Some historical values of the mean motions of the superior planets

	Daily mean motion of Saturn in longitude (°/d)	Daily mean motion of Jupiter in longitude (°/d)	Daily mean motion of Mars in longitude (°/d)
al-Khwārizmī	0; 2, 0,22,57	0; 4,59, 9, 8	0;31,26,28, 6
Ibn al-Kammād	0; 2, 0,25,36	0; 4,59, 6,43	0;31,26,31,40
Toledan Tables	0; 2, 0,26,35,17	0; 4,59, 7,37,19	0;31,26,32,15,17
Ibn al-Raqqām	0; 2, 0,27,46,42,52	0; 4,59, 7,36,24,31, 1	0;31,26,31, 9, 4,52
Ptolemy	0; 2, 0,33,31,28,51	0; 4,59,14,26,46,31	0;31,26,36,53,51,33
Vimond	0; 2, 0,35,17,31	0; 4,59,15,27,12	0;31,26,38,40, 9
Parisian Alfonsine T.	0; 2, 0,35,17,40,21	0; 4,59,15,27, 7,23,50	0;31,26,38,40, 5
al-Battānī	0; 2, 0,35,51,48, 3	0; 4,59,16,55,54,57	0;31,26,40,15,11,13
Tables of Barcelona	0; 2, 0,38,26,21	0; 4,59,13,30,40	0;31,26,40,28,51

Table 5.1G: Some historical values of the mean motions of the inferior planets

	Daily mean motion of Venus in anomaly (°/d)	Daily mean motion of Mercury in anomaly (°/d)
Ptolemy	0;36,59,25,53,11,28	3; 6,24, 6,59,35,50
Vimond*	0;36,59,27,23,51	3; 6,24, 7,42,25
Ibn al-Raqqām	0;36,59,27,23,58,51	3; 6,24, 7,42,40,23,56
Parisian Alfonsine Tables	0;36,59,27,23,59,31	3; 6,24, 7,42,40,52
Ibn al-Kammād	0;36,59,29,21	3; 6,24, 7,19
Toledan Tables	0;36,59,29,27,29	3; 6,24, 7,39,31
Tables of Barcelona	0;36,59,29,27,29	3; 6,24, 7,54,5
al-Battānī	0;36,59,29,28,42,45	3; 6,24, 7,45,53,33
al-Khwārizmī	0;36,59,34,13	3; 6,24, 8, 6

* In Vimond's tables, the mean motions are given as 1;36,7,35,47,21°/d (Venus) and 4;5,32,16,5,55°/d (Mercury). These values are the sum of their mean motions in anomaly and the solar mean motion (0;59,8,8,23,30°/d); the underlying daily mean motions are those we have included in the list.

(the difference is about 0;0,0,0,0,0,30°/d), another clear indication of the Andalusian character of the Parisian Alfonsine Tables that has not previously been recognized.

2. *Radices*

The radix of an astronomical argument is its value at epoch, that is, at an instant taken as the temporal origin, generally noon on the date of the epoch. The radix of a particular argument is usually found in the table for its mean motion. As the radix depends not only on time but, in the case of the Moon, also on the geographical longitude of the observer, it is relatively frequent to find radices of a specific argument for various localities in the same table, thus making that table valid for users located in different places. Sometimes, the radices for the planets, the Sun, and the Moon are collected in a single table. This is the case in Bonn, Universitätsbibliothek, MS S 498, a manuscript which was once in Paris, for the seal of the Bibliothèque nationale is clearly visible on f. 1r. On f. 63r there is a comprehensive list of radices for mean motions for Prague for 1400, also including the apogees of the planets and the Sun (see Table 5.2A).

It is even more uncommon to find sets of radices for various epochs. However, this is the case for the Alfonsine Tables, which display radices for 17 quantities related to the planets, the Sun, and the Moon,

Table 5.2A: Radices for Prague for 1400

	(s)	(°)
Fixed stars	0	10;17, 8,34
Equation 8th sphere	2	11;12,33,58,28
Mean motion of the Sun	9	18;35, 6,44
Solar apogee	3	10;13,33
Solar argument	6	18;21,34
Mean motion of the Moon	3	21;31,41,59,42
Lunar center	0	5;53,10,32
Lunar anomaly	3	10;10,41, 7,46
Mean motion of Saturn	9	12;38,31,44
Center of Saturn	1	0;26,40
Apogee of Saturn	8	12;11,52
Anomaly of Saturn	0	5;57,35
Mean motion of Jupiter	6	27;39,25,29
Center of Jupiter	1	5;14,15
Apogee of Jupiter	5	22;25,10
Anomaly of Jupiter	2	20;55,41,15
Mean motion of Mars	6	3;31,21,18
Center of Mars	1	19;30,58
Apogee of Mars	4	14; 0,23
Anomaly of Mars	3	15;3,45,26
Anomaly of Venus	0	24;39,33,53
Center of Venus	6	18;21,34
Center of Mercury	6	29;27,43
Anomaly of Mercury	2	10; 7,24
Lunar node	5	20; 5,27,15
True motion of the lunar node	6	9;54,32,45

for each of 10 different epochs (cf. Table 4.3D). Table 5.2B displays an excerpt of this presentation taken from Madrid, Biblioteca Nacional, MS 7856, f. 4v. The radices, given in the manuscript for only 8 epochs (the two missing here are for Philippus and Diocletianus, as in Table 4.3D), refer to quantities associated with the Moon and were computed for Toledo, as is usual in most manuscripts containing Alfonsine Tables.

Table 5.2B: Alfonsine radices related to the Moon (excerpt)

Longitude of the Moon (°)	
Diluvii	4,47;49,43,52, 3
Nabugodo.	0,26;46,43,14,51
Alex. Magni	2, 4;21, 1,10,38
Cesaris	1,54;25,20,23,27
Incarnationis	2, 2;46,50,16,40
Alhighera	2, 2; 1,16,23,53
Iesdaiert	5,53;16,32, 5, 2
Alfonsii regis	5,36; 5,21,11,50

Lunar anomaly (°)	
Diluvii	3,42;41, 4,38
Nabugod.	4,13; 3,50, 0
Alex. Magni	4,25;47,30,18,25
Cesaris	4,56;57,51,30, 2
Incarnationis	3,19; 0,14,31,17
Alhighera	1,47;21,27,42,28
Iesdayert	4,54;52,33, 4
Alfonsii regis	4,10;51,40, 9

Argument of lunar latitude (°)	
Diluvii	2,24;31, 4,49,28
Nabugod.	1,13;17,42, 0,42
Alex. Magni	5,15;56,34, 1,12
Cesaris	3,11; 6,54, 9,18
Incarnationis	3,34;28,42,38,20
Alhighera	5,55; 7,51,55,18
Iesdaiert	0,58;17,30, 6,10
Alfonsii	2,32; 4, 7, 3, 0

Lunar node (°)	
Diluvii	3,36;55,21,17
Nabugod.	0,46;44,59, 5
Alex. Magni	3,11;49,33,21
Cesaris	1,11;55,34, 5*
Incarnationis	1,31;55,52,41
Arabum	3,53;20,35,51
Persarum	1, 5;14,58,20
Alfonsii	2,56;12,46,11

* Instead of 1,16;55,34,5, as in the printed editions of the Alfonsine Tables.

CHAPTER SIX

EQUATIONS

As defined in the preceding section, the difference between the true longitude and the mean longitude of a celestial body at time t is its correction, also called its equation. In order to compute the true position of the Sun, the Moon, and the planets, it is necessary to determine the equation corresponding to the mean position and, thus, in all sets of tables one finds, explicitly or implicitly, tables for this purpose. In some cases the equation is a function of one argument, but in others it is a function of two arguments. In some tables, the entries are given to degrees and minutes, but in others seconds are also displayed.

For the Sun we display the eccentric version of Ptolemy's model, whereas for his first lunar model we display both the eccentric and epicyclic versions. In *Almagest* III.1 (Toomer 1984, pp. 146–153) Ptolemy demonstrated the equivalence of the eccentric and epicyclic models, where the radius of the epicycle is equal to the eccentricity, and the motion on the epicycle takes place in the direction opposite to the motion on the deferent (the circle that carries the epicycle).

1. *Sun*

In Ptolemy's model for the motion of the Sun (see Figure 10), O is the observer, D is the center of circle ASP (whose radius is 60) on which the Sun moves, OD is the eccentricity, A and P are the solar apogee and perigee, respectively, and OV indicates the direction of Aries 0°. The mean Sun is in the direction $O\overline{S}$ and has a mean longitude $\overline{\lambda}$. The Sun is at S with true longitude, λ, and $O\overline{S}$ and DS are parallel. The solar equation, c, is the difference between the true and the mean longitudes of the Sun,

$$c = \lambda - \overline{\lambda},$$

and it is a function of $\overline{\kappa}$, the argument of center, defined as

$$\overline{\kappa} = \overline{\lambda} - \lambda_A,$$

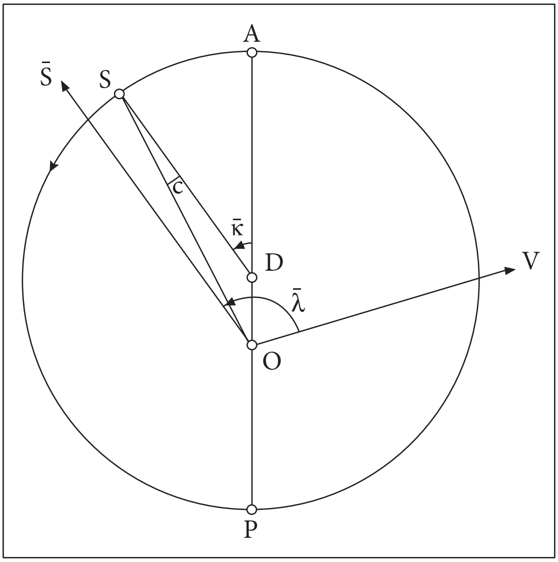

Figure 10: The model for solar motion

where λ_A is the longitude of the solar apogee; $\overline{\kappa}$ is also called "solar anomaly." Note that $c \leq 0$ for $0° \leq \overline{\kappa} \leq 180°$ and $c \geq 0$ for $180° \leq \overline{\kappa} \leq 360°$.

Using modern notation, the aforementioned function can be written as

$$c(\overline{\kappa}) = \arctan \left(\frac{e \sin \overline{\kappa}}{60 + e \cos \overline{\kappa}} \right),$$

where e is the eccentricity. This expression is easily deduced from Figure 10 by drawing a perpendicular from O to SD, and determining $\tan c(\overline{\kappa})$ in the resulting right triangle. Note that the following symmetry holds:

$$c(360 - \overline{\kappa}) = -c(\overline{\kappa}),$$

indicating that any table representing this function only needs the argument to be given from 0° to 180° (or 0° to 3,0°). As an example of a table representing $c(\overline{\kappa})$, Table 6.1A shows an excerpt of the Alfonsine solar equation as found in Madrid, Biblioteca Nacional, MS 4238, ff. 46v–47v, where the maximum entry, 2;10°, corresponds to arguments of center 1,32°–1,34° (92°–94°).

Table 6.1A: Alfonsine solar equation (excerpt)

Argument (°)	(°)	Solar equation (°)
0 1	5 59	0; 2,10
0 2	5 58	0; 4,19
. . .		
0 30	5 30	1; 2,54
. . .		
1 0	5 0	1;50,54
. . .		
1 30	4 30	2; 9,57
1 31	4 29	2; 9,59
1 32	4 28	2;10, 0
1 33	4 27	2;10, 0
1 34	4 26	2;10, 0
1 35	4 25	2; 9,57
. . .		
2 0	4 0	1;54,57
. . .		
2 30	3 30	1; 7, 7
. . .		
2 59	3 1	0; 2,29*
3 0	3 0	0; 0; 0

* Instead of 0;2,24, as in the printed editions of the Alfonsine Tables.

In the Ptolemaic tradition the solar equation is characterized by a single parameter, its maximum value (c_{max}). It is related to the eccentricity, e, and the radius of the deferent, R = 60, by the modern expression: $\sin c_{max} = e/R$. In *Almagest* III.6, Ptolemy used an eccentricity of 2;30, corresponding to a maximum solar equation of 2;23°, and constructed a table giving the solar equation as a function of the mean solar anomaly, $\bar{\kappa}$. It is noteworthy that this function, given at intervals of 6° in the first half of the table and of 3° in the second, reaches its maximum, 2;23°, for values of the argument slightly greater than 90°, in the interval 90°–96°. For the most part this model remained unchanged in subsequent centuries but for the basic underlying parameter. However, in the tradition based on Indian sources, the maximum solar equation corresponds to argument 90°, and the remaining values in the table are computed by an interpolation function rather than by the geometry of a model. So, in the case of al-Khwārizmī's table for the solar equation, the entries were computed by the formula

$$c(\kappa) = c_{max} \cdot (\delta(\kappa) / \varepsilon)$$

where c_{max} is equal to 2;14°, δ is the solar declination corresponding to the argument, κ, and ε is the obliquity of the ecliptic, here taken to be 23;51° (Neugebauer 1962a, pp. 95–96).

In Table 6.1B we present a list of eccentricities and maximum values of the solar equation found in various sets of tables, showing the range of these parameters.

We note that Ibn Muʿādh, despite using al-Khwārizmī's maximum equation, applies it in a Ptolemaic model (see Samsó 1992, p. 157). On the other hand, Andalusian and Maghribi astronomers, beginning with Azarquiel, constructed models with variable eccentricities (see § 4.2, above), whose minimum and maximum values are:

Ibn al-Kammād	1;50,30–2;29,30
Ibn al-Hāʾim	1;51,30–2;29,30
Ibn al-Bannāʾ	1;50,59–2;29,33
Ibn Isḥāq	1;50,56–2;29,30

Table 6.1B: Some historical values of the maximum solar equation

	Eccentricity (parts)	Maximum solar equation (°)
Almanac of Azarquiel		1;52,0–1;52,50
Ibn al-Kammād		1;52,44
Ḥabash		1;59
al-Battānī	2; 4,45	1;59,10
Toledan Tables		1;59,10
Isaac Israeli		1;59,10
Levi ben Gerson (I)	2;14	2; 8, 0
Castilian Alfonsine Tables		2;10
Parisian Alfonsine Tables		2;10
al-Khwārizmī		2;14
Ibn Muʿādh		2;14
Levi ben Gerson (II)	2;23	2;17
Ptolemy	2;30	2;23

We also note that the mean value for the eccentricity in the case of Ibn al-Kammād is precisely 2;10,0, a parameter that later appears in the Alfonsine Tables (see Table 6.1A).

In some sets of tables the presentation of this table differs slightly because the argument is shifted. For example, in the Tables of Barcelona and those by Joseph Ibn Waqār (14th century, Castile), rather than having the mean solar anomaly as argument, the longitude of the solar apogee at the time has been added and the argument becomes the mean solar longitude. In both cases the table is based on Ibn al-Kammād's maximum solar equation of 1;52,44° and a longitude of 79° for the solar apogee (Chabás 1996a, pp. 489–494). On Joseph Ibn Waqār, see Castells 1996.

2. *Moon*

In Ptolemy's first lunar model the Moon moves on an eccentric deferent circle whose center, D, is placed at a distance, e, called the eccentricity, from the center of the Earth, O (see Figure 11). This model is analogous to the model used by Ptolemy for the motion of the Sun. For Ptolemy OD = e = 5;15 where the radius of the deferent, DL (parallel to $O\overline{L}$), is 60, the Moon being at L. In the epicyclic version of Ptolemy's first lunar model, the radius of the epicycle, CL in Figure 12, is equal to the eccentricity OD in Figure 11; the mean anomaly, $\overline{\alpha}$, in Figure 11 is $\angle AO\overline{L} = \angle ADL$, and in Figure 12 it is $\angle AOC = \angle \overline{L}CL$; and in both figures $\angle VO\overline{L}$ is the mean longitude, $\overline{\lambda}$. In contrast to the solar model where the apogee is fixed in longitude, here the apogee moves at the rate of the difference between the mean lunar anomaly and the mean lunar motion in longitude. In other words, the lunar apogee is found by subtracting the mean lunar anomaly from the mean lunar longitude, where:

$$\overline{\lambda} = \overline{\lambda}_0 + \Delta t \cdot \overline{v}_m.$$

In this equation $\overline{\lambda}_0$ is the mean lunar longitude at some epoch, Δt is the time since epoch in days, and $\overline{v}_m$ is the daily mean lunar motion in longitude.

The lunar equation, $c(\overline{\alpha})$, is the difference between the true and the mean longitudes of the Moon, and it is expressed as a function of a single variable, the mean lunar anomaly. In Figure 11

$$c(\overline{\alpha}) = \angle OLD = \angle LO\overline{L} = \lambda - \overline{\lambda},$$

where $\overline{\alpha}$, the mean lunar anomaly, is defined as

$$\overline{\alpha} = \overline{\alpha}_0 + \Delta t \cdot \overline{v}_a,$$

$\overline{\alpha}_0$ is the mean lunar anomaly at some epoch, Δt is the time since the epoch (in days), and $\overline{v}_a$ is the daily mean motion in lunar anomaly. Note that, as was the case for the Sun, $c \leq 0$ for $0° \leq \overline{\alpha} \leq 180°$, and $c \geq 0$ for $180° \leq \overline{\alpha} \leq 360°$.

The first lunar model was successful in accounting for lunar positions at syzygy (conjunction and opposition), but Ptolemy found it to be inadequate for other lunar phases. Therefore he introduced a second lunar model that reduces to the first model at syzygy (see Figure 13). In this model Ptolemy took the epicyclic version of the first model such that the radius of the epicycle, A_eC, is 5;15, and the radius of the deferent of the first model now comes in two parts: one part, OD = 10;19, rotates around the center of the Earth on circle DD′ such that angle DOC is twice the elongation, η, the angular distance between the mean longitude of the Moon and the mean longitude of the Sun; and the other part, DC = 49;41, is the radius of an instantaneous deferent whose center is D and whose intersection with the direction of the mean longitude of the Moon is C. A further

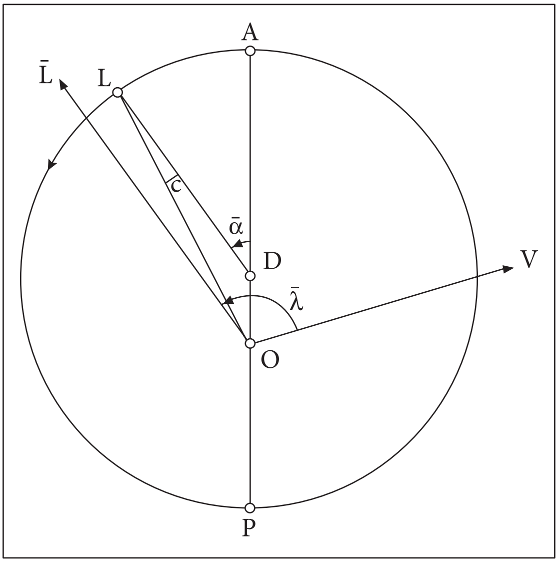

Figure 11: Ptolemy's first lunar model, eccentric version

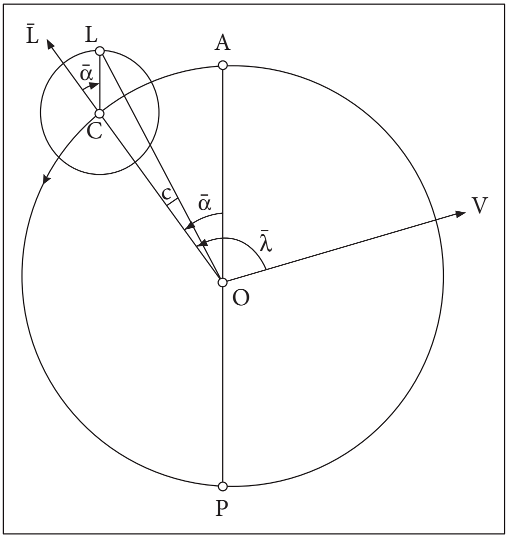

Figure 12: Ptolemy's first lunar model, epicyclic version

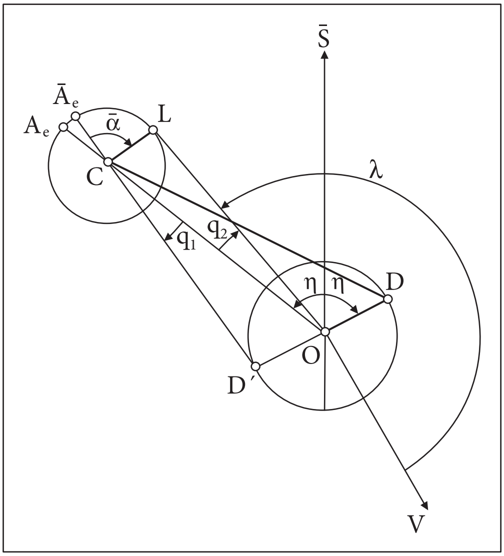

Figure 13: Ptolemy's second lunar model

complication is that the mean lunar anomaly, $\bar{\alpha}$, is counted in the direction opposite to the motion on the deferent, from a point $\bar{A}_e$ on the epicycle, where it intersects D′C (extended). By definition, point D′ is 180° from D on the circle whose radius is 10;19. At syzygy, 2η = 0°, and so the direction of the mean lunar longitude is the same as the mean solar longitude, i.e., OD and DC lie on the same line. Hence, in this circumstance OC = 60 and we have the epicyclic version of the first model.

In *Almagest* V.8 Ptolemy displays the table for the lunar equation in 7 columns (including a column for the lunar latitude). If c_i refers to the i-th column, then the general equation for the second lunar model is:

$$c = c_4(\alpha) + c_5(\alpha) \cdot c_6(2\eta)$$

where

$$\alpha = \bar{\alpha} + c_3(2\eta),$$

paying attention to the rules for the algebraic signs of the entries in the various columns. The term, $c_5(\alpha)$, is called the increment, and it represents the effect of bringing the epicycle closer to the observer in Ptolemy's second lunar model than in the epicyclic version of his first lunar model. For Ptolemy the maximum value for this increment was 2;40°. The term, c_6, serves for purposes of interpolation between extremal values, thereby reducing the number of entries needed for this table. In order to find the true longitude of the Moon, c has to be subtracted from its mean longitude if $\alpha < 180°$, but has to be added if $\alpha > 180°$. Note that in the second lunar model there are two independent arguments, the true anomaly, α, and the double elongation, 2η, whereas in the first model there is only one variable, the mean anomaly, $\bar{\alpha}$. The two equations q_1 and q_2 in Figure 13 are the equation of center and the equation of anomaly, respectively.

Ptolemy's table was standard throughout the Middle Ages, although various authors presented some of its columns in a different order. The following table displays the equivalence of the columns in several relevant sets of tables.

	argument	solar equation	equation of center	equation of anomaly	increment	minutes of proportion	latitude
Almagest	c_1 and c_2		c_3	c_4	c_5	c_6	c_7
al-Battānī	c_1 and c_2	c_3	c_5	c_4	c_7	c_6	c_8
Toledan Tab.	c_1 and c_2		c_3	c_6	c_5	c_4	c_7
Par. Alf. Tab.	c_1 and c_2		c_3	c_6	c_5	c_4	

It is noteworthy that, apart from the column for latitude which was tabulated separately, the presentation of the Parisian Alfonsine Tables agrees exactly with that of the Toledan Tables although the underlying parameters are different. Table 6.2A displays an excerpt of the Alfonsine lunar equations as they appear in Madrid, Biblioteca Nacional, MS 4238, ff. 47v–49v. Table 6.2B lists some historical values for the maximum values of the lunar equations.

Although the lunar equations were usually presented in a single table, this practice was not followed in some cases. Table 6.2C displays an excerpt of a table limited to the equation of anomaly and its increment, which comes after a table entirely devoted to the equation of center. It is found in Paris, Bibliothèque nationale de France,

Table 6.2A: Alfonsine lunar equations (excerpt)

Argument (°)	Argument (°)	Equation of center (°)	Min. prop. (′)	Increment (°)	Equation of anom. (°)
0 1	5 59	0; 9	0	0; 3	0; 4,46
0 2	5 58	0;18	0	0; 5	0; 9,31
. . .					
1 35	4 25				4;56, 0
. . .					
1 42	4 18			2;40	
. . .					
1 49	4 11			2;40	
. . .					
1 54	4 6	13; 9			
1 55	4 5	13; 9			
. . .					
2 59	3 1	0;23	60	0; 0	0; 5,40
3 0	3 0	0; 0	60	0; 0	0; 0, 0

Table 6.2B: Some historical values for the maximum values of the lunar equations

	Eq. of center	Eq. of anomaly	Increment
Almagest	13;9° (at 114°)	5; 1° (at 96°)	2;39° (at 99°–105°)
al-Battānī	13;9° (at 114°–115°)	5; 1,0° (at 95°)	2;40° (at 103°–109°)
Toledan Tables	13;9° (at 114°)	5; 1,0° (at 95°)	2;40° (at 103°–109°)
Par. Alf. Tables	13;9° (at 114°–115°)	4;56,0° (at 95°)	2;40° (at 103°–109°)

MS lat. 10262, ff. 14v–15v, containing the only known copy of the anonymous Tables of the Seven Planets for 1340. A most interesting feature of this table is that its two columns are displaced upwards compared to standard tables (e.g., Table 6.2A), to make all entries positive, thus allowing the computer to avoid dealing with a set of complicated rules for adding and subtracting various terms in an equation (these rules were later replaced by simpler rules using algebraic signs). The entries in Table 6.2C for the equation of anomaly are 4;56,0° higher that the corresponding ones in Table 6.2A; similarly, those for the increment are 2;40° higher. Since the canons to these tables are not preserved, we cannot say how the unknown compiler of these tables explained its use in computing the equation of lunar anomaly. For the expression, "displaced tables," see Kennedy 1977b, p. 16; for the occurrence of these tables in Arabic and Hebrew zijes, see Salam and Kennedy 1967, p. 497; Kennedy 1971, p. 69; Goldstein 1974, pp. 139–140, 229–238; Samsó and E. Millás 1994, pp. 19–20.

As shown in Table 6.2A, the maximum value for the equation of anomaly in the Parisian Alfonsine Tables is 4;56°, a parameter which had already appeared in Indian texts, and it was transmitted to the Islamic world (see the zij of al-Khwārizmī: Suter 1914, p. 134; Neugebauer 1962a, p. 96). In the Iberian Peninsula this parameter was used by Ibn Muʿādh (d. 1093) in a Ptolemaic lunar model in his *Tabule Jahen* (Samsó 1992, p. 157) and the table was reproduced in the almanac of Ferrand Martines, *ca*. 1391 (Chabás 1996b, p. 271). The "increment" refers to the maximum difference between the equation of anomaly at quadrature (when $\eta = 90°$) and the equation of anomaly at syzygy, and it is usually called *diversitas diametri* in Latin texts.

Ptolemy's model was, with few exceptions, unchallenged throughout the Middle Ages. Levi ben Gerson was an astronomer who introduced a new lunar model for which he produced tables (Goldstein 1974, pp. 53–74, 212–217). In the 13th century Naṣīr al-Dīn al-Ṭūsī (in

Table 6.2C: Displaced lunar equations (excerpt)

Argument (s)	Argument (°)	Equation of anomaly (°)	Increment (°)
0	0	4;56, 0	2;40
0	1	4;51,14	2;37
. . .			
3	4	0;..0, 5	
3	5	0;..0, 0	
3	6	0;..0, 4	
. . .			
3	12		0; 1
3	13		0; 0
. . .			
3	19		0; 0
3	20		0; 1
. . .			
8	10		5;19
8	11		5;20
. . .			
8	17		5;20
8	18		5;19
. . .			
8	24	9;51,56	
8	25	9;52, 0	
8	26	9;51,55	
. . .			
11	28	4; 5,31	2;45
11	29	4; 0,56	2;43

Iran) introduced other lunar models, based on a mathematical lemma now called the "Ṭūsī couple," that were supposed to produce the same results as those which follow from Ptolemy's model, and the eastern Arabic astronomers who pursued this research are usually called the "Marāgha School," after the observatory where al-Ṭūsī was the director (see Roberts 1957, Hartner 1969, Ragep 1993, Saliba 1996).

3. *Planets*

Ptolemy introduced an equant model for four planets (Saturn, Jupiter, Mars, and Venus), and a different, more complicated, model for Mercury (O. Pedersen 1974, pp. 261–328). For the model for Mars, see Figure 14, where the observer is at O, the direction to the vernal point is OV, the center of the deferent is D, and the equant point is E such that OD = DE. Point A is the apogee of the deferent, and one argument, the mean center, $\bar{\kappa}$, is measured about point E counted from the direction of EA. The second argument is the mean anomaly, $\bar{\alpha}$, and it is measured from the epicyclic mean apogee, $\bar{A}_e$, which lies on the extension of the line from the equant through the center of the epicycle. In contrast to the lunar model, motion on the planet's epicycle takes place is the same direction as that on the deferent. Mars is at point M and its true longitude, λ, is angle VOM which in turn is the algebraic sum of the mean longitude VO$\bar{M}$, q_1, and q_2, where q_1 is the equation of center, q_2 is the equation of anomaly, and $c(\bar{\kappa}, \bar{\alpha}) = q_1 + q_2$ is the total correction. In Fig. 14 CE is parallel to O$\bar{M}$; hence angle ECO = angle CO$\bar{M}$. Note that the argument for q_2 is the true anomaly, α, angle MCA_e, which is the algebraic sum of $\bar{\alpha}$ and q_1. Moreover, MC is always parallel to the direction from the observer to the mean Sun, O$\bar{S}$, a characteristic feature of Ptolemy's models for the outer planets.

In *Almagest* XI.11 there are tables for computing the longitudes of each of the planets, all of which are treated alike. The first step is to compute $\bar{\kappa}$ and $\bar{\alpha}$, both of which depend on mean motions that in

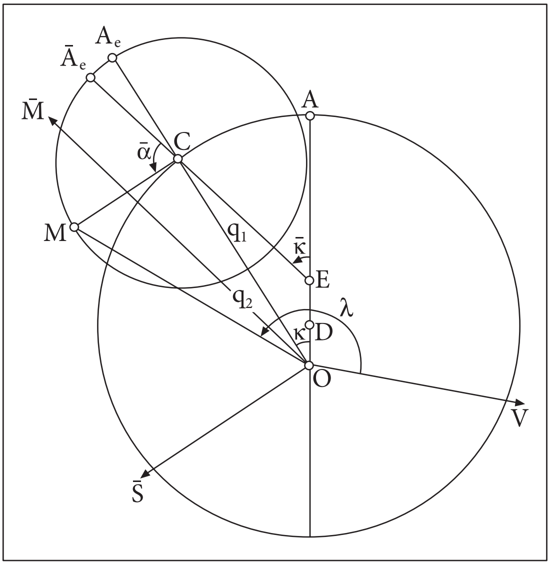

Figure 14: Ptolemy's model for the longitude of Mars, configured for November 17, 1447 at 18;40h, counted from noon

turn depend on the time since epoch. The second step is to compute $c_3(\overline{\kappa})$—in al-Battānī's numbering of the columns—which is the equation of center and the correction to the mean anomaly $\overline{\alpha}$, to get the true anomaly, $\alpha = \overline{\alpha} + c_3(\overline{\kappa})$. It follows that:

$$q_1 = c_3(\overline{\kappa}),$$

where $c_3(\overline{\kappa})$ can be positive or negative.

Then, with α as argument, one enters column 6 and either column 5 or column 7, and one enters column 4 (for purposes of interpolation) with $\overline{\kappa}$ as argument (for details, see Neugebauer 1975, pp. 183–186). The algorithm for finding q_2, the equation of anomaly, is then:

$$q_2 = c_6(\alpha) + c_4(\overline{\kappa}) \cdot c_5(\alpha), \text{ if } c_4 \leq 0$$

or

$$q_2 = c_6(\alpha) + c_4(\overline{\kappa}) \cdot c_7(\alpha), \text{ if } c_4 \geq 0.$$

At maximum distance of the epicycle from the observer (or apogee),

$$q_2 = c_6(\alpha) - c_5(\alpha),$$

whereas at mimimum distance (or perigee),

$$q_2 = c_6(\alpha) + c_7(\alpha).$$

Then, the total correction is

$$c(\overline{\kappa}, \overline{\alpha}) = q_1 + q_2.$$

With very few exceptions, these planetary correction tables were reproduced in later sets of astronomical tables, although in several cases some of the underlying parameters were modified. However, the planetary equation tables in al-Khwārizmī's zij are based on Indian models that differ from Ptolemy's models and have no equant (see Neugebauer 1962a, pp. 23–30, 98–101). The following table displays the equivalence of the columns in several standard sets of tables, where c_i refers to the i-th column. Note that the values for the equation of center in the *Almagest* are not explicitly given but result from combining the "equation in longitude," c_3, and the "difference in equation," c_4 (see Neugebauer 1975, p. 183).

	argument	equation in longitude	difference in equation	subtractive difference	equation of anomaly	additive difference	minutes of proportion
Almagest	c_1 and c_2	c_3	c_4	c_5	c_6	c_7	c_8
Handy Tables	c_1 and c_2	equation of center: c_3		c_5	c_6	c_7	c_4
al-Battānī	c_1 and c_2	equation of center: c_3		c_5	c_6	c_7	c_4
Toledan Tab.	c_1 and c_2	equation of center: c_3		c_5	c_6	c_7	c_4
Par. Alf. Tab.	c_1 and c_2	equation of center: c_3		c_5	c_6	c_7	c_4

Table 6.3A displays the maximum values of the equations of center and anomaly in several important sets of tables, showing the main traditions and some of their variations. Table 6.3B displays the equation of Jupiter in the Toledan Tables.

Most tables for the planetary equations consist of a single table for each planet, as is the case for all the sets mentioned previously in this section. But there were other forms of presentation, taking into account the fact that the planetary equations depend on two variables, its mean center and its mean anomaly. For example, in Erfurt, Biblioteca Amploniana, MS Q 362, a manuscript containing material from the 11th to the 14th century, there is a set of tables for the equations of the planets in the framework of the Parisian Alfonsine Tables, where for each planet we are given two different tables: one for $\bar{\kappa}$, displaying the equation of center, the differences between two consecutive entries, and the minutes of proportion; and the other for α (the true anomaly), displaying the equation of anomaly, the differences between two consecutive entries, and the two columns to interpolate between apogee and perigee.

John Vimond seems to be responsible for the first occurrence in Latin of this separation of the functions that depend on the mean argument of center from those that depend on the mean argument of anomaly, probably following a tradition among Andalusian and Maghribī astronomers (see Mestres 1999; Chabás and Goldstein 2004, pp. 236–256). Tables 6.3C and 6.3D display excerpts of Vimond's tables for Jupiter, taken from Paris, Bibliothèque nationale de France, MS lat. 7286C, ff. 6v–7r. These tables have other unprecedented features, e.g., in Table 6.3C, instead of displaying the equation of center, Vimond displayed what he labeled *motus completus*, that is, the argument of center corrected for the equation of center (for a detailed account of the rest of the columns, see Chabás and Goldstein 2004, p. 239). Note

Table 6.3A: Some historical values of the planetary equations

Saturn	Eq. of center	Eq. of anomaly
Almagest	6;31° (at 90°–93°)	6;13° (at 96°)
Handy Tables	6;31° (at 90°–94°)	6;13° (at 94°–99°)
al-Khwārizmī	8;36° (at 90°)	5;44° (at 95°–98°)
al-Battānī	6;31° (at 90°–94°)	6;13° (at 94°–99°)
Toledan Tables	6;31° (at 90°–94°)	6;13° (at 94°–99°)
Par. Alf. Tables	6;31° (at 90°–94°)	6;13° (at 94°–99°)

Jupiter	Eq. of center	Eq. of anomaly
Almagest	5;15° (at 90°–96°)	11; 3° (at 102°)
Handy Tables	5;15° (at 89°–96°)	11; 3° (at 99°–102°)
al-Khwārizmī	5; 6° (at 89°–91°)	10;52° (at 98°–103°)
al-Battānī	5;15° (at 92°–97°)	11; 3° (at 99°–102°)
Toledan Tables	5;15° (at 88°–96°)	11; 3° (at 99°–102°)
Par. Alf. Tables	5;57° (at 90–96°)	11; 3° (at 99°–102°)

Mars	Eq. of center	Eq. of anomaly
Almagest	11;25° (at 93°–96°)	41; 9° (at 132°)
Handy Tables	11;25° (at 93°–96°)	41;10° (at 131°)
al-Khwārizmī	11;13° (at 90°)	40;31° (at 128°–129°)
al-Battānī	11;25° (at 93°–96°)	41; 9° (at 130°–132°)
Toledan Tables	11;24° (at 91°–97°)	41; 9° (at 130°–132°)
Par. Alf. Tables	11;24° (at 92°–96°)	41;10° (at 130°–132°)

Venus	Eq. of center	Eq. of anomaly
Almagest	2;24° (at 90°)	45;59° (at 135°)
Handy Tables	2;24° (at 85°–89°)	45;59° (at 135°–136°)
al-Khwārizmī	2;14° (at 90°)	47;11° (at 135°)
al-Battānī	1;59° (at 84°–97°)	45;59° (at 135°–136°)
Toledan Tables	1;59° (at 90°–93°)	45;59° (at 135°–136°)
Par. Alf. Tables	2;10° (at 87°–98°)	45;59° (at 135°–136°)

Table 6.3A (*cont.*)

Mercury		
	Eq. of center	Eq. of anomaly
Almagest	3; 2° (at 93°–99°)	22; 2° (at 111°)
Handy Tables	3; 2° (at 93°–97°)	22; 2° (at 111°–112°)
al-Khwārizmī	4; 2° (at 90°)	21;30° (at 112°–113°)
al-Battānī	3; 2° (at 93°–97°)	22; 2° (at 111°–112°)
Toledan Tables	3; 2° (at 93°–97°)	22; 2° (at 111°–112°)
Par. Alf. Tables	3; 2° (at 93°–97°)	22; 2° (at 111°–112°)

Table 6.3B: Equations for Jupiter in the Toledan Tables (excerpt)

Argument (s, °)	Argument (s, °)	Equation of center (°)	Min. prop. (′)	*Long. long.* (°)	Equation of anom. (°)	*Long. prop.* (°)
0 1	11 29	0; 6	60	0; 0	0;10	0; 0
0 2	11 28	0;11	60	0; 1	0;20	0; 1
0 3	11 27	0;16	60	0; 1	0;29	0; 1
. . .						
2 27	9 3	5;14	2			
2 28	9 2	5;15	1			
2 29	9 1	5;15	1			
3 0	9 0	5;15	2			
. . .						
3 6	8 24	5;15				
3 7	8 23	5;14				
3 8	8 22				11; 2	
3 9	8 21				11; 3	
. . .						
3 12	8 18				11; 3	
3 13	8 17				11; 2	
. . .						
3 16	8 14			0;29		
3 17	8 13			0;30		
3 18	8 12					0;32
3 19	8 11					0;33
. . .						
3 28	8 2			0;30		
3 29	8 1			0;29		
4 0	8 0					0;33
4 1	7 29					0;32
. . .						
5 28	6 2	0;12	60	0; 2	0;29	0; 2
5 29	6 1	0; 6	60	0; 1	0;15	0; 1
6 0	6 0	0; 0	60	0; 0	0; 0	0; 0

Table 6.3C: Equation of center for Jupiter according to John Vimond (excerpt)

Argument		*Motus Completus*		Increment	Daily velocity	Min. prop.	First station	
s	(°)	s	(°)	min	min	min	s	(°)
0	6	0	11;43	58; 0	5	23	4	5;19
0	12	0	17;31	57;20	5	20	4	5; 9
. . .								
2	12	2	12;59	44;10	4	0	4	4; 6
2	18	2	18;23	44; 0	4	0	4	4; 5
2	24	2	23;48	44; 0	4	0	4	4; 5
3	0	2	29;12	44;10	4	0	4	4; 6
. . .								
5	18	5	12; 5	59;40	4	28	4	5;36
5	24	5	18; 3	60;10	4	32	4	5;44
6	0	5	24; 4	60;50	4	34	4	5;52
. . .								
8	12	8	10;55	66;20	6	59	4	7;10
8	18	8	17;33	66;30	6	60	4	7;11
8	24	8	14;14	66;20	6	60	4	7;11
9	0	9	0;52	66;20	6	60	4	7;10
. . .								
11	12	11	17;55	60;20	5	34	4	5;55
11	18	11	23;57	59;50	5	32	4	5;47
11	24	11	29;56	59;10	5	29	4	5;39
12	0	12	0;51*	58;40	5	26	4	5;29

* Instead of 5;51.

Table 6.3D: Equation of anomaly for Jupiter according to John Vimond (excerpt)

Argument				*Motus completus*	Increment	Daily veloc.	Subtr. & add. diff.	Increment	Veloc.
s	(°)	s	(°)	(°)	min	min	min	sec	sec
0	6	11	24	0;57	9	8	0; 4	1	1
0	12	11	18	1;52	9	8	0; 8	1	1
. . .									
3	6	8	24	10;33	0	0	0;59	0	0
3	12	8	18	10;34	1	1	1; 1	0	0
3	18	8	12	10;29	2	2	1; 2	0	0
3	24	8	6	10;15	3	3	1; 3	0	0
4	0	8	0	9;54	5	4	1; 2	0	0
. . .									
5	24	6	6	1;21	13	12	0; 9	1	1
6	0	6	0	0; 0	13	12	0; 0	1	1

that the maximum equation of center is 5;57°, and it occurs at 5s 24° (5s 24°–5s 18;3° = 5;57°) and the minimum at 11s 18° (11s 18°–11s 23;57° = –5;57°). This parameter, unknown in the previous astronomical literature, was adopted in the Parisian version of the Alfonsine Tables. Another new feature is that the maximum equation of center is reached at an argument of 174° (5s 24°), indicating that in this particular table the entries are shifted by about 80° as compared with other tables for the same purpose, such as those in the zij of al-Battānī and the Toledan Tables (see Table 6.3A). This shift results from the difference between the apogee of Jupiter and that of the Sun and is intended to make the table more user-friendly.

In Table 6.3D column 2 gives, for each value of the mean argument of anomaly in column 1, the difference between the equation of anomaly and the correction at maximum distance, here also called *motus completus*. Its highest value, 10;34°, is reached at 3s 12°, and it is equivalent to a maximum equation of anomaly of 11;3°, as found in all other tables for the same purpose (zij of al-Battānī, Toledan Tables, among others), where the correction at maximum distance is 0;29°. For a detailed account of the rest of the columns, see Chabás and Goldstein 2004, pp. 250–252.

Vimond's tables certainly have a structure that differs from any other tables for the planetary equations. But the main improvement was to present them in the form of double argument tables which greatly simplified computations, for they only require linear interpolation. John of Lignères seems to have been the first astronomer in the Latin West to draw up a double argument table, combining the equations of center and anomaly for each planet. Such tables, set up in different ways, were also constructed by later astronomers, such as William Batecombe, Judah ben Verga, Abraham Zacut, and Bianchini, among others (North 1977; Goldstein 2001, pp. 248–250, 270–272; for Bianchini's tables, see Chabás and Goldstein 2009b). There are also double argument tables for planetary and lunar velocities, for planetary positions and latitudes, and for the time from mean to true syzygy (see §§ 7.3, 8.2, 9.2, 13.2 and 13.3, and ch. 14, below).

In Table 6.3E we reproduce an excerpt of the double argument table for the equation of Jupiter, as found in Erfurt, Biblioteca Amploniana, MS CA 2° 388, ff. 8r–10v, a manuscript containing the *Tabule magne* ascribed to John of Lignères (see § 5.1). Both $\overline{\alpha}$ (given as the first column) and $\overline{\kappa}$ (given as the horizontal heading) are given in integer degrees at 6°-intervals.

Table 6.3E: Double argument table for the equation of Jupiter (excerpt)

$\overline{\kappa}$ / $\overline{\alpha}$ (s, °)	0s 6° (°)	0s12° (°)	. . .	8s18° (°)	8s24° (°)	9s 0° (°)	. . .	11s24° (°)	12s 0° (°)
	M	M		A	A	A		A	A
0 0	0;30	0;59	. . .	4;55	4;58	4;59	. . .	0;30	0; 0
	A								
0 6	0;26	0; 3	. . .	5;54	5;58	5;57	. . .	1;27	0;56
		A							
0 12	1;22	0;52	. . .	6;52	6;56	6;55	. . .	2;23	1;53
. . .									
3 12	9;58	9;22	. . .	16;58	16;59	16;56	. . .	11;10	10;34
3 18	9;52	9;16	. . .	16; 0*	17; 1	16;58	. . .	11; 6	10;29
3 24	9;37	9; 0	. . .	16;56	16;57	16;53	. . .	10;53	10;15
. . .									
5 18	1;56	1;13	. . .	10; 2	10; 6	10; 4	. . .	7;44	7; 7
		M							
5 24	0;36	0; 7	. . .	8;42	8;46	8;45	. . .	7;44	7; 7
	M								
6 0	0;44	1;27	. . .	7;17	7;22	7;21	. . .	0;39	0; 0

M stands for *Minue* and A for *Adde*.
* Instead of 17;1.

In some cases, as in London, British Library, MS add. 24070, ff. 25r–42v, these double argument tables also display successive differences, both for $\overline{\alpha}$ and $\overline{\kappa}$.

CHAPTER SEVEN

TRUE POSITIONS

Some sets of tables, rather than giving the mean motion of a celestial body and its equations at a certain time, just list true positions for specific dates, thus facilitating the task of the computer. These tables are commonly, but not exclusively, found in almanacs.

1. *Sun*

The table giving the true position of the Sun for each day of a year is usually part of a group of four tables covering a period of four consecutive years. This is the case in the Almanac of Azarquiel (epoch: September 1, 1088), the Almanac of Jacob ben Makhir (March 1, 1301), the Almanac of 1307 (March 1, 1307), and Abraham Zacut's *Ḥibbur* (March 1, 1473 or January 1, 1473, depending on the manuscript), just to mention a few. In other cases, the table is isolated. The entries are usually given in degrees and minutes, but sometimes they are only given to degrees, more rarely to seconds. Table 7.1A is not integrated in a 4-year cycle; it is taken from the Tables for 1321 by John of Murs and gives, to seconds, the true positions of the Sun at Toledo for that year (Lisbon, MS Ajuda 52-XII-35, ff. 57v–58v). We note that physical signs of 60° are used here (Chabás and Goldstein 2009a).

Table 7.1A: True positions of the Sun at Toledo for 1321 (excerpt)

Day	January (°)	February (°)	March (°)	. . .	November (°)	December (°)
1	4,49;47, 5	5,21;17,33	5,49;19,14	. . .	3,47;12,50	4,17;47,50
2	4,50;48,22	5,22;18, 9	5,50;18,43	. . .	3,48;13,45	4,18;49,22
3	4,51;49;34	5,23;18,43	5,51;18, 9	. . .	3,49;14,39	4,19;50,49
. . .						
12	5, 1; 0,38	5,32;21,50	0, 0;11,43	. . .	3,58;23,50	4,29; 4, 6
. . .						
30	5,19;16,19	–	0,17;49,21	. . .	4,16;46,33	4,47;29,43
31	5,20;16,59	–	0,18;46,49	. . .	–	4,48;30,58

Table 7.1B: Solar correction

4-year cycle	Correction (°)	4-year cycle	Correction (°)
1	0; 1,46	18	0;31,46
2	0; 3,32	19	0;33,32
3	0; 5,18	20	0;35,18
4	0; 7, 4	21	0;37, 4
5	0; 8,50	22	0;38,50
6	0;10,36	23	0;40,36
7	0;12,22	24	0;42,22
8	0;14, 8	25	0;44, 8
9	0;15,54	26	0;45,54
10	0;17,40	27	0;47,40
11	0;19,25	28	0;49,25
12	0;21,11	29	0;51,11
13	0;22,57	30	0;52,57
14	0;24,43	31	0;54,43
15	0;26,29	32	0;56,29
16	0;28,15	33	0;58,15
17	0;30, 0	34	1; 0, 0

In order to use Table 7.1A for times in the future or the past, it is sometimes associated with another table, containing the corrections to be added to the true positions of the Sun after successive 4-year cycles. The joint use of both tables makes them work as a perpetual almanac. The table for the solar correction is found, for example, in Zacut's *Ḥibbur* (see Table 7.1B, taken from Segovia, Biblioteca de la Catedral, MS 110, f. 19r).

A different way of presenting the true positions of the Sun for specific times is to list the times of entry of the Sun into the zodiacal signs. Table 7.1C is taken from Oxford, Bodleian Library, MS Can. Misc. 27, f. 118r, and was computed from the *Tabulae Resolutae* that reached Salamanca no later than 1461 (see Chabás and Goldstein 2000, pp. 47–49).

The last two rows in Table 7.1C represent the entry of the Sun into two specific degrees of a particular sign, Scorpio, but the canons for this table do not explain what was of interest for those two dates in November 1461.

Even more unusual is a table labeled "entry of the Sun into months," giving the true position of the Sun at the beginning of each month for a particular year. Table 7.1D is taken from Florence, Biblioteca

Table 7.1C: Entry of the Sun into zodiacal signs at Salamanca for 1461

Sign	Month	*Tempus diebus non equatis* (d)	(h)	*Tempus diebus equatis* (d)	(h)	*Ascensiones ascendentis* (°)
Aqr	Jan	10	0; 9,10, 0	10	0;11,58	35;12,30
Psc	Feb	8	14;54,58,23	8	14;54,22	285;57,30
Ari	Mar	10	18; 0,30, 1	10	18; 8,38	2; 9,30
Tau	Apr	10	11; 6,47, 0	10	11;24,43	289; 3,45
Gem	May	11	16;45,33, 0	11	17; 6,53	44;30,15
Cnc	Jun	12	52; 8,38, 0	12	5;45, 6	266;16,30
Leo	Jul	13	18;24,50,42	13	18;36,19	131;17,45
Vir	Aug	14	0;36,13,46	14	0;50,42	254;47,30
Lib	Sep	13	18;25,44,33	13	18;49,44	192;26, 0
Sco	Oct	13	22;17,42,48	13	22;49, 7	280; 9,45
Sgr	Nov	12	13;38,16,34	12	14; 6,57	179;31,15
Cap	Dec	11	21;54, 4,36	11	22; 9,43	332;25,45
Sco 20°	Nov	1	17;59,37, 0	1	18;30,49	234;13,15
Sco 21°	Nov	2	17;59,25,40	2	18;30,22	

Table 7.1D: True position of the Sun at the beginning of each month for 1252

Month	Sun (°)	
March	Psc	17; 3,26
April	Ari	17;29,46
May	Tau	16;30,32
June	Gem	16; 8,46
July	Cnc	14;44,10
August	Leo	14;26, 1
September	Vir	14;30,45
October	Lib	14; 7,31
November	Sco	15;15,26
December	Sgr	15;45,37
January	Cap	17;23,18
February	Aqr	18;48,47

Nazionale Centrale, MS Conv. Soppr. J.V.6 (San Marco 189), f. 83v, a manuscript compiled in the 13th and 14th centuries containing the Toledan Tables and tables for the city of Cremona. The heading of the table, at the lower right margin, specifies that it is valid for the year 1252 (see F. S. Pedersen 2002, p. 1608).

2. *Moon*

The most extensive table giving the true positions of the Moon is in Zacut's *Ḥibbur* (see Table 7.2A, Segovia, Biblioteca de la Catedral, MS 110, f. 21v) for it contains entries for 11,325 consecutive days (31 Julian years and 2 days). For each day we are given, to minutes, the true position of the Moon for mean noon at Salamanca beginning in 1473 and a correction (in minutes) to be applied to the corresponding lunar longitude after one cycle of about 31 years has elapsed (Chabás and Goldstein 2000, pp. 110–114). Note that in this Latin version of the *Ḥibbur* the year begins in January whereas in the *Almanach Perpetuum* it begins in March.

Of course, in addition to those tables intended for precise computations and displaying accurate lunar positions of the Moon, there were others where the user only sought a rough value for the position of the Moon. This is the case for the table by Peter (Philomena) of Dacia, *fl.* 1292–1303, probably compiled in Paris, that displays the zodiacal sign of the Moon for each day of the year (F. S. Pedersen 1983–1984, p. 360). It consists of 14 × 12 cells; the 12 columns are for the months, starting in March, and each of the 14 rows is for a group of two or three days, from 1 to 30 days. Each cell is assigned a zodiacal sign, which is written along a diagonal within the cell, from left to right or, alternatively, from top to bottom or from bottom to top. This table is not infrequent in the astronomical literature and it is usually found in manuscripts containing calendars, such as the calendar of Nicholas de Lynn, *fl.* 1386 (Eisner 1980, pp. 182–183), the calendar of John Somer, *ca.* 1340–1409 (Mooney 1998, p. 146), and isolated in manuscripts

Table 7.2A: True positions of the Moon at Salamanca for 1473 (excerpt)

		January			February			March		. . .
Day	(s)	(°)	(′)	(s)	(°)	(′)	(s)	(°)	(′)	
1	10	23;53	40	0	10;49	35	0	19;13	36	
2	11	7;16	39	0	23;13	34	1	1;26	35	
3	11	20;10	37	1	5;25	33	1	13;23	34	
. . .										
28	10	19; 0	40	0	6;45	36	0	15;35	36	
29	11	2;29	39	–			0	27;53	36	
30	11	15;22	38	–			1	10; 1	35	
31	11	28;14	36	–			1	21;58	35	

Table 7.2B: Zodiacal sign of the Moon

Day	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	A	C	A	U	G	C	L	R	L	O	S	P
	Q	S	R	A	E	N	E	I	I	C	A	A
2	U	P	I	T	M	C	O	V	B	S	G	C
3 4	C	A	U	G	C	L	R	L	O	S	P	A
	S	R	A	E	N	E	I	I	C	A	A	Q
5	P	I	T	M	C	O	V	B	S	G	C	U
6	A	U	G	C	L	R	L	O	S	P	A	C
	R	A	E	N	E	I	I	C	A	A	Q	S
7	I	T	M	C	O	V	B	S	G	C	U	P
8 9	U	G	C	L	R	L	O	S	P	A	C	A
	A	E	N	E	I	I	C	A	A	Q	S	R
10	T	M	C	O	V	B	S	G	C	U	P	I
11	G	C	L	R	L	O	S	P	A	C	A	U
	E	N	E	I	I	C	A	A	Q	S	R	A
12	M	C	O	V	B	S	G	C	U	P	I	T
13 14	C	L	R	L	O	S	P	A	C	A	U	G
	N	E	I	I	C	A	A	Q	S	R	A	E
15	C	O	V	B	S	G	C	U	P	I	T	M
16	L	R	L	O	S	P	A	C	A	U	G	C
	E	I	I	C	A	A	Q	S	R	A	E	N
17	O	V	B	S	G	C	U	P	I	T	M	C
18 19	R	L	O	S	P	A	C	A	U	G	C	L
	I	I	C	A	A	Q	S	R	A	E	N	E
20	V	B	S	G	C	U	P	I	T	M	C	O
21	L	O	S	P	A	C	A	U	G	C	L	R
	I	C	A	A	Q	S	R	A	E	N	E	I
22	B	S	G	C	U	P	I	T	M	C	O	V
23 24	O	S	P	A	C	A	U	G	C	L	R	L
	C	A	A	Q	S	R	A	E	N	E	I	I
25	S	G	C	U	P	I	T	M	C	O	V	B
26	S	P	A	C	A	U	G	C	L	R	L	O
	A	A	Q	S	R	A	E	N	E	I	I	C
27	G	C	U	P	I	T	M	C	O	V	B	S
28 29	P	A	C	A	U	G	C	L	R	L	O	S
	A	Q	S	R	A	E	N	E	I	I	C	A
30	C	U	P	I	T	M	C	O	V	B	S	G

Table 7.2C: Table of the zodiacal signs

Zodiacal sign	Longitude
Aries	0° – 30°
Taurus	30° – 60°
Gemini	60° – 90°
Cancer	90° – 120°
Leo	120° – 150°
Virgo	150° – 180°
Libra	180° – 210°
Scorpio	210° – 240°
Sagittarius	240° – 270°
Capricorn	270° – 300°
Aquarius	300° – 330°
Pisces	330° – 360°

containing diverse items, e.g., Vienna, Nationalbibliothek, MS 5371*, f. 86r. A similar table for the same purpose, which is much older, begins in January (Aquarius), rather than March (Aries), and has 12 rows instead of 14 (see F. S. Pedersen 1983–1984, pp. 14, 393; 2002, p. 1610). It is reproduced in Table 7.2B from Vienna, Nationalbibliothek, MS 2367, f. 9v. In each cell we have written the standard three-letter abbreviation of the zodiacal sign rather than the full name that appears in the manuscript. This table is schematic and does not work in detail. Note, for example, that all months are assumed to have 30 days (even February!). The distribution of two-day and three-day durations of the Moon in a zodiacal sign cannot be the same in all months. Note also that for January the Moon passes through 12 signs, from Aquarius to Capricorn (see Table 7.2C), but the first sign in February is not Aquarius which comes after Capricorn, but Pisces, which means that the Moon has skipped a sign from the end of January to the beginning of February!

3. *Planets*

The tables giving the true positions of the planets are mostly found in almanacs. It is assumed that the positions of a given planet will repeat (but for a small correction) after a specified period. Values for the planetary periods were already determined by Babylonian astronomers in antiquity (Neugebauer 1975, pp. 150–151), and were known to Ptolemy as well (*Almagest* IX.3; Toomer 1984, p. 424). The earliest known almanac compiled in the West is the Almanac of Azarquiel (Millás 1943–1950, pp. 72–237), and the periods used in these tables are almost the same as those of Ptolemy. The epoch of the almanac is September 1, 1088 (beginning of year 1400, Era of Alexander), and the true positions of the planets are given in sidereal coordinates. The Almanac of Azarquiel initiated a long tradition in the systematic computation of planetary positions, of which the Almanac of 1307 and those compiled by Profatius Judaeus, John of Lignères, and John of Saxony are remarkable examples. In Tables 7.3A–D we present specimens of the true longitudes of Mars taken from these four 14th-century almanacs. The fact that they all list true positions for 79 years in the case of Mars, facilitates a direct comparison of the entries; in the tables below we show some entries for the first year of each almanac, and for year 1361, which was arbitrarily selected for purposes of comparison. Note, in particular, that in all four almanacs Mars completes its retrogradation arc at the beginning of January 1361. Even though the samples displayed are limited, they clearly show the differences between the four almanacs in the frequency and accuracy of the entries as well as in other characteristics, such as the time chosen for the beginning of the year and the type of coordinates involved.

The almanac by Profatius Judaeus (Jacob ben Makhir) was composed in Hebrew for the city of Montpellier. The epoch for the planets is March 1300. A Latin version of it was published by Boffito and Melzi d'Eril in 1908, based on Florence, Biblioteca Laurenziana, MS S. Crucis, Plut. XVIII.1, from which Table 7.3A is taken. The entries are given in tropical coordinates.

The Almanac of 1307 has been preserved in Latin, Catalan, Castilian, and Portuguese, but not in its original Arabic version. In all of them the epoch is March 1307. It has also been called "Almanac of Tortosa" because of its city of origin, when only one version—copied in that Mediterranean Spanish city—was known. Table 7.3B displays the entries found in Paris, Bibliothèque nationale de France, MS 7403, ff. 19r–28r.

The almanac compiled by John of Lignères seems to be uniquely preserved in an incomplete copy: Philadelphia, Free Library, MS Lewis E.3, ff. 3r–10r. The set of tables begins abruptly in the middle of a table for Saturn, and there are no tables for the positions of the Sun, the Moon, and the lunar node, as in other almanacs, but the part

Table 7.3A: True positions of Mars in Jacob ben Makhir's almanac (excerpt)

1300 [year 1300/1301]			60 [year 1361]		61 [year 1361]	
March		. . .	. . .		March	
10	Psc 11; 2				10	28;25
20	18;53				20	Cnc 2;44
31	27;27				31	7;52
. . .					. . .	
January		. . .	January			
10	Leo 25;34		10	Gem 14;43		
20	22;26		20	15;50		
31	18;38		31	16;59		
February		. . .	February			
10	13;33		10	19; 9		
20	11; 0		20	21;51		
28	9; 8		28	24; 9		

Table 7.3B: True positions of Mars in the Almanac of 1307 (excerpt)

1 [year 1307/1308]		. . .	54 [year 1361]		55 [year 1361]	
March		. . .	. . .		March	
1	Sco 29				1	Gem 10
6	Sgr 2				6	12
11	4				11	14
16	7				16	17
21	9				21	19
26	11				26	23
. . .					. . .	
January		. . .	January			
1	Psc 12		1	Tau 27		
6	15		6	26		
11	19		11	26		
16	22		16	27		
21	26		21	27		
26	Ari 0		26	28		
February		. . .	February			
1	3		1	29		
6	7		6	Gem 1		
11	10		11	2		
16	13		16	4		
21	17		21	6		
26	20		26	8		

Table 7.3C: True positions of Mars in the almanac of John of Lignères (excerpt)

1 [year 1341]		...	21 [year 1361]	
January		...	January	
10	Sgr 5		10	Gem 17
20	12		20	16
31	19		31	17
February		...	February	
10	26		10	19
20	Cap 3		20	22
28	11		28	25
March		...	March	
10	16		10	28
20	23		20	Cnc 3
31	Aqr 2		31	8
...			...	

corresponding to Mars is complete. The starting date of this almanac is 1341, and the entries were derived from the Parisian Alfonsine Tables: see Table 7.3C.

John of Saxony also composed an almanac which is extant in various manuscripts. The entries displayed in Table 7.3D are taken from Oxford, Bodleian Library, MS Rawlison D.1227, f. 19v. In this manuscript the planetary positions range from 1361 to 1380. As was the case for the almanac of John of Lignères, the entries, here given to the minutes, were derived from the Parisian Alfonsine Tables.

In the almanacs of Azarquiel and Jacob ben Makhir, as well as in many others, we are told to add fixed quantities to the longitude of the planet when it has completed a cycle. In the case of Mars, these values are +1;0° (Almanac of Azarquiel) or +1;40° (Almanac of Jacob ben Makhir) after 79 years have elapsed. However, more refined tables were also compiled for these corrections. Instead of making them constant, some computers made them depend both on the daily velocity of the planet and its position at the end of the cycle. This is the case for the tables for Mars, Venus, and Mercury in Zacut's *Ḥibbur* and in the *Almanach Perpetuum*. Table 7.3E gives an excerpt for Mars (Chabás and Goldstein 2000, pp. 74 and 143).

In addition to displaying the equations of the planets (see Table 6.3E), double argument tables were also used to show the true positions of the planets, greatly facilitating computations. In Table 7.3F we

Table 7.3D: True positions of Mars in the almanac of John of Saxony (excerpt)

1361	
January	
6	Gem 14;10
12	14; 0
18	14;10
24	14;49
31	15;54
February	
6	17;11
12	18;46
18	20;34
24	22;42
28	24; 9
March	
6	26;31
12	29; 1
18	Cnc 1;42
24	4;27
31	7;46
...	

Table 7.3E: Corrections for Mars according to Abraham Zacut (excerpt)

Daily motion (°/d)		Correction (°) Leo	Cnc	Gem	Tau	Ari	Psc	Aqr
0;39		1;33	1;37	1;40	1;44	1;48	1;52	1;55
...								
0; 1		2;21	2;46	3;10	3;35	3; 0*	4;24	4;47
0; 0		2;24	2;48	3;13	3;37	4; 1	4;26	4;51
0; 1	*Retro*	2;26	2;51	3;16	3;41	4; 6	4;31	4;56
0; 3		2;29	2;54	3;20	3;46	4;12	4;38	5; 4
...								
0;25		3; 4	3;33	4; 3	4;33	5; 3	5;33	6; 3

* *Sic.*

Table 7.3F: True positions of Saturn in the *Tabule Anglicane* (excerpt)

$\bar{\alpha}$ $\bar{\kappa}$ (s, °)			0s 6° (°)	0s12° (°)		3s 0° (°)		6s 0° (°)		9s 0° (°)		11s24° (°)	12s 0° (°)
0	6	Sgr	17;39	18;12	...	22;54	...	16;46	...	11; 9	...	16;31	17; 5
0	12	Sgr	23; 4	23;38	...	28;16	...	22;14	...	16;31	...	21;56	22;30
...													
3	0	Psc	6;23	6;58	...	11;24	...	4;21	...	Aqr 29; 6	...	Psc 5;13	5;48
...													
6	0	Gem	12;18	12;55	...	18;15	...	18;29	...	5; 5	...	11;40	18;18
...													
9	0	Vir	18; 7	18;44	...	24;15	...	18;59	...	11;56	...	16;57	17;32
...													
11	24	Sgr	6;49	7;23	...	12;11	...	6;23	...	0;26	...	5;41	6;15
12	0	Sgr	12;14	12;47	...	17;33	...	11;40	...	5;47	...	11; 6	11;40

present an excerpt of the table corresponding to the true positions of Saturn, taken from the *Tabule Anglicane*, as found in Florence, Biblioteca Laurenziana, San Marco 185, ff. 125r–127v. The full table has 3,600 positions of Saturn. It is worth noting that the arguments at the head of the rows and columns (mean anomaly and mean center) in the *Tabule Anglicane* have been interchanged with respect to those in John of Lignères's *Tabule magne* for the planetary equations.

As pointed out by John North (1977, p. 281), the entries for the planetary true positions in the *Tabule Anglicane* do not directly give the true longitude of the planet. Rather, a difference that he called δω has to be added to the entry in the table to account for the change in the position of the aux (i.e., the apogee) since epoch (1348 in the *Tabule Anglicane*), and he adds that "the difference (δω) between the new and old positions is simply the movement of the eighth sphere," a quantity that could be found in an appropriate set of tables.

VELOCITY

1. *Sun and Moon*

The velocities of the Sun and the Moon vary as they complete their revolutions. To illustrate the solar velocity, consider Figure 10 representing an eccentric model of solar motion. $O\bar{S}$ (and DS) progress at a constant velocity, the mean solar motion, but OS progresses at a variable velocity, which depends on $\bar{\kappa}$. The variable velocity of the true Sun has its minimum when it is at apogee (near summer solstice) and reaches a maximum when at perigee (near winter solstice); in both cases the solar equation, c, vanishes. In contrast, when OS is perpendicular to AP, and the solar equation is maximum or minimum, the solar velocity equals the solar mean motion. This variability of the solar and lunar velocities plays a crucial role in the determination of the phases of eclipses; thus, in addition to tables for mean motions, one also finds tables for the true solar and lunar velocities when they are near syzygy, which are needed to compute the times of true syzygy, as well as the times and durations of eclipses (see § 13.2, below). Most common is a table where the hourly velocities of the two luminaries are given as functions of their anomalies at intervals of 6°. In Table 8.1A we reproduce an excerpt of such a table in Madrid, Biblioteca Nacional, MS 4238, f. 63v, part of a set of Parisian Alfonsine Tables, where the entries are in minutes of arc per hour. The entries for lunar velocity in this table are based on Ptolemy's first model (see § 6.2, above).

This table, with the same extremal values, is already found, but for variant readings, in the zij of al-Battānī (Nallino 1903–1907, 2:88), as well as in the Toledan Tables (F. S. Pedersen 2002, pp. 1409–1412), based on a solar eccentricity of about 2;4,45 and a lunar eccentricity of 5;15 (for a set of recomputed values corresponding to those in the zij of al-Battānī, see Goldstein 1974, pp. 108–113). Note that for Ptolemy the solar eccentricity is 2;30 and the lunar eccentricity is 5;15. In the *editio princeps* of the Alfonsine Tables (Ratdolt 1483, ff. g6r–g7r) the entries at intervals of 1° are in the range 0;2,23°/d–0;2,34°/d for the Sun and 0;30;21°/d–0;36,25°/d for the Moon (for discussion of the underlying

Table 8.1A: Hourly velocities of the Sun and the Moon (excerpt)

Argument (°)	Sun (′/h)	Moon (′/h)
0 360	2;23	30;18
6 354	2;23	30;19
. . .		
330	2;24	30;34
. . .		
60 300	2;26	31;24
. . .		
90 270	2;28	32;42
. . .		
120 240	2;31	34;14
. . .		
150 210	2;33	35;31
. . .		
174 186	2;33	36; 2
180 180	2;33	36; 4

parameters, see Goldstein 1980). In the tradition of al-Khwārizmī the lunar velocities lie in the range 0;30,12°/h–0;35,40°/h (Suter 1914, pp. 175–180; F. S. Pedersen 2002, pp. 1419–1420). Tables for solar and lunar velocities in this tradition can be found, for example, in Ibn al-Kammād's *Muqtabis* (Chabás and Goldstein 1994, pp. 10–13) and in the Tables of Barcelona (Millás 1962, pp. 32–233; Chabás 1996a, p. 508).

Table 8.1B lists some historical values that appear in tables for the true velocities of the Sun at apogee and perigee, as well as the central value (for 90°) in each case. We have also indicated the interval for the argument in each table. The entries are given in °/h, except those in italics, which are in °/mn, where "mn" means 1/60 of a day.

For the Moon there were other tables composed in the 14th century based on Ptolemy's second lunar model: see Goldstein 1992; Goldstein 1996; Chabás and Goldstein 2009b, pp. 116–118. In a table ascribed to John of Genoa (see Table 8.1C), but probably due to John of Lignères, one column displays the entries in °/h in the range 0;29,37,13°/h–0;36,58,54°/h and in another column they are given, equivalently, in °/mn, in the range 0;11,50,53°/mn–0;14,47,33°/mn (Paris, Bibliothèque nationale de France, MS 7282, f. 129r: for discussion of the problems with the values for 180°, see Goldstein 1992, pp. 12–14). These tables

Table 8.1B: Some historical values for the true velocities of the Sun

	Interval	Minimum (apogee)	90°	Maximum (perigee)
al-Battānī	6°	0; 2,23°/h	0; 2,28°/h	0; 2,33°/h
al-Khwārizmī	1°	0; 2,22	0; 2,28	0; 2,34
Toledan Tables	6°	0; 2,23	0; 2,28	0; 2,33
Levi ben Gerson	6°	0; 2,23	0; 2,28	0; 2,33
Alfonsine Tables	1°	0; 2,23	0; 2,27 *	0; 2,34 **
(eds. 1483 and 1492)	3°	*0; 0,57°/mn*	*0; 0,59°/mn*	*0; 1, 2°/mn*

* Santritter 1492: 0;2,28 (as in the Toledan Tables).
** Santritter 1492: 0;2,33 (as in the Toledan Tables).

reflect an awareness that in computing the lunar velocity at syzygy Ptolemy's second model cannot be ignored, even though its contribution to the lunar equation under these circumstances is zero. In computing his table for the time from mean to true syzygy, Nicholaus de Heybech (*ca*. 1400) used a table for lunar velocity based on Ptolemy's second lunar model that was compiled in the 14th century (see Chabás and Goldstein 1992; Kremer 2008, p. 153). Levi ben Gerson produced two tables for lunar velocity, one with the range 0;30,18°/h–0;36,4°/h (as in Table 8.1A), based on his own lunar model, whose entries differ slightly from those the zij of al-Battānī (see Goldstein 1974, pp. 113–114); and another with the range 0;29,35°/h–0;36,56°/h which takes into account Ptolemy's second lunar model as tabulated in the zij of al-Battānī (see Goldstein 1974, p. 182; Goldstein 1992, p. 10). Moreover, there is one double argument table for true lunar velocity (where the arguments are the corrected lunar anomaly and the double elongation) in a set of tables by Judah ben Asher II (Vatican, MS Heb. 384, ff. 375a–384b): see Goldstein 1998, pp. 180–181.

Table 8.1D lists some historical values that appear in tables for the true velocities of the Moon at apogee and perigee, as well as the central value (for 90°) in each case. We have also indicated the interval for the argument in each table. The entries are given in °/h, except those in italics, which are in °/mn.

It is worth noting that the table found in the zij of al-Khwārizmī contains columns for the true velocities of the Sun and the Moon, as well as columns for the radii of the Sun, the Moon, and the Earth's shadow. This two-fold feature was not followed by most table-makers, who treated the velocities separately in a single table for both luminaries

Table 8.1C: John of Genoa's table for lunar velocity at syzygy (excerpt)

Argument (°)	lunar velocity (°/h)
0	0;29,37,13
12	0;29,42, 6
24	0;29,51,51
30	0;29,59,31
...	
90	0;32,39,45
...	
150	0;36,13,37
168	0;36,46,22
174	0;36,53,15 *
180	0;36,58,54 **

* Read: 0;36,51,15. ** Read: 0;36,53,20.

Table 8.1D: Some historical values for the true velocities of the Moon

	Interval	Minimum (apogee)	90°	Maximum (perigee)
al-Battānī *	6°	0;30,18°/h	0;32,41°/h	0;36, 4°/h
al-Khwārizmī	1°	0;30,12	0;32,56	0;35,40
Toledan Tables	6°	0;30,18	0;32,42	0;36, 4
Levi ben Gerson (I)	6°	0;30,18	0;32,41	0;36, 4
Levi ben Gerson (II)	6°	0;29,35	0;32,40	0;36,56
Alfonsine Tables **	1°	0;30,21	0;32,44	0;36,25
(eds. 1483 and 1492)	1°	*0;12, 9°/mn*	*0;13, 4°/mn*	*0;14,25°/mn*

* In contrast to his tables, the extreme values in al-Battānī's text are given as 0;30,12°/h and 0;36,10°/h (Nallino 1903–1907, 1:58).
** Santritter 1492: 0;30,18°/h, 0;32,42°/h, and 0;36,4°/h (as in the Toledan Tables).

(or in two different tables with one for the Sun, and one for the Moon) and put their radii in another table (see § 15.1). However, we have found a table where the velocities and the radii are displayed together. As mentioned above, this mixture by itself is uncommon, but even more peculiar are the extremal values: 0;2,22,30°/h and 0;2,33,40°/h (with 0;2,27,46°/h for 90°), in the case of the Sun, and 0;29,37,13°/h and 0;36,58,54°/h (with 0;32,39,45°/h for 90°), in the case of the Moon. Moreover, we know of two copies of this table, that have the same title (*Tabula semidiametrorum solis et lunae et umbrae et pertinet ad eclipses*) and have the same short text next to it (Inc: *Cum argumento solis invenies*…): Madrid, Biblioteca Nacional, MS 4238, f. 60r (not f. 63v, as in Table 8.1A), and Paris, Bibliothèque nationale de France, MS lat. 7286C, f. 56v. They belong to sets of tables associated with John of Lignères (manuscript in Paris) or "alie tabule" (manuscript in Madrid), but both manuscripts contain tables by John Vimond. In any case, the column for the Moon in this table is the same as that ascribed to John of Genoa, (see Table 8.1C). F. S. Pedersen (2002, p. 1421) presents an excerpt of this table in the manuscript in Paris.

2. *Planets*

In most medieval astronomical treatises the computation of planetary velocities (i.e. their true, or unequal, motions), needed for determining the time and the longitude of planetary conjunctions, was made to depend on computing the positions of the planets at intervals of one day: see, for example, John of Saxony's canons for the Parisian Alfonsine Tables (Poulle 1984, pp. 96–101). However, at least one general table was constructed for this purpose. It is of Andalusian origin and is described in the Castilian canons of the Alfonsine Tables (Goldstein, Chabás, and Mancha 1994). It appears in many manuscripts associated with the Toledan Tables, the Parisian Alfonsine Tables as well as in 16th-century printed collections of tables (see Chabás and Goldstein 2003, pp. 170–182, where a complete edition of this table can be found). In Table 8.2A we reproduce an excerpt of it, from Florence, Biblioteca Nazionale Centrale, MS Conv. Soppr. J.V.6 (San Marco 189), f. 57r–v (note that the minus signs do not appear in the manuscript).

The velocity of a planet is a function of two variables: the corrected argument of center and the corrected argument of anomaly. Table 8.2A treats each variable separately, and the texts accompanying this table explain how to combine the entries in each of the two columns for a given planet. Note that Table 8.2A also includes two columns for the Moon: a column giving the argument in days, and a column giving multiples of lunar motion in degrees and minutes. The underlying parameter, 13;53°/d, is explained in § 18.5, where a full account of these two columns is given.

Table 8.2A: Daily unequal motion of the planets (excerpt)

Arg.	Saturn		...	Mercury		Moon	Lunar progress	
	center	anomaly		center	anomaly			
(°)	(′)	(′)	...	(′)	(′)	(′/h)	Days	(°)
6	1;44	5;43		56;10	51;10	29; 0	1	13;53
12	1;45	5;36		56;15	51; 5	29; 4	2	27;46
18	1;46	5;24		56;21	50;58	29; 8	3	41;39
24	1;46	5;12		56;28	50;30	29;13	4	55;32
30	1;47	5; 1		56;36	49;50	29;22	5	99;28
36	1;48	4;46		56;46	49; 0	29;33	6	83;17
42	1;48	4;32		56;57	47;30	29;46	7	97;10
48	1;49	3;18		57; 8	45;30	30; 2	8	111; 3
54	1;50	3;50		57;22	43; 0	30;19	9	124;56
60	1;51	3;20		57;34	40; 0	30;37	10	138;49
66	1;52	2;52		57;46	37; 0	30;58	11	152;42
72	1;53	2;22		58; 0	34; 0	31;18	12	166;38
78	1;54	1;50		58;14	31; 0	31;39	13	180;28
84	1;56	1;18		58;28	28; 0	32; 3	14	194;21
90	1;58	0;36		58;44	25; 0	32;26	15	208;14
96	2; 0	0; 0		59; 0	19; 0	32;49	16	222; 7
102	2; 1	−0;36		59;16	12; 0	33;15	17	236; 0
108	2; 3	−1;20		59;34	5; 0	33;44	18	249;52
114	2; 4	−2; 0		59;42	−3; 0	34;17	19	263;45
120	2; 5	−2;40		60;10	−11; 0	34;49	20	277;38
126	2; 6	−3;10		60;28	−19; 0	35;17	21	291;31
132	2; 7	−3;50		60;44	−31; 0	35;44	22	314;24
138	2; 8	−4;30		61; 0	−44; 0	36;11	23	319;27
144	2; 9	−5; 0		61;14	−58; 0	36;38	24	333;10
150	2;10	−5;20		61;26	−71; 0	37; 1	25	347; 3
156	2;11	−5;50		61;36	−83; 0	37;19	26	0;56
162	2;12	−6;15		61;45	−84; 0	37;33	27	14;48
168	2;12	−6;40		61;53	−93; 0	37;44	28	28;41
174	2;13	−7; 0		62; 0	−108; 0	37;52	29	30;32
180	2;14	−7;15		62; 5	−120; 0	37;46	30	–

Double argument tables for the daily velocities of the planets are found in a set of tables by Judah ben Asher II, uniquely preserved in Vatican, MS Heb. 384, ff. 364a–374a (Goldstein 1998, pp. 179, 185 n. 27), with specific tables for each planet. The names of the planets here are unusual: Saturn is called "old man" (*zaqen*); Jupiter, "judge" (*shofeṭ*); Mars, "killer" (*horeg*); Venus, "woman" (*ishshah*); and Mercury, "scribe" (*sofer*). For this name of Mercury in Hebrew, see *Babylonian Talmud, Shabbat*, 156a. In Table 8.2B we reproduce an excerpt of the table for Mercury (ff. 372a–374a).

Table 8.2B: Daily unequal motion of Mercury (excerpt)

corrected center		0, 6°	. . .	1,54°	. . .	2,54°	3, 0°
		5,54°	. . .	4, 6°	. . .	3, 6°	3, 0°
		(°)		(°)		(°)	(°)
corrected anomaly							
0, 6°	5,54°	1;42,27		1;53, 2		1;54, 2	1;54, 2
. . .							
2,25°	3,35°			0; 1,38			
				Retrograde			
2,26°	3,34°			−0; 5,17			
2,27°	3,33°	0; 0,38				0; 3,25	0; 3,19
		Retrograde				Retrograde	Retrograde
2,30°	3,30°	−0; 1,42				−0; 7,20	−0; 7, 7
. . .							
2,58°	3, 2°	−0;34,23		−1; 5, 6		−0;56,27	−0;56, 8
3, 0°	3, 0°	−0;31,22		−1; 6, 8		−0;58,19	−0;58, 0

Much the same table, but with different intervals and using zodiacal signs of 30° instead of physical signs of 60°, is also found in Abraham Zacut's *Ḥibbur* and appeared in print in the *Almanach Perpetuum* (Chabás and Goldstein 2000, pp. 75 and 145–146). The entries in Zacut's table are rounded to minutes, and were not copied directly from Vatican, MS Heb. 384. We have succeeded neither in recovering the method, nor the parameters used to compute the entries, and we cannot explain why Zacut presented a table for only one planet, Mercury.

CHAPTER NINE

LATITUDE

1. *Moon*

As indicated in ch. 6, some sets of tables include the lunar latitude in their tables for the lunar equation, as is the case in the *Almagest*, the zij of al-Battānī, and the Toledan Tables. Other sets have a separate table for the lunar latitude.

The parameter that characterizes this particular table is the value of the maximum latitude. It offers a nice example of the twofold tradition in medieval Spain: (1) the Greek/Ptolemaic tradition represented by Ptolemy's *Almagest* and *Handy Tables* and exemplified by al-Battānī (Nallino 1903–1907, 2:78–83), and (2) the Indian tradition exemplified by al-Khwārizmī (Suter 1914; Neugebauer 1962a).

1.1 *Ptolemaic tradition*

In *Almagest* V.8, Ptolemy took 5;0° as the maximum lunar latitude at 180° of the argument counted from the northern limit (Toomer 1984, p. 238), a value that also appears in the *Handy Tables* (Stahlman 1959, pp. 261–262). This value was accepted by a great many medieval astronomers—al-Battānī, Azarquiel, the Toledan Tables, the Parisian Alfonsine Tables, and many others—where the argument was usually counted from the ascending node such that the maximum occurs at argument 90°. The entries in this table can be recomputed by means of the modern formula

$$\beta = \arcsin (\sin i \cdot \sin \omega),$$

where β is the latitude, ω is the argument of latitude, and the i is the inclination of the lunar orb, taken here as 5;0°. In Figure 15, strictly speaking, the argument of lunar latitude is NM, the angular distance between the lunar ascending node, N, and the true position of the Moon on its orb, M, and the lunar latitude is ML, perpendicular to NL. However, most medieval computers took the argument of lunar

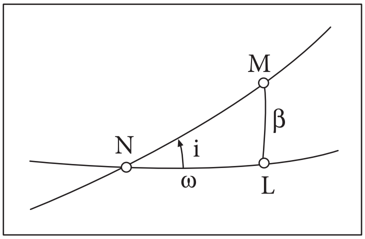

Figure 15: Lunar nodes

Table 9.1A: Lunar latitude in the Ptolemaic tradition (excerpt)

Argument (°)		0s (°)	1s (°)	2s (°)	. . .
1	29	0; 5,13	2;34,24	4;22,22	
2	28	0;10,27	2;38,52	4;24,51	
3	27	0;15,40	2;43,57	4;27,14	
. . .					
10	20	0;57, 9	3;12,39	4;41,52	
. . .					
20	10	1;42,27	3;49,40	4;55,25	
. . .					
28	2	2;20,40	4;14,22	4;59,50	
29	1	2;25,17	4;17, 7	4;59,58	
30	0	2;29,52	4;19,47	5; 0, 0	
		11s	10s	9s	. . .

latitude to be the difference on the ecliptic between the true longitude of the Moon and that of the lunar ascending node, making this equation slightly incorrect (see O. Pedersen 1974, pp. 199–200), even though the differences between using one or the other formula amount to less than 30 seconds of arc. Table 9.1A displays an excerpt of the table for lunar latitude in the Ptolemaic tradition, as represented in the Parisian Alfonsine Tables (Ratdolt 1483, f. h1r).

1.2 *Indian tradition*

In al-Khwārizmī's zij the lunar latitude, β, is a function of the argument of lunar latitude, ω, and its maximum value is 4;30° (Suter 1914, pp. 132–134; Neugebauer 1962a, pp. 95–98). Kennedy and Ukashah

(1969, pp. 95–96) have shown that the entries in this table were computed according to the "method of sines" given by the formula

$$\beta = 4;30 \cdot \sin \omega.$$

Kennedy (1956a, p. 146) mentions that a similar table, with the same maximum value, appears in the zij of Yaḥyā ibn Abī Manṣūr.

The same table is also found in Ibn al-Kammād's zij, *al-Muqtabis*; we consider the maximum value of 4;29° as a variant reading of 4;30° (Chabás and Goldstein 1994, pp. 20–22). It is noteworthy that *al-Muqtabis* includes another table for the lunar latitude, that of the Ptolemaic tradition. Both tables, with the same maximum values of 4;29° and 5;0°, are also found among the Tables of Barcelona (Millás 1962, pp. 194–195, 234; Chabás 1996a, p. 504) and in the tables of Solomon Franco (*fl.* 1370), extant in Vatican, MS Heb. 498, ff. 40r and 60r. Levi ben Gerson also used 4;30° as the maximum lunar latitude in his table where the entries, given to sexagesimal seconds, were computed by means of the formula

$$\sin \beta = \sin 4;30 \cdot \sin \omega$$

(see Goldstein 1974, pp. 132–134, 212–217). The entries in Levi's table were reproduced in the *Ḥibbur*, compiled by Abraham Zacut, as well as in other tables derived from it, the *Tabule Verificate* and the

Table 9.1B: "Adjusted" lunar latitude

Argument (°)		0s/6s (°)	1s/7s (°)	2s/8s (°)
1	29	0; 5	2;19	3;56
2	28	0; 9	2;22	3;58
3	27	0;14	2;27	4; 0
...				
10	20	0;47	2;53	4;13
...				
20	10	1;32	3;27	4;26
...				
28	2	2; 6	3;48	4;29
29	1	2;11	3;51	4;29
30	0	2;15	3;54	4;29
		5s/11s	4s/10s	3s/9s

Almanach Perpetuum (Chabás and Goldstein 2000, pp. 32, 60, 62, and 130–131). This latitude is called "precise," "adjusted," or "exact" in the headings of the tables we have located. Table 9.1B displays an excerpt of the table for the lunar latitude in the Tables of Barcelona (Ripoll, MS Lambert Mata 21, f. 147r, with the heading *Taula della latitut delle luna endressade*).

2. *Planets*

As was the case for the lunar latitude, there are two different traditions for the presentation and contents of the tables for planetary latitudes: the Greek/Ptolemaic tradition and the Indian tradition (Goldstein and Chabás 2004).

2.1 *Ptolemaic tradition*

In *Almagest* XIII Ptolemy gave a full treatment of planetary latitudes and, in the case of the inferior planets, he referred to three components which we call inclination (*declinatio*), slant (*reflexio*), and deviation (*deviatio*). However, Ptolemy's tables for the latitudes of Venus and Mercury in *Almagest* XIII.5 display only the first two of these components (see O. Pedersen 1974, pp. 355–386; Neugebauer 1975, pp. 216–226; Riddell 1978; Toomer 1984, pp. 632–634; and Swerdlow 2004); in *Almagest* XIII.6 Ptolemy gave instructions for computing the third component (see Toomer 1984, p. 636; see also Neugebauer 1975, p. 224). For Ptolemy's model for the latitude of the inferior planets, see Neugebauer 1975, pp. 212–216.

In the case of the superior planets Ptolemy's models for planetary latitudes include a deferent with a fixed inclination to the ecliptic, as is the case for the Moon (see Figure 15). The plane of the planet's deferent intersects the ecliptic at two points, called the ascending node (crossing from south to north) and the descending node (crossing from north to south); in Latin texts these points are often called *geuzahar* (Arabic: *jawzahar*), or variants of it. The nodal line joining the two nodes divides the deferent into two unequal parts, where the northern sector is greater than the southern sector. The deferent carries an epicycle, where the planet is at its apogee when farthest from the observer or at its perigee when closest. The inclination of the epicycle to the plane of the deferent varies. At the ascending and descending nodes,

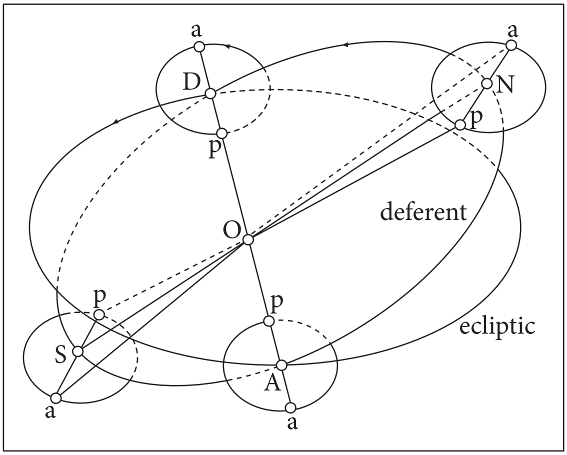

Figure 16: Ptolemy's model for the latitude of a superior planet

the epicycle lies in the plane of the ecliptic, whereas at the northern and southern limits the epicycle reaches its maximum inclinations from the plane of the deferent. At the northern limit the latitude of the epicyclic apogee is smaller than the latitude of the epicyclic perigee. Figure 16 shows the Ptolemaic model for the latitude of a superior planet, where O is the observer, ANDS is the planet's deferent, A is the ascending node, D is the descending node, N is the northern limit, and S is the southern limit of the deferent. AODNA is the northern sector and AODSA is the southern sector. On the epicycle a is the apogee and p is the perigee.

In the sets of tables considered here, the positions of the planetary nodes are not usually listed; rather, they are scattered in various planetary tables. In Table 9.2A we reproduce a list of the planetary ascending and descending nodes found in Florence, Biblioteca Nazionale Centrale, MS Conv. Soppr. J.V.4 (San Marco 192), f. 42v. This is a 14th-century manuscript that includes copies of the Tables of Novara (a variant of the Toledan Tables) and the Parisian Alfonsine Tables. The list of planetary nodes, here called *zenzar*, is located between these two sets but it does not belong to either of them.

Note that in each case the two entries for a given planet add up to 360°, and thus *medius cursus eius* refers to the other node of the planet. Note also that the entries are exactly the same as those in the zij of al-Khwārizmī, and in one set of values in the Toledan Tables (Suter

Table 9.2A: Planetary nodes

Cnc	13;12, 0, 0	Saturn
Sgr	16;48, 0, 0	*medius cursus eius*
Gem	22; 1, 0, 0	Jupiter
Cap	7;59, 0, 0	*medius cursus eius*
Ari	21;54, 0, 0	Mars
Psc	8; 6, 0, 0	*medius cursus eius*
Tau	29;27, 0, 0	Venus
Aqr	0;33, 0, 0	*medius cursus eius*
Ari	21;10, 0, 0	Mercury
Psc	8;50, 0, 0	*medius cursus eius*

1914, pp. 138–167; Neugebauer 1962a, p. 103; Toomer 1968, p. 45; F. S. Pedersen 2002, pp. 1230–1233).

The mainstream tradition found in most medieval sets of astronomical tables dealing with planetary latitudes certainly derives from the *Almagest* and it was transmitted to the West primarily via the zij of al-Battānī (*ca.* 900) and the Toledan Tables. For the zij of al-Battānī, see Nallino 1903–1907, 1:115–116, 2:140–141; for the Toledan Tables, see Toomer 1968, pp. 71–72, and F. S. Pedersen 2002, pp. 1322–1326. This tradition was later represented in the Parisian Alfonsine Tables (Ratdolt 1483, f. h1v: see Table 9.2B). As was the case in the *Almagest*, none of these tables display columns for the deviation for Venus and Mercury.

In Table 9.2B the argument for all columns under the names of the planets is the true anomaly, α, on the epicycle of the planet, counted from the apogee of the epicycle (point a in Figure 16), whereas the argument for the column of the minutes of proportion is the argument of latitude, ω, counted from the northernmost point of the deferent (point N in Figure 16).

In the course of time, from the middle of the 2nd century to the end of the 15th century, some parameters for planetary latitudes were modified, most likely due to copyists' errors (see Table 9.2C where positive entries are northern, and negative entries are southern).

In the *Handy Tables*, composed after the *Almagest*, Ptolemy made some changes in the theory of the planetary latitudes and introduced new tables with a different presentation. The latitude tables in the *Handy Tables* had relatively little influence on subsequent astronomers. However, they were the source for parameters in the corresponding tables in the *Mumtaḥan* zij of Yaḥyā ibn Abī Manṣūr. For the *Handy Tables*, see Stahlman 1959, pp. 143–155, 325–334; Neugebauer 1975,

Table 9.2B: Planetary latitudes in the *Almagest* tradition (excerpt)

Argument		Saturn		Jupiter		Mars		Venus		Mercury		Min.
		N	S	N	S	N	S	Incl.	Slant	Incl.	Slant	prop.
(°)		(°)	(°)	(°)	(°)	(°)	(°)	(°)	(°)	(°)	(°)	(′)
6	354	2; 4	2; 2	1; 7	1; 5	0; 7	0; 3	1; 2	0; 8	1;45	0;11	59;36
12	348	2; 5	2; 3	1; 8	1; 6	0; 9	0; 4	1; 1	0;16	1;44	0;22	58;36
18	342	2; 6	2; 4	1; 8	1; 6	0;11	0; 5	1; 0	0;24	1;43	0;33	57; 0
. . .												
30	330	2; 8	2; 6	1;10	1; 8	0;14	0; 7	0;57	0;41	1;36	0;55	52; 0
. . .												
60	300	2;16	2;15	1;16	1;16	0;28	0;22	0;36	1;20	0;59	1;44	30; 0
. . .												
90	270	2;30	2;30	1;30	1;30	0;52	0;49	0; 0	1;57	0; 0	2;20	0; 0
. . .												
120	240	2;45	2;45	1;45	1;45	1;34	1;37	0;59	2;25	1;25	2;29	30; 0
. . .												
150	210	2;57	2;58	2; 0	2; 0	2;55	3;29	3; 3	2;22	3; 7	1;45	52; 0
. . .												
168	192	3; 1	3; 3	2; 6	2; 6	4; 0	5;43	5;24	1;27	3;54	0;48	58;36
174	186	3; 2	3; 4	2; 7	2; 7	4;14	6;26	6;24	0;48	4; 2	0;28	59;36
180	180	3; 3	3; 5	2; 8	2; 8	4;21	7;30	7;12	0; 0	4; 5	0; 0	60; 0

Table 9.2C: Some extremal values of planetary latitudes in the *Almagest* tradition

	Almagest	al-Battānī	Toledan T.	Parisian Alf. T.
Mercury	4; 5°	4; 5°	4; 5°	4; 5°
	−2;30°	−2;30°	−2;30°	−2;30°
Venus	6;22°	6;22°	7;22°	7;12°
	−2;30°	−2;30°	−2;30°	−2;30°
Mars	4;21°	4;21°	4;21°	4;21°
	−7; 7°	−7; 7°	−7;30°	−7;30°
Jupiter	2; 4°	2; 4°	2; 5°	2; 8°
	−2; 8°	−2; 8°	−2; 8°	−2; 8°
Saturn	3; 2°	3; 2°	3; 2°	3; 3°
	−3; 5°	−3; 5°	−3; 5°	−3; 5°

pp. 1006–1016; and Swerdlow 2004. For the *Mumtaḥan* zij, see Kennedy 1956a, pp. 145–147, 173; and Vernet 1956. In the Iberian Peninsula this tradition is represented by Ibn al-Kammād whose zij provides the only previously known example of a set of astronomical tables where the planetary latitudes follow the *Handy Tables* for the inferior planets (Chabás and Goldstein 1994, pp. 31–32). In Table 9.2D we

Table 9.2D: Planetary latitudes of the inferior planets in the *Handy Tables* tradition (excerpt)

						Venus					Mercury		
Argument				Min	Min	Mean*	Min	Min	Min	Min	Mean	Min	Min
(s, °)		(s, °)		(′)	(′)	(°)	(′)	(′)	(′)	(′)	(°)	(′)	(′)
0	6	11	24	60	1	0;28	1	60	60	11	1;46	5	60
0	12	11	18	59	1	0;30	1	59	58	11	1;47	5	59
0	18	11	12	57	1	0;32	1	58	57	12	1;48	6	57
...													
1	0	11	0	53	1	0;35	2	53	51	13	1;49	6	52
...													
2	0	10	0	30	2	0;45	2	30	14	15	1;58	8	32
...													
3	0	9	0	2	4	1; 2	4	0	38	18	2;18	11	0
...													
4	0	8	0	30	8	2;52	8	30	60	28	2;49	16	30
...													
5	0	7	0	52	17	4;35	19	52	48	38	3;31	21	52
...													
5	16	6	12	59	29	7;35	34	59	41	41	3;42	28	59
5	24	6	6	59	33	8;24	38	60	40	40	3;47	29	60
6	0	6	0	60	36	8;35	40	60	40	40	3;52	30	60

* The degrees in the first half of this column for the mean latitude of Venus differ systematically from those in the *Handy Tables* (see Stahlman 1959, p. 331, col. III).

reproduce this particular table for the latitude of the inferior planets, as presented in Madrid, Biblioteca Nacional, MS 10023, f. 45v.

2.2 *Indian tradition*

The tables in this tradition are not based on Ptolemaic models. Rather, their roots lie in Indian astronomy as transmitted in the zij of al-Khwārizmī. For each planet a table is displayed for their equations, their stations (see ch. 10, below), and their latitudes. The last two columns on the right side of the table are called first and second latitudes (see Suter 1914, pp. 138–167; Neugebauer 1962a, pp. 101–103). On the rules for computing planetary latitudes in the zij of al-Khwārizmī, see Kennedy and Ukashah 1969, espec. p. 89. For a detailed discussion of the models, see Neugebauer 1962, pp. 34–41. A variant of this tradition for treating planetary latitudes is found in the zij of Ibn ᶜAzzūz (Fez, 14th century), where the entries in one column for each planet are the same as in the zij of al-Khwārizmī and those in the other are the reciprocals of the corresponding entries in the earlier zij: see Samsó 1999, pp. 114; and Samsó 1997, p. 92. Later on, these two columns for the latitude of each planet became two tables, one for each component of latitude, headed *Tabula bipartialis numeri* and *Tabula quadripartialis numeri* in the Toledan Tables (see Toomer 1968, pp. 69–70; Richter-Bernburg 1987; and F. S. Pedersen 2002, pp. 15, 1309–1321). In both tables the range of the argument is 1°–360°; in one case, the argument is presented in two columns (Table 9.2E), and in the other, in four columns (Table 9.2F), which accounts for the respective denominations of the tables (see F. S. Pedersen 2002, p. 1317). It may be worth noting that in Naples, Biblioteca Nazionale, MS VIII.C.49, f. 54r–56v, the headings of the tables are *Tabula latitudinum planetarum ad epiciclum* and *Tabula latitudinum planetarum ad excentricum*, and all entries are expressed in minutes, e.g., the first entry in the first table is 93′ (= 1;33°) and the last entry in the second table is 375′ (= 6;15°). Tables 9.2E and 9.2F are taken from the Tables of Marseilles, an early adaptation of the Toledan Tables to the Julian calendar (see d'Alverny *et al.* 2009), as found in Paris, Bibliothèque nationale de France, MS lat. 14704, ff. 133v–134v, where the titles are *Tabula binarii numeri ad sciendam latitudinem planetarum* and *Tabula quaternarii numeri ad sciendam latitudinem planetarum*, respectively.

Table 9.2E: *Tabula bipartialis numeri* (excerpt)

Argument (°)		Saturn (°)	Jupiter (°)	Mars (°)	Venus (°)	Mercury (°)
1	359	1;33	1;36	2;14	2;24	1;44
2	358	1;32	1;35	2;14	2;23	1;44
. . .						
30	330	1;24	1;28	2; 9	2;18	1;41
. . .						
60	300	1;22	1;24	1;55	2; 3	1;33
. . .						
90	270	1;18	1;18	1;36	1;41	1;21
. . .						
120	240	1;15	1;10	1;10	1;13	1; 7
. . .						
150	210	1;12	1; 4	0;44	0;42	0;53
. . .						
180	180	1;10	0;57	0;27	0;22	0;48

Table 9.2F: *Tabula quadripartialis numeri* (excerpt)

Argument (°)				Saturn (°)	Jupiter (°)	Mars (°)	Venus (°)	Mercury (°)
1	179	181	359	0; 5	0; 3	0; 4	0; 5	0; 7
2	178	182	358	0;10	0; 5	0; 8	0;10	0;13
...								
30	150	210	330	2;30	1;15	1;52	2;30	3; 7
...								
60	120	240	300	4;20	2;10	3;15	4;20	5;25
...								
90	90	270	270	5; 0	2;30	3;45	5; 0	6;15

In Table 9.2F the columns for the 5 planets follow a sine-like function (note that the values for 30° are equal to half the values for 90°). Let c_s, c_j, c_{ma}, c_v, and c_{me} be the entries for Saturn, Jupiter, Mars, Venus, and Mercury, respectively, for a given argument. Then the following simple relations hold:

$$^1/_2 \cdot c_s = c_j = {}^2/_3\, c_{ma} = {}^1/_2 \cdot c_v = {}^2/_5 \cdot c_{me},$$

indicating that the entries in the various columns are not independent of one another, for they are related by an expression of the type:

$$c_i(\alpha) = 1;15\ k \cdot \sin \alpha,$$

where *i* represents one of the five planets, α the argument, and k a coefficient whose values are 2, 3, 4, and 5. The relations of the entries in the columns depend on the values assigned to the angle between the deferent and the ecliptic in the models used in the zij of al-Khwārizmī: 5;0° (Saturn), 2;30° (Jupiter), 3;45° (Mars); 5;0° (Venus), and 6;15° (Mercury).

2.3 *Further developments*

Beginning in the 14th century a few tables for planetary latitudes were presented as double argument tables, and those we have found belong to the mainstream tradition, that of the *Almagest*. In John of Murs's Tables for 1321, for example, the superior and inferior planets are treated differently (Chabás and Goldstein 2009a). For the superior planets, the entries are presented in 7 columns and 31 rows, and the vertical argument is the argument of center of the planet, shifted +50° in the case of Saturn, –20° for Jupiter, and with no shift for Mars, in

Table 9.2G: John of Murs's table for the latitude of Saturn (excerpt)

Center (°)		0°/360° (°)	hd* (′)	30°/330° (°)	. . .	150°/210° (°)	hd (′)	180°/180° (°)
310	310	2; 3	3	2; 8	. . .	2;57	3	3; 2
316	304	2; 1	3	2; 7	. . .	2;55	3	3; 0
. . .								
34	226	0;13	0	0;14	. . .	0;19	0	0;19
40	220	0; 0	0	0; 0	. . .	0; 0	0	0; 0
46	214	0;13	0	0;13	. . .	0;19	0	0;20
. . .								
124	136	2; 0	2	2; 3	. . .	2;55	4	3; 3
130	130	2; 1	2	2; 5	. . .	2;57	4	3; 5

* "hd" stands for half the difference (in minutes of arc and rounded, up or down, to the nearest minute) between the entries of two successive columns for a fixed argument of center (i.e., entries in the same row).

accordance with the instructions given in *Almagest* XIII.6. Table 9.2G reproduces an excerpt of the table for Saturn found in Lisbon, MS Ajuda 52-XII-35, f. 60v, and Oxford, Bodleian Library, MS Can. Misc. 501, f. 88v.

For each of the inferior planets, we are given two double argument tables for latitude, in 7 columns and 16 rows: one for the inclination (here called *declinatio*) and one for the slant (*reflexio*). The argument, here given in four columns, is shifted +60° in the table for the inclination of Venus and +90° in the table for the inclination of Mercury. Tables 9.2H and 9.2I display excerpts for the latitude of Venus, as found in Lisbon, MS Ajuda 52-XII-35, f. 63r, and Oxford, Bodleian Library, MS Can. Misc. 501, f. 91r.

Most notable is the presentation of a column for the 3rd component of latitude, or deviation, in the tables for the slants of both inferior planets. This is indeed a very unusual feature in medieval tables. The values for the deviation were mentioned, but not tabulated, in *Almagest* XIII.6, with a limit of 0;10° (north) for Venus and a limit of 0;45° (south) for Mercury. It is significant that the canons to the Castilian Alfonsine Tables explicitly addressed this particular problem, giving instructions to take into account the third component of latitude that appeared in the (lost) tables to which they refer. Unfortunately, we do not know the values assigned in the Castilian Alfonsine Tables to these limits. In particular, the limit for Mercury would be a valuable clue for the transmission of these tables to Paris, for this parameter is given as

Table 9.2H: John of Murs's table for the latitude of Venus, inclination (excerpt)

Center (°)				0°/360° (°)	hd (′)	30°/330° (°)	. . .	150°/210° (°)	hd (′)	180°/180° (°)
300	300	120	120	1; 3	3	0;57	. . .	3; 3	129	7;22
306	244	114	126	1; 3	3	0;57	. . .	3; 2	127	7;16
. . .										
342	258	78	162	0;47	3	0;42	. . .	2;15	95	5;25
. . .										
24	216	36	204	0; 7	0	0; 6	. . .	0;18	15	0;47
30	210	30	210	0; 0	0	0; 0	. . .	0; 0	0	0; 0

Table 9.2I: John of Murs's table for the latitude of Venus, slant (excerpt)

Center (°)				0°/360° (°)	hd (′)	30°/330° (°)	. . .	150°/210° (°)	hd (′)	180°/180° (°)	3rd lat. (′)
0	360	180	180	0; 0	20	0;41	. . .	2;22	70	0; 0	10
6	354	174	186	0; 0	20	0;41	. . .	2;20	70	0; 0	10
. . .											
42	318	138	222	0; 0	15	0;30	. . .	1;46	53	0; 0	8
. . .											
84	276	96	264	0; 0	2	0; 4	. . .	0;15	7	0; 0	1
90	270	90	270	0; 0	0	0; 0	. . .	0; 0	0		

0;45° (south) in the Tables of John Vimond (Chabás and Goldstein 2004, p. 257), 0;23° (south) in John of Murs's Tables for 1321 (Chabás and Goldstein 2009a, p. 309), and 0;13° (south) in the Tables of the Seven Planets for 1340 (Paris, Bibliothèque nationale de France, MS lat. 10262, ff. 44r–46r.).

In Table 9.2J we present an excerpt of another double argument table for the latitude of Venus (where the arguments are the mean arguments of anomaly, $\bar{\alpha}$, and the mean arguments of center, $\bar{\kappa}$), as found in the *Tabule Anglicane*, where there is a single table for the latitude of each planet. The entries in Table 9.2J are taken from Florence, Biblioteca Laurenziana, MS San Marco 193, ff. 42r–43r, a 14th-century manuscript containing Alfonsine material. The horizontal argument ($\bar{\alpha}$) is given at intervals of 0;15° from 0s 0° to 4s 0° and from 8s 0° to 12s 0°, and at intervals of 0;10° from 4s 0° to 8s 0° (except in the vicinity of 6s 0° where it is given at intervals of 0;2°). The full table contains 1,800 entries. It is noteworthy that in the *Tabule Anglicane*,

Table 9.2J: Latitude of Venus in the *Tabule Anglicane* (excerpt)

$\bar{\alpha}$ / $\bar{\kappa}$ (s, °)		0s15° (°)	1s 0° (°)	. . .	5s20° (°)	5s28° (°)	6s 0° (°)	6s 2° (°)	6s10° (°)	. . .	11s15° (°)	12s 0° (°)
		S	S		S	M	M	M	M		M	S
0	6	0;37	0;57	. . .	0;48	0;27	0;37	0;51	1;37	. . .	0; 4	0;16
											S	
0	12	0;42	1; 2	. . .	0;16	1;10	1;20	1;35	2; 6	. . .	0; 4	0;22
. . .												
					M							
2	24	1; 5	1; 3	. . .	5;41	7;15	6;54	6;35	5;20	. . .	1; 0	1; 4
3	0	1; 2	0;58	. . .	5;52	7;22	6;56	6;35	5;15	. . .	1; 1	1; 4
3	6	1; 0	0;54	. . .	5;54	7;20	6;53	6;29	5; 5	. . .	1; 3	1; 4
. . .												
		M	M						S			
5	24	0; 3	0;25	. . .	1;40	0;51	0;37	0;19	0;48	. . .	0;37	0;16
							S	S				
6	0	0;10	0;31	. . .	1; 4	0; 6	0;10	0;26	1;24	. . .	0;30	0;10
						S						
6	6	0;17	0;37	. . .	0;28	0;39	0;57	1;11	1;59	. . .	0;24	0; 4
. . .												
					S						M	M
8	24	1; 2	1; 0	. . .	5; 6	6;32	6;57	7;21	5;54	. . .	0;57	1; 2
9	0	1; 1	0;58	. . .	5;15	6;36	6;59	7;22	5;50	. . .	1; 0	1; 3
9	6	0;57	0;53	. . .	5;25	6;37	6;55	7;16	5;41	. . .	1; 1	1; 1
. . .												
		S	S						M			S
11	24	0;24	0;45	. . .	1;59	1;11	1; 7	0;43	0;28	. . .	0;27	0; 4
								M				
12	0	0;30	0;51	. . .	1;24	0;26	0;10	0; 6	1; 4	. . .	0;10	0;10

M stands for *Meridionalis* (South) and S for *Septentrionalis* (North).

the tables for the true positions (Table 7.3F) and the latitudes of the planets (Table 9.2J) have both arguments arranged in the same way (at the head of the rows and columns).

Note that Table 9.2J also takes the deviation into account for, when $\bar{\alpha}$ = 180° or 360° and $\bar{\kappa}$ = 180° or 360°, the latitude is 0;10°, that is, the maximum deviation of Venus. For other double argument tables for planetary latitudes in the same tradition, see Chabás and Goldstein 2000, pp. 138–140.

Another astronomer who incorporated the deviation into his tables for the latitudes of the inferior planets was Giovanni Bianchini (see Chabás and Goldstein 2009b, pp. 96–99). Although his tables were presented in a new way (see Table 9.2K for the latitude of Venus), they conform strictly to the *Almagest* tradition.

Table 9.2K: Bianchini's table for the latitude of Venus (excerpt)

Arg. (°)	Inclination (°)	Slant (°)	Min. prop. Inclination (′)	Min. prop. slant (′)	Deviation (°)
1	1; 3	0; 1	1; 4	59;56	9;59
2	1; 3	0; 2	2; 8	59;52	9;59
...					
45	0;49	1; 1	42;12	42;12	7; 3
...					
90	0; 0	1;57	60; 0	0; 0	0; 0
...					
135	1;45	2;30	42;12	42;12	7; 2
...					
180	7;22	0; 0	0; 0	60; 0	10; 0
...					
225	1;41	2;30	42;12	42;12	7; 3
...					
270	0; 0	1;57	60; 0	0; 0	0; 0
...					
315	0;49	1; 1	42;12	42;12	7; 3
...					
360	1; 3	0; 0	0; 0	60; 0	10; 0

CHAPTER TEN

STATIONS AND RETROGRADATIONS

All five planets generally move from west to east, called "direct motion" (in the direction of increasing longitude), but sometimes they move from east to west, called "retrograde motion" (opposite the direction of increasing longitude). The place where the motion of a planet shifts from direct to retrograde motion is called "first station," and the place where its motion shifts from retrograde motion to direct motion is called "second station." The retrograde arc is the distance in longitude between first and second station. *Almagest* XII.8 displays a table for the first and second stations of the five planets, where the argument, $\bar{\kappa}$, the mean center of the planet, is given at intervals of 6°, and the entries in the table are values for α, the true anomaly (Toomer 1984, p. 588; see also MacMinn 1998, and O. Neugebauer 1975, pp. 190–206, for a full explanation of the Ptolemaic theory of retrogradation). Figure 17 represents the situation of the stationary points P and R in the model for the motion of Mars (see also Figure 14). The first station, P, and the second, R, are symmetrical with respect to OC, and PR is the retrograde arc.

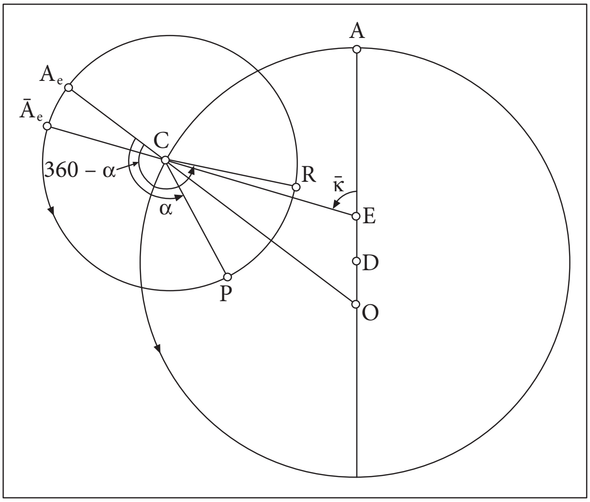

Figure 17: Stations

In his later work, the *Handy Tables*, Ptolemy presented similar tables with the argument at intervals of 3°, but with a slightly different definition of the argument (Neugebauer 1975, pp. 1005–1006): in the *Almagest*, it is the longitude of the center of the epicycle measured from the apogee of the deferent ($\bar{\kappa}$ = angle AEC in Figure 17), whereas in the *Handy Tables* it is the true longitude of the center of the epicycle measured from the apogee of the deferent (angle AOC in Figure 17). We display selected values of both tables for arguments 90° and 180° to show the small differences between the two approaches (see Table 10A).

Table 10A: Some historical values of the planetary stations

Saturn	*Almagest*	*Handy Tables*
First station (90°)	114; 1°	114;11°
First station (180°)	115;29°	115;29°
Second station (90°)	245;59°	245;59°
Second station (180°)	244;31°	244;31°
Jupiter	*Almagest*	*Handy Tables*
First station (90°)	125;32°	125;42°
First station (180°)	127;11°	127;11°
Second station (90°)	234;28°	234;18°
Second station (180°)	232;49°	232;49°
Mars	*Almagest*	*Handy Tables*
First station (90°)	162;18°	163;25°
First station (180°)	169; 9°	169;14°
Second station (90°)	197;42°	196;35°
Second station (180°)	190;51°	190;46°
Venus	*Almagest*	*Handy Tables*
First station (90°)	167; 7°	167;11°
First station (180°)	168;21°	168;21°
Second station (90°)	192;53°	192;49°
Second station (180°)	191;39°	191;39°

Table 10A (*cont.*)

Mercury	*Almagest*	*Handy Tables*
First station (90°)	144;40°	144;38°
First station (180°)	144;40°	144;40°
Second station (90°)	215;20°	215;22°
Second station (180°)	215;20°	215;20°

Note that for any value of the argument the entry for the second station is found by subtracting the entry for the first station from 360°. This is why later compilers of tables often only tabulated the first station. This is the case in the zij of al-Khwārizmī, where the values for first station of a planet are presented as a column in the table for its equation; the argument is given at intervals of 1° and the entries are derived from those in the *Handy Tables* (Suter 1914, pp. 138–167; Neugebauer 1962, p. 101; Stahlman 1959, pp. 335–339). In the zij of al-Battānī we find a single table for all planets with entries for both stations in each case, very similar to those in the *Handy Tables* (Nallino 1903–1907, 2:138–139). In the Toledan Tables, only the first station is listed in columns included in the tables for planetary equations, as in the zij of al-Khwārizmī. In some sets of tables we are given selected values: only four values (first and second stations both at apogee and perigee of the deferent) are given in the zij *al-Muqtabis* of Ibn al-Kammād (Chabás and Goldstein 1994, p. 33), which we reproduce in Table 10B from Madrid, Biblioteca Nacional, MS 10023, f. 46r.

Table 10B: Planetary stations in *al-Muqtabis*

	Saturn (°)	Jupiter (°)	Mars (°)	Venus (°)	Mercury (°)
First station at apogee	3s 22;44	4s 4; 5	5s 7;28	5s 15;51	4s 27;14
Second station at apogee	8s 7;16	7s 25;55	6s 22;32	6s 14; 9	7s 2;46
First station at perigee	3s 25;30	4s 7;11	5s 19;15	5s 18;21	4s 24;42
Second station at perigee	8s 4;30	7s 22;49	6s 10;45	6s 11;39	7s 5;18

Table 10C: Planetary stations in the Alfonsine corpus (excerpt)

Argument		Saturn		Jupiter		Venus		Mercury	
		1st st.	2nd st.	1st st.	2nd st.	. . .	2nd st.	1st st.	2nd st.
(°)	(°)	(°)	(°)	(°)	(°)		(°)	(°)	(°)
6	354	112;45	247;15	124; 5	235;55	. . .	194; 8	147;12	212;48
12	348	112;47	247;13	124; 7	235;53	. . .	194; 6	147; 8	212;52
. . .									
84	276	114; 4	245;56	125;32	234;28	. . .	192;58	144;43	215;17
90	270	114;11	245;49	125;42	234;18	. . .	192;49	144;37	215;23
96	264	114;19	245;41	125;50	234;10	. . .	192;42	144;34	215;26
. . .									
174	186	115;29	244;31	127;11	232;49	. . .	191;40	144;44	215;19
180	180	115;30	244;30	127;12	232;47	. . .	191;39	144;44	215;18

There is no table for the planetary stations in the *editio princeps* of the Parisian Alfonsine Tables, but such tables are often found in the Alfonsine corpus: see, e.g., Table 10C, excerpted from Paris, Bibliothèque nationale de France, MS lat. 7287, ff. 4v–5r.

In *Almagest* XII.2–6 Ptolemy gives information on planetary retrogradation but he does not display it in the form of a table. The Toledan Tables have such a table based on Ptolemy's account (Toomer 1968, p. 77; F. S. Pedersen 2002, pp. 1538–1541, where variants are listed). Table 10D displays such a list taken from Florence, Biblioteca Laurenziana, MS San Marco 194, f. 45v, a 13th/14th-century manuscript containing the Toledan Tables and other tables for Italian localities. The entries give the retrograde arcs (in degrees) and the time (in days) needed by each planet to go through them.

Table 10D: Retrogradation in the Toledan Tables

	Maximum distance (°)	Time (days)	Mean distance (°)	Time (days)	Minimum distance (°)	Time (days)
Retrog. Saturn	7;14,10	147;47	7;16,20	138	7;18, 0	136
Retrog. Jupiter	9;49,14	123	9;12,16	121	9;54,40	118
Retrog. Mars	19;53,32	80	16;18,44	73	11;12,14	64;30
Retrog. Venus	16;25,26	43	15;17,34	41;40	14;50,48	40;40
Retrog. Mercury	7;54,22	21	12;17,10	22;30	15;12, 0	23

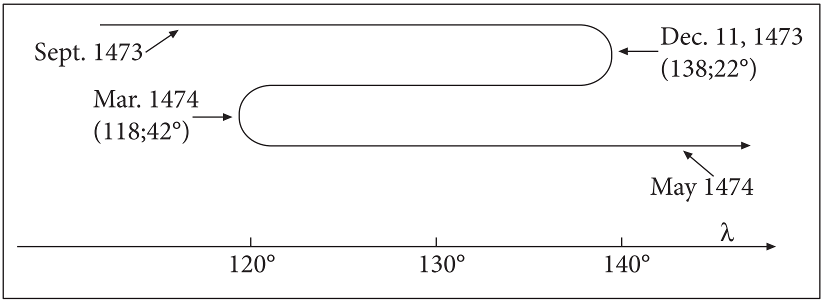

Figure 18: The retrograde arc of Mars

Figure 18 shows the retrograde arc of Mars for the period Dec. 11, 1473 to March 1, 1474, corresponding to data in the table for the true positions of Mars in the *Almanach Perpetuum*.

CHAPTER ELEVEN

VISIBILITY OF THE MOON AND THE PLANETS

1. *Moon*

According to Neugebauer 1975, p. 829, the theory of first and last visibility of the Moon represents the most refined part of Babylonian mathematical astronomy, but no trace of it has been found in Greek astronomy. On the other hand, this was a topic of major concern in Islamic astronomy because the beginning of the Islamic months is determined by the visibility of the lunar crescent. Through Islamic astronomy, the issue of the visibility of the lunar crescent entered astronomy in the West and, for example, one of the chapters of the Castilian Alfonsine Tables is devoted to this subject (Chabás and Goldstein 2003, pp. 199–200).

Tables for this purpose are extant in the zij of al-Khwārizmī (Suter 1914, p. 168; see also Neugebauer 1962a, pp. 42–43), and are common in Islamic astronomy. For the theory of lunar crescent visibility and an analysis of a large number of tables constructed for this purpose, see King 1987 and 1988, and Kennedy 1997. Table 11.1A reproduces one such table found in Madrid, Biblioteca Nacional, MS 10053, f. 18v, with the title *Tabula apparitionis lune de sub radiis*. Although it reproduces the values in the zij of al-Khwārizmī, the table is presented differently and contains many scribal errors. As explained in Kennedy and Janjanian 1965, the entries were probably computed on the basis of an Indian visibility theory for the midpoints of the three faces (or decans) of each zodiacal sign (i.e., 5°, 15°, and 25°) and at a latitude valid for Northern Spain. Hogendijk (1988, pp. 32–35) also recomputed this table and tentatively concluded that is was computed for the city of Zaragoza (lat. 41;35° according to one medieval source) and an obliquity of 23;35°, but the mathematical analysis does allow for other slightly different values. The zij of Ibn Isḥāq al-Tūnisī (beginning of the 13th century) ascribes this table to an otherwise unknown al-Qallās (Mestres 1996, p. 429, and 1999, p. 287).

Table 11.1A: Lunar visibility in Madrid, MS 10053

Facies	*Prima* (°)	*Secunda* (°)	*Tercia* (°)
Ari	9;26	9;15	9;21
Tau	9;29	9;18	9;21
Gem	9;32	9;57	10;35
Cnc	11;29	21;48	14;15
Leo	15;58	17;31	19;11
Vir	20;20	21; 4	21;57
Lib	21;25	20; 4	15;58
Sco	21;27	20; 4	15;58
Sgr	19;11	17;31	11;29
Cap	18;15	21;48	9;33
Aqr	9;21	9;18	9;19
Psc	9;21	9;25	9;26

Table 11.1B: Lunar visibility in the Toledan Tables

	1st face (°)	2nd face (°)	3rd face (°)
Ari	9;26	9;25	9;21
Tau	9;19	9;18	9;21
Gem	9;33	9;57	10;37
Cnc	11;29	12;48	14;15
Leo	15;58	17;31	19;11
Vir	20;20	21; 4	21;17
Lib	21;17	21; 4	20;20
Sco	19;11	17;31	15;58
Sgr	14;15	12;48	11;29
Cap	10;37	9;57	9;33
Aqr	9;21	9;18	9;19
Psc	9;21	9;25	9;26

Tables for the visibility of the lunar crescent in the Toledan Tables are also reported by F. S. Pedersen (2002, pp. 1482–1488). Table 11.1B reproduces one of them, displayed by F. S. Pedersen (2002, p. 1483), and it is essentially the same as that of al-Khwārizmī and Table 11.1A. Note the symmetrical features of Table 11.1B and one of many errors in Table 11.1A (the entries for the second decan for Cancer and Capricorn, which are underlined).

2. *Planets*

In *Almagest* XIII.10 Ptolemy provides tables for planetary visibility, otherwise called tables of planetary phases. For each of the superior planets, there are two phases (morning rising and evening setting), and for each of the inferior planets, four (evening rising, morning setting, morning rising, and evening setting). For each of the 14 phases, Ptolemy gives entries for the beginning of the 12 zodiacal signs. The entries, in degrees and minutes, represent the elongation between the true longitudes of the Sun and the planet at the time under consideration. Later, in the *Handy Tables* Ptolemy extends his tables to each of the seven climates (Stahlman 1959, pp. 340–346): see also Table 2.7B for a list of the climates. The corresponding tables in the zij of al-Battānī and the Toledan Tables seem to derive from the table for climate IV, for a longest daylight of 14½h and a terrestrial latitude of

Table 11.2: Planetary phases (excerpt)

	Saturn		Jupiter		Mars	
	M.R. (°)	E.S. (°)	M.R. (°)	E.S. (°)	M.R. (°)	E.S. (°)
Aries	29;28	13;46	19;33	9;28	29;10	14;12
Taurus	26;26	14; 7	18;21	9;38	27;11	15; 8
. . .						
Aquarius	21;16	14;40	14;14	10;16	22;14	16; 7
Pisces	26;46	14; 0	18;11	9;38	27;11	15; 8

Venus

	E.R. 1–137 (°)	M.S. 224–360 (°)	M.R. 180–223 (°)	E.S. 138–180 (°)
Aries	15;31	4;25	3;36	2;27
Taurus	13;48	4;29	4; 9	3;30
. . .				
Aquarius	12;47	8;29	1;36	1;14
Pisces	15;28	7;43	2;43	1;31

Mercury

	E.R. 1–152 (°)	M.S. 249–360 (°)	M.R. 180–248 (°)	E.S. 153–180 (°)
Aries	24;10	12;24	23;43	12; 9
Taurus	21;15	12;18	24;23	12;12
. . .				
Aquarius	20;25	14; 7	14;25	11;32
Pisces	24;38	12;14	18;22	11;47

36°, in the *Handy Tables* (Nallino 1903–1907, 2:142–143; F. S. Pedersen 2002, pp. 1530–1537). As noted by Toomer (1968, p. 75), the headings of the columns in the *Handy Tables* (as well as in the zij of al-Battānī and the Toledan Tables) are associated with the wrong columns of entries. See Table 11.2 for an excerpt of the table for the planetary phases found in a copy of the Toledan Tables in Naples, Biblioteca Nazionale, MS VIII.C.49, f. 53v (cf. Toomer 1968, pp. 73–76; F. S. Pedersen 2002, pp. 1530–1537). In this table the numbers (in degrees) in the headings are diplayed as they appear in the manuscript and have not been corrected. The following abbreviations have been used in the headings: E.R. (evening rising), M.S. (morning setting), M.R. (morning rising), and E.S. (evening setting).

The *editio princeps* of the Parisian Alfonsine Tables (Ratdolt 1483) does not include any tables for the planetary phases.

CHAPTER TWELVE

PARALLAX

In lunar theory there are three kinds of lunar positions: mean, true, and apparent. Both mean and true lunar positions assume that the Earth is a point with respect to the distance of the Moon or, alternatively, that the "observer" is at the center of the Earth. Parallax is the angular correction between the true lunar position and its apparent position for an observer at a specific location on Earth, and it depends on the distance from the center of the Earth to the center of the Moon as well as on the altitude of the Moon at a given time. A similar definition holds for the Sun, but its parallax is much smaller than that for the Moon (Ptolemy took the maximum solar parallax to be about 0;3°, which is much too big from a modern point of view), since it is much farther away from the Earth.

Let T be the center of the Earth, O the observer, Z the zenith for that observer, and M the position of the Moon. The true zenith distance of the Moon is z (= ∠ZTM) and the apparent zenith distance of the Moon is z′ (= ∠ZOM). The lunar parallax, π, is defined as $\pi = z' - z$ (see Figure 19).

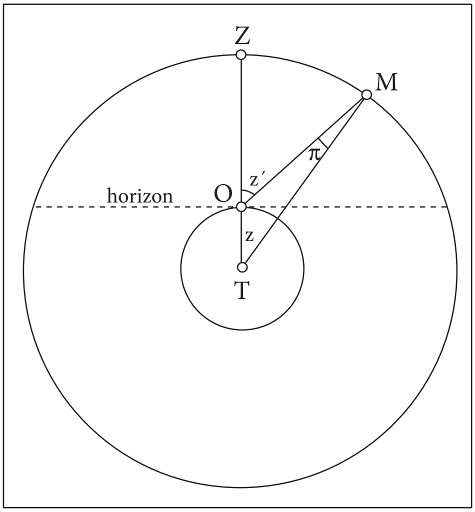

Figure 19: Parallax

When the Moon is at the horizon, π reaches its maximum value, π_0, called horizontal parallax. In computing solar eclipses, it was common to use the difference between the lunar parallax at syzygy and the solar parallax, called adjusted parallax.

In the Ptolemaic tradition the distance from the center of the Earth to the center of the Moon (measured in terrestrial radii) was assumed to vary in the same proportion as the variation in distance in the lunar model (measured in units where the radius of the deferent at syzygy is taken to be 60). The lunar distance in Ptolemy's model at a given time depends on the elongation of the Moon from the Sun as well as on the lunar anomaly. In fact, Ptolemy's model produces too great a variation in lunar distance, for it implies that the diameter of the Moon is twice as great at quadrature (half-Moon phase) than it is at opposition (full-Moon phase). But no one before the 14th century called attention to this issue, and parallax was only rarely computed for lunar positions other than at the time of a solar eclipse. In the 14th century both Levi ben Gerson and Ibn al-Shāṭir noticed that the lunar distances produced by Ptolemy's model varied too much. And in the 15th century Regiomontanus also mentioned this problem with Ptolemy's lunar model. In the *Almagest* Ptolemy computed the lunar parallax for several observations, presumably using his 9-column table in V.18, but he does not give the details. For instance, in *Almagest* IX.10 (Toomer 1984, p. 461) Ptolemy describes an observation he made in 139 AD of Mercury with respect to the apparent Moon; in this case he computed lunar parallax in order to confirm that the true position of the Moon at the time of the observation agreed with the true lunar position that follows from his lunar theory. For an analysis of Ptolemy's lunar observations, see Britton 1992. For an example of a modern recomputation of parallax, according to the rules in the *Almagest*, for one of Ptolemy's observations, see Toomer 1984, pp. 652–653.

The parallax tables offer a very clear example of the twofold tradition in medieval Spain: (1) the Greek/Ptolemaic tradition represented by Ptolemy's *Almagest* and *Handy Tables* and exemplified by al-Battānī (Nallino 1903–1907), and (2) the Indian tradition exemplified by al-Khwārizmī (Suter 1914; Neugebauer 1962a).

1. *Ptolemaic Tradition*

In *Almagest* V.18 Ptolemy considered parallax in altitude and provided a table in 9 columns to compute it (Toomer 1984, p. 265; cf.

Table 12.1A: Parallax in altitude (excerpt)

1 (°)	2 (°)	3 (°)	4 (°)	5 (°)	6 (°)	7 (′)	8 (′)	9 (′)
2	0; 7	1;54	0;23	0; 3, 0	0;50	0;14	0;11	0;15
4	0;13	3;48	0;45	0; 7, 0	1;40	0;28	0;22	0;30
...								
30	0;13	27; 9	5;18	0;40, 0	12;30	14; 0	13;33	17;18
...								
60	2;29	46;46	9; 3	1; 8, 0	22; 0	44; 0	43;24	47;21
...								
88	2;51	53;33	10;17	1;18,20	24;50	59;47	59;45	59;50
90	2;51	53;34	10;17	1;19, 0	25; 0	60; 0	60; 0	60; 0

Neugebauer 1975, pp. 112–115). This table takes into account Ptolemy's second lunar model (see § 6.2). Table 12.1A displays an excerpt of Ptolemy's table as found in a 13th-century manuscript in Latin: Madrid, Biblioteca Nacional, MS 10053 (f. 9r). Column 1 gives the argument from 2° to 90° at intervals of 2°, and it serves as the argument for all the remaining columns. The zenith distance of the Sun serves as argument in column 2, which lists the values of solar parallax, with a maximum of $\pi_0 = 0;2,51°$. Columns 3–6 apply to the Moon and depend exclusively on its zenith distance, z (col. 3 is for syzygy and true anomaly $\alpha = 0°$; col. 4 is for syzygy and $\alpha = 180°$; col. 5 is for quadrature and $\alpha = 0°$; and col. 6 is for quadrature and $\alpha = 180°$). Columns 7–9 are for linear interpolation (cols. 7 and 8 depend on α and are used for syzygies and quadratures, respectively, whereas col. 9 depends on double elongation, 2η, and is to be used between syzygy and quadrature). The lunar parallax is finally obtained by means of the modern formula

$$\pi = (c_5(z) + c_6(z) \cdot c_8(\alpha)) \cdot c_9(2\eta) + (c_3(z) + c_4(z) \cdot c_7(\alpha)) \cdot (1 - c_9(2\eta)).$$

This table is also found in the zij of al-Battānī (Nallino 1903–1907, 2:94), and it spread in the Iberian Peninsula to authors writing in Arabic, such as Azarquiel, in whose Almanac it is partially reproduced (only columns 3 and 4; see Millás 1943–1950, p. 232), and to those writing in Hebrew, such as Abraham bar Ḥiyya (Millás 1959, p. 113). This table was not included in the Toledan Tables, but it is preserved in various manuscripts related to them (Paris, Bibliothèque nationale de France, MS lat. 16208; Edinburgh, Observatory, MS Cr.2.5; Oxford, Bodleian Library, MS 613; and London, Royal Astronomical Society, MS Add. 1, Vol. 2: see F. S. Pedersen 2002, p. 1407). It was certainly part of the Castilian Alfonsine Tables, for it is described in detail in chapter 32 of the canons explaining their use (Chabás and Goldstein 2003, pp. 191–193). As mentioned above, a copy of it is found in Madrid, Biblioteca Nacional, MS 10053. However, we have not found this particular table among those produced by the Parisian astronomers in the early 14th century.

It is noteworthy that in the 14th century Levi ben Gerson constructed a similar table displaying the differences $c_3 - c_2$, but this seems to be a unique case, because the entries depart from Ptolemy's and were computed according to a different model (see Goldstein 1974, pp. 116–117, 183). Levi also computed tables for adjusted parallax for different geographical latitudes that have the same layout as those in Ptolemy's *Handy Tables*, but the entries were all recomputed and displayed to minutes and seconds (see Table 12.1C). Moreover, Levi made observations of the Moon and of solar eclipses for which he computed parallax (see Goldstein 1979a, pp. 123–150). In fact, it is quite unusual to find computations of parallax for a specific dated observation in the Middle Ages. (For another such example, see the observations of the solar eclipses of 1333 and 1337 described by John of Murs: Beaujouan 1975.) Levi's parallax table for his own location at geographical latitude 44° was included in one copy of Immanuel Bonfils's set of planetary tables (see Munich, MS Heb. 386, f. 35a).

Although not among the Parisian Alfonsine Tables, Ptolemy's 9-column table can be found in the Alfonsine corpus: see, e.g., the *Tabule Astronomice Elisabeth Regine* compiled by Alfonso de Córdoba *ca*. 1500 (Chabás 2004, pp. 200–201).

In the *Handy Tables* Ptolemy offered a different approach to the computation of parallax and presented it in the form of various tables arranged for the 7 climates (see Table 2.7B): 16;27° (13h), 23;51° (13;30h), 30;22° (14h), 36;0° (14;30h), 40;56° (15h), 45;1° (15;30h), and 48;32° (16h). For each of the climates there are 12 sub-tables, one for the true longitude of the Moon at the beginning of each zodiacal sign, displaying the longitudinal and latitudinal components of the adjusted parallax to minutes (see Stahlman 1959, pp. 268–283). These tables only apply at the times of solar eclipses and they take advantage of the fact that the lunar distance varies much less at conjunction (and opposition) than at other lunar phases. There is still a variation in lunar distance at conjunction, but Ptolemy ignored it since the effect on parallax is small.

Table 12.1B displays the sub-table for Cancer corresponding to Toledo (latitude 39;54° and longest daylight 14;51h) as found in various manuscripts of the Toledan Tables (Toomer 1969, p. 98; F. S. Pedersen 2002, p. 1357). Column 1 gives the argument as an integer number of hours before or after noon; the times of sunrise and sunset (before and after noon) are given in the first and last lines, respectively. Columns 2 and 3 display the longitudinal and latitudinal components of the adjusted parallax, respectively, and are given in minutes of arc. No account is taken of the effect on parallax of lunar anomaly. In the *Handy Tables* Ptolemy included a fourth column not directly related to parallax, displaying, in degrees, the arc of the horizon between the point of the ecliptic that is setting and the point of intersection of the horizon and the arc of the great circle perpendicular to the ecliptic that passes through the beginning of each zodiacal sign (Chabás and Tihon 1993, pp. 23–141), but this arc was rarely tabulated during the Middle Ages. The usual Latin word for parallax is *diversitas aspectus*, a translation of the Arabic *ikhtilāf al-manẓar* (see, e.g., Nallino 1903–1907, 2:330). It is frequently the case that the adjusted parallax is also displayed for the time corresponding to the nonagesimal (a term coined by Kepler [see Neugebauer 1962a, p. 71 n. 3]; in the Middle Ages it was usually called "midheaven" although, strictly speaking, this refers to the place on the ecliptic crossing the meridian at a given time). The nonagesimal is the point on the ecliptic that is closest to the zenith, and it is 90° along the ecliptic from the eastern and western horizons. When the longitude of the Moon lies between the eastern horizon and the nonagesimal point, the longitudinal parallax has to be added to it but, when the Moon is to the west of the nonagesimal point, it has to be subtracted (Neugebauer 1962a, p. 71).

The set of sub-tables illustrated by Table 12.1B is also found in the zij of al-Battānī (Nallino 1903–1907, 2:95–101), and was reproduced by Abraham bar Ḥiyya (Millás 1959, p. 113), but it is not found in the Almanac of Azarquiel. In contrast, all the sub-tables are included in the Toledan Tables, as well as many similar tables for other geographical latitudes (Toomer 1968, pp. 97–112; F. S. Pedersen 2002, pp. 1351–1404). Again, as was the case for Table 12.1A, this was part of the Castilian Alfonsine Tables, for it is described in detail in chapter 31 of the canons associated with them (Chabás and Goldstein 2003, pp. 189–190).

Most astronomers copied this table, in full or in part, while others computed sub-tables for the geographical latitudes of their specific

Table 12.1B: Components of parallax for Cancer in Toledo

1 Arg. (h)	2 long. (′)	3 lat. (′)
7;27	40	36
7	41	33
6	42	31
5	41	27
4	37	23
3	30	19
2	21	17
1	11	16
Recessio	0	15
1	11	16
2	21	17
3	30	19
4	37	23
5	41	27
6	42	31
7	41	33
7;27	40	36

localities, mainly by simple interpolation between the data for two consecutive climates, or by modifying the obliquity of the ecliptic underlying those tables. However, a few astronomers, notably Levi ben Gerson, computed all sub-tables for the various climates with different parameters (Goldstein 1974, pp. 184–207); Table 12.1C displays an excerpt of one of Levi's sub-tables. A full table for all climates was included in the *editio princeps* (Ratdolt 1483) of the Alfonsine Tables, ff. l5r–l8r, as well as in the second edition (Santritter 1492), ff. i4v–k1r.

It may be of interest to note that, in some tables which were derived from Levi's table, the component in longitude is given in units of time (hours and minutes) rather than in minutes of arc. This is the case in Bonjorn's tables, *ca.* 1361 (Chabás 1992, pp. 243–248), the *Tabule Verificate* for Salamanca and the *Almanach Perpetuum* based on Abraham Zacut's tables (Chabás and Goldstein 2000, pp. 31–32, 122–124), the tables by Ben Verga (Goldstein 2001, pp. 257–258), as well as Peurbach's *Tabulae eclypsium* (first printed in 1514). Wing 5 of Immanuel Bonfils's *Six Wings* is a special case: there is no table for the parallax in latitude and, as far as we know, it is the only medieval table for

Table 12.1C: Components of parallax for Cancer for latitude 44° by Levi ben Gerson

1 Arg. (h)	2 long. (′)	3 lat. (′)
7;40	34;37	40;26
7	37;18	37;38
6	38;34	34;14
5	37;31	30; 2
4	34; 8	26;39
3	27; 0	23;30
2	19;25	20;20
1	9;43	19;16
Noon	0; 0	19; 4
1	9;43	19;17
2	19;25	20;20
3	27; 0	23;30
4	34; 8	26;39
5	37;31	30; 2
6	38;34	34;14
7	37;18	37;38
7;40	34;37	40;26

adjusted parallax that explicitly deals with the variation due to lunar anomaly, that is the longitudinal component is displayed for lunar anomaly at intervals of 30° (see Table 12.1D for an excerpt). Although the geographical latitude is not specified, it is almost certainly 44° (or close to it): see Solon 1970, p. 7.

A third table related to parallax and ascribed to Ptolemy appears in several medieval manuscripts. It is usually called "Table for correcting parallax" and it is already found in the *Handy Tables* (see Stahlman 1959, pp. 255, 257). It is used in combination with Table 12.1B, to correct the components of adjusted parallax for different positions of the Moon. In Table 12.1E we display an excerpt of it as found in the Almanac of Azarquiel (Paris, Bibliothèque de l'Arsenal, MS 8322, f. 133r; see also Millás 1943–1950, p. 233). Columns 1 and 2 give the argument, from 6° to 354°, at intervals of 6°. Column 3 is not properly related to parallax; it is actually a table of coefficients for correcting Ptolemy's eclipse tables when the Moon is between greatest and least distances which are found in *Almagest* VI.8 (see Toomer 1984, p. 308). For column 4 the argument is the anomaly, and it yields a

Table 12.1D: Parallax in longitude for Cancer by Immanuel Bonfils

Arg. (h)	0s (h)	1s/11s (h)	2s/10s (h)	3s/9s (h)	4s/8s (h)	5s/7s (h)	6s (h)
7;40							
7							
6	1;18	1;17	1;15	1;12	1; 9	1; 7	1; 6
5	1;22	1;21	1;19	1;16	1;12	1;10	1; 8
4	1;21	1;21	1;17	1;15	1;11	1; 8	1; 7
3	1;15	1;14	1;12	1; 9	1; 5	1; 2	0;59
2	0;58	0;57	0;54	0;51	0;48	0;46	0;45
1	0;32	0;31	0;30	0;28	0;26	0;25	0;24
Noon			mid-heaven (*medium celi*)				
1	0;32	0;31	0;30	0;28	0;26	0;25	0;24
2	0;58	0;57	0;54	0;51	0;48	0;46	0;45
3	1;15	1;14	1;12	1; 9	1; 5	1; 2	0;59
4	1;21	1;21	1;17	1;15	1;11	1; 8	1; 7
5	1;22	1;21	1;19	1;16	1;12	1;10	1; 8
6	1;18	1;17	1;15	1;12	1; 9	1; 7	1; 6
7							
7;40							

first corrected lunar parallax, $\pi_1 = \pi \cdot (1 + c_4)$, when the Moon is not located at apogee, whereas for column 5 the double elongation is the argument, and it yields a second corrected lunar parallax, $\pi_2 = \pi_1 \cdot (1 + c_5)$, when the Moon is not in conjunction (for a justification of these formulas, see Neugebauer 1975, pp. 994–996). Note that in the *Handy Tables* column 3 and columns 4 and 5 are presented in separate tables, where the entries in column 3 are only given to one sexagesimal place.

As in the two previous cases, this table is found in the zij of al-Battānī, including column 3 (Nallino 1903–1907, 2:89), and it was reproduced by Abraham bar Ḥiyya (Millás 1959, p. 113) and in the Almanac of Azarquiel (Millás 1943–1950, p. 233). It is also found in the Toledan Tables (Toomer 1968, pp. 116–117; F. S. Pedersen 2002, pp. 1437–1440), where columns 4 and 5 are usually labeled *circulus brevis* and *circulus egressus*. In the astronomical literature in Latin this table was commonly referred to as *tabula actatium* or *attacium* after the Arabic word *al-taqwīm*, meaning "correction" or "equation." The astronomical tables of John of Gmunden, an early 15th-century Viennese astronomer and a notable collector of tables, also include this

Table 12.1E: Table for correcting parallax (excerpt)

1	2	3		4	5
(°)	(°)	(′)	(″)	(′)	(′)
6	354	0	21	0	0
12	348	0	42	0	0
18	344*	1	42	0	1
...					
60	300	14	0	3	9
...					
120	240	44	0	9	26
...					
168	192	59	21	12	32
174	186	59	41	12	32
180	180	60	0	12	32

* Instead of 342.

table (Porres 2003, pp. 363, 613). The same table, with all its columns, appears in the *editio princeps* of the Alfonsine Tables (Ratdolt 1483 f. 18v), as well as in the second edition (Santritter 1492, f. k2r).

Variants of this table, or adaptations of columns 4 and 5, are frequent in the astronomical literature: see, e.g., Isaac Ibn al-Ḥadib's tables (later reproduced by F. Mithridates; see Goldstein and Chabás 2006); and Abraham Zacut's *ha-Ḥibbur ha-gadol* as well as in its Latin version, the *Almanach Perpetuum* (Chabás and Goldstein 2000, pp. 33–34, 62, 118–122).

2. *Indian Tradition*

This tradition is represented in al-Andalus by the zij of al-Khwārizmī, which includes a table for the parallax of the Moon in 3 columns. The first is for the argument and the other two represent the components of parallax in longitude, π_λ, in units of time, and latitude, π_β, in minutes and seconds of arc (Suter 1914, pp. 191–192; Neugebauer 1962a, pp. 121–126). Column 2 has a maximum of 1;36h at 66° of argument, and ranges up to 150° but, as pointed out by Neugebauer (p. 125), there is no reason for the argument to go beyond 90°. In contrast, column 3 is only tabulated from 1° to 90°, where it reaches a maximum of 0;48,45°. According to Kennedy (1956a, pp. 44–51; 1956b, p. 150), the two components are given by the modern expressions

$\pi_\beta = 0;48,45° \sin h_e$

and

$\pi_\lambda = 1;36 \sin \theta(t)$,

where h_e is the zenith distance of the nonagesimal point. In Islamic astronomy h_e was called "the latitude of the visible climate" (Neugebauer 1962a, p. 72 n. 6). The formula for π_β depends upon the fact that this component of parallax is constant for all points on the ecliptic at the time when the nonagesimal point is fixed (see Neugebauer 1962a, pp. 72, 123). We note that the coefficient, 1;36 (= 24/15), contains the standard Indian value for the obliquity of the ecliptic, 24°. Table 12.2A displays the components of parallax in al-Khwārizmī's zij.

In the zij *al-Muqtabis* by Ibn al-Kammād (Madrid, Biblioteca Nacional, MS 10023, f. 53r–v; Chabás and Goldstein 1994, pp. 19–23) these components are presented in two separate tables: see Tables 12.2B and 12.2C.

Table 12.2A: Components of parallax in the zij of al-Khwārizmī (excerpt)

Arg. (°)	π_λ (h)	π_β (′)
1	0; 2,53	0;51
2	0; 5,46	1;42
3	0; 8,38	2;33
. . .		
30	1;11, 5	24;22
. . .		
60	1;35,27	42;13
. . .		
66	1;36, 0	44;27
. . .		
90	1;28,51	48;45
. . .		
120	1; 6, 1	
. . .		
148	0;37, 5	
149	0;35,58	
150	0;34,51	

Tables 12.2B and 12.2C: Components of parallax in the zij *al-Muqtabis* of Ibn al-Kammād (excerpts)

Component in longitude (f. 53v)

Arg. (h)	π_λ (h)
0;15	0;10
0;30	0;19
...	
2; 0	1;11
...	
4; 0	1;35
4;15	1;35
4;30	1;36
4;45	1;35
...	
6; 0	1;29
...	
8; 0	1; 6
...	
8;45	0;54
9; 0	0;50

Component in latitude (f. 53r)

Arg. (°)	π_β (′)
1	0;50
2	1;41
...	
30	23;35
...	
60	40;44
...	
89	48;27
90	48;32

The argument in Ibn al-Kammād's table for the parallax in longitude, in contrast to al-Khwārizmī's, is given here in hours and minutes, at intervals of 15 min; as in the zij of al-Khwārizmī, π_λ reaches a maximum of 1;36h. In the table for the parallax in latitude, the entries differ slightly from al-Khwārizmī's and the maximum is also different (0;48,32° vs. 0;48,45°). This value may not be just a variant reading, for it seems to be related to Crd (2ε). Using the obliquity of the ecliptic in the *Almagest* (ε = 23;51,20°) and the table of chords in *Almagest* I.11, we find Crd (2ε)/60 = 0;48,31,54 ≈ 0;48,32°.

Kennedy discovered one copy of Table 12.2C in the East, in the zij of Ibn al-Shāṭir, *ca*. 1350 (Kennedy 1956b, p. 48; Kennedy and Faris 1970, p. 190). In the Iberian Peninsula, this Indian tradition was represented by Ibn al-Kammād and it was quite successful, for there were at least three other sets of tables which included Tables 12.2B and 12.2C: the Tables of Barcelona (Millás 1962, pp. 234–235; Chabás 1996a, pp. 508–510), the tables of Solomon Franco (Vatican, MS Heb. 498,

f. 59v), and the tables of Juan Gil (Madrid, Biblioteca Nacional, MS 23078, f. 90a; formerly London, Jews College, MS Heb. 135). It is thus clear that this table circulated widely in the Peninsula, especially in the Jewish astronomical community, and we find echoes of this tradition even in Abraham Zacut's *ha-Ḥibbur ha-gadol* (Chabás and Goldstein 2000, p. 123).

CHAPTER THIRTEEN

SYZYGIES

A conjunction or an opposition of the Sun with the Moon occurs when the elongation between the two luminaries is 0° or 180°, respectively. Syzygy is an astronomical term that refers both to conjunction and opposition. At mean syzygy, the double elongation, $2\bar{\eta} = 0°$, where $\bar{\eta} = \bar{\lambda}_m - \bar{\lambda}_s$, and $\bar{\lambda}_m$ and $\bar{\lambda}_s$ are the mean longitudes of the Moon and the Sun. Analogously, at true syzygy, the double elongation, $2\eta = 0°$, where $\eta = \lambda_m - \lambda_s$, and λ_m and λ_s are the true longitudes of the Moon and the Sun. The determination of the time from mean to true syzygy, Δt, an essential step in the calculation of eclipses, was historically one of the major problems considered in computational astronomy (Chabás and Goldstein 1992 and 1997).

Figure 20 represents a mean conjunction of the Sun and the Moon (indicated by the direction of $\bar{S}$ and $\bar{M}$) and its corresponding true conjunction (indicated by the direction of S´ and M´). In this case, the mean conjunction (when $\bar{\lambda}_s = \bar{\lambda}_m$), which takes place at time t, comes

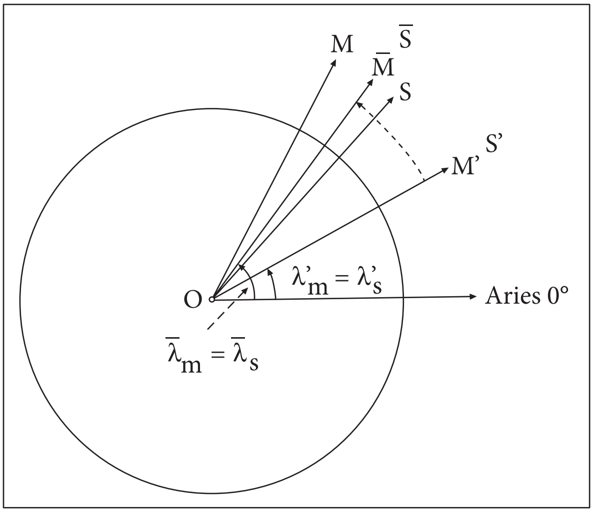

Figure 20: Mean and true syzygies

Table 13.1A: Some historical values of the mean synodic month

Mean synodic month	
29;31,50,7,37,27,8,25d	Parisian Alfonsine Tables
29;31,50,7,54,25,3,32d	Levi ben Gerson
29;31,50,8,9,20d	al-Ḥajjāj's Arabic trans. of the *Almagest*, Copernicus
29;31,50,8,9,24d	Ibn Yūnus, al-Biṭrūjī
29;31,50,8,14,38d	Ibn al-Kammād
29;31,50,8,19,50d	al-Battānī
29;31,50,8,20d	*Almagest*, Toledan Tables

after the true conjunction ($\lambda_s{}' = \lambda_m{}'$), which occurs at time t′, so that $\Delta t = t' - t < 0$.

The mean synodic month is the time between two consecutive mean syzygies of the same kind, that is, between two successive mean conjunctions or two successive mean oppositions.

The length of the mean synodic month was determined in antiquity by the Babylonians. The standard parameter in the *Almagest* is 29;31,50,8,20d, but many variants of it are found in the astronomical literature (see Goldstein 2003; see also Mancha 2002–2003). A few medieval values (some given explicitly and others derived from medieval tables by modern computation) are displayed in Table 13.1A.

1. *Mean Syzygies*

In *Almagest* VI.3 Ptolemy displayed tables for mean conjunctions and oppositions of the Sun and the Moon (see Toomer 1984, pp. 278–280). In fact, there are four different tables, all of them with the same structure. The first is for mean conjunctions, the second for mean oppositions, and the other two for the yearly and monthly increments for mean syzygies. The tables proceed in steps of 25 Egyptian years (of 365 days each) and have five columns. The first column is for the argument (periods of 25 years, single years, or months); the second indicates the number of days elapsed for a certain mean syzygy in the periods used as argument; the third is the mean solar anomaly (in degrees); the fourth is the mean lunar anomaly (in degrees); and the fifth is the mean argument of lunar latitude (in degrees). With this tabulated information one can easily derive the time of mean syzygy for any specific date.

Table 13.1B: List of mean conjunctions (excerpt)

Year	Day	hours	points
1270	Jan 23	19	849
	Feb 22	8	562
	Mar 23	21	275
	Apr 22	9	1069
	May 21	22	781
	Jun 20	11	494
	Jul 20	0	207
	Aug 18	12	1000
	Sep 17	1	713
	Oct 16	14	426
	Nov 15	3	139
	Dec 14	15	932
1271	Jan 13	4	645
	Feb 11	17	358
	. . .		

Many astronomers followed Ptolemy when constructing their own tables for mean syzygies, adapting them to their respective calendars, but maintaining the structure and displaying the same quantities. Others offered tables with the results of their computations for specific dates. As an example of a simple list of successive mean conjunctions for specific dates, we partially reproduce one that appears in a 13th-century manuscript (Brussels, Bibliothèque Royale, MS 2910–20, f. 101r–v) for the period 1270–1278: see Table 13.1B. In this case only the times of the mean conjunctions are given in hours and "points" (*puncti*) of which there are 1080 in an hour. This "point" is a unit of time commonly used by medieval Jewish astronomers who called it *ḥeleq* (lit. "part"; plural: *ḥelaqim*).

Note that the constant difference between successive conjunctions is 29d 12h 793p (where p stands for points), that is, 29;30d + $^{793}/_{(24 \cdot 1080)}$d = 29;31,50,8,20d, which is the value used by Ptolemy in the Babylonian tradition. The same value, 29d 12h 793p, is also used in the Jewish calendar (see, e.g., Neugebauer 1956, p. 114).

In some manuscripts we find longer lists of syzygies, as in the case of John of Murs's *Patefit*, where we are given data for mean conjunctions and oppositions for a period of 76 years from 1321 to 1396, thus covering 1880 (= 940 × 2) syzygies. Table 13.1C reproduces an excerpt of this work as found in London, British Library, MS Royal 12.C.XVII,

Table 13.1C: John of Murs's list of mean conjunctions and oppositions for 1321–1396 (excerpt)

	1321 Conjunctions								1321 Oppositions								
	Day	Time	*motus*		*arg. lune*		*arg. lat.*		Day	Time	*motus*		*arg. lune*		*arg. lat.*	
		(h)	(s)	(°)	(s)	(°)	(s)	(°)		(h)	(s)	(°)	(s)	(°)	(s)	(°)
Jan	28	17;14	10	16;21	10	17; 5	0	20;40	13	22;52	10	1;48	4	4; 9	6	5;20
Feb	27	5;58	11	15;28	11	12;54	1	21;20	12	11;36	11	0;54	4	29;59	7	6; 1
Mar	28	18;42	0	14;34	0	8;43	2	22; 1	14	0;20	0	0; 1	5	25;48	8	6;41
. . .																
Oct	21	11;51	7	8;19	6	9;26	9	26;42	6	17;28	6	23;46	11	26;31	3	11;22
Nov	20	0;35	8	7;25	7	5;15	10	27;22	5	6;13	7	22;52	0	22;20	4	12; 2
Dec	19	13;19	9	6;32	8	1; 4	11	28; 2	4	18;57	8	21;58	1	18; 9	5	12;42

	1322 Conjunctions								1322 Oppositions								
	Day	Time	*motus*		*arg. lune*		*arg. lat.*		Day	Time	*motus*		*arg. lune*		*arg. lat.*	
		(h)	(s)	(°)	(s)	(°)	(s)	(°)		(h)	(s)	(°)	(s)	(°)	(s)	(°)
Jan	18	2; 3	10	5;38	8	26;53	0	28;42	3	7;41	9	21; 5	2	13;58	6	13;22
Feb	16	14;47	11	4;44	9	22;42	1	29;22	1	20;25	10	20;11	3	9;47	7	14; 3
Mar	18	3;31	0	3;51	10	18;31	3	0; 3	3	9; 9	11	19;18	4	5;36	8	14;43
. . .																
. . .																

	1396 Conjunctions								1396 Oppositions								
	Day	Time	*motus*		*arg. lune*		*arg. lat.*		Day	Time	*motus*		*arg. lune*		*arg. lat.*	
		(h)	(s)	(°)	(s)	(°)	(s)	(°)		(h)	(s)	(°)	(s)	(°)	(s)	(°)
Jan	10	13;49	9	28;16	4	9;15	0	12;15	25	8;11	10	12;49	10	22;10	6	27;33
. . .																
Dec	0	9;53	8	18;27	1	23;14	11	19;34	15	15;32	8	3;53	7	10;20	5	4;15
Dec	29	22;48	9	17;33	2	19; 5	0	20;---	15	4;16	9	3; 0	8	6; 9	6	4;55

ff. 155v–168r. The headings in Latin correspond to the mean solar longitude (*motus*), the mean argument of lunar anomaly (*arg. lune*), and the mean argument of lunar latitude (*arg. lat.*) at the time of mean syzygy.

However, this was not the most common presentation for tables of mean syzygies. Rather, the presentation in the zijes of al-Battānī (Nallino 1903–1907, 2:84–87) and al-Khwārizmī (Suter 1914, pp. 183–186) was standard, as can be seen in subsequent major sets of medieval astronomical tables. The layout is analogous to that of the *Almagest*

Table 13.1D: Mean syzygies (excerpt)

Mean conjunctions

Coll. years	Time (d)	(h)	Sun & Moon (s)	(°)	Lunar anomaly (s)	(°)	Arg. of lat. (s)	(°)
1321	28	18; 2, 6	10	16;21,14	10	17; 5,20	0	20;38,41
1345	9	19;21, 9	9	21;26,35	1	8;52,13	3	9; 6,37
...								
1585	20	4; 8, 4	10	9;47,27	0	9; 2, 2	2	19;47,55
1609	24	17;11, 9	10	14;29,32	3	26;37,55	6	8;53, 4

Mean oppositions

Coll. years	Time (d)	(h)	Sun (s)	(°)	Lunar anomaly (s)	(°)	Arg. of lat. (s)	(°)
1321	13	23;40, 5	10	1;48, 2,23	4	4;10,50	6	5;18,35
1345	18	13;43,11	10	6;29,47,57	7	11;46,43	9	24;26,44
...								
1585	5	8;46, 2	9	25;14,34,16	5	26; 7,32	8	4;29,48
1609	9	22;49, 8	9	29;56,20,50	9	13;43,25	11	23;35,57

Expan. years	Time (d)	(h)	Sun & Moon (s)	(°)	Lunar anomaly (s)	(°)	Arg. of lat. (s)	(°)
1	10	15;11,23	11	19;16,50,20	10	9;48, 7	0	8; 2,45
2	21	6;22,47	11	8;33,40,39	8	19;36,14	0	16; 5,30
...								
22	2	16;18,10	11	27;11,40,43	6	2;10,38	2	2;22,25
23 b*	13	7;29,34	11	16;18,31, 3	4	11;58,45	2	10;25,10

* The "b" for year 23 stands for *bissextilis*, i.e., a leap year of 366 days.

Month	Time (d)	(h)	Sun & Moon (s)	(°)	Lunar anomaly (s)	(°)	Arg. of lat. (s)	(°)	Days
Jan.	29	12;44, 3	0	29; 6,24, 0	0	25;49, 1	1	0;40,14	31
Feb.	59	1;28, 6	1	28;12,48, 0	1	21;38, 1	2	1;20,18	59
...									
Nov.	324	20; 4,34	10	20;10,26, 0	9	13;59, 6	11	7;22,31	334
Dec.	354	8;48,37	11	19; 6,50, 0	10	9;48, 7	0	8; 2,45	365
[half]	14	18;22, 2	6	14;33,12, 0	6	12;54,30	6	14;20, 7	

and to that used for the mean motions of the celestial bodies, with sub-tables for collected and expanded years, and months (see Table 5.1A). For each value of the argument we are here given four quantities. The first is the difference in time between the beginning of the calendar year (or month) and the mean syzygy, whether a conjunction or an opposition; the other three are the mean increments, modulo 360°, in solar and lunar longitudes, in lunar anomaly, and in the argument of lunar latitude, all of them at the time of mean syzygy (complete explanations for the use of these tables are found in Neugebauer 1962a, pp. 108–115, and Toomer 1968, pp. 78–81). We note that in the zij of al-Battānī collected years are Seleucid years (cf. Table 1.1A), grouped in units of 24 years, whereas in the zij of al-Khwārizmī and the Toledan years they are Arab years, grouped in units of 30 years. In the Alfonsine corpus collected years are Julian years, also grouped in units of 24 years, beginning in 1321. Table 13.1D displays an excerpt of a table of this type that appears in Bonn, Universitätsbibliothek, MS S 498, ff. 55r–56r. Note that the mean motions of the Sun and the Moon at mean conjunction are only given to sexagesimal seconds, whereas in the rest of the table they are given to sexagesimal thirds.

In the sub-table for months, it is easy to recognize the duration of the mean synodic month (entry for January) and its half value, the entry we have labeled "half," corresponding to the time between successive syzygies.

2. *Time from Mean to True Syzygy*

Finding the time from mean to true syzygy was a major endeavor for medieval astronomers. All those addressing the problem relied on arithmetical methods, rather than on observations, and some gave tabular solutions to it (see, e.g., Chabás and Goldstein 1992 and 1997). Simply stated, the problem is as follows. Let t be the time at which a mean syzygy takes place (indicated with a bar over M and S in Figure 20). Time t is easily found using tables that were commonly available. The aim is to determine Δt, the time interval to be added to or subtracted from t in order to obtain the time, t', at which the corresponding true syzygy occurs. This time interval defined, as $\Delta t = t' - t$, can be positive or negative; when it is positive, true syzygy takes place after mean syzygy, and when it is negative, the opposite occurs.

In *Almagest* VI.4, Ptolemy presented an approximate solution—without reducing it to a table—which may be expressed in modern notation as

$$\Delta t = -13\eta \ / \ 12v_m(t),$$

where η is the true elongation between the longitudes of the Moon and the Sun at mean syzygy ($\eta = \lambda_m - \lambda_s$ for conjunction and $\eta = \lambda_m - \lambda_s + 180°$ for opposition) and $v_m(t)$ is the lunar velocity during the time interval from mean to true syzygy. Ptolemy also added a rule for computing $v_m(t)$: see Toomer 1984, pp. 277–282. This method makes the simplifying assumptions that the solar and the lunar velocities are constant over Δt and that the ratio of the lunar to solar velocity is 13 to 1. For most medieval astronomers this approach, crude as it may seem, was quite satisfactory: it was adopted by al-Battānī (Nallino 1903–1907, 1:94), and it appears in the canons of the tables of al-Khwārizmī (Neugebauer 1962a, p. 63), as well as in the principal astronomical tables composed in medieval Spain, namely the Toledan Tables and the Alfonsine Tables (F. S. Pedersen 2002, p. 451; Chabás and Goldstein 2003, pp. 57 and 189).

The earliest solution to the syzygy problem that we have found in the form of a table appears in Ibn al-Kammād's *al-Muqtabis* (Chabás and Goldstein 1994, p. 14). The solution consists of a double argument table. Its entries, $\Delta t(v_m(t) - v_s(t), \eta)$, are given in hours and minutes, and are functions of the difference between the hourly velocities of the Moon and the Sun (in minutes and seconds per hour, from 0;27,30°/h to 0;33,30°/h at intervals of 0;0,30°/h) and the elongation, η (in degrees and minutes, from 0;30° to 12;0° at intervals of 0;30°). In this approach, the relative velocity of the luminaries is also assumed to be constant during Δt. In table 13.2A we present an excerpt of Ibn al-Kammād's table taken from Madrid, Biblioteca Nacional, MS 10023, f. 52r.

This kind of table was adopted, with minor modifications, by a number of later astronomers, for it is found in various sets tables of the 14th century in the Iberian Peninsula: those of Juan Gil of Burgos and of Joseph Ibn Waqār of Seville, as well as in the Tables of Barcelona.

Among the many works attributed to John of Murs there is a text, which begins *Omnis utriusque sexus armoniam celestem*…," that explains the use of a double argument table for computing the time from mean to true syzygy (Porres and Chabás 2001). In the text the table is called *Tabulae permanentes* and we are told that it was

Table 13.2A: Time from mean to true syzygy in Ibn al-Kammād's zij (excerpt)

[$v_m(t) - v_s(t)$] [η]	0;27,30°/h (h)	0;28, 0°/h (h)	...	0;33, 0°/h (h)	0;33,30°/h (h)
0;30°	1; 6	1; 4	...	0;54	0;53
1; 0	2;11	2; 8	...	1;49	1;47
1;30	3;17	4;12	...	2;43	2;41
...					
6; 0	13; 5	12;51	...	10;45*	10;45
...					
11;30	25; 6	24;39	...	20;36	20;46**
12; 0	26;11	25;43	...	21;30	21;50**

* Read: 10;54.
** These values seem to belong to the column labeled 0;33,0, and the values there belong to this column.

Table 13.2B: Time from mean to true syzygy in the *Tabulae permanentes* (excerpt)

[κ̄] [ᾱ]	0s 0° (h)	0s 6° (h)		3s 0° (h)		6s 0° (h)		9s 0° (h)		11s 18° (h)	11s 24° (h)
0s 0°	0; 0	−0;28	...	−4;47	...	0; 0	...	4;47	...	0;57	0;28
0s 6°	1; 3	0;35	...	−3;44	...	1; 3	...	5;49	...	1;59	1;31
0s 12°	2; 5	1;37	...	−2;41	...	2; 5	...	6;51	...	3; 1	2;33
...											
2s 24°	9;40	9;15	...	5;23	...	9;44	...	14; 0	...	10;31	10; 5
3s 0°	9;40	9;15	...	5;27	...	9;44	...	13;56	...	10;31	10; 5
...											
5s 24°	0;59	0;37	...	−2;48	...	0;59	...	5;44	...	1;44	1;21
6s 0°	0; 0	−0;22	...	−3;46	...	0; 0	...	3;46	...	0;45	0;22

composed together with Firmin of Beauval. In this double argument table the horizontal argument is the mean solar anomaly (κ̄), given in degrees at interval of 6 degrees from 0s 0° to 11s 24°, and the vertical argument is the mean lunar anomaly (ᾱ), also in degrees at interval of 6 degrees from 0s 0° to 6s 0°. Note that signs of 30° are used. There are 1,800 entries, given in hours and minutes. Table 13.2B displays an excerpt of the table by John and Firmin, entitled *Tabula ostendens distantias vere coniunctionis et oppositionis a media*, as found in Erfurt, MS CA 2° 388, ff. 39v–42r. We have added a minus sign where the text reads "m(inue)" and nothing where it reads "a(dde)."

John of Saxony (c. 1330), one of the Parisian astronomers involved in adapting the Alfonsine Tables, composed canons to their Parisian version (see Poulle 1984), in which he offered a more sophisticated solution, using Ptolemy's lunar models. He made allowances for the variation in the lunar velocity in the time interval between mean and true syzygy, Δt, and introduced a method of successive approximations for determining it, first to the nearest hour, and then to the nearest minute of an hour. This method increases the accuracy of the result, but it requires a great deal of computational skill. Unfortunately, this solution cannot be displayed in tabular form. Another attempt to give more accurate solutions was successfully developed by Levi ben Gerson, who depended on his own lunar models rather than on Ptolemy's, but remained within the framework defined by Ptolemy. He presented his original solution in the form of four tables, and all of them contain only additive components to avoid calculations with negative terms (Goldstein 1974, pp. 136–144, 229–241).

Nicholaus de Heybech of Erfurt (c. 1400) seems to have been the first astronomer to reach a solution that met the criteria of improved accuracy and a "user-friendly" presentation in tabular form. To do this, he introduced two terms for the time from mean to true syzygy, to be added algebraically, one for the Sun and one for the Moon, and treated them separately. Each term can be considered as a combination of three functions, some of which only depend on the mean solar anomaly and some only on the mean lunar anomaly. The functions are presented as columns in his table. In calculating the entries in it, Nicholaus de Heybech made use of Ptolemy's second lunar model for computing the underlying lunar velocities, in contrast to his predecessors who generally depended on Ptolemy's simple lunar model. The result is a single table in 5 columns and 180 rows that gives results as good as those derived by previous methods. The rule for computing Δt from the entries in the table is as follows:

$$\Delta t = \Delta t_s - \Delta t_m = [c_1(\bar{\kappa}) - c_2(\bar{\kappa}) \cdot c_3(\bar{\alpha})] - [c_4(\bar{\alpha}) - c_5(\bar{\alpha}) \cdot c_3(\bar{\kappa})],$$

where $\bar{\kappa}$ is the mean solar anomaly, $\bar{\alpha}$ is the mean lunar anomaly, and $c_1, \ldots, c_5$ are the entries in columns 1,…, 5 of Heybech's table. Table 13.2C displays an excerpt of the table taken from Basel, Universitätsbibliothek, MS F.II.7 (ff. 36r–37v). Nicholaus de Heybech's entire table is published in Chabás and Goldstein (1992), together with a detailed explanation of the way it was computed. Note that here the argument uses physical signs of 60°.

Table 13.2C: Nicholaus de Heybech's table for the time from mean to true syzygy (excerpt)

Argument (°)/(°)	1 *Equatio solis* (h)	2 *Diversitas eq. solis* (h)	3 *Min. prop.* (h)	4 *Equatio lune* (h)	5 *Diversitas eq. lune* (min)
1/5,59	0; 5	0; 1	0	0;11	0
2/5,58	0;10	0; 2	0	0;22	0
3/5,57	0;15	0; 3	0	0;33	0
...					
30/5,30	2;19	0;30	4	4;59	2
...					
1, 0/5, 0	4; 4	0;51	15	8;31	3
...					
1,30/4,30	4;47	1; 1	31	9;40	4
...					
2, 0/4, 0	4;14	0;54	47	8;15	3
...					
2,30/3,30	2;29	0;32	56	4;42	2
...					
2,58/3, 2	0;12	0; 2	60	0;20	0
2,59/3, 1	0; 6	0; 1	60	0;10	0
3, 0/3, 0	0; 0	0; 0	60	0; 0	0

The approach taken in the *Tabulae permanentes* was retained by early modern astronomers, such as John of Gmunden and Georg Peurbach. In the *Tabulae eclypsium*, an extensive set of astronomical tables compiled by Peurbach and first printed in 1514, the forty-eight-page table entitled *Tabula distantie vere coniunctionis aut oppositionis a media* (ff. a3v–d3r) is a variant of the table by John and Firmin, but the total number of entries was increased to 32,400.

The same presentation in a double argument table is also found in an earlier work on eclipses, known as *Six Wings,* by Immanuel ben Jacob Bonfils of Tarascon in the mid-fourteenth century: see Solon 1970. The table that Bonfils called "Wing 2" is for determining the differences both in longitude and time from mean to true syzygy, that is, $\Delta\lambda$ and Δt. Actually, the tabulated entries for the latter represent Δt + 24;16h, for they take into account the equation of time and have 24h added to avoid calculations with negative terms. In Bonfils's table the positions of the arguments are interchanged with respect to

Table 13.2D: Levi's true lunar motion for each day after mean syzygy (excerpt)

Days	0s 0° (°)	0s 10° (°)	. . .	6s 0° (°)	. . .	11s 10° (°)	11s 20° (°)
1	12; 7,11	12; 9,18	. . .	15;27,10	. . .	12; 9,46	12; 7,27
2	24;14, 0	24;19,58	. . .	30;40,30	. . .	24;14,24	24;11,42
. . .							
13	168;35,52	170,23,34	. . .	172; 9,11	. . .	165;10,50	166;51,51
14	183;57,28	185;44,17	. . .	184;13,53	. . .	180;28,35	182;11,30

those of John of Gmunden but, more importantly, they can be derived from al-Battānī's tables, and thus do not follow the Alfonsine tradition, in contrast to the tables of Nicholaus de Heybech and John of Gmunden.

Levi ben Gerson also constructed a double argument table which displays the arc of longitude traveled by the Moon between successive syzygies, where the one argument is the number of days after mean syzygy and the other argument is the mean motion in anomaly at mean syzygy. The entries are to be added to the mean longitude of the Moon at syzygy in order to find the true position of the Moon (Goldstein 1974, pp. 148–149, 246–254). Table 13.2D shows an excerpt of it.

Within the astronomical tradition in the Iberian Peninsula, there were other tables where the computation of time from mean to true syzygy relied on separating the effects of the Sun and the Moon. Such is the case of Isaac Ibn al-Ḥadib's astronomical tables (of which there exists a Latin version compiled by Flavius Mithridates in the second half of the 15th century), which are in the Ptolemaic tradition, but do not depend on the Toledan Tables or the Alfonsine Tables (Goldstein and Chabás 2006). In Tables 13.2E and 13.2F we reproduce excerpts of two of al-Ḥadib's tables as presented by Mithridates (Vatican, Biblioteca Apostolica, MS Urb. lat. 1384). Table 13.2E displays the correction for the solar position (expressed in hours) at syzygy as a function of the solar anomaly given for each degree, whereas Table 13.2F displays the correction for the lunar position (expressed in hours) at syzygy as a function of the lunar anomaly given for each degree. In both tables "S" and "A" stand for "subtract" and "add."

Table 13.2E: Al-Ḥadib's solar equation in time at syzygy (excerpt) (f. 44r)

S (°)	0[s] (h)	1[s] (h)	2[s] (h)	3[s] (h)	4[s] (h)	5[s] (h)
1	0; 4, 1	1;57,10	3;21,11	3;54,22	3;23, 0	1;56,55
2	0; 7,56	2; 0,16	3;23,15	3;54,20	3;21,55	1;53,13
3	0;11,52	2; 4, 0	3;25,17	3;54,18	3;20, 8	1;49,25
4	0;15,48	2; 7,22	3;27,15	3;54, 8	3;18, 1	1;45,35
5	0;19,41	2;10,43	3;29, 9	3;53,51	3;15,51	1;41,45
. . .						
10	0;39,20	2;26,41	3;37,28	3;51,56	3; 3,44	1;22,21
. . .						
20	1;16,35*	2;55,32	3;49,21	3;41,15	2;33, 0	0;42, 5
. . .						
30	1;53,52	3;19, 6	3;54, 2	3;24,40	2; 0,41	0; 0, 0
A	11[s]	10[s]	9[s]	8[s]	7[s]	6[s]

* Read: 1;17,35.

Table 13.2F: Al-Ḥadib's lunar equation in time at syzygy (excerpt) (f. 44v)

S (°)	0[s] (h)	1[s] (h)	2[s] (h)	3[s] (h)	4[s] (h)	5[s] (h)
1	0; 9,20	4;38,25	8; 9, 5	9;40, 1*	8;39,46	5; 5,30
2	0;18,40	4;46,52	8;14,25	9;41,35	8;36,15	4;55,47
3	0;28, 2	4;55, 0	8;18,36	9;41,54	8;31,18	4;46,20
4	0;37,20	5; 3, 6	8;22,46	9;42, 4	8;26, 9	4;36,37
5	0;46,40	5;11,12	8;27,50	9;42, 5	8;20,49	4;26,53
6	0;56, 0	5;19,19	8;32, 2	9;42, 6	8;15,14	4;17, 3
. . .						
10	1;33, 0	5;50,32	8;49,41	9;40,11	7;51,29	3;36,47
. . .						
20	3; 3,49	7; 1,20	9;22,55	9;21,53	6;40,13	1;50,30
. . .						
30	4;30,15	8; 1,44	9;40,11	8;45,42	5;14,26	0; 0, 0
A	11[s]	10[s]	9[s]	8[s]	7[s]	6[s]

* Probably a mistake for 9;41,0h; the same mistake is found in Vatican, MS Heb. 379.

3. *True Syzygies*

The tables for true syzygies require a vast amount of computation by the compiler of the tables and are aimed at offering the user a final product requiring no further work. The tables of Jacob ben David Bonjorn, for instance, were computed for Perpignan (Southern France), for a geographical latitude of 42;30°, with epoch 1361. Nothing in the canons accompanying Bonjorn's tables points to a new method for computing the time from mean to true syzygies; rather, as we have shown elsewhere (see Chabás 1991), the calculation of the 767 true syzygies relies on Levi's tables for mean syzygies as well as for the correction to the time of syzygy. However, what is innovative in Bonjorn's approach is the introduction of a correction to determine the time of true syzygy for periods before and after the basic cycle 1361–1391 (see column headed "Corr." in Table 13.3A), thus rendering the table valid for a long span of time and enormously facilitating the task of an astronomical practitioner. We note that the entries in that column are given in time, in minutes and parts of them (that we have separated by a colon), such that 17 parts = 1 min. The division of a minute into 17 parts is most unusual.

Table 13.3A: Bonjorn's table for true syzygies (excerpt)

Year	Date		Week-day	Time (h)	Corr. (min)	Sun & Moon (s)	Sun & Moon (°)	Arg. of lat. (s)	Arg. of lat. (°)	
1361	Mar	7	1	4;57	32:12	11	25;37	3	25;45	[C]
	Mar	21	1	20;28	21: 1	0	10; 4	4	10;58	[O]
	Apr	5	2	13;21	36: 7	0	24;25	4	26; 6	[C]
	Apr	20	3	12;56	26: 3	1	8;55	5	11;24	[O]
	May	4	3	21; 4	38: 0	1	22;40	5	25;54	[C]
	May	20	5	3;32	30: 5	2	7;15	6	11;18	[O]
	Jun	3	5	4;59	37: 0	2	20;36	6	25;24	[C]
	Jun	18	6	16; 4	32: 5	3	5;16	7	10;53	[O]
	Jul	2	6	14; 2	33: 2	3	18;27	7	24;48	[C]
	Jul	18	1	2;54	32:12	4	3;13	8	10;23	[O]
...										
1391	Mar	6	2	2;45	19: 8	11	24;18	11	4;35	[C]
	Mar	20	2	11;42	33: 1	0	8;29	11	19;32	[O]
	Apr	4	3	19; 5	26: 2	0	23;26	0	5;17	[C]
...										
	Jan	24	4	9; 0	15: 9	10	13; 7	10	10;34	[C]
	Feb	8	5	18;40	28: 6	10	28;43	10	26;59	[O]
	Feb	23	6	2;36	17: 3	11	13; 6	11	12; 7	[C]

[C] = conjunction; [O] = opposition

Abraham Zacut was one of the followers of Bonjorn. In addition to praising Bonjorn in his *ha-Ḥibbur ha-gadol,* he also adapted Bonjorn's tables for syzygies for Salamanca, Zacut's hometown, and made them start in March 1478. This table by Zacut appeared in the Latin version of the *Ḥibbur*, and in the *Almanach Perpetuum*. The table displays the time of 767 consecutive true syzygies as well as the corresponding correction in each case, but lacks the two columns associated with the lunar position. Note also that the entries for the correction are rounded to minutes. However, this was not the only table for computing true syzygies proposed by Zacut in the *Ḥibbur,* which was later reproduced in the *Almanach Perpetuum*. He also composed a double argument table whose title is "Table for correcting the time of conjunction and opposition and quarters of the month and all [astrological] aspects of the Moon with all the planets" (Lyon, MS Heb. 14, f. 142r). It has the elongation between the Sun and the Moon, in degrees, as vertical argument and the daily increment of elongation, from 10;36° to 16;0°, as horizontal argument. The 868 entries in this table do not give the time from mean to true syzygy but the time at which the true syzygy takes place, counted from mean noon in his city, Salamanca (Chabás and Goldstein 2000, pp. 128–130). Table 13.3B displays an excerpt of this table, as presented in the *Almanach Perpetuum*.

The tradition of tables for true syzygies represented by Bonjorn and Zacut, gave rise to a very popular astronomical genre, the *lunaria*, consisting of annual tables displaying the time of successive true

Table 13.3B: Table for the equation of syzygies in the *Almanach Perpetuum* (excerpt)

	10;36 (h)	10;48 (h)	11; 0 (h)	. . .	15;36 (h)	15;48 (h)	16; 0 (h)
0; 5	0;11	0;11	0;11	. . .	0; 8	0; 8	0; 8
0;10	0;23	0;22	0;22	. . .	0;16	0;15	0;15
0;20	0;46	0;44	0;44	. . .	0;31	0;30	0;30
0;30	1; 8	1; 7	1; 6	. . .	0;46	0;45	0;45
. . .							
1; 0	2;16	2;13	2;11	. . .	1;32	1;31	1;30
1;30	3;24	3;20	3;17	. . .	2;16	2;16	2;15
. . .							
12; 0	0; 0	0; 0	0; 0	. . .	18;29	18;12	18; 0
12;30	0; 0	0; 0	0; 0	. . .	19;14	18;57	18;45
13; 0	0; 0	0; 0	0; 0	. . .	20; 0	19;45	19;30

Table 13.3C: True syzygies for 1485 in Granollachs's *Lunari*

Date	Type of syzygy	Time (h)
Jan 16	Conjunction	5;49
Jan 30	Opposition	7;57
Feb 14	Conjunction	16;54
Mar 1	Opposition	1;43
Mar 16	Conjunction	2;46
Mar 30	Opposition	1; 4
Apr 19	Conjunction	10;55
Apr 29	Opposition	11;11
May 13	Conjunction	18;32
May 29	Opposition	1;31
Jun 12	Conjunction	2;31
Jun 27	Opposition	13;55
Jul 11	Conjunction	11;50
Jul 27	Opposition	0;43
Aug 10	Conjunction	23;24
Aug 25	Opposition	10;36
Sep 8	Conjunction	13;41
Sep 23	Opposition	19;53
Oct 8	Conjunction	6;33
Oct 23	Opposition	5;12
Nov 7	Conjunction	1; 9
Nov 21	Opposition	15;10
Dec 6	Conjunction	20; 8
Dec 21	Opposition	2;10

conjunctions and oppositions of the Sun and the Moon for a given locality, with indication of the circumstances of eclipses. No doubt, the most widely diffused example of *lunaria* was that first published in 1485 by a member of the lesser nobility (*ciutadà honrat*) of Barcelona, Bernat de Granollachs (Chabás and Roca 1985, 1998). It consists of 66 annual tables covering the period 1485–1550. The tables for true syzygies computed by Granollachs for his hometown have a direct ancestor in the tables of Jacob ben David Bonjorn, who had also been active in Barcelona more than a century earlier. Table 13.3C displays the first table (1485) in Granollachs's *Lunari*. In this case, the column for the correction found in Bonjorn's and Zacut's tables has disappeared.

Granollachs's *Lunari* was not a thick book, no more than 68 pages and, perhaps due to this fact, it became a best-seller in the early years of printing astronomical literature: we know of more than 60 editions in 40 years in Spain, France, and Italy. It has been suggested that several tens of thousands of copies of that concise and practical book circulated in Southern Europe at the time, and the readers, no longer confined to practitioners of astronomy, could find directly in it, with no further computation, all they needed to know in advance about new and full moons, as well as solar and lunar eclipses.

CHAPTER FOURTEEN

PLANETARY CONJUNCTIONS

1. *Sun–Planets*

The mathematical procedure for determining the circumstances of conjunctions between the Sun and any of the planets is the same as that for the conjunctions of the Sun and the Moon. It is therefore not surprising that an astronomer who devoted various works to the computation of syzygies, e.g., John of Murs (see §13.1), also focused on the conjunctions of the Sun and the planets, for which he used the same approach. But he was among the few astronomers who made tables for this purpose. As in the case of conjunctions of the Sun and the Moon, John of Murs presented two tables for the conjunctions of each planet with the Sun: a *tabula principalis* listing the dates of successive mean conjunctions of the Sun and the planet from 1320 onwards and a *contratabula* to find the true longitude of each planet when not in conjunction with the Sun (see Chabás and Goldstein 2009a). These tables belong to a set called "The Tables of 1321" which, as far as we can determine, are only extant in two manuscripts: Lisbon, MS Ajuda 52-XII-35, and Oxford, Bodleian Library, MS Can. Misc. 501. Tables 14.1A and 14.1B give excerpts for the conjunctions of the Sun and Saturn, reproduced from these manuscripts. In Table 14A the first column displays the years from 1320 to 1359 where conjunctions occur (in the case of Saturn, 58 years are listed, for no such conjunction took place in years 21 and 50 after the radix); the second column displays the number of days in that year (counted from the last day of the previous year) that have elapsed; the third column replaces the number of days in the second column by the date; the fourth column is the time of the mean conjunction; and the rest of the columns give the *motus* (mean motion), *centrum* (mean argument of center), and *verus locus* (true longitude) of the planet at that time. Below the table are the amounts to be added to the first entry in a given column to arrive at the final entry in that column.

Table 14.1A: Mean conjunctions of the Sun and Saturn (excerpt)

Year	day	month	time (h)	*motus* (°)	*centrum* (°)	*verus locus* (°)
1320	100	Apr 10	15;17	0,27;14	2,15;51	0,22;59
1	113	Apr 23	17;29	0,39;54	2,28;31	0,36;43
2	126	May 6	19;41	0,52;34	2,41;10	0,50;35
. . .						
59	102	Apr 11	20;53	0,29; 8	2,17;15	0,24;59
	add	1	5;36	1;54	1;24	

Table 14.1B: Correction for Saturn when not in conjunction with the Sun (excerpt)

w	d	days	hours	0,0° (°)	hd (′)	0,12° (°)	. . .	2,12° (°)	hd (′)	2,24° (°)	. . .	5,36° (°)	hd (′)	5,48° (°)	hd (′)
1	5	12	14;48	1;29	34	0;21	. . .	2;52	28	1;55	. . .	3;51	34	2;44	37
3	4	25	4;57	2;56	35	1;46	. . .	1;16	29	0;17	. . .	5;19	34	4;11	37
. . .															
27	0	189	1; 6	5;35	43	4;10	. . .	1;13	38	2;29	. . .	8;30	42	7; 5	45
. . .															
52	1	365	11;43	9;53	33	8;46	. . .	7;23	33	8;29	. . .	12;17	35	11; 8	37
54	0	378	2;12	11;24	33	10;18	. . .	9; 7	33	10;13	. . .	13;44	35	12;34	35

In order to find the true longitude of the planet when not in conjunction with the Sun, another table, called *contratabula*, is needed (see Table 14.1B for Saturn). The entries in this double argument table give, in degrees and minutes, the correction to be added, or subtracted, to the mean motion of the planet to obtain its true longitude, as a function of the mean argument of center of the planet, given at intervals of 0;12°. In the heading, "w" stands for week, "d" for day of the week, and "hd" for half the difference (in minutes of arc) between two successive values of the mean argument of center of the planet.

2. *Superior Planets*

In historical astrology planetary conjunctions play an important role: see, e.g., Goldstein 1964; Goldstein and Pingree 1990; and Yamamoto and Burnett 2000, 1:11–43, 582–589. Very few tables for computing the circumstances of planetary conjunctions are found among sets of astronomical tables in the West. Vienna, Nationalbibliothek, MS 2288, f. 38r–v, a 14th-century manuscript, has one such table for the mean conjunctions of the superior planets presented as three sub-tables, one for each of the pairs Saturn–Jupiter, Saturn–Mars, and Jupiter–Mars (Table 14.2A). In contrast to the true conjunctions of the Sun and the planets in Tables 14.1A and 14.1B, the conjunctions of the superior planets addressed in Table 14.2A are mean conjunctions. Column 1 supplies the argument for columns 2 and 3: it displays the number of mean conjunctions of each kind, up to 600 accumulated conjunctions for the pair Saturn-Jupiter, and 1,200 for the other two pairs. Although not specified, column 2 is the time, in days, in sexagesimal multiples and submultiples of it, between the initial conjunction and a subsequent conjunction; and column 3 is the distance, in degrees, traversed by the slower planet (Saturn for the first and second sub-tables and Jupiter for the third one).

To show that the first entry in column 2 in the first sub-table, 2,0,53;37,29d = 7253;37,29d (about 20 years), is the time between two successive conjunctions of Saturn and Jupiter, let v_S and v_J be the daily velocities of Saturn and Jupiter, respectively. In the Parisian Alfonsine Tables v_S = 0;2,0,35,17,40,21°/d and v_J = 0;4,59,15,27,7,24°/d (see Table 5.1F). The relative velocity is their difference, 0;2,58,40,9,27,3°/d, and the time sought, $t = 360/(v_J - v_S)$, is 7253;37,29d, in agreement with the text. To show that the first entry in column 3 in the first sub-table, 4,2;58,21,52° = 242;58,21,52°, is the distance traversed by Saturn during time t, one has to compute $v_S \cdot t$, which amounts to 242;58,21,52°, also in agreement with the text. The entries in rows 2, 3, etc. just display multiples of the entries in the first row. The same applies for the other sub-tables.

In order to compute the exact moment of any conjunction, as well as the positions of the planets at that time, radices are needed. And indeed the text provides this information (f. 38r–v):

> *Radix coniunctionum Saturni et Iovis post Incarnationem Christi in anno imperfecto 14:* 1,25,7;5,10 4,5;9,34,44
> *Radix coniunctionum Saturni et Martis post Incarnationem Christi in primum annum:* 1,6;35,10,11 1,16;19,9,25
> *Radix coniunctionum Iovis et Martis secundum medium cursum post Incarnationem Christi:* 5,15;40,58,22,30 3,26;51,51,12,45

In each case we are given the time since the Incarnation (i.e., the number of days in sexagesimal form) and the mean position of both planets

Table 14.2A: Planetary conjunctions of the superior planets in Vienna, MS 2288 (excerpt)

Saturn and Jupiter

	[time] (d)	[distance] (°)
1	2, 0,53;37,29	4, 2;58,21,52
2	4, 1,47;14,58	2, 5;56,43,44
3	6, 2,40;52,27	0, 8;55, 5,36
...		
240	8, 3,34,29;56, 0	5,53;27,28, 0
360	12, 5,21,44;54, 0	5,50;11,12, 0
600	20, 8,56,14;50, 0	5,53;38,40, 0

Saturn and Mars

1	12,13;50,18	24;34,52,17
2	24,27;40,37	29; 9,44,34*
3	36,41;30,55	1,13;44,36,51
...		
600	2, 2,18,23; 4, 0	5,48;42,50, 0
960	3,15,41,24;56, 0	3,17;56,32, 0
1200	4, 4,36,46;10, 0	5,37;25,40, 0

* Read: 49;9,44,34°.

Jupiter and Mars

1	13,36;26, 9,58	1, 7;52,58,21,52
2	27,12;52,19,56	2,15;44, 9,18,32
3	40,49;18,29,54	3,23;36,13,57,48
...		
600	2,16, 4,21;39,40, 0	0,40;46,32,40, 0
960	3,37,42,58;39,28, 0	5,53;14,28,16, 0
1200	5,32, 8,43;19,20, 0	1,21;33, 5,20, 0

at that time. So, the first statement indicates that a mean conjunction of Saturn and Jupiter occurred 5,107 days and 2;4h after the Incarnation (Dec. 25, 14 AD at 2;4h after noon, where *anno imperfecto 14* refers to the current year, 14 AD, i.e., 13 full years have elapsed) and the mean position of Saturn was 245;9,34,44°. According to recomputations using a program for the Parisian Alfonsine Tables prepared by E. S. Kennedy and H. Mielgo, there was a mean conjunction of Saturn and Jupiter on Dec. 25, 14 AD at about 2h after noon, at longitude 245;9,34°, in close agreement with the text. Similarly, there was a mean conjunction of Saturn and Mars on Mar. 7, 1 AD at about 14h after noon, at longitude 76;19,9°, in close agreement with the text (Mar. 7 is day 66 = day 1,6 of the year). And for the conjunction of Jupiter and Mars, recomputation yields a mean conjunction on Nov. 11, 1 AD at about 16h after noon, at about longitude 206;52°, in close agreement with the text (Nov. 11 is day 315 = 5,15 of the year).

The only other table known to us that gathers such information, although presented in a different way, is found in the second edition of the Parisian Alfonsine Tables (Santritter 1492, ff. h8r–i2v). The heading of the first sub-table is *Tabula coniunctionum Saturni et Jovis post Incarnationem Christi, secundum medium motum: per tabulas Alfonsi notate*. Table 14.2B displays excerpts of all sub-tables for the three pairs. In each case column 1 gives the date of the mean conjunction (year, month, day, hour, minute, and second). Note that the year and the month refer to periods of time that have already passed. Column 2 gives the number of accumulated conjunctions; column 3 displays the number of days, in sexagesimal notation both for integers and fractions, that have elapsed since the Incarnation; column 4 displays the mean positions of the planets at that time; and column 5 displays the symbols of the corresponding zodiacal signs of the mean positions.

Table 14.2B: Planetary conjunctions of the superior planets in the second edition of the Parisian Alfonsine Tables: Santritter 1492 (excerpts)

Saturn and Jupiter

Year	Date (M)	(d)	Time (h)	Conj.	Days	Position (s)	(°)	Sign
13	11	25	2; 4, 0	1	0, 1,25, 7; 5,10	4	5; 9,34,44	Sgr
33	10	3	17; 3,36	2	0, 3,26, 0;42,39	2	8; 7,56,36	Leo
...								
1563	0	4	19;32,48	79	2,38,34,49;48,52	1	57; 2, 0,20	Cnc
1582	10	14	10;32,24	80	2,40,35,43;26,21	0	0; 0,22,12	Ari

Saturn and Mars (*anni collecti*)

Year	Date (M)	(d)	Time (h)	Conj.	Days	Position (s)	(°)	Sign
0	2	7	14; 0,24	Radix	0, 0, 1, 6;35, 1	1	16;19, 9,25	Gem
60	5	15	17;42,24	30	0, 6, 8, 1;44,16	1	33;45,18,10	Cnc
...								
1567	3	23	14;12,24	780	2,39, 1, 0;35,31	2	49;38,56,53	Vir
1627	7	1	17;54,24	810	2,45, 7,55;44,46	3	7; 5, 5,38	Lib

Saturn and Mars (*anni expansi*)

Year	Date (M)	(d)	Time (h)	Conj.	Days	Position (s)	(°)	Sign
2	0	3	20; 7,12	1	0, 0, 2,13;50,18	0	24;34,52,17	Ari
4	0	6	16;14,48	2	0, 0,24,27;40,37	0	49; 9,44,35	Tau
...								
58	3	7	7;34,24	26*	0, 5,54,41;18,56	5	52;51,16,27	Psc
60	3	10	3;42, 0	30	0, 6, 6,55; 9,15	0	17;26, 8,45	Ari

* Instead of 29.

Jupiter and Mars (*anni collecti*)

Year	Date (M)	(d)	Time (h)	Conj.	Days	Position (s)	(°)	Sign
0	10	11	16;23,12	Radix	0, 0, 5,15;40,58	0	26;51,51,13	Ari
45	6	27	9;43,12	20	0, 4,37,24;24,18	2	4;13,24,18	Leo
...								
1833	9	17	6;52,24	820	3, 6, 3,13;17,11	0	58;35,27,51	Tau
1878	6	2	0;12, 0	840	3,10,35,22; 0,30	5	35;57, 0,57	Psc

Jupiter and Mars (*anni expansi*)

Year	Date (M)	(d)	Time (h)	Conj.	Days	Position (s)	(°)	Sign
2	2	26	10;28, 0	1	0, 0,13,36;26,10	1	7;52, 4,39	Gem
4	5	20	20;56, 0	2	0, 0,27,12;52,20	2	15;44, 9,19	Leo
. . .								
42	5	21	6;51,36	19	0, 4,18,32;17, 9	3	29;29,28,26	Lib
44	8	14	17;19,36	20	0, 4,32, 8;43,19	4	37;22,23, 5	Cap

CHAPTER FIFTEEN

ECLIPSES

A standard topic in ancient and medieval astronomy was the prediction of eclipses, and many techniques were developed to do so, including the construction of various astronomical tables. A lunar eclipse occurs when the Earth casts its shadow on the Moon. A solar eclipse occurs when the Moon casts its shadow on the Earth. In both cases, the Moon is at syzygy (full Moon, i.e., opposition, for a lunar eclipse, and new Moon, i.e., conjunction, for a solar eclipse); in both cases, the Moon has to be close to one of its nodes, that is, the argument of lunar latitude is restricted to a zone near one of the nodes, and the lunar latitude has to be small. According to the *Almagest*, the distance of the Sun from the Earth does not vary, whereas the distance of the Moon from the Earth varies even at syzygy (see *Almagest* V.13,15; Toomer 1984, pp. 247–257).

In a lunar eclipse, the circular shadow of the Earth projected onto the Moon is surrounded by a larger circle, the penumbra. In some cases a lunar eclipse only takes place in the penumbra, but medieval astronomers did not consider them. In Figure 21 we display the situation of a total lunar eclipse where the Moon, traveling from M_1 to M_5, is totally obscured by the Earth's shadow centered at S'. The first contact occurs at M_1 and the second at M_2 (first inner contact), the central position M_3 is mid-eclipse, and M_4 and M_5 are the second inner contact and the last contact, respectively. Medieval astronomers used the expressions "half-duration of the eclipse" or "immersion" (*minuta casus*) for the arc M_1M_3 (= M_3M_5) or, less frequently, for the time the Moon takes to traverse this arc. Another quantity displayed in medieval tables is the "half-duration of totality" (*minuta more* or *dimidium more*), when the eclipse is total, as depicted in Figure 21. This quantity is the arc M_2M_3 (= M_3M_4) or, less frequently, the time the Moon takes to traverse it.

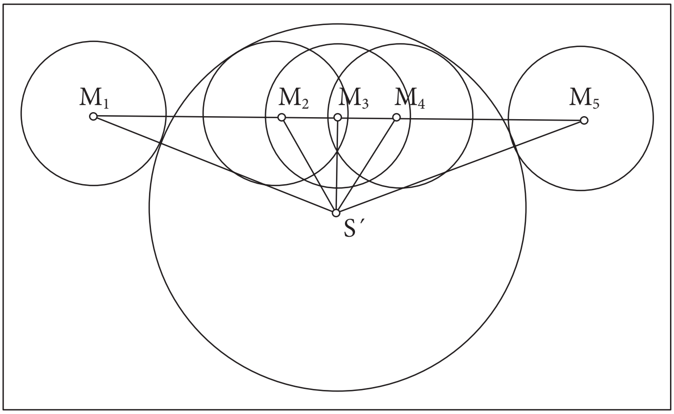

Figure 21: Total lunar eclipse

1. *Diameters of the Luminaries and the Earth's Shadow*

In *Almagest* V.14, Ptolemy devoted an entire chapter to the apparent diameters of the Sun, the Moon, and the Earth's shadow (at the distance of the Moon) at syzygies, although he did not include any specific table to determine them (Toomer 1984, pp. 251–254; see also Neugebauer 1975, pp. 103–108). In contrast, the zij of al-Khwārizmī displays information on the three radii and it is inserted into a table for the true motion of the Sun and the Moon (for the tables, see Suter 1914, pp. 175–180; for an analysis of them, see Neugebauer 1962a, pp. 105–107; see also Goldstein 1967, pp. 226–230). There are a few tables of this type that differ from one another, associated with the Toledan Tables, depending either on al-Khwārizmī's table or on the parameters given by al-Battānī in his zij (see Toomer 1968, pp. 83–84, 94–95, 157–158; F. S. Pedersen 2002, pp. 1409–1425). Although no such table is found in the *editio princeps* of the Parisian Alfonsine Tables, the second edition (Santritter 1492) included a table of which we give an excerpt in Table 15.1. It is similar in format to the one reproduced by F. S. Pedersen (2002, p. 1425), with slightly different entries. The argument is the solar anomaly (in physical signs and degrees) for the column displaying the radius of the Sun, whereas the argument is the lunar anomaly for the remaining columns; for a full explanation of the column for the variation of the shadow, see Toomer 1968, pp. 157–158.

For a table combining the three radii together with the velocities of the Sun and the Moon, see § 8.1.

Table 15.1: Radii of the Sun, the Moon, and the Earth's shadow in the Parisian Alfonsine Tables: Santritter 1492, f. k1v (excerpt)

Argument (°)	(°)	Radius Sun (′)	Radius Moon (′)	Radius shadow (′)	Variation shadow (′)
0	6, 0	15;40	14;30	37;42	0; 0
6	5,54	15;41	14;31	37;45	0; 0
12	5,48	15;41	14;32	37;48	0; 0
. . .					
1,30	4,30	16;15	15;59	41;35	0;26
. . .					
2,48	3,12	16;53	18; 0	46;49	0;55
2,54	3, 6	16;54	18; 3	46;55	0;56
3, 0	3, 0	16;55	18; 4	46;57	0;56

2. *Digits of Eclipse*

2.1 *Lunar eclipses*

In *Almagest* V.15 and V.17 (Toomer 1984, pp. 257, 259) Ptolemy argues that at syzygy the lunar distance from the center of the Earth varies from 53;50 to 64;10 terrestrial radii, whereas he considers the solar distance fixed at 1210 terrestrial radii. Then, in *Almagest* VI.8, Ptolemy presents four eclipse tables for computing the basic features of eclipses: (i) lunar eclipses at greatest and least distances of the Moon from the Earth, (ii) solar eclipses at greatest and least distances of the Moon from the Earth, (iii) interpolation schemes for the eclipses between greatest and least lunar distances (see § 15.3), and (iv) eclipsed part of the solar or lunar disks (see § 15.4). In this section we focus on the first two features, i.e., the tables for digits of eclipse (Toomer 1984, pp. 306–307). In both of them one enters with the argument of lunar latitude, ω (the difference between the lunar longitude and the longitude of the lunar ascending node), whereas in Ptolemy's *Handy Tables* the lunar latitude, β, is the argument in these two tables (Stahlman 1959, pp. 258–259). As indicated in ch. 9, these two quantities are related by the modern expression:

$$\sin \beta = \sin i \cdot \sin \omega,$$

where i is the inclination of the lunar orb, i.e., the angle between it and the ecliptic (see Figure 15).

As in the *Almagest*, al-Khwārizmī compiled tables for the digits of eclipse with ω, the lunar argument of latitude, as the argument (Suter 1914, pp. 187–190, 193), but he used parameters that differ from those of Ptolemy. In contrast, al-Battānī adhered to the tradition of the *Handy Tables*, where one enters the table with β, the lunar latitude (Nallino 1903–1907, 2:90–91), but he too based his tables on parameters that differ from Ptolemy's. Note that Ptolemy used i = 5° for the inclination, both in the *Almagest* and the *Handy Tables*, whereas for al-Khwārizmī i = 4;30°. In Tables 15.2A and 15.2B we present some values of historical interest for the limits of eclipses, as well as the maximum values for other quantities that characterize eclipses, both lunar and solar (digits, times of immersion, and half-totality). Note that the times of immersion and half-totality are given in minutes of arc.

These two traditions converged in the Toledan Tables, where we find identical tables, but for copyists' errors, with those in the zijes of al-Khwārizmī and al-Battānī (Toomer 1968, pp. 86–96 and F. S. Pedersen 2002, pp. 1458–1478). However, the *editio princeps* of the Alfonsine Tables only include tables using the argument of lunar latitude (Ratdolt 1483, ff. m1v–m2v), and they are the same as those in the Toledan Tables in the tradition of al-Khwārizmī. Both forms of presentation persisted throughout the Middle Ages, and it is not uncommon to find them used contemporaneously. For example, both are represented in Oxford, Bodleian Library, MS Can. Misc. 27, ff. 93v–95r, a miscellaneous 15th-century manuscript with a wealth of tables associated with the Parisian Alfonsine Tables. We reproduce excerpts of the tables in that manuscript: the first two (Tables 15.2C and 15.2D) deal with lunar eclipses and are in the tradition of al-Khwārizmī: *ad longitudinem longiorem* (greatest distance, when the Moon is at the apogee of its epicycle) and *ad longitudinem propiorem* (least distance, when the Moon is at the perigee of its epicycle); the third (Table 15.2E), also for lunar eclipses, is in the tradition of al-Battānī (i.e., the argument used is the lunar latitude) and is presented as two sub-tables, one for the Moon at apogee and one for the Moon at perigee. This third table is also found in the Almanac of Azarquiel (Millás 1943–1950, p. 231), which probably explains why it is given the title "Tabula zachelis" or "Tabula Toletana" in some manuscripts. The columns labeled "digits," often called *puncti eclipsis*, give the eclipsed fraction of the diameter of the disk and it is expressed in digits such that 12 digits is the diameter of the eclipsed body. The columns headed "immersion"

Table 15.2A: Some historical values of the principal parameters for lunar eclipses

	Greatest distance				Least distance			
	ω or β (°)	Digits	Immersion	Half-total.	ω or β (°)	Digits	Immersion	Half-total.
Almagest	ω = 10;48	> 21	31;20	25; 4	ω = 12;12	> 21	35;20	28;16
al-Khwār.	ω = 10;50	20;46	29;16	21;22	ω = 13;17	21;31	34;34	27;27
Handy T.	β = 0;57,0	21½	31	25	β = 0;64,0	21½	35	28
al-Batt.	β = 0;53,0	21;31,30	29;30	23;30	β = 0;63,36	21;36	35;20	28;56
P. Alf. Tables	ω = 11	20;46	29;16	21;22	ω = 13	21;31	34;35	27;27

Table 15.2B: Some historical values of the principal parameters for solar eclipses

	Greatest distance				Least distance			
	ω or β (°)	Digits	Immersion	Half-total.	ω or β (°)	Digits	Immersion	Half-total.
Almagest	ω = 6	12	31;20	–	ω = 6;24	12 ⅘	33;20	–
al-Khwār.	ω = 6;37	10;48	30;55	–	ω = 7;11	12;44	33;34	–
Handy T.	β = 0;32,0	12	31; 0	–	β = 0;34,0	12 ¼	33; 0	–
al-Batt.	β = 0;31,0	11;23,30	31; 0	–	β = 0;34,0	12;33	34; 0	–
P. Alf. Tables	ω = 6;37	10;45	30;55	–	ω = 7;1	12;44	33;34	–

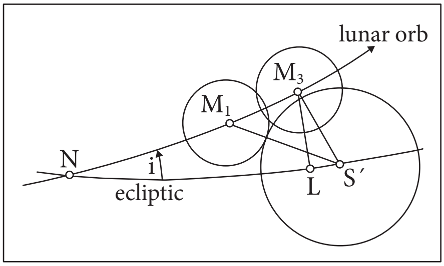

Figure 22: Lunar eclipse

and “half-totality” are given in minutes of arc. Figure 22 shows two positions of the Moon in the case of a lunar eclipse; S′ is the center of the shadow (180° in longitude from the center of the Sun), N is the ascending node, M_1 is the position of the Moon at the beginning of immersion, M_3 its position at mid-eclipse, NL is the longitude of the Moon at mid-eclipse, LM_3 is the latitude of the Moon at mid-eclipse. The number of linear digits of eclipse, not labeled here, is the amount of the overlap on line $S'M_3$ of the two circles at mid-eclipse, where the diameter of the Moon is taken to be 12.

In his *Kalendarium solis et lune* for 1321 John of Murs presents one table for lunar eclipses, and one for solar eclipses, at any lunar distance, that is, each of these two tables merges two tables otherwise presented separately: one for greatest lunar distance and another for least lunar distance. This presentation is most uncommon in eclipse tables (indeed, we have not found another example), and it shows John of Murs’s ability to recast tables in new ways (Chabás and Goldstein 2012, in press). In Table 15.2F the three columns for digits, immersion, and half-totality correspond to greatest lunar distance, whereas the three columns labeled differences represent the digits or minutes of arc to be added in each case to the entry in the previous column in order to obtain the value corresponding to least distance. Table 15.2F is taken from Brussels, Bibliothèque Royale, MS 1086–1115, f. 30v.

2.2 *Solar eclipses*

The situation for solar eclipses is similar to that of lunar eclipses, but when computing them, the true argument of lunar latitude has to be corrected for parallax (in this case one needs the adjusted parallax:

see ch. 12). Note also that, as the solar and lunar disks have similar sizes, the magnitude of the solar eclipse cannot go much beyond the maximum number of linear digits, 12 (see Table 15.2H), whereas in a lunar eclipse it may reach 21 (see Tables 15.2D and 15.2E), because the diameter of the cross-section of the Earth's shadow at the distance of the Moon is much bigger than the lunar diameter. In Figure 23 M_1 is the position of the Moon at the beginning of the eclipse, and M_3 at mid-eclipse (where S is the position of the Sun, and NL is the argument of lunar latitude at M_3 (for most purposes the difference in

Table 15.2C: Lunar eclipses *ad longitudinem longiorem* (greatest distance) (excerpt)

Arg. of latitude North (°)		Digits	Immersion (′)	Half-totality (′)
11; 0	169; 0	0; 0	0; 0	0; 0
10;30	169;30	0;40	12;10	0; 0
10; 0	170; 0	1;40	19;30	0; 0
. . .				
4;30	175;30	12;11	46; 4*	0; 0
4; 0	176; 0	13; 9	36;42	10;21
3;30	176;30	14; 7	34; 1	13;47
. . .				
1; 0	179; 0	18;53	29;52	20;52
0;30	179;30	19;50	29;19	21;16
0; 0	180; 0	20;46	29;16	21;22

* Instead of 41;4.

Arg. of latitude South (°)		Digits	Immersion (′)	Half-totality (′)
360; 0	180; 0	20;46	29;16	21;22
359;30	180;30	19;50	29;19	21;16
359; 0	181; 0	18;53	29;52	20;52
. . .				
356;30	183;30	14; 7	34; 1	13;47
356; 0	184; 0	13; 9	36;42	10;22
355;30	184;30	12;11	41; 4	0; 0
. . .				
349;30	190;30	0;40	12;30	0; 0
349; 0	191; 0	0; 0	0; 0	0; 0
348;30	191;30	0; 0	0; 0	0; 0

Table 15.2D: Lunar eclipses *ad longitudinem propiorem* (least distance) (excerpt)

Arg. of latitude North (°)		Digits	Immersion (′)	Half-totality (′)
13; 0	167; 0	0;26	12;25	0; 0
12;30	167;30	1;13	20;52	0; 0
12; 0	168; 0	2; 2	26; 7	0; 0
. . .				
6; 0	174; 0	11;43	55;20	0; 0
5;30	174;30	12;35	47;14	0; 7
5; 0	175; 0	13;27	43;53	14; 9
. . .				
1; 0	179; 0	19;54	34;49	27; 2
0;30	179;30	20;43	34;40	27;16
0; 0	180; 0	21;31	34;35	27; 7*

* Instead of 27;27.

Arg. of latitude South (°)		Digits	Immersion (′)	Half-totality (′)
360; 0	180; 0	21;31	34;45*	27;27
359;30	180;30	20;43	34;40	27;16
359; 0	181; 0	19;54	34;44	27; 2
. . .				
355; 0	185; 0	13;27	43;53	14; 9
354;30	185;30	12; 5	47;14	9; 7
354; 0	186; 0	11;43	55;20	0; 0
. . .				
348; 0	192; 0	2; 2	26; 7	0; 0
347;30	192;30	1;13	20;52	0; 0
347; 0	193; 0	0;26	12;25	0; 0

* Instead of 34;35.

Table 15.2E: Lunar eclipses at greatest and least distances (excerpt)

Arg. of lat. (°)	Latitude (′)	Digits	Immersion (′)	Half-totality (′)	Arg. of lat. (°)	Latitude (′)	Digits	Immersion (′)	Half-totality (′)
9;11*	52; 0	0	0; 0	0; 0	12;25	63;36	0	0; 0	0; 0
8;43	50;33	1	15;36	0; 0	11;41	60;39	1	19; 9	0; 0
8;18	48; 5	2	22;29	0; 0	11; 7	57;43	2	27;20	0; 0
. . .									
3;30	23;30	12	47;30	0; 0	5;25	28;19	12	56;29	0; 0
3; 2	21; 3	13	48;11	10;27	4;51	25;23	13	55;47	12;35
2;24	18;35	14	35;14	14;23	4;17	21;26	14	42;15	17;16
. . .									
0;44	3;50	20	29;41	23;11	0;24	4;43	20	35;35	27;52
0;16	1;23	21	29;31	23;28	0;20	1;46	21	35;23	28;13
0; 0	0; 0	21	29;20	23;30	0; 0	0; 0	21	35;20	28;16

* The arguments of latitude should be: 10;11, 9;43, 9;18,... 4;30, 4;2, 3;24,... (see F. S. Pedersen 2002, p. 1478).

Table 15.2F: Lunar eclipses at any distance

Arg. of latitude (°)	Digits	Diff. digits	Immersion (′)	Diff. (′)	Half-totality (′)	Diff. (′)
13; 0	0	0	0	12	0	0
12;30	0	1		21	0	0
12; 0		2		26		
11;30		3		30		
11; 0	0	4	0	26		
10;30	1		12	25		
10; 0	2		19	22		
9;30	3	4	25	18		
9; 0	4	3	28	17		
8;30	5		31	16		
8; 0	5		34	15		
7;30	6		36	15		
7; 0	7		39	14		
6;30	8	3	40	14		
6; 0	9	2	42	13		0
5;30	10		44	3		9
5; 0	11		45	0		14
4;30	12		41	0	0	17
4; 0	13		37	2	10	10
3;30	14	2	34	4	14	8
3; 0	15	1	33	4	16	8
2;30	16		32	4	18	7
2; 0	17		31	5	19	7
1;30	18		30	5	20	7
1; 0	19		30	5	21	6
0;30	20		29	6		6
0; 0	21	1	29	6	21	6

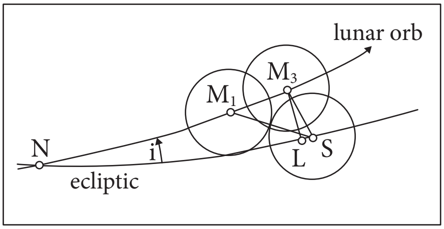

Figure 23: Solar eclipse

longitude between L and S is ignored), and M_3 L is the lunar latitude at mid-eclipse. The amount of the (linear) digits of eclipse is the length of the overlap on SM_3, where the solar diameter is taken to be 12.

The three other tables from which excerpts are reproduced below are also taken from Oxford, Bodleian Library, MS Can. Misc. 27(ff. 93v–94r), and they all deal with solar eclipses. The first two (Tables 15.2G and 15.2H) are in the tradition of al-Khwārizmī: *ad longitudinem longiorem* (greatest distance, when the Moon is at the apogee of its epicycle) and *ad longitudinem propiorem* (least distance, when the Moon is at the perigee of its epicycle). The third (Table 15.2I), also for eclipses of the Sun, is in the tradition of al-Battānī and it is presented as two sub-tables, one for the Moon at apogee and one for the Moon at perigee. Here "immersion" represents the difference in minutes of arc between the center of the Moon at the beginning of the eclipse (first contact) and at mid-eclipse.

3. *Interpolation Schemes*

As the tables for digits of eclipse give entries for the extremal positions of the Moon at apogee and perigee of its epicycle, one needs another table, displaying the coefficients of interpolation (*minuta proportionalia*), when the Moon is at intermediary positions. We find a specific table for this in *Almagest* VI.8, where the argument is that of lunar anomaly and it is given at 6°-intervals (Toomer 1984, p. 308). The same table is found in the zij of al-Battānī (Nallino 1903–1907, 2:89). However, in the zij of al-Khwārizmī it is given at 2°-intervals and is inserted as a column in the table for the digits of eclipse (Suter 1914, pp. 187–189). Morever, the entries do not fully agree with those in the *Almagest*, as pointed out by Neugebauer: "The values of al-Khwārizmī's table are at the beginning sometimes smaller than in the table of the

Table 15.2G: Solar eclipses *ad longitudinem longiorem* (excerpt)

Argument of latitude (°)		Digits	Immersion (′)
6;37	173;32	0; 0	0; 0
6;30	173;30	0;11	5;30
6; 0	174; 0	1; 5	13; 7
. . .			
0;30	179;30	10;32	30;21
0; 0	180; 0	10;45	30;55
159;30*	180;30	11;32	30;51

* Instead of 359;30.

Argument of latitude (°)		Digits	Immersion (′)
359;30	180;30	11;32	30;51
359; 0	181; 0	9;29	30;27
358;30	181;30	8;28	30; 7
. . .			
354; 0	186; 0	1; 5	13; 7
353;30	186;30	0;11	5;30
353; 0	187; 0	0; 0	0; 0

Table 15.2H: Solar eclipses *ad longitudinem propiorem* (excerpt)

Argument of latitude (°)		Digits	Immersion (′)
7; 1	172;40	0; 0	0; 0
7; 0	173; 0	0;17	7;56
6;30	173;30	1; 9	14;11
1; 0	179; 0	10;48	33;15
0;30	179;30	11;30	33;30
0; 0	180; 0	12;44	33;34

Argument of latitude (°)		Digits	Immersion (′)
359;30	180;30	11;30	33;30
359; 0	181; 0	10;38	33;15
358;30	181;30	9;47	32;45
353;30	186;30	1; 9	14;16
353; 0	187; 0	0;17	7;16
352;30	187;30	0; 0	0; 0

Table 15.2I: Solar eclipses at greatest and least distances (excerpt)

Latitude (′)	Digits	Immersion (′)
31; 0	0	0; 0
28;18	1	12;39
25;35	2	17;30
. . .		
3;55	10	30;45
1; 3	11	30;59
0; 0	12	31; 0

Latitude (′)	Digits	Immersion (′)
34; 0	0	0; 0
31;18	1	11;16
28;35	2	18;25
4;13	11	33;44
1;30	12	33;48
0; 0	12	34; 0

Table 15.3: Interpolation between apogee and perigee (excerpt)

Argum. (°)		Min. prop.	Argum. (°)		Min. prop.	Argum. (°)		Min. prop.
2	358	0; 2	62	298	14;55	122	238	44; 0
4	356	0; 6	64	296	15;45	124	236	45; 0
6	354	0;12	66	294	16;41	126	234	46; 7
. . .								
54	306	11;33	114	246	41; 0	174	186	59;11*
56	304	12;21	116	244	42; 0	176	184	59;56
58	302	13;10	118	242	43; 0	178	182	59;58
60	300	14; 0	120	240	44; 0	180	180	60; 0

* Instead of 59;51.

Almagest, toward the end occasionally larger. In the majority of cases the numbers are identical" (Neugebauer 1962, p. 118; see also F. S. Pedersen 2002, pp. 1449 and 1444–1445). This table was reproduced, in one version or another, in almost all late medieval sets of astronomical tables. In particular, both versions are found in the corpus of the Toledan Tables (see Toomer 1968, pp. 116–117; and F. S. Pedersen 2002, pp. 1437–1446). Table 15.3 displays an excerpt of such a table, in the version of al-Khwārizmī, found in Oxford, Bodleian Library, MS Can. Misc. 27, f. 93r.

In the *editio princeps* of the Alfonsine Tables the corresponding table for interpolation is in the tradition of al-Battānī (Ratdolt 1483, f. 18v).

4. *Eclipsed Part of the Solar and Lunar Disks*

These two tables, often found merged into one, give the eclipsed part of the solar and the lunar disks as a function of the linear digits of the diameters of the luminaries, such that 12 linear digits is equal to the diameter of the eclipsed body. The eclipsed part can also be specified in area digits, such that 12 area digits are equal to the area of the eclipsed body. The table is already found in *Almagest* VI.8, as well as in the *Handy Tables*, the zij of al-Battānī, the zij of al-Khwārizmī, the Almanac of Azarquiel, and the Toledan Tables (see Toomer 1984, p. 308; Stahlman 1959, p. 258; Nallino 1903–1907, 2:89; Suter 1914, 190; Toomer 1968, pp. 113–114; and F. S. Pedersen 2002, pp. 1448–1452). In all these cases the tradition of the *Almagest* was almost unchanged.

Table 15.4: Eclipsed part of the solar and the lunar disks in Vienna, MS 2488

Sun Linear dig.	Sun Area dig.		Moon Linear dig.	Moon Area dig.	
1	0;20		1	0; 0	(a), (b): 0;30
2	1; 0		2	0; 0	(a), (b): 1;10
3	1;45	(a): 1;50	3	2;45	(a): 2;5 (b): 2;8
4	2;40		4	3;10	
5	3;40	(a): 3;20	5	4;20	
6	4;40		6	5; 1	(a), (b): 5;30
7	5;50		7	6:45	(a): 6;42
8	7; 0		8	8; 0	
9	8;20		9	9;20	(a), (b): 9;10
10	9;40		10	10;20	
11	10;50		11	11;20	
12	12; 0	(b): 12;20	12	12; 0	

Both Toomer and F. S. Pedersen distinguish between two different traditions in the Tables of Toledo: (a) a tradition that is perhaps dependent on al-Battānī, and (b) the common tradition. In Table 15.4 we reproduce a copy of this table found in Vienna, Nationalbibliothek, MS 2488, f. 71v, a manuscript containing the *Almanaque Perpetuo* by Ferrand Martines (1391) (see Chabás 1996b). We have also indicated, in the right margin of the tables, the variants in traditions (a) and (b).

It would therefore seem that the table in Vienna, Nationalbibliothek, MS 2488, adheres to the common tradition despite four scribal errors, where (a) and (b) agree on a reading that differs from the reading in this MS. Note that no such table appears in the *editio princeps* of the Alfonsine Tables (Ratdolt 1483).

5. *Reflexio tenebrarum*

This table has a misleading title, which in Latin is *Tabula reflexionis tenebrarum in utraque eclipsi*. It appears already in *Almagest* VI.12 (Toomer 1984, p. 319) and the *Handy Tables* (Stahlman 1960, p. 256); the entries are given to minutes in the *Almagest*, but only to degrees in the *Handy Tables*. This is a table to compute the "prosneusis," that is, the angle between the ecliptic and the great circle passing through the centers of the Sun and the Moon (or the shadow) at different

Table 15.5: *Tabula reflexionis tenebrarum* (excerpt)

Dig.	(°)	(°)	(°)
0	90	90	0
1	67	73	0
2	57	60	0
. . .			
11	6	29	0
12	2	36	90
13	0	23	64
. . .			
19	0	7	16
20	0	4	10
21	0	2	4

phases of solar and lunar eclipses, as a function of the digits of eclipse (col. 1). So the term *tenebrarum* in the title is intended to indicate phases of eclipses. The angles in columns 2, 3, and 4 correspond, respectively, to the beginning and the end of a solar eclipse, the beginning and the end of a lunar eclipse, and the beginning and the end of totality in a lunar eclipse. This table was reproduced unmodified in most zijes throughout the Middle Ages and is, according to F. S. Pedersen (2002, p. 1453), "probably a left-over from Albattani, and not intended for practical use." It appeared in print in the second edition of the Alfonsine Tables (Santritter 1492, f. k2r), from which we reproduce an excerpt of it in Table 15.5.

6. *Colors*

It is common to find tables of colors for solar and lunar eclipses sets of in medieval astronomical tables. We are not aware, however, of any medieval observations of eclipses where the color was noted. From a modern point of view, there is no basis for assigning colors to solar eclipses; for lunar eclipses, see Minnaert [1937] 1954, pp. 295–296. Moreover, this topic is not considered in Ptolemy's *Almagest*. The discussion of colors of eclipses derives from Indian astronomy, and such a table probably featured in the lost original version of al-Khwārizmī's zij. Indeed, in his commentary on al-Khwārizmī's zij, Ibn al-Muthannā (10th century) stated that the color changes during a single eclipse (Goldstein 1967, pp. 119, 234–235), whereas for Ibn Ezra (Millás

1947, p. 167), who generally depended on Ibn al-Muthannā, color is a function of lunar latitude. In *al-Qānūn al-Masʿūdī* al-Bīrūnī (*d.* 1048) has a chapter devoted to the colors of solar and lunar eclipses: for details, see Chabás and Goldstein 2003, pp. 196–197. Moreover, *Kitāb al-ʿAmal bi'l-Asṭurlāb*, chapter 151, by the Persian astronomer, Abu l-Ḥusain ʿAbd al-Raḥmān ibn ʿUmar al-Ṣūfī (903–986), contains a list of the colors of lunar eclipses, based on the argument of lunar latitude at opposition; both the arguments and the colors differ from those in Table 15.6 (see Kennedy and Destombes [1966] 1983, p. 413). For additional information on tables for colors of eclipses (mainly in medieval Hebrew astronomical texts), see Goldstein 2005.

Tables for the colors of eclipses appear in several medieval Latin astronomical texts, notably, the Latin version of Ibn al-Kammād's *al-Zīj al-Muqtabis* (Chabás and Goldstein 1994, pp. 18–19; see Table 15.6) and the Tables of Barcelona, extant in Hebrew, Latin, and Catalan versions (Millás 1962, p. 238; Chabás 1996a, pp. 512–514; Goldstein 2005, pp. 14, 23–24).

Table 15.6 is arranged in 4 columns: column 1 displays the argument of lunar latitude; col. 2 gives the color of a solar eclipse as a function of the argument of lunar latitude, in degrees; col. 3 displays lunar latitude in minutes; col. 4 gives the color of a lunar eclipse as a function of lunar latitude: see Madrid, Biblioteca Nacional, MS 10023, f. 52v, cols. 1 to 4; cols. 5 and 6 (not shown here) display the magnitudes of solar eclipses in area digits as a function of the magnitude of the eclipse in linear digits.

In the canons to the Toledan Tables the colors of eclipses are discussed, but there is no table for them in this zij (F. S. Pedersen 2002, pp. 535, 652–653). The *Libro de las tablas alfonsies*, chapter 35, "On the colors of eclipses" (*De que color sera ell eclipsy*), gives two different rules for the colors of lunar eclipses (Chabás and Goldstein 2003, pp. 70, 196–199); these rules show similarities with those in the above-mentioned tables, but do not fully agree. Tables for the colors of eclipses are also found in the *editio princeps* of the Parisian Alfonsine Tables (Ratdolt 1483, f. l8v).

It is noteworthy that in a text by al-Battānī, colors of eclipses are also associated with the planets (Kennedy *et al.* 2009–2010, p. 84).

Table 15.6: Color of eclipses in *al-Zīj al-Muqtabis*

Arg. lat. (°)	Solar eclipse	Lat. (′)	Lunar eclipse
1	valde niger	10	niger valde
2	niger clarus		in nigredine
3	turbatus rubeus	20	niger
4	turbatus croceus		cum rubedine
5	turbatus clarus	30	niger
6	turbatus cinereus		cum rubedine
7	cinereus	40	niger
8	cinereus		cum croceo
9	cinereus	50	turbatus
10	cinereus		
11	croceus	60	cinereus
12	rubeus albus		

7. *Table of the* Samt *for Solar Eclipses*

This is a very unusual type of table of which few examples are known, but they are always associated with solar eclipses in connection with the tables related to the lunar parallax in the Indian tradition. For instance, it is found in the zij of Yaḥyā ibn Abī Manṣūr (see Kennedy and Faris 1970, pp. 21–24) with the heading "The table of *samt* for determining solar eclipses (*jadwal al-samt li-ʿilm kusūf al-shams*), as well as in Ibn al-Kammād's *al-Zīj al-Muqtabis* and the Tables of Barcelona (see Chabás and Goldstein 1994, pp. 14–17; Millás 1962, p. 236; Chabás 1996, pp. 510–511). The Arabic term *samt* (plural: *sumūt*) means "direction," "azimuth," and "zenith."

In Table 15.7 we present a copy of this table found in Madrid, Biblioteca Nacional, MS 3349, f. 2v, a codex written in the first half of the 14th century, for the most part in Portuguese (see Chabás and Goldstein, 2010). The title of the table is *Tavoa dos çomutes en Burgos*, where *çomutes* is probably a rendering of the Arabic *sumūt*. For each degree of each zodiacal sign we are given a value in degrees and minutes; the maximum value, +23;51°, occurs at Vir 29°–Lib 1° and the minimum, –23;51°, at Psc 29°–Ari 1°. The entries in this table are not distributed symmetrically, for zero occurs at Sgr 10° and Cnc 20°. This table gives the declination of upper midheaven (i.e., the intersection of the meridian and the ecliptic above the local horizon) as a function of the longitude of the horoscope (i.e., the intersection of the ecliptic

Table 15.7: Table of *samt* for Burgos

(°)	(°)	Lib N	Sco N	Sgr N	Cap S	Aqr S	Psc S
1	29	23;51°	18;40°	5; 0°	10;12°	19;30°	22;58°
2	28	23;50	18;22	4;25	10;24	19;40	23; 2
3	27	23;49	18; 4	3;50	10;56	19;50	23; 6
4	26	23;48	17;46	3;15	11;18	19;59	23;10
5	25	23;45	17;28	2;39	11;39	20; 8	23;14
6	24	23;40	17;10	2; 4	12; 1	20;17	23;18
7	23	23;37	16;48	1;33	12;25	20;26	23;21
8	22	23;33	16;26	1; 2	12;49	20;35	23;24
9	21	23;25	15;59	0;32	13;13	20;43	23;26
10	20	23;17	15;34	0; 0	13;37	20;51	23;28
11	19	23; 9	15; 9	0;24	14; 1	20;59	23;30
12	18	22;59	14;44	0;48	14;25	21; 7	23;32
13	17	22;40*	14;22	1;16	14;46	21;14	23;34
14	16	22;40*	13;57	1;44	15; 6	21;22	23;36
15	15	22;40*	13;29	2;12	15;35	21;29	23;38
16	14	22;41*	12;59	2;40	15;42	21;37	23;39
17	13	22;11*	12;30	3; 8	16; 0	21;44	23;40
18	12	21; 1*	12; 1	3;37	16;18	21;51	23;42
19	11	21;50	11;32	4;12	16;33	21;56	23;43
20	10	21;40	11; 3	4;48	16;48	22; 1	23;44
21	9	21;28	10;34	5;23	17; 3	22; 7	23;46
22	8	21;17	10; 4	5;59	17;18	22;12	23;47
23	7	21; 6	9;35	6;59	17;32	22;16	23;48
24	6	20;55	9; 5	7;10	17;46	22;20	23;49
25	5	20;34	8;31	7;40	18; 2	22;25	23;49
26	4	20;14	7;56	8;10	18;18	22;30	23;49
27	3	19;55	7;21	8;37	18;34	22;35	23;50
28	2	19;36	6;46	9; 3	18;50	22;40	23;50
29	1	19;18	6;11	9; 6	19; 5	22;46	23;51
30	0	18;58	5;36	9;50	19;20	22;53	23;51
		N Vir	N Leo	S Cnc	S Gem	S Tau	S Ari

* The entries for Lib 13° to 18° seem to be corrupt since the differences are not smooth. In particular, although the manuscript reading is secure, the entry for Lib 18° should be 22;1° (instead of 21;1°).

and the eastern horizon), and can be recomputed according to a 3-step procedure reconstructed by Neugebauer, as indicated in a footnote on page 24 of Kennedy and Faris 1970. The 3-step procedure requires the use of 3 tables: one for the solar declination (as in *Almagest* I.15), another for the longitude of the horoscope (as in al-Khwārizmī's zij: see Suter 1914, pp. 171–173), and a table for oblique ascensions for the appropriate geographical latitude. Table 15.7 seems to be computed for an obliquity of 23;51° and a latitude of 42°–43°.

8. *Lists of Eclipses*

The tables considered above were to help in the computation of the different circumstances of eclipses. However, as the demand for calendars increased, there appeared lists of eclipses for a given locality specifying the major features of these events. In Table 15.8A we transcribe such a list for solar eclipses, found in Vienna, Nationalbibliothek, MS 5371*, which also contains a similar list for lunar eclipses. In this manuscript these lists follow a calendar structured in 19-year cycles beginning in 1330, indicating the meridian altitude of the Sun for each day of the year. The maximum altitude is 64;43°, and the minimum, 17;37°. From these values we derive the latitude for which the table is valid: $\varphi = 90° - (h_{max} + h_{min})/2 = 48;50°$, which is the latitude of Paris according to many medieval sources.The same lists for solar and lunar eclipses are also found in Vienna, Nationalbibliothek, MS 15343, f. 2r, again after a few pages of a calendar of the same kind.

For the solar eclipses, the list in Vienna, Nationalbibliothek, MS 5371* gives in each case the year; the year within a cycle of 19 years beginning in 1330; the date (month and day), and the time of true conjunction, in hours, minutes, and seconds; the magnitude, in digits, minutes, and seconds of a digit; the half-duration and the total duration, both in hours, minutes, and seconds. In the last column we have added the Oppolzer number (Oppolzer 1887), here labelled [Opp.], for purposes of identification.

Close inspection of the entries indicates that the table has quite a number of copyists' errors: in several instances the total duration is not twice the corresponding half duration, and in two cases (eclipses for 1376 and 1379) the time for the eclipse does not agree with the time recomputed with modern tools.

Table 15.8A: Solar eclipses listed in Vienna, MS 5371*

Year	Year in cycle	Date		Time (h)	Magnitude (dig.)	Half-duration (h)	Total duration (h)	[Opp.]
1330	1	Jul	17	4;32,30	10;15,18	1; 2, 0	2; 4, 0	6051
1333	4	May	15	3; 3, 9	9;28, 3	1; 6,43	2; 7,26	6058
1338	9	Feb	20	21;29,57	11;11, 3	0;34,40	1; 9,16	6068
1339	10	Jul	7	2;27,31	11;58,51	1; 7, 8	2;14,18	6072
1341	12	Dec	9	21;31,43	6;17,28	1; 6,43	1;44, 8	6077
1342	13	May	2	23;35,37	0;25, 0	0;16, 1	0;32, 3	6078
1344	15	Oct	7	17;54,26	10;42,45	1; 5, 0	2;10, 0	6083
1345	16	Sep	27	0;54,21	4;10,58	0;49,23	1;38,46	6086
1352	4	May	14	20;52, 5	6; 0,22	0;48,19	1;52,36	6101
1354	6	Sep	17	21; 2,35	6;44,33	0;59,56	1;56,52	6106
1355	7	Mar	15	5;17,19	7;33,35	1; 3,33	2; 7, 6	6107
1361	13	May	5	20;52,42	5;48,14	0;42,29	1;51,58	6122
1362	14	Oct	14	3;28,36	2;30, 0	0;20, 9	1;40,18	6125
1364	16	Mar	4	23; 9,31	8;49, 1	1; 9,55	2; 4,50	6128
1365	17	Feb	21	23;30,21	2;47,10	0;36,19	1;48,26	6130
1366	18	Aug	7	16;49,16	5;57,21	0;44,34	1;54, 8	6133
1370	3	May	26	5;54,13	3;52,52	0;44,39	1;29,18	6142
1371	4	Oct	9	21;47,49	9;30,22	0;52,14	1;44,28	6145
1376	9	Jul	17	18;28,41	3;20,27	0;45,47	1;31,34	6156
1377	10	Jan	10	23;19,11	2;23,17	0;37,11	1;14,22	6157
1379	12	May	17	7;33, 1	8; 1,19	0;56,37	1;53,14	6163
1384	17	Aug	16	1;18, 1	10; 0, 0	1; 2,51	2; 5,42	6174
1386	19	Jan	1	22;30,36	7;36, 5	0;57,32	1;55, 4	6178

Such lists as that displayed in Table 15.8A are associated with calendars and, indeed, they contain a column for the "year in the cycle," that is, the cycle of 19 years characteristic of medieval calendars. In this particular case, the list covers 3 cycles. This tradition was continued in subsequent calendars, including those by Nicholas de Lynn and John Somer (see Eisner 1980 and Mooney 1998) which extend over (normally four) 19-year cycles beginning in 1387, and add specific illustrations for each eclipse, displaying the partially (or fully) covered eclipsed body with some numerical data: see, e.g., Eisner 1980, pp. 154–163 (*figure eclipsis lune*). Lists of solar and lunar eclipses are also found in other astronomical texts, such as almanacs, but in that case no reference is made to 19-year cycles. In Table 15.8B we transcribe the list of lunar eclipses in Segovia, Biblioteca de la Catedral, MS 110, a Latin version of Zacut's *Ḥibbur* (see Chabás and Goldstein 2000, pp. 53, 66–67). For each eclipse we are given the year; the date, and the time of true opposition, in hours and minutes; the half-duration and the total duration of the eclipse, both in hours and minutes; the magnitude, in digits and minutes of a digit; the half-duration and the duration of totality, both in hours and minutes. As above, we have added the Oppolzer number.

Table 15.8B: Lunar eclipses listed in Segovia, MS 110

Year	Date			Time (h)	Half-dur. (h)	Dur. (h)	Magn. (dig.)	Half-dur. (h)	Total dur. (h)	[Opp.]
1475	Mar *a*									
1475	Sep	Thu	14	18; 0	1;42	3;24	15;7*g*	0;36	1;12	4147
1476	Mar	Sun	10	6;20	1;42	3;24	15	0;36	1;12	4148
1476	Sep	Tue	3	10;50	1;43	3;26	16	0;41	1;22	4149
1478	Jul	Tue	14	13;50	1; 7	2;14	5	0; 0	0; 0	4151
1479	Jul	– *c*	3	15; 0	1;48	3;36	21	0;54	1;48	4153
1479	Jul *b*	Tue	28	11;40	1;42	3;24	15	0;36	1;12	4154
1482	Oct	Sat	26	4;28*f*	1;37	3;14	13	0;22	0;44	4157
1483	Apr	Tue	22	10; 0	1;44	3;28	17	0;45	1;30	4158
1483	Oct	Wed	15	12;17	1;47	3;34	19	0;50	1;40	4159
1487	Feb	Wed	7	14;40	1;43	3;26	16	0;41	1;22	4164
1489	Dec	Mon	7	16;22	1;43	3;26	16	0;41	1;22	4168
1490	Jun	Wed	2	9;40	1;49	3;38	23	0;55	1;50	4169
1490	Nov	Fri	26	16;45	1;46	3;32	18	0;48	1;36	4170
1493	Apr	Mon	1	13;11	1;43	3;26	16	0;41	1;22	4173
1494	Mar	Fri	21	13;40	1;43	3;26	16	0;41	1;22	4175
1494	Sep	Mon *d*	14	18;49	1;44	3;28	17	0;45	1;30	4176
1497	Jan	Wed	18	5;34	1;44	3;28	17	0;45	1;30	4178
1500	Nov	Thu	5	11;54	1;37	3;14	13	0;22	0;44	4182
1501	May	Mon	2	17;20	1;47	3;34	19	0;50	1;40	4184
1502	Oct	Sat	15	11;12	0;57	1;54	3 *h*	0; 0	0; 0	4186
1504	Feb	Thu	29	12;30	1;43	3;26	16	0;41	1;22	4187
1505	Aug	Thu	14	7;24	1;42	3;24	15	0;38	1;16	4190
1508	Jun	Tue *e*	12	17;10	1;49	3;38	23	0;55	1;50	4194
1509	Jun	Sat	21	10;46	1;17	2;34	7	0; 0	0; 0	4196
1511	Oct	Mon	6	10;48	1;37	3;14	13	0;22	0;44	4199
1515	Jan	Mon	29	14;20	1;43	3;26	16	0;42*j*	1;24	4205
1516	Jan	Sat	19	5; 0	1;42	3;26	15;7*i*	0;38	1;16	4207
1516	Jul	Sun	13	10;50	1;40	3;20	14	0;30	1; 0	4208
1519	Nov	Sun	6	5; 0	1;48	3;36	20	0;52	1;44	4212
1522	Sep	Fri	5	11;22	1;42	3;24	15	0;36	1;12	4216
1523	Mar	Sun	1	7;30	1;44	3;28	17	0;45	1;30	4217
1523	Aug	Tue	25	14;17	1;44	3;28	17	0;45	1;30	4218

a. The rest of the line is blank. This line is missing entirely in the Hebrew version.
b. Instead of Dec, as in the Hebrew version.
c. 7 (= Sat) in the Hebrew version.
d. Instead of 1 (= Sun), as in the Hebrew version.
e. Instead of 2 (= Mon), as in the Hebrew version.
f. 4;30h in the Hebrew version.
g. 15 in the Hebrew version.
h. 4 in the Hebrew version.
i. 15 in the Hebrew version.
j. 0;43 in the Hebrew version.

CHAPTER SIXTEEN

FIXED STARS

After an explanation of the method for determining star positions in *Alm*agest VII.4, Ptolemy presented a catalogue of the fixed stars in *Alm*. VII.5 and VIII.1. The catalogue gives the name or description, the ecliptic coordinates (longitude and latitude), and the magnitude of stars grouped in 48 constellations. In his catalogue Ptolemy listed 1,025 stars, of which 3 stars ascribed to the constellation of Leo (a group called *Plókamos*, meaning "lock" or "tuft of hair" [of Berenice]) were not included in the count of 1,022 stars (see Kunitzsch 2002, p. 22). Moreover, 3 stars are listed twice (not those in Leo), so that the number of entries in his catalogue is 1,028. Although subject to some controversy, the epoch of Ptolemy's star catalogue is 137 AD (Toomer 1984, p. 340, n. 91).

This catalogue was mostly known to western astronomers through the translation into Latin of the Arabic versions of the *Almagest* made by Gerard of Cremona around 1175. A modern edition of this Latin translation can be found in Kunitzsch 1990. Another line of transmission to the West came through the star catalogue for 964 AD compiled by al-Ṣūfī which, in turn, depended on the star catalogue in Ptolemy's *Almagest* (see Comes 1990). The total precession in al-Ṣūfī's text is 12;42° (Samsó and Comes 1988, p. 69), and this is the amount to be added to the star's longitude given by Ptolemy to obtain that of al-Ṣūfī. This work was adapted into Castilian in 1256 by Judah ben Moses ha-Cohen with the help of Guillén Arremón Daspa, both in the service of King Alfonso X of Castile, and it was later revised (1276) by the king himself, in collaboration with other astronomers in his *scriptorium*. The title of this text is *Libro de las estrellas de la ochaua espera* and it is also known as *Libro de las XLVIII figuras de la VIII spera* (Rico Sinobas 1863–1867, vol. 1, pp. 5–145). The total precession in this text is 17;8°.

Ptolemy's star catalogue gave rise, directly or indirectly, to a wealth of other star tables, mostly much shorter. The number of stars and their names vary widely, and in many cases quantities other than ecliptic longitude and latitude were tabulated, e.g., meridian altitude (*altitudo*) or coordinates related to the equator, such as mediation (*medium celi*; i.e., the longitude of the point on the ecliptic that has the same right ascension as the star when it culminates), as well as the names of the planets associated astrologically with each star (*natura*).

These star tables can be grouped into three categories:

(i) short lists of about 20–40 stars, usually listing the stars to be used on astrolabes or other instruments;
(ii) long lists of about a few hundred stars, much more uncommon and too long to be used on an astronomical instrument;
(iii) catalogues of about 1,000 stars that depend on the *Almagest*.

(i) As an example of short list we have chosen that appearing in Oxford, Bodleian Library, MS Can. Misc. 27, f. 129v (see Table 16A). At the right margin of the table, a note reads: *Ista tabula fuit composita anno 1225*. For each of the 34 stars we are given the longitude on the ecliptic, without any indication of its sign, specified here between square brackets, and mediation, m, which is the co-culminating degree of the ecliptic. Mediation (m) and right ascension (α) are related by means of the following equation:

$$\tan m = \tan \alpha \,/\, \cos \varepsilon,$$

where ε is the obliquity of the ecliptic. Now, the right ascension is obtained from the given ecliptic coordinates of the star, λ, and latitude, β, by means of the modern equations

$$\tan \alpha = \tan \lambda \cdot \cos (\theta + \varepsilon) \,/\, \cos \theta$$

and

$$\tan \theta = \tan \beta \,/\, \sin \lambda,$$

where θ is an auxiliary angle. At the far right of Table 16A we have added the modern designation of each star.

Comparison with the coordinates in Ptolemy's star catalogue shows an increment of 14;55° in longitude. Thus, this list should be related to Kunitzsch's type XIII (35 stars with a mixture of increments of 14;55° and 15;7°) and XIV (32 stars with increments of 14;55°), both associated with the Toledan Tables (Kunitzsch 1966, pp. 87–97; see also F. S. Pedersen 2002, pp. 1494–1501). The list displayed in Table 16A does not quite match either of them: on the one hand, it only gives longitude and mediation (both found in type XIII lists, among other

Table 16A: List of stars in Oxford, MS Can. Misc. 27

		Longitude (°)	Mediation (°)	[Modern]
Ratalmara .i. caput mulieris	[Ari]	2;25	350; 4	α And
Calbalhor .i. caput cor piscium	[Ari]	18;48	4;33	β And
Alnac	[Ari]	22;35	17; 5	β Ari
Aldebaran .i. vesp[er]a	[Tau]	27;35	51;24*	α Tau
Razalgob .i. caput catene	[Tau]	14;35	32;44	β Per
Rasalgaude .i. pes geminorum	[Gem]	4;45	64;49	β Ori
Alaayor	[Gem]	9;55	63;28	α Aur
Mencabalgauze .i. humerus geminorum	[Gem]	16;55	78;30	α Ori
Asurcobalamen .i. cavilla dextra	[Gem]	10;35	67;35	β Tau
Moniralhenaar	[Gem]	29;35	89;39	ξ Gem
Razalgaude .i. caput geminorum	[Gem]	11;55	74;25	λ Ori
Balsare alabor	[Cnc]	2;35	92; 5	α CMa
Alsare gumente algomaice	[Cnc]	14; 5	102;17	α CMi
Rosalbum .i. caput bubonis	[Cnc]	8;15	99;42	α Gem
Cedrealceratan .i. pectus cancri	[Cnc]	25;15	117;26	ε Cnc
Caput bubonis	[Cnc]	11;35	103;15	β Gem
Calbalacet .i. cor leonis	[Leo]	17;25	140;13	α Leo
Huncalgorab .i. colum corvi	[Vir]	29;15	170;30	ε Crv
Alsarfa	[Vir]	9;25	156;25**	β Leo
Alcumecalasal	[Lib]	11;35	189;36	α Vir
Calbalacrab .i. cor scorpionis	[Sco]	27;35	234;24	α Sco
Rasalm .i. caput circundantes (?)	[Sgr]	9;25	253; 0	α Oph
Sautelalacob .i. crines scorpionis	[Sgr]	12;25	248;47	λ Sco
Alnasaralarq .i. aquila cadens	[Cap]	2;15	271;20	α Lyr
Alnasara altayr .i. aquila volans	[Cap]	17;48	285;35	α Aql
Rocubesarn .i. genua sagitarii	[Cap]	1;55	272;26	α Sgr
Huldebaran .i. cavilla sagitarii	[Cap]	2;40	273;35	β^1 Sgr
Humaran .i. occulus sagitarii	[Cap]	0; 5	270; 4	ν^2 Sgr
Fomalat .i. os piscis	[Aqr]	21;55	334;21	α PsA
Alradif	[Aqr]	24; 5	302; 7	α Cyg
Iafalesalferaz	[Aqr]	20; 8	315;20	ε Peg
Rocuben algega .i. genua galine	[Aqr]	27; 5	302; 4	ω^2 Cyg
Mencabalferaz .i. humerus equi	[Psc]	17; 5	334;41	β Peg
Denabcaitoz	[Psc]	20;35	0;51(?)	β Cet

* Instead of 56;24. ** Instead of 164;40.

variables) but with a mixture of increments; and on the other hand, the coordinates listed in Table 16A differ from those in type XIV lists, which only give longitude and latitude.

(ii) In Table 16B we display a long list of 225 stars in the Tables of John Vimond that appear in Paris, Bibliothèque nationale de France, MS lat. 7286C, f. 8r–v (see Chabás and Goldstein 2004 for details). For each star and nebula we are given its longitude, latitude (where + and − represent North and South, respectively), and magnitude (where "n" means nebulous) but, in general, the names are omitted. At the far right of Table 16B we have added the modern designation of each star.

The stars are divided into three groups, in turn divided into several subgroups according to the associated planets, a feature which is certainly not common. Group I has 137 stars that belong to the zodiacal constellations arranged in 52 subgroups, group II has 44 stars in northern constellations (19 subgroups), and group III has 44 stars in southern constellations (19 subgroups); the total number of subgroups is thus 90. We note the balanced representation of the stars on both sides of the zodiac. The order and the grouping of the stars in this list is peculiar, for they do not follow the pattern of the catalogue in Ptolemy's *Almagest* that was generally adopted in medieval star lists and catalogues. Rather, this list is organized according to Ptolemy's *Tetrabiblos*, a handbook on astrology written by Ptolemy after the *Almagest* (Robbins 1940), the Latin translation of which was known in the West as the *Quadripartitum*. In *Tetrabiblos* I.9, Ptolemy grouped the stars into three main categories (zodiacal, northern, and southern constellations), following an order differing from that in the *Almagest* where the northern constellations precede the zodiacal constellations, and grouped the stars within each category according to their associated planets. For example, in *Tetrabiblos* I.9 (Of the Power of the Fixed Stars) Ptolemy says: "The stars in the head of Aries, then, have an effect like the power of Mars and Saturn, mingled; those in the mouth like Mercury's power and moderately like Saturn's; those in the hind foot like that of Mars, and those in the tail like that of Venus" (Robbins 1940, p. 47). These are indeed the "associated planets" in the four cases (see Table 16B). The power of the planets is addressed in *Tetrabiblos* I.4.

The star positions generally agree with those in Gerard of Cremona's version of Ptolemy's star catalogue in the *Almagest* with an increment in longitude of 17;52° for precession. As indicated in Chabás

Table 16B: List of stars in the Tables of John Vimond

[Constellation]	Longitude (s)	(°)	Associated planets / Latitude (°)	Magn.	Name	[Modern]
[Zodiacal constellations]						
1 [Aries]			Mars, Saturn			
	0	24;32	+7;20	3		γ Ari
	0	25;32	+8;20	3		β Ari
2 [Aries]			Mercury, Saturn			
	0	28;52	+7;40	5		η Ari
	0	29;22	+6; 0	5		θ Ari
3 [Aries]			Mars			
	1	2;52	−5; 5	4		μ Cet
	1	5;52	−1;30	5		σ Ari
	1	7;32	−1;20	5		ρ Ari
4 [Aries]			Venus			
	1	9;12	+4;50	5		ε Ari
	1	11;42	+1;40	4		δ Ari
	1	13;12	+2;30	4		ζ Ari
	1	14;52	+1;50	4		τ Ari
5 [Taurus]			Venus, Jupiter			
	1	17;32	−9;30	5		30(e) Tau
	1	21;32	−8; 0	3		λ Tau
6 [Taurus, The Pleiades]			Moon, Mars			
	1	20; 2	+4;30	5		19 Tau
	1	20;22	+4;40	5		23 Tau
	1	20;32	+5; 5	5		27 Tau*
	1	21;32	+5;20	5		BSC 1188*
7 [Taurus]			Mars			
	2	0;32	−5;10	1	*aldebaran*?	α Tau
8 [Taurus]			Saturn, Mercury			
	1	26;52	−5;45	3		γ Tau
	1	28;42	−5;50	3		θ^1 Tau
	1	29;42	−3; 0	3		ε Tau
	2	3;32	−4; 0	4		τ Tau
9 [Taurus]			Mars			
	2	7;52	−3;30	5		106(l) Tau
	2	8;12	−5; 0	5		104(m) Tau
	2	13;32	+5; 0	5		β Tau
	2	15; 2	−2;30	3		ζ Tau
10 [Gemini]			Mercury, Venus			
	2	24;22	−1;30	4		η Gem
	2	26; 2	−1;15	4		μ Gem
	2	28; 2	−3;30	4		ν Gem

Table 16B (*cont.*)

[Constellation]			Associated planets			
	Longitude (s)	(°)	Latitude (°)	Magn.	Name	[Modern]
	2	29;52	−7;30	3		γ Gem
	3	2;32	−10;30	4		ξ Gem
11 [Gemini]			Saturn			
	3	9;32	−5;30	3		δ Gem
12 [Gemini]			Mars			
	3	11;12	+9;40	2	( )*annai*?	α Gem
13 [Gemini]			Mars			
	3	14;32	[..]6;15	[..]	*hercules*?	β Gem
14 [Cancer]			Mercury, Mars			
	3	20;32	+1; 0	5		μ Cnc
	3	25; 2	−7;30	4		β Cnc
15 [Cancer]			Saturn, Mercury			
	3	26;12	+11;50	4		ι Cnc
	4	4;22	+5;30	4		α Cnc
16 [Cancer]			Moon, Mars			
	3	28;12	+0;40	n	*meollef*?	Galaxy M44
17 [Cancer]			Mars, Sun			
	3	28;12	+2;40	4	*assinis*?	γ Cnc
	3	29;12	+0;10	4		δ Cnc
18 [Leo]			Saturn, Mars			
	4	12; 2	+9;30	3		ε Leo
	4	12; 2	+12; 0	3		μ Leo
19 [Leo]			Saturn, Mars			
	4	18; 2	+11; 0	3		ζ Leo
	4	18;32	+4;30	3		η Leo
	4	20; 2	+7;30	2		γ Leo
20 [Leo]			Mars, Jupiter			
	4	20;22	+0;10	1	*almalak*?	α Leo
21 [Leo]			Venus, Saturn			
	4	29;12	+13;15	5		60(b) Leo
	5	2; 2	+13;40	2		δ Leo
	5	2;12	+11;30	5		81 Leo*
	5	4;12	+9;40	3		θ Leo
	5	12;22	+12;50	1	??	β Leo
22 [Leo]			Venus, Mercury			
	5	8;12	+5;50	3		ι Leo
	5	8;22	−3; 0	5		υ Leo
	5	9;32	+0;50	4		τ Leo
	5	9;32	+1;15	4		σ Leo

Table 16B (*cont.*)

[Constellation]			Associated planets			
	Longitude (s)	(°)	Latitude (°)	Magn.	Name	[Modern]
23 [Virgo]			Mercury, Mars			
	5	14;12	+4;35	5		ν Vir
	5	14;52	+5;40	5		ξ Vir
	5	17; 2	+6; 0	3		β Vir
	5	18; 2	+5;30	5		π Vir
24 [Virgo]			Mercury, Venus			
	5	26; 7	+1;10	3		η Vir
	6	1; 2	+2;50	3		γ Vir
25 [Virgo]			Saturn, Mercury			
	6	0; 2	+15;10	3		ε Vir
26 [Virgo]			Venus, Mercury			
	6	14;32	−2; 0	1	*almure*?	α Vir
27 [Virgo]			Mercury, Mars			
	6	24;32	+7;30	4		ι Vir
	6	25;12	+2;40	4		κ Vir
	6	7;52	+0;30	4		λ Vir
	7	0;32	+9;50	4		μ Vir
28 [Libra]			Jupiter, Mercury			
	7	5;52	+0;40	2		α Lib
	7	10; 2	+8;30	2		β Lib
29 [Libra]			Saturn, Mercury			
	7	9;12	[..]1;15	[..]		ν Lib
	7	11;52	[..]1;40	[..]		ι Lib
	7	15;22	[..]3;45	[..]		γ Lib
	7	20;52	[..]4;30	[..]		θ Lib
30 [Scorpius]			Mars, Saturn			
	7	23;32	−1;40	3		δ Sco
	7	23;32	−5; 0	3		π Sco
	7	24;12	+1;20	3		β Sco
31 [Scorpius]			Mars, Jupiter			
	8	0;29	−4; 0	2		α Sco
32 [Scorpius]			Saturn, Venus			
	8	5;52	−15; 0	4		$\mu^1 + \mu^2$ Sco
	8	11; 2	−19;30	3		η Sco
	8	16; 2	−18;50	3		θ Sco
	8	15;52	−15;10	3		κ Sco
	8	18;22	−16;40	3		ι^1 Sco
33 [Scorpius]			Mercury, Mars, Moon			
	8	15;52	−23;30	4		υ Sco
	8	14;22	−13;20	3		λ Sco

Table 16B (*cont.*)

[Constellation]	Longitude (s)	Longitude (°)	Associated planets / Latitude (°)	Magn.	Name	[Modern]
34 [Scorpius]			Mars, Moon			
	8	19; 2	−13;15	n		G Sco*
35 [Sagittarius]			Saturn, Moon			
	8	22;22	−6;30	3		γ Sgr
	9	0;52	−3;50	4		ϕ Sgr
36 [Sagittarius]			Jupiter, Mars			
	8	24;32	+2; 7	4		μ Sgr
	8	26;52	−1;30	3		λ Sgr
37 [Sagittarius]			Mercury, Jupiter, Sun, Mars			
	9	3; 2	−7;45	n		$\nu^1 + \nu^2$ Sgr
38 [Sagittarius]			Jupiter, Mercury			
	9	4;12	−6;45	3		ζ Sgr
	9	5;32	−2;30	4		τ Sgr
	9	7;52	−2;30	5		ψ Sgr
39 [Sagittarius]			Jupiter, Saturn			
	9	4;52	−18; 0	2		α Sgr
	9	5;32	−23; 0	2		$\beta^1 + \beta^2$ Sgr
	9	14;32	−13; 0	3		η Sgr
40 [Sagittarius]			Venus, Saturn			
	9	16;32	−5;50	5		59(b) Sgr
	9	16;32	−4;50	5		60(A) Sgr
	9	16;42	−4;50	5		ω Sgr
	9	17;32	−6;30	5		62(c) Sgr
41 [Capricornus]			Mars, Venus			
	9	25;12	+2;20	3		$\alpha^1 + \alpha^2$ Cap
	9	25;12	+5; 0	3		β Cap
	9	26;42	+1;30	6		ρ Cap
	9	26;52	+0;45	6		ο Cap
42 [Capricornus]			Mars, Mercury			
	9	29;32	−8;40	4		ω Cap
	10	4;32	−7;40	4		24(A) Cap
	10	8; 2	−6;50	4		ζ Cap
	10	8;12	−6; 0	5		36(b) Cap
43 [Capricornus]			Saturn, Mercury			
	10	12;42	+2;10	3		γ Cap
	10	14;12	+2; 0	3		δ Cap
	10	14;42	−0;20	4		42(d) Cap
	10	15;32	−2;50	5		λ Cap
44 [Aquarius]			Saturn, Mercury			
	10	2;32	+8;40	3		ε Aqr
	10	4; 2	+8; 0	4		μ Aqr

Table 16B (*cont.*)

[Constellation]		Associated planets			
Longitude (s)	(°)	Latitude (°)	Magn.	Name	[Modern]
10	14;22	+8;50	2		β Aqr
10	24;12	+11;15	4		α Aqr
45 [Aquarius]		Mercury, Saturn			
10	19;12	−5; 0	4		τ Aqr
10	19;32	−7;30	3		δ Aqr
10	22;32	−5;40	5		53(f) Aqr
46 [Aquarius]		Saturn, Jupiter			
11	5;32	−1; 0	4		83(h) Aqr
11	6;52	−7;30	4		ψ^1 Aqr
11	7;52	−0;30	4		ϕ Aqr
11	8;12	−1;40	4	*a[n]phora*	χ Aqr
47 [Pisces]		Mercury, Saturn			
11	9;32	+9;15	4		β Psc
11	12; 2	+7;30	4		γ Psc
11	13;52	+9;20	4		7(b) Psc
48 [Pisces]		Jupiter, Mercury			
11	13;52	+4;30	4		κ Psc
11	17;32	+2;30	4		λ Psc
49 [Pisces]		Saturn, Mercury			
11	23;52	+6;20	4		ω Psc
11	28;52	+5;45	6		41(d) Psc
50 [Pisces]		Jupiter, Venus			
0	17;12	+15;20	4		ϕ Psc
0	20; 2	+17; 0	4		υ Psc
51 [Pisces]		Saturn, Jupiter			
0	13;32	+14;20	4		ψ^1 Psc
0	14;12	+13; 0	4		ψ^2 Psc
0	15;32	+12; 0	4		χ Psc*
52 [Pisces]		Mars, Mercury			
0	20;22	−8;30	3		α Psc

Table 16B (*cont.*)

[Constellation] Longitude (s)	(°)	Associated planets Latitude (°)	Magn.	Name	[Modern]
[Northern constellations]					
1 [Ursa Minor]		Saturn, Venus			
4	5; 2	+72;50	2	*aliedin*	β UMi
4	14; 2	+74;50	2	*alforcami*	γ UMi
2 [Ursa Maior]		Moon, Venus			
5	0; 2	+53;30	2		ε UMa
5	5;52	+55;40	2	*benezna*	ζ UMa
5	17;42	+54; 0	2		η UMa
3 [Draco]		Saturn, Mars			
5	26;22	+84;50	3		ζ Dra
5	27;52	+88; 0	3		η Dra
4 [Cepheus]		Saturn, Jupiter			
0	4;32	+69; 0	3		α Cep
0	25;22	+71;10	4		β Cep
11	27;12	+72; 0	4		η Cep
5 [Hercules]		Saturn, Mars			
7	28; 2	+53;30	4		ε Her
7	29;42	+54;10	3		ζ Her
8	1;52	+59;50	3		π Her
8	3;12	+60;20	4		69(e) Her
6 [Corona Borealis]		Venus, Mercury			
6	29;32	+46;10	4		β CrB
7	2;32	+44;30	2	*alfeca*	α CrB
7	5; 2	+44;45	4		γ CrB
7	7; 2	+44;50	4		δ CrB
7 [Bootes]		Mercury			
6	9;12	+28; 0	3		η Boo
8 [Lyra]		Venus, Mercury			
9	5;12	+62; 0	1	*lilurah*	α Lyr
9 [Perseus]		Saturn, Jupiter			
1	17;29	+23; 0	2	*eiumezuz*?	β Per
10 [Perseus]		Mars, Mercury			
1	22;42	+30; 0	2		α Per
11 [Auriga]		Mars, Mercury			
2	12;52	+22;30	1	*alhaioch*	α Aur
12 [Ophiuchus]		Saturn, Venus			
8	12;42	+36; 0	3	*alhanue*	α Oph
13 [Serpens]		Saturn, Mars			
7	12;12	+25;30	3		α Ser
7	12;42	+36;30	4		λ Ser
7	14;12	+24; 0	3		ε Ser
7	16;32	+16;30	4		μ Ser

Table 16B (*cont.*)

[Constellation]	Longitude (s)	Longitude (°)	Associated planets / Latitude (°)	Magn.	Name	[Modern]
14 [Sagitta]			Mars, Venus			
	9	24;32	+39;10	6		ζ Sge
	9	28; 2	+39;20	4		γ Sge
15 [Aquila]			Jupiter, Mars			
	9	21;42	29;10	2	*vultur*	α Aql
16 [Delphinus]			Saturn, Mars			
	10	6;22	+32; 0	3		β Del
	10	8; 2	+33;50	3		α Del
	10	9;12	+32; 0	3		δ Del
	10	11;22	+32;10	3		γ Del
17 [Pegasus]			Mars, Mercury			
	0	15; 2	+12;31	3		γ Peg
	11	20; 2	+31; 0	2		β Peg
18 [Andromeda]			Mars, Venus			
	0	13;32	+15; 7	3		η And
	0	19;42	+30; 0	3		μ And
	0	19;52	+32;30	3		ν And
	0	25;42	+26;20	3		β And
	1	4;42	+23; 0	3		γ And
19 [Triangulum]			Mercury			
	0	28;52	+16;30	3		α Tri
	1	3;52	+20;40	3		β Tri

[Constellation]	Longitude (s)	Longitude (°)	Associated planets / Latitude (°)	Magn.	Name	[Modern]
[Southern constellations]						
1 [Piscis Austrinus]			Mars, Venus, Mercury			
	10	9;42	−16;30	4		θ PsA
	10	16; 2	−15; 0	4		η PsA
	10	16;42	−14; 0	4		λ PsA
	10	18;32	−20;20	4		β PsA
2 [Cetus]			Saturn			
	0	12;52	−20; 0	2		ζ Cet
3 [Orion]			Mars, Mercury			
	2	19;52	−17; 0	1		α Ori

Table 16B (*cont.*)

[Constellation]	Longitude (s)	Longitude (°)	Associated planets Latitude (°)	Magn.	Name	[Modern]
4 [Orion]			Jupiter, Saturn			
	2	7;42	−31;30	1		β Ori
	2	13;12	−24;10	2		δ Ori
	2	15;12	−24;50	2		ε Ori
	2	16; 2	−25;40	2		ζ Ori
5 [Eridanus]			Jupiter			
	0	18; 2	−53;30	1		θ Eri
6 [Eridanus] [..]						
	2	5;12	−31;50	4		λ Eri
7 [Lepus]			Saturn, Mars			
	2	12;42	−44;20	3		β Lep
	2	13;22	−41;30	3		α Lep
8 [Canis Maior]			Venus			
	2	13;52	−57;40	2		α Col
	2	16;52	−59;40	2		β Col
9 [Canis Maior]			Jupiter, Mars			
	3	5;32	−39;10	1		α CMa
	3	7;32	−35; 0	4		θ CMa
10 [Canis Minor]			Mercury, Mars			
	3	13;22	−14; 0	4		β CMi
	3	17; 2	−16;10	1		α CMi
11 [Hydra]			Saturn, Jupiter			
	4	17;52	−20;30	2		α Hya
	4	23;52	−26;30	4		κ Hya
	4	26;32	−26; 0	4		υ^1 Hya
12 [Crater]			Venus, Mercury			
	5	17;52	−18; 0	4		δ Crt
	5	17;52	−18;30	4		ζ Crt
	5	20;22	−19;30	4		γ Crt
13 [Corvus]			Saturn, Mercury			
	6	2;12	−19;40	3		ε Crv
	6	6;22	−14;50	3		γ Crv
14 [Argo]			Saturn, Jupiter			
	3	5; 2	−69; 0	1		α Car
15 [Centaurus]			Mars, Venus			
	6	24; 2	−25;40	3		ι Cen
	7	3;32	−22;30	3		θ Cen
16 [Centaurus]			Venus, Jupiter			
	6	26;13	−41;10	1		α Cen
	6	27;52	−51;10	2		γ Cru
	6	29; 2	−55;20	2		α Cru

Table 16B (*cont.*)

[Constellation]		Associated planets			
Longitude (s)	(°)	Latitude (°)	Magn.	Name	[Modern]
7	3;12	−51;40	2		β Cru
7	12; 2	−45;20	2		β Cen
17 [Lupus]		Venus, Mars			
7	13;42	−29;10	3		α Lup*
7	15;52	−24;10	3		β Lup
18 [Corona Australis]		Saturn, Mercury			
9	4;22	−15;20	4		γ Cr A
9	4;42	−16; 0	4		α Cr A
9	5;12	−17;10	4		β Cr A
19 [Ara]		Jupiter, Mercury			
8	8;32	−30;20	4		ε^1 Ara
8	12;52	−33;20	4		β Ara
8	13; 2	−34;10	4		γ Ara

* This symbol indicates that Kunitzsch 1986b and Kunitzsch 1991, pp. 187–200, give a modern designation other than that in Toomer 1984, our source for the modern names of the stars.

and Goldstein 2004, the star list compiled by John Vimond is extant in at least five copies: Cambridge, Gonville and Caius College, MS 141 (191), pp. 377–382; Erfurt, Universitätsbibliothek, MS Amplon. 2° 395, ff. 104v–105v; Madrid, Biblioteca Nacional, MS 4238, ff. 65v–66v; Munich, Bayerische Staatsbibliothek, MS Clm 26667, ff. 46v–47v; and Paris, Bibliothèque nationale de France, MS lat. 7286C, f. 8r–v. We can now add to this list Segovia, Biblioteca de la Catedral, MS 84, ff. 46r–51v, and Paris, Bibliothèque nationale de France, MS lat. 7482, ff. 61v–69v.

Part of Vimond's star list was included in the *Repertorium prognosticum de mutatione aeris* by Firmin of Beauval (1338). This text is preserved in at least six manuscripts and was printed twice (Firmin 1485 and 1539). Erhard Ratdolt, who had edited the *editio princeps* of the Alfonsine Tables in 1483, then published Firmin's text two years later with the title *Opusculum repertorii pronosticon in mutationes aeris tam via astrologica quam metheorologica*. As a matter of fact, Vimond, the author of the table, is mentioned in chapter 17 in the first of the seven parts (f. 12v:10–11), as "vnimundus" (*sic*), just before the tables (ff. 13r–15v). We note that in the edition of 1485 the star list was reproduced in part: only the zodiacal constellations were included. This amounts to 137 stars arranged in 51 subgroups, rather than 52, for subgroups 37 and 38 were merged (see Table 16B). The title indicates that that the stars were *verificate ad annum domini 1312*, but this is most probably an error for year "1321 incomplete" (see Thorndike 1934, p. 274, n. 24). The date agrees with that of Vimond's tables.

John of Lignères, a contemporary of John Vimond, seems to have compiled another list containing data for 276 stars, where the longitudes are Alfonsine, i.e., Ptolemy's values plus 17;8°: it is found in Paris, Bibliothèque nationale de France, MS lat. 7316A, ff. 182r–183v, and MS lat. 10264, ff. 36v–38v; and Florence, Biblioteca Nazionale Centrale, MS Conv. Soppr. J.IV.20 (San Marco 182), ff. 214v–216r.

Al-Battānī lists 533 stars (Nallino 1903–1907, 2:144–177), where longitude, latitude, and magnitude are given for each star, as in Ptolemy's catalogue from which it derives. It is most likely that some of the lists mentioned above derive in turn from Latin translations of al-Battānī's list.

(iii) In medieval literature there are not many catalogues of stars other than Ptolemy's. In all cases they depend on the *Almagest* and have more or less the same number of stars. We already mentioned that composed by al-Ṣūfī for 964 AD with a total precession of 12;42°. Then there is the Latin version of the *Almagest*, mentioned above. A century later another such catalogue was compiled with a quite different presentation but with the same data as the *Almagest*: the Alfonsine *Libro de las estrellas de la ochaua espera*, where the total precession is 17;8°. In addition to these examples, the star catalogue in the Parisian Alfonsine Tables (see Ratdolt 1483 and Santritter 1492) also has a total precession of 17;8°, but it does not derive from the *Libro de las estrellas de la ochaua espera*. Rather, it follows closely the translation made by Gerard of Cremona.

In the early 15th century the Paduan astronomer Prosdocimo de' Beldomandi compiled another catalogue of 1,022 stars (Chabás 2007). As in Gerard of Cremona's version of Ptolemy's star catalogue, for each star and nebula we are given its longitude (in signs, degrees, and minutes), latitude (in degrees, and minutes), and magnitude. The nature of the stars, or groups of stars, is also indicated, outside the framework of the table. Comparing the entries for longitude with those in the *Almagest*, one finds a total precession of 17;8°, exactly the Alfonsine precession. This is probably why the headings of the various copies we have located indicate that the "fixed stars were verified by

King Alfonso *annis Ihesu Christi 1251 completis et mensibus 5*." The grouping in constellations (48) and the ordering are the same as in the Latin translation, and the names in Prosdocimo's catalogue agree, but for a few variants, with those in the Latin translation by Gerard of Cremona, indicating that this was the source used by Prosdocimo for his compilation. In turn, the star catalogue published by Ratdolt in 1483 in the *editio princeps* of the Parisian Alfonsine Tables seems to be a version of the catalogue of stars by Prosdocimo. In Table 16C we display an excerpt of the star catalogue in the Tables of Prosdocimo de' Beldomandi that appears in Bologna, University, MS 2284 (San Salvatore 192), ff. 35v–47r.

Table 16C: Star catalogue in the Tables of Prosdocimo de' Beldomandi (excerpt)

Stelle fixe verificate per regem Alfonsium annis Ihesu Christi 1251 completis et mensibus 5.

Denomination	Long. (°)	Lat. (°)	Magn.	Associated Planets
Et primo urse minoris & sunt 8				
Illa que est super extremitatem caude	1,17;18	66; 0	3	
Illa que est post istam super caudam	1,19;38	70; 0	4	Sat. & Ven.
Illa que est post eam in origine caude	1,33;18	74; 0	4	
Meridiana que est a latere antecedente laterum	1,46;48	75;40	4	
Septentrionalis ab hoc latere et loco	1,50;48	77;40	4	
Meridiana duarum que sunt in latere sequente	2, 4;18	72;50	2	
Septentrionalis ab hoc loco	2,13;18	74;50	2	
Illa que est inter eas et non est in forma	0, 0; 0	0; 0	0	
Meridiana que est secundum rectitudinem duarum stellarum que sunt in latere sequente	2, 0; 8	71;10	4	
Stelle urse maior & sunt 35				
Illa que est super extremitatem muside	1,42;28	39;50	4	Mars
Antecedens duarum que sunt in duobus oculis	1,42;58	43; 0	5	
Sequens earum	1,43;28	43; 0	5	
Antecedens duarum que sunt in fronte	1,43;18	47;10	5	
. . .				
. . .				
. . .				
Stelle piscis meridionalis & sunt 17				
. . .				
Media earum	4,58; 8	22;10	3	
Sequens harum trium	5, 1; 8	21; 0	3	
Occulta antecedens hanc	4,59; 8	20;50	5	
Meridionalis duarum reliquarum	5, 0;18	16; 0	4	
Decliuior earum ad septentrionem	5, 0;58	14;50	4	

CHAPTER SEVENTEEN

GEOGRAPHICAL LISTS

In his *Geography* Ptolemy listed the geographical coordinates of thousands of places of the known inhabited world at his time (see Berggren and Jones 2000). He counted geographical latitude from the equator, and geographical longitude from a prime meridian, namely, the Canary Islands, also called the "Fortunate Islands." This tradition was reflected in the geographical lists that are usually included in sets of astronomical tables compiled during the Middle Ages. By contrast, geographical lists were not included in Ptolemy's *Almagest* or in his *Handy Tables*. Lists of geographical coordinates abound in Arabic sources (see the monumental survey in Kennedy and Kennedy 1987). In the West most of the geographical lists depend on the list in the Toledan Tables which, in turn, is based on the traditions in earlier Arabic zijes. The large number of geographical lists in western medieval manuscripts normally consist of some dozens up to a few hundred places; no two lists are identical, mainly because the copyists eliminated places unknown to them, while incorporating familiar names, or miscopied the numerical information attached to those names. Another source of variation was the fact that sometimes a different prime meridian was used, notably the so-called "meridian of the water" at a distance of 17;30° west of the Canary Islands (Laguarda 1990, p. 75; Comes 1994; Chabás and Goldstein 2010).

Table 17A displays the geographical coordinates in Vienna, Nationalbibliothek, MS 2288, f. 37v, a 14th-century manuscript of Italian origin containing a version of the Parisian Alfonsine Tables. For each longitude we are also given entries for the supplement in 180° in longitude, but we have not displayed them. Most of the 51 places listed are European cities, in contrast to those in the Toledan Tables (see Toomer, pp. 134–139; F. S. Pedersen 2002, pp. 1512–1513), from which many of the non-Italian names in this list may have been taken.

In the 15th century some lists of geographical coordinates give the longitude in time-degrees (rather than in arc-degrees), counted from a specific city, taken as a "prime" meridian. Indeed, among all categories of tables those for geographical coordinates are possibly the

Table 17A: Geographical coordinates in Vienna, MS 2288

	Longitude (°)	Latitude (°)		Longitude (°)	Latitude (°)
Corduba	9;20	37;30	Neapolis	43;20	40; 0
Sedes francorum	23;45	45;50	Pisis	33; 0	43; 0
Cartago	27; 0	37; 0	Florencia	33;25	43;30
Tunis	29; 0	38; 0	Mediolanum	31; 0	45; 0
Yspane	75; 0	34;30	Ravenna	34;44	44; 0
Armenia	77; 0	41; 0	Verona	31;30	44;30
Sardinia	31; 0	38; 0	Cremona	31; 0	45; 0
Roma	34;25	41;50	Panormun	36; 0	38;26
Insula cecilie	36; 0	39; 0	Constaninopolis	49; 0	41; 0
Alexandria	51;20	31; 0	Tolosa	40;45	49; 6
Damiath	54; 0	31; 0	Ferraria	32; 0	45; 0
Egiptus	55; 0	30; 0	Pistorium	33; 0	43;30
Ascalon	55;40	22; 0	Beneventum	41; 0	41;30
Yerusalem	56; 0	32; 0	Ephesus	57;30	35; 0
Damascus	60; 0	27; 0	Bononia	32;30	44;23
Venecia	38;30	45;15	Monspesulanus	32; 0	44; 0
Valencia	30;20	39;36	Padua	32;35	44;20
Sibilia	25;35	37;30	Aquilegia	34; 0	45; 0
Toletum	11; 0	25;23*	Siracusa	39;20	37; 0
Bertagna	25;23	39;30	Capua	41; 0	41;20
Navarra	30;15	45; 0	Marsilia	23;30	54; 0
Janua	29; 0	43;53	Lugdunum	19; 0	58; 0
Anchona	36;30	43; 1	Parixius	24;15	48; 0
Vicencia?	32;22	–	Oxonia	17;56	51;50
(?)dertum	32;15	–	Catha?	90; 0	0; 0
Mantua	30;30	–			

* The latitude for Toledo in medieval tables is usually 39;54°.

most varied. For a review of meridians used in copies of the Alfonsine Tables, see Kremer and Dobrzycki 1998. In Table 17B we present a list computed for Nuremberg, Regiomontanus's residence at the time he composed this list. This list of geographical coordinates for 62 localities appeared in Regiomontanus's *Kalendarium* (1483), which was published about two months after the *editio princeps* of the Alfonsine Tables (both were printed in Venice). Note that names of cities are followed by "m" (*minue*) or "a" (*adde*), which signifies that they are located West or East, respectively, of the meridian of Nuremberg.

Table 17B: Geographical coordinates in Regiomontanus's *Kalendarium*

		Longitude (h)	Latitude (°)			Longitude (h)	Latitude (°)
Hybernia	m	1;16	59	Madeburgum	a	0;16	54
Scotia	m	0;36	59	Erfordia	a	0; 4	51
Oxonium	m	0;52	53	Lips	a	0;10	51
Compostellum	m	1;40	45	Ingelstadium	a	0; 4	49
Lysibona	m	1;40	41	Nuremberga	a	0; 0	49
Toletum	m	1;24	41	Ratisbona	a	0; 6	49
Corduba	m	1;27	38	Ulma	a	0; 0	47
Cesaraugusta	m	1; 6	41	Praga	a	0;24	50
Rhotomagus	m	0;43	50	Vratislavia	a	0;40	51
Parisius	m	0;30	48	Cracovia	a	0;56	51
Lugdunum	m	0;31	45	Caschovia	a	0;56	50
Burdigalia	m	0;52	45	Buda	a	0;50	47
Avinio	m	0;32	44	Segnia	a	0;32	45
Tolosa	m	0;43	43	Vienna Pannonie	a	0;15	48
Viena provincie	m	0;30	44	Patavia	a	0;10	48
Massilia	m	0;28	43	Salcerburgum	a	0;12	48
Prugis	m	0;36	52	Judeburgum	a	0;14	47
Gandavum	m	0;24	52	Villacum	a	0;13	46
Traiectum	m	0;12	55	Brixina	a	0; 8	45
Colonia agrippina	m	0;13	51	Venetie	a	0;10	45
Machilinia	m	0;24	51	Ancon	a	0;14	44
Maguntia	m	0;15	50	Roma	a	0;20	42
Herbipolis	m	0; 4	50	Tarentum	a	0;44	40
Argentina	m	0;12	47	Brundusium	a	0;40	39
Constantia	m	0;10	46	Neapolis	a	0;36	41
Augusta Vindelicorum	a	0;10	46	Florentia	a	0;10	43
Dacia	a	0;36	58	Mediolanum	a	0; 0	44
Suetia	a	0;28	62	Taurinum	m	0; 2	43
Lubeca	a	0;16	56	Genua	m	0; 4	43
Dausticum	a	0;56	56	Sardinia	a	0; 2	38
Prunsuiga	a	0; 0	53	Sicilia	a	0;30	37

CHAPTER EIGHTEEN

ASTROLOGY

In his essay on astrological houses as defined by medieval Islamic astronomers, E. S. Kennedy maintained that astrology "attracted numerous serious and able practitioners. This alone makes it worthy of study by historians" (Kennedy 1996, p. 535). There are indeed a great many tables in astrological texts, some of which involve sophisticated mathematical techniques encountered nowhere else in medieval literature, as well as lists with no mathematical structure; we will focus our attention for the most part on mathematical astrology, that is, the part of astrology that relates to astronomical concepts and requires the use of mathematical tools.

1. *Horoscopes and Astrological Houses*

Astrologers claimed to be able to make predictions about an individual and about the world based on the configuration at a particular time of the five planets, the Sun, the Moon, and the lunar ascending node. For an individual astrologers focussed their attention on the time of birth, and a key element for their predictions was the arrangement of the twelve astrological houses at that time for a specified locality. The ecliptic was usually divided into 12 consecutive unequal sections, called astrological houses, numbered from 1 to 12 in the direction of increasing longitude, beginning at the ascendant (called *ascendens* or *horoscopus* in Latin), that is, the point on the ecliptic that is rising at the eastern horizon at a particular time. These astrological houses had names that, broadly speaking, indicated the significance they had for the "native": (future) life, business, brothers, parents, children, illness, marriage, death, travel, honors, friends, and enemies, or variants of them. Thus, future life was affected by the planets in the first house, business by those in the second house, and so on, with many variations (North 1986, p. 1). In addition to the first house, three others begin at points with astronomical significance. The 4th house begins at the lower midheaven (Latin: *imum medium caelum*), i.e., the intersection of the ecliptic and the meridian below the horizon; the 7th house begins at the descendant, i.e., the intersection of the ecliptic and the western horizon; and the 10th house begins at midheaven (Latin: *medium caelum*), i.e., the intersection of the ecliptic and the meridian above the horizon. For a discussion of the difficult historical questions concerning the various mathematical rules for defining these astrological houses, see North 1996, pp. 579–582. In Figure 24 H is the ascendant, M is midheaven, and the cusps, that is, the initial points of the houses on the ecliptic, are indicated by black dots. Note that the numbers correspond to the houses and that the back of the sphere is not shown. There were several medieval systems for domification, that is, the division of the zodiac into astrological houses, and the data required for each procedure were the date, the time of day, a value for the obliquity of the ecliptic, and a geographical latitude. Each system generated a different set of 12 houses for the same data (North 1986, especially pp. 46–49; Kennedy 1996, especially pp. 535–545).

Generally, the longitudes of the cusps were given (in degrees and minutes) as a function of the longitude of the ascendant (in integer degrees). This type of table is usually called a table for the equation of the houses and it is presented as 12 sub-tables, one for each zodiacal sign (see Table 18.1A). There are only a few different tables of this

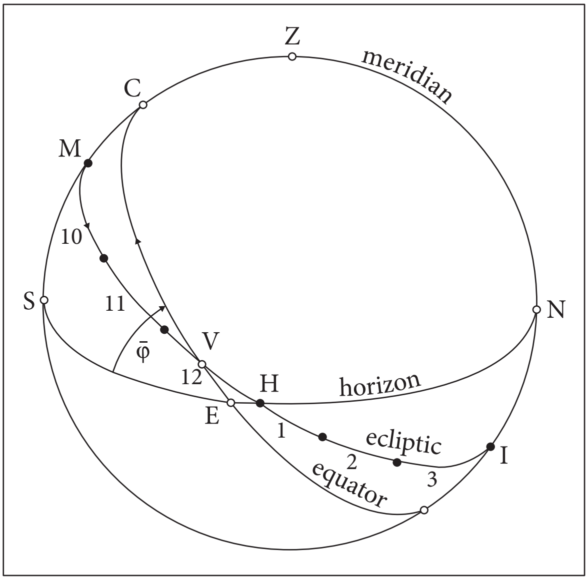

Figure 24: The astrological houses

kind. The two most common are the following: (i) one is associated with Maslama al-Majrīṭī's adaptation of the zij of al-Khwārizmī and it is based on an obliquity of the ecliptic of 23;35°, which is al-Battānī's value and not al-Khwārizmī's, and it is valid for the latitude of Córdoba, 38;30°, although many manuscripts assign it erroneously to Toledo (see Suter 1914, pp. 194–20; Toomer 1968, pp. 140–143; F. S. Pedersen 2002, pp. 1075–1092, who suggests an obliquity of 23;33°); (ii) the other is a composite table where the 6 sub-tables for Aries, Taurus, Gemini, Cancer, Aquarius, and Pisces are based on an obliquity of 23;35° and are valid for Toledo, whereas the first five columns of the sub-table for Leo coincide with those mentioned above for Córdoba, while the rest of the sub-tables, including the last column of the sub-table for Leo, are probably based on the Ptolemaic value for the obliquity (23;51°) and valid for an unknown locality at a latitude of about 41;11°, as pointed out by North (1986, p. 17; see also Toomer 1968, pp. 144–146; and F. S. Pedersen 2002, pp. 1093–1108). As for the unknown locality, we have examined Florence, Biblioteca Nazionale Centrale, MS Conv. Soppr. J.V.5 (San Marco 191), ff. 79r–83v, containing a complete table for the equation of the houses for Zaragoza (latitude 41;30°, as indicated in the heading), and found that the sub-tables for the unknown locality in the composite table are in very good agreement with those for Zaragoza. As indicated in § 2.4 (right ascension), MS San Marco 191 contains other tables associated with the tradition of al-Khwārizmī. Table 18.1A displays an excerpt of a table for the equation of the houses for Toledo found in Vienna, Nationalbibliothek, MS 2488, ff. 77v–83r, a 14th-century manuscript in Castilian uniquely containing the *Almanaque Perpetuo* by Ferrand Martines. As far as we can determine, this is the only fully consistent set of equations of the houses for Toledo (see Chabás 1996b, pp. 288–294). The longitudes of the ascendant and the other cusps (labeled 2 to 6 in the headings) depend on time, that is, on the day of the year and the time of the day, as well as on the geographical latitude of a given locality. Consequently, other tables were needed to convert the time to the longitude of the ascendant.

In Table 18.1B we reproduce an excerpt of a table that appears in the *Almanach Perpetuum* printed in 1496, computed for Salamanca, for all days in a year beginning in March. This table is also found in Zacut's *Ḥibbur* (see Chabás and Goldstein 2000, pp. 63 and 100–102). The numbers in the heading for columns 3 to 8 refer to the astrological houses. In each column the entries are the longitudes,

Table 18.1A: The equation of the astrological houses for Toledo (excerpt)

Ari (°)	2 Tau	3 Gem	4 Cnc	5 Cnc	6 Leo
1	3; 3	2;50	0;30	28;30	28;34
2	3;52	3;28	1; 4	29; 8	29; 2
...					
...				Leo	Vir
...					
29	26; 7	21;23	16; 8	18; 8	23;12
30	26;57	22; 4	16;40	18;53	24; 8

Tau (°)	2 Tau	3 Gem	4 Cnc	5 Leo	6 Vir
1	27;47	22;46	17;46	19; 8	25;25
2	28;36	23;23	17;23	20;26	26;46
...					
...	Gem	Cnc	Leo	Vir	Lib
...					
29	21;36	13;15	6;36	13;37	22;34
30	22;24	14;36	7;24	14;35	23;34

Gem (°)	2 Gem	3 Cnc	4 Leo	5 Vir	6 Lib
1	23;26	15;25	8;14	15;34	24;38
2	24;18	16;16	9; 4	16;34	25;49
...					
...	Cnc	Leo	Vir	Lib	Sco
...					
29	20; 8	12;10	5;44	15;57	24;11
30	21; 8	13;14	6;51	17; 7	25;14

Cnc (°)	2 Cnc	3 Leo	4 Vir	5 Lib	6 Sco
1	22;10	14;21	8; 2	18;18	26;18
2	23;14	15;28	9;13	19;29	27;22
...					
...	Leo	Vir	Lib	Sco	Sgr
...					
29	22;44	18; 5	14; 7	21;30	25;34
30	23;53	19;22	15;29	22;40	26;35

Leo (°)	2 Leo	3 Vir	4 Lib	5 Sco	6 Sgr
1	25; 2	20;40	16;50	23;49	27;34
2	26;12	21;58	18;11	24;58	28;35
...					
...	Vir	Lib	Sco	Sgr	Cap
...					
29	27;35	26;20	23;22	24;53	25;46
30	28;45	27;35	24;40	25;58	26;46

Vir (°)	2 Vir	3 Lib	4 Sco	5 Sgr	6 Cap
1	29;54	28;47	25;27	27; 2	27;47
	Lib	Sco			
2	1; 3	0; 0	27;28	29; 7	28;48
...					
...	Sco		Sgr	Cap	Aqr
...					
		Sgr			
29	1; 8	1; 2	28;51	26;48	26;45
			Cap		
30	2;13	2; 7	0; 0	27;52	27;47

Table 18.1A (*cont.*)

Lib (°)	2 Sco	3 Sgr	4 Cap	5 Cap	6 Aqr
1	3;16	3;11	1; 9	28;58	28;51
				Aqr	
2	4;19	4;15	2;18	0; 4	29;56
. . .					
. . .	Sco*	Cap	Aqr	Psc	Psc
. . .					
					Ari
29	2;10	2;59	4; 6	1;14	0; 8
30	3;11	4; 3	5;19	2;26	1;16

* MS: Sco, instead of Sgr.

Sco (°)	2 Sgr	3 Cap	4 Aqr	5 Psc	6 Ari
1	4;11	5; 7	6;36	3;41	2;26
2	5;12	6;11	7;49	4;56	3;35
. . .					
. . .	Cap	Aqr	Psc	Ari	Tau
. . .					
29	2;27	6;11	13;11	9;20	4;58
30	3;28	7;20	14;33	10;37	6; 7

Sgr (°)	2 Cap	3 Aqr	4 Psc	5 Ari	6 Tau
1	4;30	8;30	15;54	11;54	7;38
2	5;31	9;40	17;14	13;11	8;21
. . .					
. . .	Aqr	Psc	Ari	Tau	Gem
. . .					
29	3;42	11;43	21;58	15;38	7;48
30	4;46	12;54	23; 9	16;45	8;50

Cap (°)	2 Aqr	3 Psc	4 Ari	5 Tau	6 Gem
1	5;49	14; 4	24;16	17;48	9;51
2	6;52	15;13	25;24	18;52	10;52
. . .					
. . .	Psc	Ari	Tau	Gem	Cnc
. . .					
29	5;24	14;26	21;45	14;36	7;38
30	6;26	15;26	22;36	15;22	8;32

Aqr (°)	2 Psc	3 Ari	4 Tau	5 Gem	6 Cnc
1	7;27	16;22	23;23	16;15	8;25
2	8;28	17;18	24;10	17; 4	9;18
. . .					
. . .	Ari	Tau	Gem	Cnc	Leo
. . .					
29	5;56	10;20	12;44	7;14	2;15
30	6;58	11; 7	13;21	7;56	3; 4

Psc (°)	2 Ari	3 Tau	4 Gem	5 Cnc	6 Leo
1	6;57	11;51	13;57	8;37	3;53
2	7;42	12;35	14;34	9;18	4;43
. . .					
. . .	Tau	Gem			
. . .					
29	1;32	1;26	29;28	26;23	26;57
			Cnc		
30	2;15	2; 6	0; 0	26;53	27;44

λ_i (for $1 \leq i \leq 6$), of the cusps. For the 7th to 12th houses, the following relation holds: $\lambda_j = \lambda_{j-6} + 180°$ (for $7 \leq j \leq 12$). In contrast to Table 18.1A, the use of Table 18.1B no longer requires any auxiliary tables for converting the time of day on a given day of the year to the longitude of the ascendant at that moment. As we learn from the canons, the date in the first column refers to noon on that day. This is the first unusual feature of this table, since the ascendant is generally the argument for the entries in the other columns. The second unusual feature is the column labeled time which, together with the date in the first column, is intended to allow the user to determine the cusps of all the houses at any time on any day of the year, without requiring any other tables. This is certainly "user-friendly" since the user can read off the information he seeks from this table directly without any other computations (except, perhaps, linear interpolation between entries, if greater precision is sought). This procedure was not at all obvious to us, and it needs some discussion to explain how it works.

We note that the entries in the column, labeled time, increase at the rate of about 0;4h per day from 0;0h on March 1 to 24;0h on February 29, that is, the entries accumulate to 1 day in the course of a year. We interpret this to mean that the configuration of the astrological houses on March 2 at noon was the same as their configuration on March 1 at 0;4h after noon. To justify this interpretation, note that in a day the Sun advances in longitude by about 1°, and this corresponds to about 1° in right ascension or, equivalently, about 0;4h (360° = 24h, or 15° = 1h, or 1° = 0;4h). The daily rotation takes place in the direction opposite to the Sun's increase in longitude; hence, the interval from the Sun's crossing the meridian to its next such crossing is longer than the interval for a fixed star's crossing the meridian to its next such crossing. For the Sun this is called a solar day, and for a fixed star it is called a sidereal day. In units where the solar day is 24;0h, a sidereal day is about 23;56h. This means that the entries in the column labeled time gives the accumulated difference between solar and sidereal time. The canons give a worked example for 4h after noon on December 15 (Zacut 1496, Latin version, f. 3a–b): the entry in the column labeled time for (noon) December 15 is 18;49h (f. 20r) to which one adds 4h, yielding 22;49h. The date in the first column corresponding most closely to 22;49h is February 10. So the entries for the cusps for (noon) February 10 are those sought for 4h after noon on December 15. Zacut has cleverly taken advantage of the fact that, due to the daily rotation, all points on the ecliptic cross the horizon within the course of a single day. Therefore, one can identify configurations of the astrological houses for noon over the course of a year with configurations in the course of a single day. For this purpose Zacut chose March 1 as the epoch for the table. Table 18.1B displays an excerpt of the table for March; there are subsequent tables for each month, up to February, for which an excerpt is also displayed. The parameters that come closest to reproducing the entries for the cusps are an obliquity of 23;33° and a geographical latitude of 41;19° (Salamanca): see Chabás and Goldstein 2000, pp. 100–101. We are not aware of any other European table for the astrological houses, based on the principle introduced by Zacut.

Table 18.1B: The cusps of the astrological houses for each day of the year (excerpt)

March day	Time (h)	1/7 Cnc	2/8 Leo	3/9 Leo	4/10 Vir	5/11 Sco	6/12 Sgr
1	0; 0	12	3	26	20	0	7
2	0; 4	13	4	27	21	1	8
. . .							
		Leo		Vir	Lib		
. . .							
30	1;46	3	27	23	19	26	29
31	1;49	4	28	24	20	27	30

. . .

Febr. day	Time (h)	1/7 Gem	2/8 Cnc	3/9 Leo	4/10 Leo	5/11 Lib	6/12 Sco
1	22;13	17	8	1	22	2	11
2	22;17	18	9	2	23	3	12
. . .							
					Vir		
10	22;48	26	17	9	1	21 *	21
. . .							
		Cnc	Leo			Sgr	
. . .							
28	23;56	11	2	25	19	29	6
29	24; 0	12	3	26	20	30	7

* Instead of 12 (as in the *Ḥibbur*: Lyon, MS Heb. 14, f. 135r).

2. *Projection of the Rays*

According to standard astrological practice, a celestial body projects or casts its rays on seven points of the ecliptic located at a distance of 180°, 120°, 90°, and 60° from it, whether to the right or to the left. The configurations generated by these points are the astrological aspects (opposition, trine, quartile, and sextile, respectively) of a planet. The points are found opposite the planet (opposition) or at the other vertices of an equilateral triangle (trine), a square (quartile), or a regular hexagon (sextile). In the standard theory the latitude of the planet is ignored and these regular polygons are inscribed in the ecliptic (see Figure 25), but in some cases, when the latitude of the planet is taken into account, the aspects are measured on the celestial equator and then projected onto the ecliptic. In the former case, no tables were needed but, in the latter case, extensive tables were computed for the determination of the longitudes of these points from the longitude of the planet, the latitude of a given terrestrial location, and the longitude of the ascendant at some given time on a given day. There are two types of tables for the projection of the rays: one is represented by the table attributed to al-Khwārizmī, constructed for a geographical latitude of 33° (Baghdad) and an obliquity of the ecliptic of 23;51°;

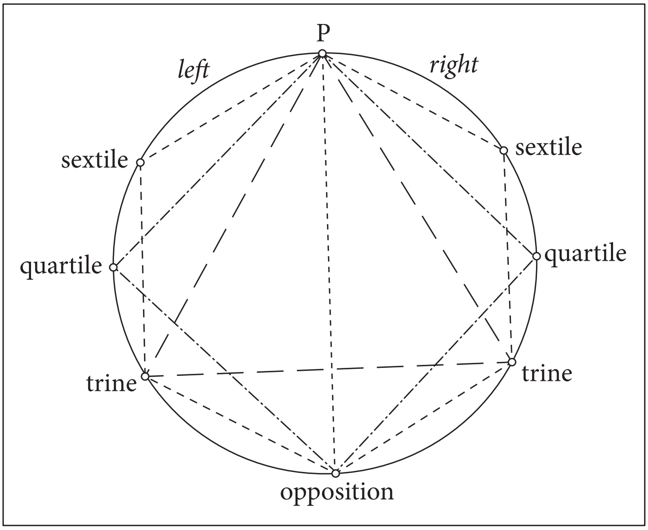

Figure 25: Planetary aspects

the other is represented by the table attributed to Maslama al-Majrīṭī and is valid for the latitude of Córdoba, 38;30°. Al-Khwārizmī's table for this purpose is not preserved in the Latin translation of Maslama's adaptation of his zij.

(i) In his *Book on Astrology*, the Christian scholar Ibn Hibintā, living in Baghdad at about the same time as al-Khwārizmī, attributed to the latter the compilation of a table usually presented as an array of 36 sub-tables, one for each "face" or decan (one third of a zodiacal sign, 10°), of the zodiac (Kennedy and Krikorian-Preisler 1972, p. 4). To each of the 36 faces a planet or luminary, a "lord of the face," is associated: e.g., Mars, Sun, and Venus for the three faces of Aries, respectively, and so on, according to a pattern described in al-Birūnī's *Tafhīm* (Wright 1934, p. 263) and found in tabular form, for example, in Zacut's *Ḥibbur* (see Chabás and Goldstein 2000, p. 88, n. 6). This table for the projection of the rays has not survived in Arabic, but it is extant in Latin with the title *Tabula proiectionis radiorum planetarum*, or *Tabula de faciebus signorum*, or variants of them. In particular, it is found among the Toledan Tables (Toomer 1968, pp. 147–151; F. S. Pedersen 2002, pp. 1520–1528). In Table 18.2 we reproduce an excerpt of it (only six sub-tables, where it is assumed that the ascendant is in Aries or Cancer). For explanations of the structure and use of this table, see Toomer 1968, Kennedy and Krikorian-Preisler 1972, and Hogendijk 1989; see also Samsó and Berrani 1999, pp. 302–306.

The entries in Table 18.2 depend both on the longitude of the planet and the longitude of the ascendant at some given moment in time. The first three sub-tables shown here, for instance, are valid when the longitude of the ascendant of the planet is in any of the three faces of Aries (zodiacal sign 1), which can be designated by [1, 1], [1, 2], and [1, 3], following Hogendijk's notation. For each of the 36 faces (one for each sub-table) of the ascendant and each longitude of the planet we are given two numbers and a letter. The first number, in degrees and minutes, is called F(d, m) in Hogendijk's notation, where "d" stands for face and "m" for the zodiacal sign where the planet is located. The letters S, Q, and T stand for sextile, quartile, and trine, respectively. The second number is displayed in two rows (one for the degrees and another one for the minutes), and it is called S(d), Q(d), or T(d) in Hogendijk's notation. Note that the numbers S(d), Q(d), and T(d) are always in the proportion 1:1;30:2 (or 60:90:120). Thus, for example, in the second sub-table (second face of Aries), 27;52° is

Table 18.2: Projection of the rays, attributed to al-Khwārizmī (excerpt)

Aries Sign	First face (°)	Mars	
Ari	20; 5	53	S
Tau	25;41	18	S
Gem	31;24		S
Cnc	32;16		S
Leo	27;47	79	Q
Vir	22;40	57	Q
Lib	20; 5		Q
Sco	25;41		Q
Sgr	31;24	106	T
Cap	32;16	35	T
Aqr	27;47		T
Psc	22;40		T

Aries Sign	Second face (°)	Sun	
Ari	21;15	53	S
Tau	27; 2	52	S
Gem	31;59		S
Cnc	32;16		S
Leo	27;52	80	Q
Vir	22;40	48	Q
Lib	19;52		Q
Sco	25;58		Q
Sgr	31;59	107	T
Cap	32;16	44	T
Aqr	27;24		T
Psc	22;40		T

Aries Sign	Third face (°)	Venus	
Ari	22;24	54	S
Tau	28;23	26	S
Gem	32;33		S
Cnc	32;16		S
Leo	27;56	81	Q
Vir	22;40	40	Q
Lib	19;39		Q
Sco	26;17		Q
Sgr	32;33	108	T
Cap	32;16	53	T
Aqr	27; 1		T
Psc	22;40		T

. . .

Cancer Sign	First face (°)	–	
Cnc	34;52	61	S
Leo	34; 7	51	S
Vir	30;25		S
Lib	27;50		S
Sco	27;47	92	Q
Sgr	30;32	47	Q
Cap	34;52		Q
Aqr	34; 7		Q
Psc	30;25	123	T
Ari	27;50	42	T
Tau	27;47		T
Gem	30;32		T

Cancer Sign	Second face (°)	–	
Cnc	35;19	62	S
Leo	33;59	54	S
Vir	30;25		S
Lib	27;50		S
Sco	27;56	94	Q
Sgr	31;41	21	Q
Cap	36;54		Q
Aqr	34;54		Q
Psc	30;25	125	T
Ari	27;50	47	T
Tau	28;28		T
Gem	31;41		T

Cancer Sign	Third face (°)	–	
Cnc	35;47	63	S
Leo	33;50	57	S
Vir	30;25		S
Lib	27;50		S
Sco	28; 6	95	Q
Sgr	32;51	55	Q
Cap	38;56		Q
Aqr	35;40		Q
Psc	30;25	127	T
Ari	27;50	53	T
Tau	29; 9		T
Gem	32;51		T

. . .

the number F([1, 2], 5) associated with a planet in Leo (zodiacal sign 5: longitude between 120° and 150°) when its ascendant is in the second face of Aries ([1, 2]: longitude between 10° and 20°), and 80;48° is the number Q(d). In order to compute the longitudes of the extremes of the rays from the entries F(d, m) together with S(d), Q(d), and T(d) in the table, one should follow al-Khwārizmī's complex instructions, transmitted by Ibn Hibintā and translated by Kennedy and Krikorian-Preisler, or similar instructions in some other medieval text.

(ii) The second type of table is attributed to Maslama al-Majrīṭī for the latitude of Córdoba, 38;30°, and is found in the Latin version of al-Khwārizmī's zij (Suter 1914, pp. 206–229; Neugebauer 1962a, pp. 78–80, 129–131). It is composed of 72 sub-tables, six for each sign of the zodiac at steps of 5°. The entries in this table differ from those in (i) although in this case too the numbers S(d), Q(d), and T(d) are in the proportion 1:1;30:2 (or 60:90:120).

Another type of table for the same purpose was provided by Ibn Azzūz for the latitude of Fez (33;40°) and an obliquity of the ecliptic of 23;33° (Casulleras 2007) but, as far as we know, this table is only found in Arabic and did not reach Western Europe. For a list of methods for casting the rays in Arabic astrology, see Casulleras 2004 and 2010.

3. *Planetary Dignities*

The main astrological attributes of the planets (or dignities)—domiciles, exaltations, triplicities, terms, and faces (decans)—are usually found in specific tables, but often they are grouped together in a single table, as in the zij of al-Khwārizmī (Suter 1914, p. 231). We have found such a table in various Latin manuscripts (Lisbon, Torre do Tombo, MS 1711, f. 19r; Segovia, Biblioteca de la Catedral, MS 110, f. 84v; and Naples, Biblioteca Nazionale, MS VIII.C.49, f. 57v), as well as in Hebrew manuscripts containing Abraham Zacut's *Ḥibbur* (Chabás and Goldstein 2000, pp. 70, 72). A very similar table is found in Firmin of Beauval 1485, ff. 6v–7r. Table 18.3A displays the entries in Lisbon, Torre do Tombo, MS 1711, f.19r.

These attributes are already defined in Ptolemy's *Tetrabiblos* I.17-21. As for domiciles, also called houses, each of the planets is associated with two zodiacal signs, whereas the Sun and the Moon are related to only one sign, following the distribution displayed in Figure 26.

The exaltations are the places on the zodiac where the celestial bodies reach their maximum influence. For this purpose, according to *Tetrabiblos* I.19 a zodiacal sign is associated with each of the planets, the Sun, the Moon, and the ascending and the descending lunar nodes. In al-Bīrūnī's *Book of Instruction* (Wright 1934, p. 258), a degree in the corresponding sign is also given (e.g., Aries 19° for the Sun). The entries for the exaltations displayed in Table 18.3A, below, agree with al-Bīrūnī's list.

The four possible equilateral triangles connecting the 12 zodiacal signs are described in *Tetrabiblos* I.18 in relation to the celestial bodies associated with them (triplicities). The entries for the triplicities in Table 18.3A, above, agree with Ptolemy's list. In al-Bīrūnī's *Book of Instruction* (Wright 1934, p. 259), each triplicity corresponds to one

Table 18:3A: Planetary dignities

Sign	Domicile	Exaltation		Triplicity	. . . (cont.)
Ari	Mars	Sun	19	Sun, Jupiter, Saturn	
Tau	Venus	Moon	3	Venus, Moon, Mars	
Gem	Mercury	Asc. node	3	Saturn, Mercury*, Jupiter	
Cnc	Moon	Jupiter	15	Venus, Mars, Moon	
Leo	Sun			Sun, Jupiter, Saturn	
Vir	Mercury	Mercury	15	Venus, Moon, Mars	
Lib	Venus	Saturn	21	Saturn, Mercury, Jupiter	
Sco	Mars			Venus, Mars, Moon	
Sgr	Jupiter	Desc. node	3	Sun, Jupiter, Saturn	
Cap	Saturn	Mars	28	Venus, Moon, Mars	
Aqr	Saturn			Saturn, Mercury, Jupiter	
Psc	Jupiter	Venus	27	Venus, Mars, Moon	

Sign	. . .	Terms [according to Ptolemy]**										Faces					
Ari	. . .	Jup.	6	Ven.	8	Merc.	7	Mars	5	Sat.	4	Mars	10	Sun	20	Ven.	30
Tau	. . .	Ven.	8	Merc.	7	Jup.	7	Sat.	2	Mars	6	Merc.	10	Moon	20	Sat.	30
Gem	. . .	Merc.	7	Jup.	6	Ven.	7	Mars	6	Sat.	4	Jup.	10	Mars	20	Sun	30
Cnc	. . .	Mars	6	Jup.	7	Merc.	7	Ven.	7	Sat.	4	Ven.	10	Merc.	20	Moon	30
Leo	. . .	Sat.	6	Merc.	7	Mars	5	Ven.	6	Jup.	6	Sat.	10	Jup.	20	Mars	30
Vir	. . .	Merc.	7	Ven.	6	Jup.	5	Sat.	6	Mars	6	Sun	10	Ven.	20	Merc.	30
Lib	. . .	Sat.	6	Ven.	5	Merc.	5	Jup.	8	Mars	6	Moon	10	Sat.	20	Jup.	30
Sco	. . .	Mars	6	Ven.	7	Jup.	8	Merc.	6	Sat.	3	Mars	10	Sun	20	Ven.	30
Sgr	. . .	Jup.	8	Ven.	6	Merc.	5	Sat.	6	Mars	5	Merc.	10	Moon	20	Sat.	30
Cap	. . .	Ven.	6	Merc.	6	Mars	5	Jup.	8	Sat.	5	Jup.	10	Mars	20	Sun	30
Aqr	. . .	Sat.	6	Merc.	6	Ven.	5	Jup.	5	Mars	5	Ven.	10	Merc.	20	Moon	30
Psc	. . .	Ven.	8	Jup.	6	Merc.	6	Mars	5	Sat.	5	Sat.	10	Jup.	20	Mars	30

* Missing in Segovia, MS 110.

** Cnc (5th column): "Saturn 4" instead of "Saturn 3"; Aqr (3rd column): "Venus 5" instead of "Venus 8."

of the four sublunar elements: fire (Ari, Leo, Sgr), earth (Tau, Vir, Cap), air (Gem, Lib, Aqr), and water (Cnc, Sco, Psc); in Table 18.3A this characteristic is not mentioned.

As for the terms, the underlying principle is the following: each zodiacal sign is divided into 5 unequal parts, called terms, and each of the 5 planets is the lord of one term in each zodiacal sign. For each zodiacal sign the sum of the terms must equal 30°. In the *Tetrabiblos* Ptolemy lists terms in two systems, one associated with the Egyptians (*Tetrabiblos*, I.20), and another introduced by Ptolemy in his own name (*Tetrabiblos*, I.21). In Table 18.3A the terms follow the system

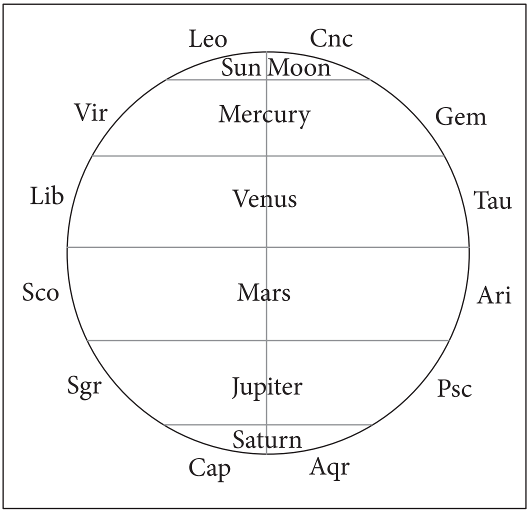

Figure 26: Domiciles

according to Ptolemy, as in al-Battānī (Kennedy *et al.* 2009–2010, p. 83, n. 2).

According to al-Bīrūnī (Wright 1934, p. 262): "Each third of a sign—10 degrees—is called a 'face' and the lords of these faces according to the agreement of the Persians and the Greeks are as follows: the lord of the first face of Aries is Mars, of the second the Sun, of the third Venus, of the first face of Taurus, Mercury, and so on in the order of the planets from above downwards till the last face of Pisces." (see § 18.2 above).

The information contained in this table is presented in many different ways, whether totally or partially, sometimes with different entries, and mostly as separate sub-tables for the various dignities (see examples in F. S. Pedersen 2002, pp. 1590–1595). But there is a peculiar presentation, in the form of a circular table, or "rose," which is found among the tables in the zij of al-Battānī (Nallino 1903–1907, 2:309), where the dignities are displayed as concentric circles (exaltations, domiciles, faces, triplicites, and terms, from the inside to the outside). Sometimes the information is not explicitly given; instead, a code from 1 to 5 is used to designate the astrological dignities of the planets: faces (1), terms (2), triplicities (3), exaltations (4), and domiciles (5). This is the same order, but from right to left, as in Table 18.3A. We are aware of only two examples of a table for astrological

Table 18.3B: Dignities in a "rose"

	Ari	Tau	Gem	Cnc	Leo	Vir	Lib	Sco	Sgr	Cap	Aqr	Psc
Sat.	2	1,2,3	2	2	1,2	3,2	1,3,2,4	2	1,2	5,2	5,2	1,2
Jup.	2,3	2	2, 1	4,2	3,1,2	2	1,2	2	5,2,3	1,2	2	1,2,5
Mars	1,5,2	2	2, 1	2,3	1,2,3	2	2	1,5,3	3,2	4,1,3,2	2	1,2,3
Sun	3,4,1	0	1	1	5,3	1	0	1	3	1	0	0
Venus	2,1	2,5	3,2	1,2	2	1,2,3	5,2	1,2	2	3,2	3,1,2	2,3,4
Merc.	2	1,2,3	5,2	1,2	2	4,1,2,5	2	2	1,2	3,1	3,1,2	2
Moon	0	4,1	1	5, 1	0	3	1	3	1	0	1	3

Gem, Moon: 1. Read: 0 (as in Pedersen 2002, p. 1599).
Cnc, Sun: 1. Read: 0 (as in Pedersen 2002, p. 1599).
Lib, Sat.: 3,2,4. Read: 1,3,2,4 (Pedersen 2002, p. 1599: 4,2,1).
Cap, Merc.: 3,1. Read: 3,2 (as in Pedersen 2002, p. 1599).
Sco, Mars: 1,5,3. Read: 1,5,2 (as in Pedersen 2002, p. 1599).
Psc, Venus: 2,3. Read 2,3,4 (Pedersen 2002, p. 1599: 4,2).

dignities that uses the codes 1 to 5 in this way: one is a non-circular table (F. S. Pedersen 2002, p. 1599, Table RE21), and the other is a circular table found in Madrid, Biblioteca Nacional, MS 3349, f. 11v, which we reproduce in Table 18.3B (where the circular form has been replaced by a rectangle).

4. *Excess of Revolution*

In chapters 10 and 11 of his canons to the Parisian Alfonsine Tables, John of Saxony refers to one such table for finding the "revolution of the world years or of someone's nativity" and addresses its astrological applications. Thus, for example, he indicates that "to find the revolution of the ascendant is no more than finding the ascendant degree, that is, the degree which is on the eastern horizon when the revolution of the year has been completed" (Ratdolt 1483, f. a4r–v; cf. Poulle 1984, pp. 47–51, for a version in French; and Martínez Gázquez 1989, pp. 44–47, for a version in Spanish). This is a topic that has a long tradition in Arabic astronomy: see, e.g., Kennedy, Pingree, and Haddad 1981, p. 261, where we are told that "The precise amount of the excess [of revolution] was of interest to astrologers because [they were concerned with] the change in horoscopes cast at a fixed locality at the beginning of, and at the expiration of a single year, or an integer number of years; it is essentially a function of the excess of revolution." See also Kennedy and Pingree 1971, pp. 75, 77, and 121; and Goldstein 1967, pp. 143–144, 242.

The excess of revolution is the difference between a tropical or sidereal year and a year of 365 days (see Table 5.1D for some historical values of the length of the year). This difference and its multiples were frequently displayed in a table, as time, measured in time-degrees or in hours, where 1h = 15°. For a tropical year the excess of revolution in one year is always less than 6h or 90°, whereas for a sidereal year it is greater than 6h or 90°; that is, a tropical year is less than 365;15d, whereas a sidereal year is greater than 365;15d. Table 18.4A displays an excerpt of the two sub-tables for the excess in revolution in the zij *al-Muqtabis* by Ibn al-Kammād (Madrid, MS 10023, f. 54v), where the years are sidereal. For a list of parameters for the excess of revolution in Islamic sources, see Kennedy 1956a, p. 147.

An example of the two sub-tables for the excess in revolution for tropical years is shown in Table 18.4B. It is taken from Paris,

Table 18.4A: Excess of revolution in sidereal years (excerpt)

Years	Angle (°)
1	92;36*
2	187;12
3	280;48
. . .	
10	216; 0
20	72; 1
. . .	
100	0; 5

Years	Time (h)
1	6;14
2	12;28
3	18;43
. . .	
10	14;23
20	4;47
. . .	
100	23;58

* Read: 93;36.

Table 18.4B: Excess of revolution in tropical years (excerpt)

Years	Angle (°)
1	87;19, 6
2	174;38,12
3	261;57,18
. . .	
20	306;22, 0
. . .	
100	91;50,24
. . .	
1000	198;20, 0
. . .	
4000	73;10, 0

Years	Time (h)
1	5;49,16
2	11;38,32
3	17;27,48
. . .	
20	20;25,20
. . .	
100	6; 6,40
. . .	
1000	13; 6,40
. . .	
4000	5;26,40

Bibliothèque nationale de France, MS 7286C, f.55v, a 14th-century manuscript associated with the Parisian Alfonsine Tables.

The sub-tables displayed in Table 18.4B, but only up to 800 years, were printed in the *editio princeps* of the Alfonsine Tables (Ratdolt 1483, f. d3r) to sexagesimal thirds, and in the second edition (Santritter 1492, ff. k5r–v) to sexagesimal fourths. Note, however, that the basic parameters, that is, the entries for 1 year, differ from those in the printed editions (Santritter 1492: 87;18,55,55,30° and 5;49,15,43,52h), and thus all entries in the respective sub-tables differ from those in Table 18.4B.

Following the pattern of Table 18.4B, Bianchini computed other tables for the excess of revolution for the Sun and the correction to be applied to the revolution of the Sun when it returns to the same longitude after approximately one year, as well as tables for the excess of revolution for the Moon and the planets, unprecedented in the astronomical literature (Chabás and Goldstein 2009b, pp. 119–129). As an example, in Table 18.4C we reproduce an excerpt of the table for the excess of revolution for the Moon and the ascending node as presented by Bianchini (Nuremberg, Stadtbibliothek, Cent V 57, f. 97r).

The entry for 1y, 7d 0;48h, means that in a year of 365 days 5;49,16h there is an integer number of anomalistic months plus 7d 0;48h and, indeed, 7d 0;48h = 365d 5;49,16h–(13 · 27d 13;18,36h). The entries for the other four quantities (double elongation, mean lunar longitude, mean argument of lunar latitude, and longitude of the lunar node) have to be understood as the increments in these quantities between the beginning of the year and this precise moment (7d 0;48h before the completion of a year of 365d 5;49,16h).

Table 18.4C: Excess of revolution for the Moon and the ascending node in expanded years (excerpt)

Years	Time (d, h)		Double elongation (°)	Longitude (°)	Arg. latitude (°)	Lunar node (°)
1	7	0;48	93;41	39;54	58;52	5,40;41
2	14	1;36	187;21	1,19;49	1,57;45	5,21;19
. . .						
39	26	7;22	339;50	2,24; 0	2,56;55	5,25;42
40	5	18;51	25;20	3, 6;58	4, 0;19	5, 6;21

5. *Revolution of the Months*

One often finds a table for the revolution of the months (*Tabula revolutionis mensium*) associated with tables for the excess of revolution. Table 18.5A shows an excerpt of a representative of this kind of table, as found in Paris, Bibliothèque nationale de France, MS 7286C, f. 55v, in the framework of the Parisian Alfonsine Tables.

Note that the entry for 13 "months" is the length of the tropical year in the Alfonsine corpus (see § 4.3), whereas a "month" here is a little more than 28 days. The same table was printed in the first edition of the Alfonsine Tables (Ratdolt 1483, f. d3r) with entries that are also to sexagesimal fifths, as well as in the second edition (Santritter 1492, f. k5r), to sexagesimal fourths. Surprisingly, the heading in Santritter 1492 refers to radices valid for year 1372 complete (i.e., 1373 in modern usage) thus indicating that the printer used a manuscript reporting information about 120 years old.

In contrast to tables for the excess of revolution which make perfect sense astronomically, this table, which is based on dividing the year into 13 "months," has no astronomical meaning. The idea may go back to a passage in Ptolemy's *Tetrabiblos* IV.10 (Robbins 1940, p. 453) and to a passage in Pseudo-Ptolemy's *Centiloquium* (a text that circulated in the Middle Ages in Greek, Arabic, Latin, and Hebrew: see, e.g., Sela 2003, pp. 321–323), aphorism 87: "Monthly revolutions are made in twenty-eight days, two hours and about eighteen minutes" (*Mensium conuersiones fiunt ex diebus uiginti octo, horis duabus, ac minutis circiter decem & octo*); see Ashmand 1822, p. 234; Gemusaeus 1541, p. 504. The revolution of the months is discussed in several medieval astrological texts, and it is related to motion in 13 zodiacal

Table 18.5A: Revolution of the months (excerpt)

Months	(d)	(h)
1	28	2;17,38, 3,11,51 *
2	56	4;35,16, 9,23,42 **
. . .		. . .
12	337	3;31,37,50,22,12
13	365	5;49,15,59,34, 3

* Instead of 2;17,38, 9,11,51.
** Instead of 4;35,16,18,23,42.

signs, or 390° (e.g., Ibn Ezra, in Sela 2009, p. 252; and al-Bīrūnī, ed. Wright 1934, p. 325). Ibn Ezra has the parameter 28d and 2h and close to a third of an hour; al-Bīrūnī has 28d 1;51h, a parameter based on a different length of the year, for 365d/13 = 28d 1;50,46h, or about 28d 1;51h. In his *Book of Reasons* (Sela 2007, pp. 249–250; see also p. 328) Ibn Ezra offers the following "explanation" of the revolution of the months (rounding its value to the nearest hour): "When you divide 365 and a quarter days, which is approximately the length of the year, by the signs, with the first sign serving at [both] the beginning and the end [for a total of 390°], each month turns out to have 28 days and 2 hours. The explanation for what I have just mentioned is that the Sun moves each month the [same] number of the degrees in each sign.…" It may be relevant to note that in a synodic month the Moon advances by about 390°.

Both Ibn Ezra and al-Bīrūnī define the revolution of the months with respect to a motion in 13 zodiacal signs, and this has led us to a new interpretation of a table that has hitherto been unexplained. This table (see Table 18.5B) was first published in Goldstein, Chabás, and Mancha 1994, p. 93, based on a single 13th-century Latin manuscript (Madrid, Biblioteca Nacional, MS 10053, f. 10r), and later republished in Chabás and Goldstein 2003, p. 182, based on several manuscripts. This table is usually inserted as two columns in another table, used for the unequal motion of the planets (see § 8.2, Table 8.2A), where it is headed "Daily progess of the Moon in the zodiacal circle" (*Diete lune in circulo signorum*); its underlying parameter is between 13;52°/d and 13;53°/d, where the entry for a given day is a simple multiple of the underlying parameter, rounded to minutes of arc. The problem was that this parameter for lunar motion has no astronomical meaning. The solution, however, is related to the table for the revolution of the months (Table 18.5A), where the basic parameter in Table 18.5B is defined by:

390° / 28d 2;17,38,3…h = 13;52,52,16,…°/d.

This is very close to the value underlying the entry for 27d, for 374;48°/27d = 13;52,53°/d. For a complete transcription of this table, see § 8.2.

Table 18.5B: Daily progress of the Moon (excerpt)

Days of the Moon	Lunar progress (°)
1	13;53
2	27;46
3	41;39
4	55;32
. . .	
25	347; 3
26	0;56
27	14;48
28	28;41
29	30;32
30	–

6. *Duration of Pregnancy*

Aphorism 51 of Pseudo-Ptolemy's *Centiloquium* (Ashmand 1822, pp. 229–230) deals with the position of the Moon at the time of child-birth and the ascendant at the time of conception, and in the *Tetrabiblos*, Book III, Ptolemy discusses the "science of nativities," but does not provide any information in tabular form. Much later, astrologers compiled tables where the duration of pregnancy was related to the position of the Moon. These were known in Latin astronomy as "tables for the *animodar*." The term, *animodar*, is a Latin calque on the Arabic word, *namūdār* (of Persian origin), for "indicator" (see, e.g., al-Bīrūnī, ed. Wright 1934, p. 329). In the medieval astrological literature there are several systems of *namūdār* to determine the time of nativity (see Kennedy 1995/96, pp. 139–144; Samsó and Berrani 1999, pp. 307–308). This tradition passed to the Latin West where there were at least three kinds of tables concerning the gestation period.

One is found in a zij preserved in Latin, *al-Muqtabis* by Ibn al-Kammād, extant in Madrid, MS 10023, ff. 62v–64r (see Chabás and Goldstein 1994). This table was known to Jacopo de Dondi (1298–1359) of Padua (the author of a set of tables of which no copy has yet been properly identified), for it is attributed to him in the 1526 edition of the Tables of Giovanni Bianchini, first printed in Venice in 1495 (see Chabás and Goldstein 2009b).The title reads *Tabula more infantis*

Table 18.6A: Animodar in *al-Muqtabis* (excerpt)

Distantia (°)	*Mora occidentalis* (d, h)		*Mora orientalis* (d, h)	
1	259	7;50	273	0; 0
2	259	9;40	273	0; 0
3	259	11;30	273	0; 0
. . .				
178	272	20;20	287	19;40
179	272	22;10	287	21;50
180	273	0; 0	288	0; 0

in utero. Per D. Jacobuum(!) *de Dondis patavinum* (ff. 394v–395v). An excerpt of Madrid, MS 10023, is displayed in Table 18.6A.

In this table the argument, called *distantia*, is the distance from the Moon to the ascendant, and it is given at intervals of 1° from 1° to 180°. The entries are distributed in two columns headed *mora occidentalis* and *mora orientalis*, both given in days, hours, and minutes. They represent the duration of pregnancy with a minimum of 259d 7;50h (in excess of 37 weeks) and a maximum of 288d 0;0h (in excess of 41 weeks). One sees immediately that the entries in the first column have constant differences of 1;50h, whereas in the second column the constant difference is 2;10h (except for the first 12 entries, which are all 273d 0;0h). We also note that for a given *distantia* greater than 12° the sum of the entries in these two columns is an integer number of days. The same table as that in Madrid, MS 10023, is found in several other MSS: Munich, MS Heb. 343, ff. 48a–49a (the related canon is on ff. 47a–47b); New York, Jewish Theological Seminary of America, MS Heb. 2597, ff. 67a–67b; and Cracow, Jagiellonian Library, MS 609, f. 82r–v.

Yet another closely related table is found in a manuscript in Arabic (of Maghribī origin): Escorial, MS Ar. 939, ff. 5v–7r. This table was identified by Vernet in a text of six chapters on astrological obstetrics, and he attributed it to Ibn al-Kammād. Vernet (1949) published the table with a Catalan translation of that text. In fact, two different tables are given there: one is for a 9-month gestation period, the other for a 7-month gestation period. Table 18.6B reproduces an excerpt of them.

The third type of table is found in Abraham Zacut's *Ḥibbur* and was reproduced in the *Almanach Perpetuum* (see Chabás and Goldstein

Table 18.6B: Duration of pregnancy (excerpt)

Moon above the horizon (9-month period)

Argum. (°)	0 signs (d, h)			5 signs (d, h)	
	259	1;20	. . .	270	10;33
1	259	3;10	. . .	270	12;23
2	259	4;59	. . .	270	14;12
. . .					
28	261	4;20	. . .	272	13;43
29	261	6; 9	. . .	272	15;23
30	261	7;59	. . .	272	17;52

Moon below the horizon (9-month period)

Argum. (°)	0 signs (d, h)			5 signs (d, h)	
	272	17;30	. . .	284	2;25
1	272	19; 1	. . .	284	4;14
2	272	20;51	. . .	284	6; 4
. . .					
28	274	20;12	. . .	286	5;29
29	274	22; 1	. . .	286	7;15
30	274	23;51	. . .	286	9; 0

Moon above the horizon (7-month period)

Argum. (°)	0 signs (d, h)			5 signs (d, h)	
	204	11;15	. . .	215	10;33
1	204	13; 4	. . .	215	12;23
2	204	15;53	. . .	215	14;12
. . .					
28	206	14; 5	. . .	218	22;22
29	206	15;54	. . .	218	0;11
30	206	17;42	. . .	218	2; 0

Moon below the horizon (7-month period)

(°)	0 signs (d, h)			5 signs (d, h)	
	218	2; 0	. . .	229	10;17
1	218	3;49	. . .	229	12; 6
2	218	5;38	. . .	229	13;58
. . .					
28	220	4;50	. . .	231	13; 7
29	220	6;39	. . .	231	14;56
30	220	8;27	. . .	231	16;45

2000, pp. 86–87 and 150–153). As in the previous case, the argument, also called *distantia*, is the distance from the Moon to the ascendant. The entries, here given in weeks, days, hours, and minutes, differ from those in Table 18.6A. In this case, the entries in the two columns for any given argument differ by a constant, 13d 16;0h (see Table 18.6C), whereas in Table 18.6A they added up to an integer number of days.

We have located some other copies of this table, and they may help in placing the table as printed in the *Almanach Perpetuum* in a specific astrological tradition. In particular, it is found in the unique copy of the zij of Juan Gil of Burgos, with radix 1310, in Madrid, Biblioteca Nacional, MS 23078, ff. 166b–167b (despite some differences in presentation and a few textual variants). Zacut's table also appeared (with many variants) in a publication of 1528 in a short set of tables computed for Leuven, Belgium, by Henri Baers (also known as

Table 18.6C: Animodar in the *Almanach Perpetuum* (excerpt)

Distantia (s, °)		*Mora occidentalis* (w)	(d)	(h)	*Mora orientalis* (w)	(d)	(h)
0s	1	37	0	15; 4	39	0	7; 4
0s	2	37	0	16;53	39	0	8;53
0s	3	37	0	18;43	39	0	10;43
. . .							
5s	28	39	0	1;36	40	6	17;36
5s	29	39	0	3;25	40	6	19;25
6s	0	39	0	5;15	40	6	21;15

Vekenstyl): for a facsimile reproduction, see Poulle and De Smet 1976, ff. Bi v–Bii v.

A much simplified table, including the motions of the Sun and the Moon, is sometimes found in manuscripts containing the Parisian Alfonsine Tables, as is the case with Bonn, Universitätsbibliothek, MS 498, f. 39v, which is reproduced in Table 18.6D.

Table 18.6D: *Mora nati in utero matris*

Days	solar mean motion (°)	lunar mean motion (°)	arg. of lunar anomaly (°)	*centrum simplex* (°)	arg. of lunar latitude (°)
258	4,14;17,48	2,39;30,36	2,10;46, 1	4,25;12,48	2,53;10,19
273	4,29; 4,54	5,57; 9,22	5,26;44,31	1,28; 4,28	0,11;48,44
288	4,43;15,58	3,14;48, 6	2,38;43, 0	4,30;58, 8	3,30; 3, 8

CHAPTER NINETEEN

MISCELLANEOUS TABLES

In addition to tables falling in the preceding categories there are many others that deal with festivities, calendaric matters, etc., which we have excluded from our survey. Normally they are specific to a given set of tables and even differ from one manuscript to another of the same set. By contrast, most sets include tables for conversion of units, multiplication, etc., that were provided by the author or the copyist to facilitate computation. In this section we present a few examples.

An example of a sexagesimal multiplication table, presented as a square matrix of 60 × 60, is found in Vienna, Nationalbibliothek, MS 2440, ff. 83v–85r, under the title *Tabula tabularum seu tabula proporcionum ad 60 minuta* (see Table 19A).

Tables 19B and 19C reproduce excerpts of tables in Madrid, Biblioteca Nacional, MS 7856, f. 3r–v, for converting sixtieths of a day (mn) into hours and vice versa. In Table 19B the coefficient to find the entries in the second column from the first is 24/60 = 0;24, whereas in Table 9C it is 60/24 = 2;30.

As we have indicated previously, several tables contain specific columns for interpolation purposes: see, e.g., Tables 6.2A, 6.3B, 9.2B, 13.2C. These columns are normally integrated into the table they are used for, but in some cases the coefficients of interpolation, usually called "minutes of proportion," are presented in a separate table. This is the case for the coefficients to be used in computing solar and

Table 19A: Sexagesimal multiplication table (excerpt)

	1	2	3		58	59	60
1	0; 1	0; 2	0; 3	. . .	0;58	0;59	1; 0
2	0; 2	0; 4	0; 6	. . .	1;56	1;58	2; 0
3	0; 3	0; 6	0;12	. . .	2;54	2;57	3; 0
. . .							
58	0;58	1;56	2;54	. . .	56; 4	57; 2	58; 0
59	0;59	1;58	2;57	. . .	57; 2	58; 1	59; 0
60	1; 0	2; 0	3; 0	. . .	58; 0	59; 0	60; 0

Table 19B: Conversion of sixtieths of a day into hours (excerpt)

(mn)	(h)
1	0;24
2	0;48
. . .	
59	23;12
60	24; 0

Table 19C: Conversion of hours into sixtieths of a day (excerpt)

(h)	(mn)
1	0; 2,30
2	0; 5, 0
. . .	
23	0;57,30
24	1; 0, 0

Table 19D: Proportion at intervals of 2° (excerpt)

Arg. (°)	Min. prop.	Arg. (°)	Min. prop.	Arg. (°)	Min. prop.
0, 2	0; 2	1, 2	14;52	2, 2	45; 0
0, 4	0; 6	1, 4	15;45	2, 4	46; 0
0, 6	0;12	1, 6	16;41	2, 6	47; 7
. . .	. . .	. . .	. . .	. . .	. . .
0,56	12;21	1,56	42; 0	2,56	59;56
0,58	13;10	1,58	43; 0	2,58	58;58*
1, 0	14; 0	2, 0	44; 0	3, 0	60; 0

* Read: 59;58.

lunar eclipses for interpolation between perigee and apogee; the coefficients are most frequently presented in a table where the argument increases at intervals of 2°. Table 19D reproduces an excerpt of it, as presented in the *editio princeps* of the Alfonsine Tables (Ratdolt 1483, f. m1r).

The function embedded in this table is not linear; rather, it is sinusoidal (see a graph of it in Neugebauer 1962, p. 117). It is an expansion of column 3 in Table 12.1E for correcting parallax, where the entries are given at intervals of 6° (see also Table 15.3). Ultimately, this table derives from the table of corrections in *Almagest* VI.8 (Toomer 1984, p. 308), where the entries are also displayed at intervals of 6°. It is found in many medieval sets of tables, e.g., as a column integrated in the table for lunar eclipses in the zij of al-Khwārizmī (Suter 1914, pp. 187–189) or as a separate table in the Toledan Tables (Toomer 1968, p. 117; F. S. Pedersen 2002, pp. 1444–1446).

In the computation of syzygies one often finds a table that displays the 12th part of the distance between the Sun and the Moon; ultimately it is a multiplication table that allows for division by 12 (see *Almagest*, VI.4; Toomer 1984, p. 281). Table 19E is an excerpt of such a table, found in the second edition of the Parisian Alfonsine Tables (Santritter 1492, f. h4r).

Table 19E: Division by 12

(′)	(′)
1	0; 5
2	0;10
. . .	
30	2;30
. . .	
59	4;55
60	5; 0

LIST OF MANUSCRIPTS

The references are to chapters or section numbers, not to page numbers.

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