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ASTRONOMY

PERIODICITIES IN LUNAR ECLIPSES

BY

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I

Astronomical literature from olden times mentions an 18 years period called the saros, which was first used by the Babylonian astronomers to predict lunar eclipses. Usually it was assumed that this is the only period that can be used for the purpose of prediction, so that the earliest cases of prediction which we meet with in the past must have been based upon knowledge of the saros. In later times, when the origin of the eclipses was exactly known, they were computed for the past and the future by means of the elements of the moon's and the sun's (apparent) orbit without making use of any period.

SCHIAPARELLI has pointed out [1]) that there are other and more simple means to forecast eclipses, regularities that must have been noticed and used at a time of more primitive knowledge. When a lunar calendar is used, with months equal to the moon's synodic period of 29.5 days, it is easy to perceive that the lunar eclipses — if, of course, the eclipses visible at night are completed by the eclipses occurring at day time, hence invisible — always follow one another with an interval of six months. In this way they form a group or series of 5 or 6 consecutive eclipses; then the series ceases. But after an interval of 17 or 23 months a new series starts, of which the eclipses occur 41 or 47 months after the corresponding ones of the former series. Each series begins with one or two partial eclipses, then in the midst come 2, 3, or sometimes 4 total eclipses, and the series ends with again one or two partial eclipses.

In a paper "The Origin of the Saros" [2]) the author has shown how the knowledge of the saros must have developed out of the knowledge of the small series. First the appearance of these series was explained by the fact that after six synodic periods the longitude of the sun and, hence, of the full moon has advanced 4°.023 relative to the moon's node. Eclipses are only possible when the distance P between full moon and node is not greater than 10°—12°, total eclipses are only possible when this distance is not larger than 5°—6°. So a series of consecutive eclipses takes place when the difference of longitude P from say — 10° every

[1]) SCHIAPARELLI G. V., I primordi dell' astronomia presso i babilonesi. (Scientia IV. p. 36).

[2]) These Proceedings 20, 943 (1917).

six months increases to nearly — 6°, — 2°, + 2°, + 6°, + 10°; then the series ceases because the next differences are + 14°, + 18° etc. But then soon the preceding full moon, with a longitude relative to the node 30°.67 smaller, comes into action and shows successive differences — 12.67, — 8.65, . . . etc.

Thus the lunar eclipses occur in an array of consecutive series, in such a way that within each series the intervals are six months, and between the series the interval is one less than a multiple of six months. Each series shows a different character of the aspect of the partial and the duration of the total eclipses, due to the different values of P fluctuating over 4°. But after five series they return to nearly the same value, being in the sixth series only 0°.48 different from the first series. Thus a larger period embracing five series appears, containing 223 lunar months. This period is the saros. Mathematically the saros is the nearest common multiple of the synodic and the draconitic revolutions of the moon. In the paper mentioned it was shown how by continued observation a knowledge of this saros must come forth out of the knowledge of the series, and that probably in Babylon it originated in this way.

The discovery of the saros-periodicity was facilitated by two circumstances. Firstly the period of 223 lunar months, i.e. 6585$\frac{1}{3}$ days, is only 11 days more than 18 years, so that the sun (and also the node that then has retrograded over nearly one circumference) returns to nearly the same longitude; this brings about that the effects caused by the excentricity of the earth's orbit return to nearly the same values after one saros. Secondly the saros is only $\frac{1}{3}$ year longer than twice the period of revolution of the moon's perigee, so that the inequalities due to the excentricity of the lunar orbit (e.g. in the lunar parallax) also return nearly to the same values. Hence the irregularities occurring in the aspects of the consecutive series return in nearly the same way after every five series.

Since the purpose of the former paper was only to show the origin of the saros out of the more simple periodicity of the series, the irregularities due to the excentricities of the orbits have not been treated there. The most important is the influence of the apparent motion of the sun. Because in the syzygies the rapidly moving moon overtakes the slowly moving sun, the place of the meeting, the longitude of the full moon, chiefly depends on the sun, whereas the time of the meeting chiefly depends on the moon. The sun at an anomaly of 90° (in spring) is 2° advanced, at an anomaly of 270° (in autumn) is 2° back relative to the mean longitude. This brings about that the distance between full moon and node then is increased or diminished by an amount of 2°.28; and thereby the character of the eclipse may be considerably changed. The effect of the excentricity of the moon's orbit, though it may change the moon's longitude by more than 5°, is diminished to circa 1/12 of this amount, 0°.43 in the longitude of the full moon.

II

Schiaparelli remarked that it happens in certain centuries that a tetrad of four total eclipses follow one another in the midst of a series, whereas in other centuries such sequences are absent. This can be easily confirmed by looking through Th. von Oppolzer's *Canon der Finsternisse*[3]). Then more irregularities present themselves. When only two eclipses in the midst of a series are total, their magnitude, expressed by the number of "digits" (Zoll, 1/12 of the moon's diameter), is high, up to more than 20, indicating a long duration of the totality. Where, however, four eclipses follow one another in the midst of a series, the first and the fourth, though remaining below 17, have a larger magnitude usually than the second and the third, just the reverse of what normally might be expected. It even happens that for the second or the third of this middle group the magnitude falls below 12 digits, hence the eclipse is

TABLE I

Number of total eclipses

Mean year	Nr. of series	Number with 4, 4 imp, 3, 2 total eclipses	Average number per series	Mean year	Nr. of series	Number with 4, 4 imp, 3, 2 total eclipses	Average number per series
– 1174	18	0 0 4 14	2.22	512	18	0 0 3 15	2.17
1111	17	0 1 4 12	2.32	575	17	0 0 4 13	2.24
1047	18	3 2 2 11	2.61	636	17	0 0 3 14	2.18
980	19	4 3 7 5	3.03	700	18	0 1 4 13	2.31
912	19	5 3 10 1	3.29	763	17	2 2 1 12	2.47
844	18	5 2 7 4	3.11	828	19	6 2 9 2	3.26
780	18	2 3 1 12	2.53	895	17	4 3 10 0	3.32
715	18	0 0 6 12	2.33	962	19	5 1 5 8	2.87
652	17	0 0 3 14	2.18	1027	17	1 3 1 12	2.44
588	18	0 0 3 15	2.17	1090	18	0 0 5 13	2.28
525	17	0 0 3 14	2.18	1153	17	0 0 3 14	2.18
461	18	1 3 3 11	2.53	1217	18	0 0 3 15	2.17
396	18	2 3 9 4	2.97	1280	17	1 1 2 13	2.32
330	19	7 1 11 0	3.39	1345	19	4 2 3 10	2.74
263	18	5 2 11 0	3.33	1412	18	4 2 9 3	3.11
197	18	4 2 5 7	2.89	1479	19	5 2 11 1	3.26
134	17	1 2 2 12	2.41	1546	18	5 1 5 7	2.86
71	18	0 0 3 15	2.17	1611	18	1 2 3 12	2.44
– 8	17	0 0 4 13	2.24	1674	17	0 0 4 13	2.24
+ 56	18	0 0 3 15	2.17	1736	17	0 0 2 15	2.12
119	17	0 1 3 13	2.26	1799	18	0 0 5 13	2.28
182	18	3 3 0 12	2.58	1863	17	1 3 2 11	2.50
249	19	7 0 8 4	3.16	1926	18	4 0 7 7	2.83
318	19	4 4 11 0	3.32	1993	19	4 4 11 0	3.32
385	18	4 2 6 6	2.94	2063	20	6 0 9 5	3.05
448	17	3 3 1 10	2.68	2132	18	3 4 0 11	2.67

[3]) Denkschriften der Kais. Akad. der Wissensch. 52 (Wien, 1887).

only partial, though it is near the centre of the series. We could call such cases mutilated or imperfect tetrads.

A closer examination of OPPOLZER's Canon reveals a remarkable periodicity in the occurrence of these tetrads. In Table I for consecutive numbers of circa 100 eclipses is indicated how many series they embrace and how many among these show in the middle part two, three or four consecutive total eclipses or imperfect tetrads. We see a regular alternating: between the years — 773 and — 472, — 111 and + 143, 475 and 729, 1050 and 1286, 1608 and 1854 the tetrads are lacking and most of the series have only two total eclipses. On the contrary about — 900, — 300, + 300, 900, 1500, 2000 the series with only two total eclipses are almost lacking and tetrads of total eclipses show a maximum frequency. To express this periodicity numerically we may for each group of years compute an average number of total eclipses per series (counting the imperfect tetrads for $3\frac{1}{2}$). These numbers are represented in Fig. 1. Though they do not run entirely regularly, we may deduce epochs of maximum and minimum:

Maximum	— 900,	— 310,	+ 300,	860,	1460,	2010
Minimum	— 600,	+ 20,	600,	1180,	1730	

from which follows a mean period of circa 580 years. At present we are in an epoch of increasing tetrads; we had one embracing four total eclipses from 1949 April 13 to 1950 Sept. 26, and the next one will consist of the four eclipses 1960 March 13 to 1961 Aug. 26.

III

In order to elucidate the origin of this periodicity a number of lunar eclipses had to be approximately computed by means of OPPOLZER's "*Syzygientafeln für den Mond*" [4]). A total eclipse takes place when the latitude of the moon's centre is smaller than the semidiameter of the shadow diminished by that of the moon itself. If this quantity is expressed by $i \sin l_0$ (l_0 indicating some distance to the node and i the inclination) then the condition of totality can be thus expressed that P, the full moon's distance to the node, must be below l_0. This limiting longitude difference l_0 varies between 5°.83 (for the perigee) and 4°.75 (for the apogee).

When (Case I) the regular part of P, which we will denote by P_0, for one eclipse is 0°, then for the next preceding and following ones it is — 4° and + 4°, both falling within the limits $\pm\, l_0$; so there are 3 total eclipses, provided that the solar correction is small (which is the case in summer and winter). When the solar correction is large, when e.g. the middle eclipse falls in autumn and the two others fall in spring, the latter are displaced + 2°.28 (at most) due to the sun's anomaly; so their relative longitudes P change into — 2° and + 6°, so that only 2 of the 3 total

[4]) Publication XVI der Astronomischen Gesellschaft (1881).

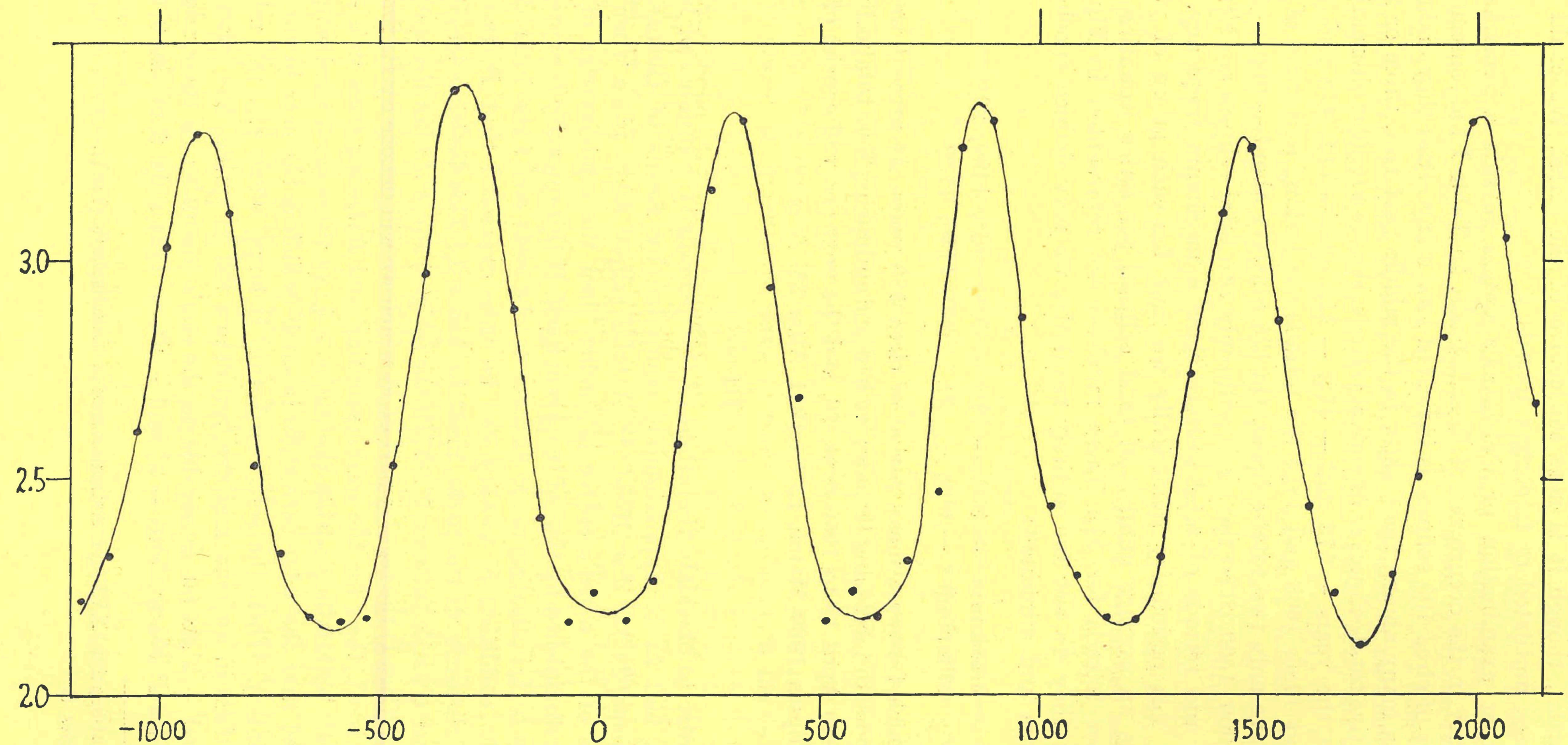

Fig. 1. Average number of total eclipses per series.

eclipses remain. The same holds for the opposite seasons with correction — 2°.28.

When (Case II) we take $P_0 = -\ 2°.01$ and $+\ 2°.01$ for the eclipses nearest to the node, and — 6°.03 and + 6°.03 for the adjacent ones, we have again two total eclipses only, if the sun's anomaly is near 0° or 180° and the solar correction is nearly zero. But the matter is different if the sun's anomaly is near 90° (spring) or 270° (autumn). In the case II*a* with

	(autumn)	(spring)	(autumn)	(spring)
$P_0 =$	— 6°.03	— 2°.01	+ 2°.01	+ 6°.03
the corrections	— 2.28	+ 2.28°	— 2.28	+ 2.28,
producing $P =$	— 8°.31	+ 0°.27	— 0°.27	+ 8°.31,

throw the first and last value farther from the node and the two middle values nearer to it, so that only two total eclipses of long duration occur. In the opposite case II*b* with

	(spring)	(autumn)	(spring)	(autumn)
$P_0 =$	— 6°.03	— 2°.01	+ 2°.01	+ 6°.03
the corrections	+ 2°.28	— 2.28	+ 2.28	— 2.28,
producing $P =$	— 3°.75	— 4°.29	+ 4°.29	+ 3°.75,

bring the first and last values within the boundary values for totality, whereas the two middle ones remain within them at a larger distance. So here we have four consecutive total eclipses, all of moderate magnitude. With somewhat different values of P_0 and the solar correction it may happen that the second or third P is thrown outside the boundary value l_0; then an imperfect tetrad is formed.

The various values which P_0 can assume for the full moons nearest to the nodes are all situated between these limiting cases I and II; hence they will present intermediate results. A tetrad further on will be indicated in short by the quantities and arguments belonging to its first member; then we know that another with negative and two others with positive P_0 will follow, in alternating seasons and with arguments of the inequalities (sun's or moon's anomaly) alternately in opposite quadrants.

In order to get an idea of the course of the phenomena we give in Tables II and III the data and results for an array of eclipses embracing two saros periods [5]), first for the years 1732—66 when the tetrads are lacking, then for the years 2010—2044, when they have a maximum frequency. The regularly varying P_0 in the 4th column was found by increasing the values of OPPOLZER's "Cyclentafel" by 2°.89 (the amount he had subtracted to make all corrections positive) and adding the values of his "Periodentafel für Vollmonde". The "Empirische Korrektionen", being small in these centuries, have been neglected. The arguments g for the corrections, (2d and 3d column) i.e. the anomaly of sun and moon

[5]) It must be remarked that a saros period has no definite beginning or end, but is only a duration (of five series); it may start with any series.

expressed in centesimal degrees (0—400) are taken from the same source, rounded to full degrees. The 5th and 6th columns give the corrections themselves which must be added to P_0. The resulting corrected P shows the character of the eclipse; when necessary for dubious cases the boundary value of l_0 is given in the next column. For comparison the last column gives the magnitude of the eclipse in digits, taken from OPPOLZER's Canon.

The table shows that in the years 1732—66 we meet repeatedly with Case II*a*, where for the first and fourth full moon the distance to the node is increased by the solar inequality, and that in the years 2010—2044 we have twice in each saros period Case II*b*, where the distance to the node is diminished by the solar inequality by such an amount as to bring about total eclipses. Moreover we see here how after five series, each with different values of P_0 and of the corrections, hence with a different aspect in the sequence of partial and total eclipses, the next five series show a repetition of nearly the same values and circumstances. P_0 has decreased by only 0°.48, and the argument of the solar inequality has increased by only 12 c.d. (centesimal degrees). The same holds for the border parts of the series with partial eclipses, which have been omitted here, and which in their aspects are still more sensitive to such differences. So we can understand how the larger period of five series forced itself into the attention of the observers and led to the discovery of the saros.

Nevertheless we can see by comparing the two consecutive saros periods how these small changes already bring about changes in the aspect: the fourth series (2021—22) drops out (2039—40) from the tetrads and in the fifth series (2043—44) a new tetrad comes into being. Thus gradually the character of the consecutive saros periods changes. After 18 such periods the argument of the solar inequality has increased by 200 c.d., and the solar correction has changed from a maximal positive to a maximal negative value, i.e. from case II*b* to case II*a*. Expressed in more exact figures: P_0 the longitude of the full moon relative to the node decreases 0°.476 per saros period; the sun's anomaly in one synodic period increases by 29°.1054, in one saros period increases by 10°.497. Hence relative to P_0 the argument of the sun's inequality increases each saros period by 10°.97. It returns to the same value after 360 : 10.97 = 32.8 saros periods = $32.8 \times 18.03 = 591$ years. This then is the theoretical value of the period for which our countings in the Canon gave an amount of nearly 580 years.

IV

Here we must go somewhat deeper into the causes which produce the sharp contrast between the maximum and minimum conditions. The five successive series of a saros follow one another rather regularly, so that their middle parts with small P_0 — because the lunar node retrogrades 350° per saros — fall upon longitudes of the node, hence of

TABLE II

Date	g ☉	g ☽	P_0	Correction ☉	☽	P	l_0	m
1732 June 8	176	289	− 3°.39	+ 0.86	+ .45	− 2°.08		t 18.3
32 Dec. 1	370	61	+ 0.64	− 1.06	− .25	− 0.67		t 21.2
33 May 28	164	233	+ 4.66	+ 1.25	+ .31	+ 6.22		p 9.1
35 Oct. 2	302	265	− 5.90	− 2.28	+ .46	− 7.72		p 6.3
36 Mrch 27	96	37	− 1.87	+ 2.28	− .14	+ 0.27		t 22.0
36 Sept. 20	290	209	+ 2.15	− 2.26	+ .09	− 0.02		t 21.7
37 Mrch 16	84	381	+ 6.17	+ 2.22	+ .06	+ 8.45		p 6.7
39 July 20	222	14	− 4.38	− 0.79	− .05	− 5.22	5.81	t 13.3
40 Jan. 13	16	186	− 0.36	+ 0.58	− .15	+ 0.07		t 21.5
40 July 9	210	358	+ 3.66	− 0.37	+ .15	+ 3.44		t 16.1
41 Jan. 1	4	130	+ 7.69	+ 0.15	− .48	+ 7.36		p 6.8
42 Nov. 12	348	390	− 6.89	− 1.69	+ .03	− 8.55		p 6.7
43 May 8	142	162	− 2.87	+ 1.83	− .35	− 1.39		t 19.2
43 Nov. 2	336	334	+ 1.16	− 1.95	+ .26	− 0.53		t 21.4
44 Apr. 26	130	106	+ 5.18	+ 2.06	− .45	+ 6.79		p 8.6
46 Aug. 30	268	138	− 5.38	− 2.02	− .47	− 7.87		p 6.1
47 Febr. 25	62	310	− 1.35	+ 1.91	+ .38	+ 0.94		t 20.3
47 Aug. 20	256	82	+ 2.67	− 1.79	− .35	+ 0.53		t 21.2
48 Febr. 14	50	254	+ 6.69	+ 1.64	+ .43	+ 8.76		p 3.6
50 June 19	188	286	− 3.86	+ 0.44	+ .46	− 2.96		t 16.4
50 Dec. 23	382	58	+ 0.16	− 0.65	− .23	− 0.72		t 21.2
51 June 9	176	230	+ 4.18	+ 0.86	+ .28	+ 5.32	4.81	p 10.9
53 Oct. 12	314	262	− 6.37	− 2.24	+ .25	− 8.36		p 5.2
54 Apr. 7	108	34	− 2.35	+ 2.27	− .12	− 0.20		t 22.4
54 Oct. 1	302	206	+ 1.67	− 2.28	+ .06	− 0.55		t 20.8
55 Mrch 28	96	378	+ 5.70	+ 2.28	+ .07	+ 8.05		p 7.6
57 July 30	233	10	− 4.86	− 1.16	− .04	− 6.06		p 11.7
58 Jan. 24	27	182	− 0.84	+ 0.96	− .19	− 0.07		t 21.8
58 July 20	221	354	+ 3.19	− 0.76	+ .06	+ 2.49		t 17.6
59 Jan. 13	15	126	+ 7.21	+ 0.55	− .48	+ 7.28		p 6.9
60 Nov. 22	360	386	− 7.38	− 1.37	+ .04	− 8.71		p 6.3
61 May 18	153	159	− 3.35	+ 1.56	− .37	− 2.16		t 17.7
61 Nov. 12	348	331	+ 0.68	− 1.69	+ .28	− 0.73		t 21.1
62 May 2	142	103	+ 4.70	+ 1.83	− .44	+ 6.09		p 10.3
64 Sept. 10	279	135	− 5.86	− 2.18	− .47	− 8.51		p 4.9
65 Mrch 7	78	307	− 1.83	+ 2.10	+ .40	+ 0.67		t 21.1
65 Aug. 30	267	79	+ 2.19	− 2.01	− .34	− 0.16		t 22.4
66 Febr. 24	61	251	+ 6.21	+ 1.89	+ .42	+ 8.52		p 4.0

TABLE III

Date	g ⊙	g ☽	P_0	Correction ⊙	☽	P	l_0	m
2010 Dec. 21	385	311	− 3°.49	− 0.55	+ .38	− 3°.66		t 15.2
11 June 15	179	83	+ 0.54	+ 0.76	− .36	+ 0.94		t 20.6
Dec. 10	373	255	+ 4.56	− 0.96	+ .44	+ 4.04		t 13.7
14 Apr. 15	111	287	− 6.00	+ 2.26	+ .46	− 3.28		t 15.4
Oct. 8	305	59	− 1.97	− 2.28	− .24	− 4.49		t 14.0
15 Apr. 4	99	231	+ 2.05	+ 2.28	+ .39	+ 4.72		t 12.3
Sept. 28	293	3	+ 6.07	− 2.27	− .02	+ 3.78		t 15.6
18 Jan. 31	31	35	− 4.48	+ 1.09	− .13	− 3.52		t 16.1
July 27	225	207	− 0.46	− 0.89	+ .07	− 1.28		t 19.4
19 Jan. 21	19	379	+ 3.56	+ 0.69	+ .07	+ 4.32		t 14.5
July 16	213	151	+ 7.59	− 0.48	− .42	+ 6.69		p 8.0
21 May 26	157	11	− 6.99	+ 1.45	− .04	− 5.58	5.82	t 12.3
Nov. 19	351	183	− 2.97	− 1.62	− .18	− 4.77	4.77	t? 11.9
22 May 16	145	355	+ 1.06	+ 1.76	+ .16	+ 2.98		t 17.1
Nov. 8	339	128	+ 5.08	− 1.88	− .48	+ 2.72		t 16.3
25 Mrch 14	77	159	− 5.48	+ 2.15	− .37	− 3.70		t 14.1
Sept. 7	271	331	− 1.45	− 2.07	+ .38	− 3.14		t 16.5
26 Mrch 3	65	103	+ 2.57	+ 1.97	− .44	+ 4.10		t 14.1
Aug. 28	259	275	+ 6.59	− 1.85	+ .47	+ 5.21	5.06	p 11.5
28 Dec. 31	396	308	− 3.96	− 0.15	+ .39	− 3.72		t 14.9
29 June 26	190	80	+ 0.06	+ 0.37	− .34	+ 0.09		t 22.5
Dec. 20	385	252	+ 4.08	− 0.55	+ .42	+ 3.95		t 13.6
32 Apr. 25	122	284	− 6.47	+ 2.16	+ .46	−3.85		t 14.3
Oct. 18	316	56	− 2.45	− 2.22	− .22	− 4.89	5.62	t 13.3
33 Apr. 14	110	228	+ 1.57	+ 2.26	+ .27	+ 4.10		t 13.2
Oct. 8	304	0	+ 5.60	− 2.28	− .00	+ 3.32		t 16.4
36 Feb. 11	42	32	− 4.96	+ 1.43	− .11	− 3.64		t 15.7
Aug. 7	236	204	− 0.94	− 1.25	+ .04	− 2.15		t 17.6
37 Jan. 31	30	376	+ 3.09	+ 1.06	+ .08	+ 4.23		t 14.6
July 27	224	148	+ 7.11	− 0.86	− .43	+ 5.82	4.91	p 10.0
39 June 6	168	8	− 7.47	+ 1.12	− .03	− 6.38		p 10.7
Nov. 30	362	180	− 3.45	− 1.31	− .20	− 4.96	4.78	p 11.5
40 May 26	156	352	+ 0.58	+ 1.48	+ .18	+ 2.24		t 18.7
Nov. 18	350	124	+ 4.60	− 1.64	− .48	+ 2.48		t 17.1
43 Mrch 25	88	156	− 5.96	+ 2.25	− .39	− 4.10		t 13.3
Sept. 19	282	328	− 1.93	− 2.21	+ .30	− 3.84		t 15.0
44 Mrch 13	76	100	+ 2.09	+ 2.14	− .43	+ 3.80		t 14.7
Sept. 7	270	273	+ 6.11	− 2.06	+ .47	+ 4.52		t 12.7

the sun, that are retrograding nearly one fifth of a circumference; every next series comes $2\frac{1}{2}$ month earlier than the preceding one. So we might expect that in one of them an eclipse with P_0 circa — 6° must fall within the spring season, so that the conditions of case II*b*, producing a tetrad of total eclipses should be present once every saros. Their complete absence about 1700 and their abundance about 2000 therefore need a further explanation.

The consecutive series are following one another with unequal intervals. Within a saros period there is twice an interval of 7, thrice an interval of 8 halfyears (the term halfyear here indicates 6 synodic periods). In 8 halfyears P_0 changes by + 1°.514, in 7 halfyears by — 2°.509; so the sequence of (rounded) values of P_0 will run as follows (taking 1735—51 as an instance):

— 6	— 4.5	— 7	— 5.5	— 4.0	— 6.5
— 2	— 0.5	— 3	— 1.5	0	— 2.5
+ 2	+ 3.5	+ 1	+ 2.5	+ 4.0	+ 1.5
+ 6	+ 7.5	+ 5	+ 6.5		+ 5.5
8 *h.y.*	7 *h.y.*	8 *h.y.*	8 *h.y.*	7 *h.y.*	

If the first of each group should fall in autumn, with a large negative solar correction there is no possibility of getting a sequence of four total eclipses; if they should fall in spring, then in the first and the fourth group a tetrad of total eclipses may result. Now with an interval of 7 halfyears the nodes of the first eclipse, hence also the seasons interchange, whereas after an interval of 8 halfyears they remain the same. So e.g. as a sequence of season dates (retrograding 73 and 67 days) and of centesimal arguments of the solar correction (decreasing 80 and 74 c.d.)

in stead of	Oct. 20	Aug. 8	June 2	Mrch 21	Jan. 7	Nov. 1
and	317	237	164	84	4	330 c.d.
we have	Oct. 20	Aug. 8	Dec. 1	Sept. 20	July 8	Nov. 1
and	317	237	364	284	204	330 c.d.,

so that the spring cases II*b* all drop out and the solar corrections are all negative. So under such conditions — as prevailed in the 18th century — there is no possibility for any tetrad of total eclipses to appear. The same, in opposite direction, takes place about 2000. When the first group of eclipses begins with a spring date, by the same occurrence twice of a 7 halfyears interval it remains in the spring, and the solar argument remains in or near the two first quadrants. Here e.g.

in stead of	April 15	Jan. 31	Nov. 25	Sept. 13	July 2	April 26
and	111	31	357	277	196	122 c.d.
we have	April 15	Jan. 31	May 26	Mrch 14	Dec. 31	April 26
and	111	31	157	77	396	122 c.d.

The result is that in every saros there are one or two series with tetrads of total eclipses.

Thus the sharp contrast between centuries with multitudes and

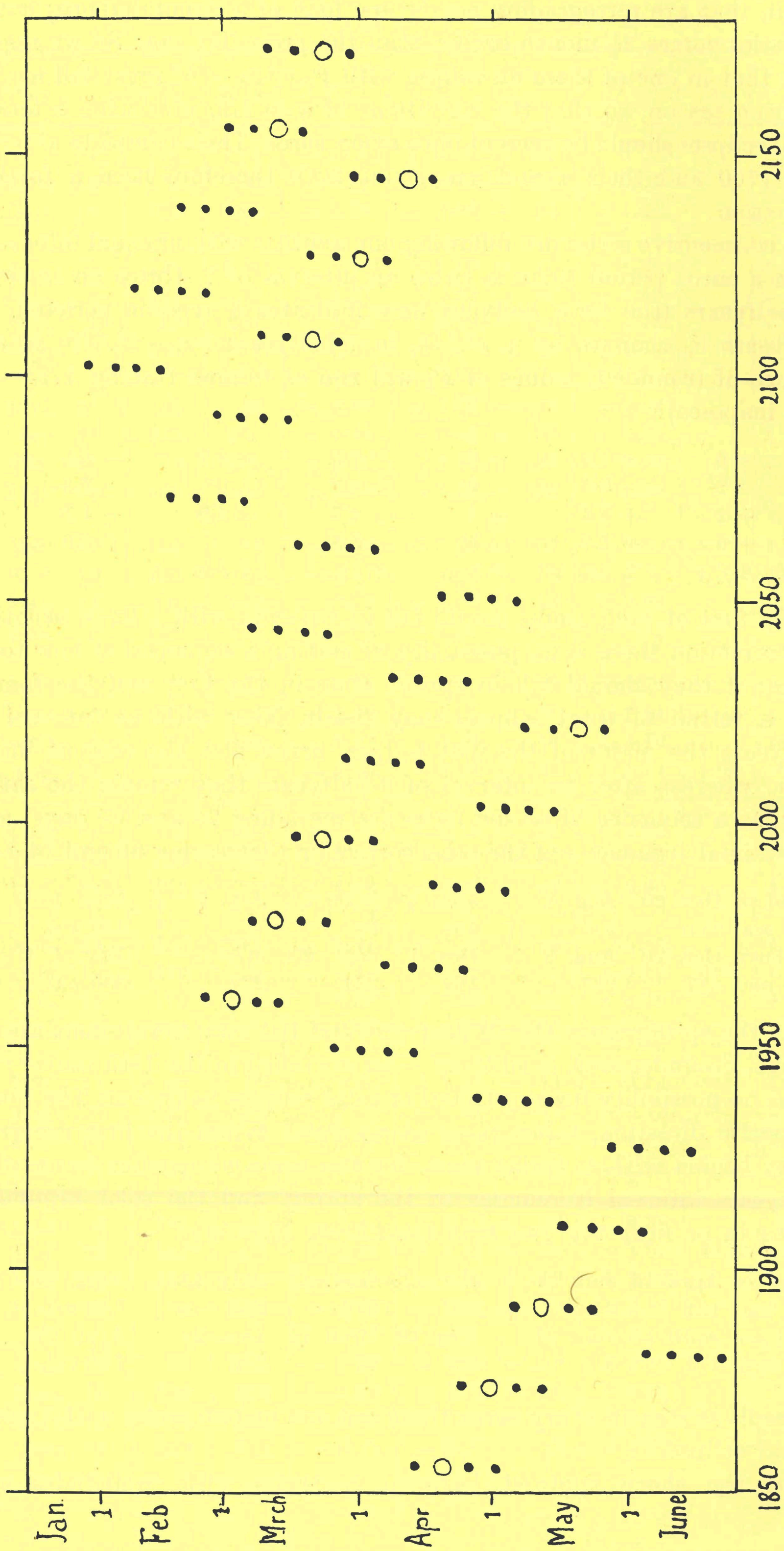

Fig. 2. Arrangement of tetrads of total lunar eclipses.

centuries with complete absence of tetrads of total eclipses is explained. Gradually, however, the conditions are changing. The (rounded) relative longitudes P_0 after one saros have decreased by 0°.5; exactly they return after 8 series (5 jumps of + 1°.5 and 3 jumps of — 2.°5) so that the jumps — 2°.5 with the 7 halfyears interval per one saros will occur on the average 15/8 instead of 2 times. Hence the 7 halfyears intervals with their reversion of season gradually will come later in the five series of a saros, the decrease of the argument into the region with opposite sign (below 200 in the first, below 400 in the second case) will not be undone by the reversion of season, and ever more among the successive series will pass to the season with a solar correction small or of opposite sign. Thus gradually after a time of abundance the conditions for tetrads of total eclipses disappear, or conversely, after a time of absence, they gradually come into being.

It will be necessary now to consider more in detail the structure of the periodically appearing multitudes of tetrads of total eclipses. When in one series we have the favourable conditions of P_0 say — 5° to + 7° and the solar argument somewhat below 100, then in each following saros period the latter will increase by 12 while the former decreases by 0°.5; thus the favourable conditions persist and an array of tetrads will appear, following one another with 18 years interval. When finally the arguments run too near to 200 this array is extinguished. Then, however, other series come into play; after 8 series the values of P_0 return and bring about a new array of tetrads, at dates 21 days earlier and with a solar argument smaller by 23 c.d. In such a way, when a first array of tetrads is extinguishing, new arrays come forth, each taking place 11 years after the former. Till at last they decline and disappear when the eclipses fall too early in the year. This arrangement of the tetrads of total eclipses in the years 1855—2174 has been reproduced in Fig. 2. The autumn eclipses have been transposed to the spring, to put them into one row with the spring eclipses of the same tetrad; open circles represent partial eclipses, hence indicate imperfect tetrads.

The irregularities shown in this arrangement of tetrads are chiefly due to the influence of the lunar terms. Though they are small (at most 0°.48) and play a secondary role only, they sometimes are operative in making eclipses near the limit total or destroying the totality. Since the argument of the lunar inequality in one saros period decreases by 3.1 c.d. only, the lunar corrections persist with nearly the same amount during an entire array of tetrads, either stabilizing or effacing it.