United Kingdom Hydrographic Office

HM NAUTICAL ALMANAC OFFICE

NAO TECHNICAL NOTE

No. 64

Algorithms for calculating the dates of Easter

by

B.D. Yallop

HM Nautical Almanac Office
UK Hydrographic Office
Admiralty Road, Taunton
TA1 2DN England

Telephone: +44 (0) 1823 484444

e-mail: hmnao@ukho.gov.uk
WWW http://astro.ukho.gov.uk/

ALGORITHMS FOR CALCULATING THE DATES OF EASTER

by

B. D. Yallop

1. Introduction

The purpose of this note is to provide a brief introduction and survey of the subject for those who are studying this fascinating subject in depth.

2. The definition of Easter Day

Easter day is the first Sunday after the first full moon that occurs on or after the vernal equinox. In the Christian calendar the date adopted for the full moon is the fourteenth day from the date of each Paschal new moon, and the date adopted for the vernal equinox in the Gregorian calendar is March 21. The possible dates of Easter range from March 22 to April 25. Tables for calculating the dates of the lunation of the Paschal moon are given in the *Encyclopaedia Britannica* (1910) and are well worth studying in order to find out how the Paschal moon (which is not the real Moon) is calculated. The reader should be warned, however, that there are occasional printing errors in this article. There is a significant one for example on page 996, Table III in the column under golden number 11, for the values 17 and 19, which are the second and third entries from the top of the column, read 19 and 18, respectively. The fundamental arguments of the tables are the year, the golden number, the epact and the dominical letter. The golden number, which depends upon the year, ranges between 1 and 19 and represents the position of the Paschal moon in the 19 year Metonic cycle. The epact, which depends upon the year and the golden number, ranges between 0 and 29 and indicates on which days of the year the new moons occur. Finally the dominical letter indicates which day of the week falls on Jan. 1 in a particular year. Other useful information will be found in the *Explanatory Supplement*, Chapter 14, The Calendar.

3. Algorithms for calculating the dates of Easter

There are two basic types of algorithm, one may be traced back to Gauss (1800), the other to Delambre (1814). It can usually be shown, after some difficult algebra, that the two algorithms or their derivatives are equivalent. The algorithm of Gauss is perhaps the most direct interpretation of the rules for calculating Easter, whilst modern versions of Delambre's algorithm are simpler to use but the purpose of each part of the calculation is often more obscure.

According to Hall (1970), Gauss claimed that he was motivated to produce a formula for the date of Easter because his mother did not know the exact date on which he was born, only that it was a Wednesday, eight days before Ascension Day. Gauss made a slight error by omitting a term of period 2500 years so that for example his algorithm fails in the year 4200, and yet he gives an example for the year 4763 in his paper, when the formula works. Several authors have reproduced Gauss's algorithm and even introduced their own errors, see for example the letters by Ross and Leigh in *Nature* (1906), **74**, 175. This is probably another reason why the preferred methods are usually based on Delambre's algorithm. Although Delambre produced a correct algorithm the original version contained printing errors, and he finally published the correct algorithm together with some criticisms of Gauss's algorithm in the *Connaissance des Temps* 1817. These problems have been discussed in more detail by Oudin (1946).

Modified versions of Delambre's algorithm will be found for example in Butcher (1877), Spencer Jones (1922), Meeus (1982), Willey (1985). Gauss's algorithm has not fared as well, a workable version will be found in the *Encyclopaedia Britannica* (1910). A version which is ideal for programming is given in Balfour and Marwick (1979) which omits the 2500 year term. Another version which describes the meaning of the various terms is given by Schocken (1976), but unfortunately two of Gauss's rules which he mentions at the end of his paper are omitted. For the sake of completeness a correct form of the Gauss algorithm is given in the next section together with some brief notes.

The simplest algorithm from the point of view of ease of programming is undoubtedly that by Willey (1985). Mr A.E. Carter (private communication) produced an empirical formula which is valid for just the 20^{th} and 21^{st} centuries and is interesting because it is short.

4. The algorithm of Gauss

In this section the following mathematical notation is used:

$[a/b]$ means take the integer part of a divided by b
a MOD b means the remainder when a is divided by b

For calculators that do not have a function key for MOD, note that

$$a \text{ MOD } b = a - [a/b]\, b$$

Step 1. Calculate M and N from the year number Y

$$\begin{aligned} S &= [Y/100] \\ k &= [(3S-5)/4] \\ N &= (k-1) \text{ MOD } 7 \\ z &= [(S-17)/25] \\ M &= S - [S/4] + 15 - [(S-z)/3] \end{aligned}$$

Step 2. Calculate A and D from

$$\begin{aligned} A &= Y \text{ MOD } 19 \\ B &= Y \text{ MOD } 4 \\ C &= Y \text{ MOD } 7 \\ D &= (19A + M) \text{ MOD } 30 \end{aligned}$$

Step 3.

If $D = 28$ and $A > 10$ then set $D = D - 1$
If $D = 29$ then set $D = D - 1$

Step 4. Calculate E and the date of Easter in March Q from

$$\begin{aligned} E &= (2B + 4C + 6D + N) \text{ MOD } 7 \\ Q &= 22 + D + E \end{aligned}$$

If $Q > 31$ then date of Easter falls in April on day $Q - 31$.

Notes on the algorithm :

1. Steps 1,2, and 4 are essentially these given by Schocken (1976) who explains the meaning of the calculation.
2. In step 1, Y is the number of the year, M with multiples of 30 removed so that it is in the range 1 to 30 is called the equation of the Moon, and N is called the equation of the Sun.
3. In step 1 the term z in the expression for M was omitted by Gauss.
4. In step 2, $A + 1$ is the golden number and D is the number of days to the next full moon after March 21.
5. The epact may be calculated at the end of step 2 from

$$\text{epact} = (53 - D) \text{ MOD } 30$$

If epact $= 25$ and $A > 10$ then set epact $= 25'$

6. Step 3 contains the two rules by Gauss which Schocken overlooked. These rules have been modified slightly to make them easier to use.
7. In step 4, $E + 1$ is the number of days after the full moon to the next Sunday and Q is the date of Easter referred to the month of March.

5. A short algorithm by Carter

This algorithm is only valid from 1900 to 2099.

Step 1. Calculate D from

$$D = 225 - 11(Y \text{ MOD } 19)$$
$$\text{If } D > 50 \text{ subtract multiples of 30 from } D \text{ until } D < 51$$
$$\text{If } D > 48 \text{ set } D = D - 1$$

Step 2. Calculate E and the date of Easter in March Q from

$$E = (Y + [Y/4] + D + 1) \text{ MOD } 7$$
$$Q = D + 7 - E$$
$$\text{If } Q > 31 \text{ then date of Easter falls in April on day } Q - 31\cdot$$

6. Mean interval between the same dates of Easter

In the Gregorian calendar the dates of Easter repeat after a period of 5 700 000 years. Using the algorithm in Section 4 it is interesting to calculate the number of times Easter falls on a particular date during one cycle and the mean interval between occurrences. The results are given in Table 1. It will be noticed that the number of occurrences falls off at the ends of the range of the dates of Easter. The mean interval for March 22 is 206·9 years and for April 25 it is 135·7 years. The date that occurs most often is April 19. The distribution is fairly flat between March 27 and April 21 and the mean interval between dates is mostly around 29 or 30 years.

An extraordinary table which lists the frequencies of all 35 possible dates of Easter and the intervals at which they reoccur appears in the exhaustive article by G. W. Walker (1945).

7. Acknowledgements

I am grateful to Dr. D. H. P. Jones who lent me his set of calendar notes which included a photocopy of the original paper by Gauss. I would like to thank Dr. B. E. J. Pagel for translating the relevant parts of Gauss's paper for me. I would also like to thank Mr. N. O'Hora for translating the paper by Oudin for me. I am also grateful to Mr. A. E. Carter who gave me his empirical algorithm for calculating the dates of Easter. This algorithm spurred me on to find out exactly how the ecclesiastical moon is calculated. Finally I would like to thank Miss. C. Y. Hohenkerk for helping me to prove that the two basic algorithms by Gauss and Delambre are equivalent, for programming these and other algorithms in FORTRAN, for producing this technical note in TEX and for carefully checking this note.

Table 1

Analysis of the number of times Easter occurs on a given day in the cycle

Date		Occurrences	Mean Interval years	Date		Occurrences	Mean Interval years
March	22	27550	206·9	April	8	192850	29·6
	23	54150	105·3		9	186200	30·6
	24	81225	70·2		10	192850	29·6
	25	110200	51·7		11	186200	30·6
	26	133000	42·9		12	192850	29·6
	27	165300	34·5		13	189525	30·1
	28	186200	30·6		14	189525	30·1
	29	192850	29·6		15	192850	29·6
	30	189525	30·1		16	186200	30·6
March	31	189525	30·1	April	17	192850	29·6
April	1	192850	29·6	April	18	197400	28·9
	2	186200	30·6		19	220400	25·9
	3	192850	29·6		20	189525	30·1
	4	186200	30·6		21	162450	35·1
	5	192850	29·6		22	137750	41·4
	6	189525	30·1		23	106400	53·6
	7	189525	30·1		24	82650	69·0
April	8	192850	29·6	April	25	42000	135·7

Total number of occurrences = 5 700 000

8. References

Balfour, A. and *Marwick, D.H.* (1979). Programming in Standard FORTRAN 77, pp 256–263. Heinemann Educational Books, London.

Butcher, S. (1877). The Ecclesiastical Calendar, p 226. McMillan and Co., London.

Delambre, M. (1814). Memoires Vol III, Astronomie théorique et pratique.

Delambre, M. (1817). Connaissance des Temps, pp 307–317.

Encyclopaedia Britannica (1910), 11th Edition. **4**, pp 987–1004. Cambridge University Press, Cambridge.

Explanatory Supplement to the Astronomical Ephemeris (1974), Chapter 14, Section E, Ecclesiastical Calendars. HMSO, London.

Gauss, C.F. (1800). Werke Vol VI, Berechnung des Osterfertes.

Hall, T. (1970). Carl Friedrich Gauss. A Biography by Tord Hall, p 60. MIT Press.

Jones, H.S. (1922). General Astronomy, 73–74. Edward Arnold and Co., London.

Meeus, J. (1982). Astronomical formulae for calculators, 2nd Edition, pp 31-33. Willmann-Bell, USA.

Oudin, M.J.-M. (1946). Bulletin Astronomique XII, 391–410.

Schocken, W.A. (1976). The Calculated Confusion of Calendars. Vantage Press, New York.

Walker, G.W. (1945). Popular Astronomy **53**, pp 162–179 and pp 218–232

Willey, D. (1985). Mathematical Gazette, **69**, No 45, 291–2.