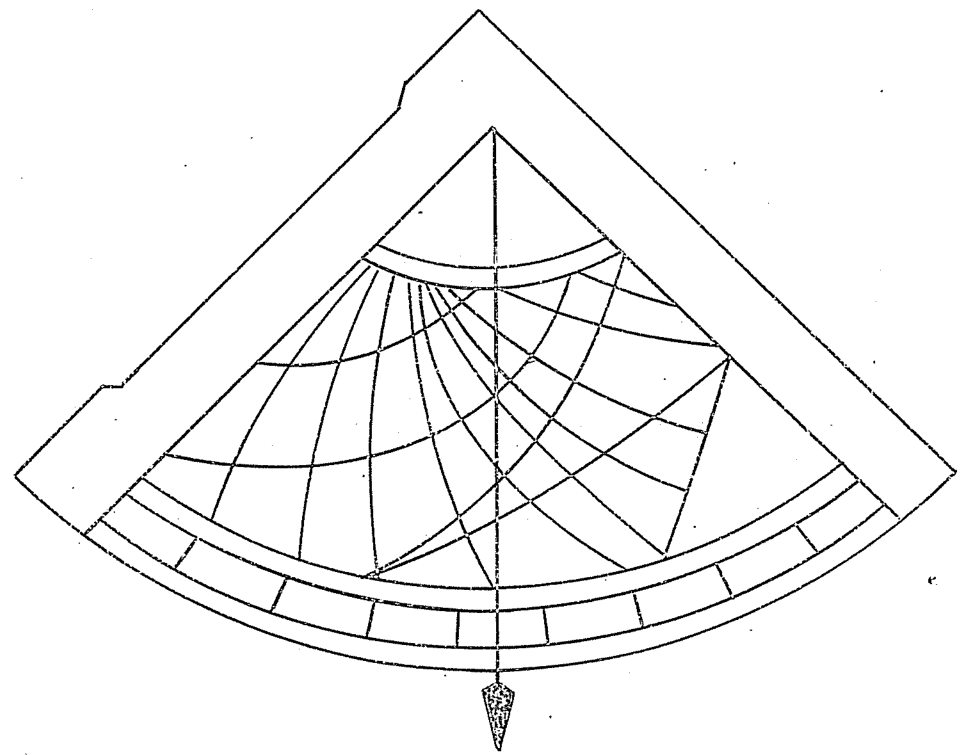
BOSPHORUS UNIVERSITY
KANDILLI OBSERVATORY

A CALCULATION METHOD FOR THE VISIBILITY CURVE OF THE NEW MOON



Muammer DİZER

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INTRODUCTION

Calendar, so called from the Roman Calends or Kalens is a method of distributing time into certain periods adopted to the purposes of civil life and religious observances, such as weeks, months and years. Three of the periods used in calendars, namely days, months and years, are based on those astronomical periods which have the greatest importance for the conditions of human life. Other measures of time, such as week and the subdivisions of the day, are artificial. In primitive societies time was conceived as a recurring cycle of natural events in sequence. Even when this sequence, such as rains, droughts, or periodic events in the life of animals and plants, was associated with recurring astronomical events such as the phases of the moon or the heliacal rising of certain stars, the astronomical vent was still chiefly a checkpoint for the beginning of certain activities rather than a time-measurement. As such checkpoints became increasingly important with the development of agriculture, however, it led to the recognition of the possibility of measuring time by use of astronomical references. When society developed into the urban stage, time - measurement became a necessity to enable both the organized co-operation of the people in economic activities and the regulation by priests and officials of duties and festivals etc. Calendars of some sort therefore soon arose in all the early urban civilizations though they varied widely in type. In most calendars there were three chief methods of time-reckoning. There was the lunar calendar which was based on observation of the phases of the moon, and resulted in a series of lunar months of some 29.5 days. There was the solar calendar which based on the earth's orbit of the sun, in approximately 365.5 days. Finally there was a civil calendar, of a conventional period of time, for the tabulation of a series of civil or religious events. One or all of these methods might be employed. The supply of light by the two great luminaries is governed by the periods known to astronomers as the solar day and the synodic month. Considerable discrepancies were bound to arise between lunar and solar calendars in particular, since 12 x 29.5 = 354. From this fact arose the difficulties with which early constructors of calendars were confronted.

The early Egyptians developed a lunar calendar as early as the Predynastic period, before 3000 B.C. Its lunar months began on the morning of the invisibility of the waning moon, though its days commenced with the rising of the sun. We have no contemporary evidence of this, but the writing of the word "month" with the moon-sign, the importance of the monthly and half-monthly festivals in later times, and the adoption of the month as a unit in the later calendar place it beyond doubt. However the Egyptians had begun to observe what is known as the helical rising of the star Sirius or Sothis, a conspicuous object in the Egyptian sky. A star is said to rise heliacally on the day on which it first appears again in the sky just before sunrise after being for some time invisible. The Egyptians noted that this rising corresponded very closely with the rise of the Nile, on which the agricultural wellare of the country depended. Small wonder then that they chose this for the first day of the year, and took the period between two such observed rising to form a unit of time which was convenient not only as being much longer than the old month, but as including a whole round of the seasons. We know from the Latin writer Censorinus that the first day of the Egyptian calendar year coincided with the rising of Sothis in A.D. 139, and it must therefore have done the same thing 1460 solar years earlier and so on, i.e., in 1321 B.C., 2781 B.C., 4241 B.C., 5701 B.C., etc. Obviously it was at one of these moments that the calendar was introduced. Now the religious texts inscribed in the pyramids of the Fifth and Sixth Dynasties show that the calendar with its five extra days was then already in existence. Egyptologists consequently date the introduction of the calendar to 4241 B.C. or to 2781 B.C., according as they believe the pyramids to be earlier or later than the latter date. A still higher date, e.g., 5701 B.C., is hardly likely.

The Babilionian calendar imposed by the kings of the First Dynasty of Babylon, on all the cities immediately under their rule, was adopted by the Assyrians at the end of the second millennium B.C., was used by the Jews on their return from exile, and was widely used in the Christian era. This calendar was equated with the Summerian calendar in use at Nippur at the time of the Third Dynasty or Ur (about 2300-2150 B.C.,). The Babylonians, as the Egyptians, also began with establishment of a lunar calendar, through their new months started with the reappearance of the moon, and the day started at sunset instead of sunrise. By about 300 B.C. the Babylonians could predict the length of the lunar months. It contained ordinarily twelve months, the beginnings of which were fixed by observations of the lunar crescent and in general their length was 30 days for economic purposes; in historical times regular watch was kept for the mew moon, and if that fell on the 30 th of the month, then the day automatically became the first of the next month, and all officials were apprised of the fact. In order to prevent too serious a derangeament of the seasons owing to the discrepancy between 12 lunar months and the solar year, a month was intercaleted; the intercalary month might be a second Elul(Ululu) or a second Adar. Such intercalations were in the late period, regularly devised within a cycle; in the Seleucid period and earlier, from 328 B.C., the cycle was 19 years; from 504-383 it was 27 years, from 528-505 it was eight years. Before the reign of Darius the intercalation was not based on any fixed cycle, but was incerted when the astronomers advised the kind that it was necessary, the object being, it has been suggested, that the first of the Nisan, with which the year always began, should not fall over o month later than the spring equinox, and not more than a month before it.

All Greek Calendars were lunar until the Roman period. Each community had a separate calerdar. Bischoff has succeded in putting together more or less complete lists of months in about a hundred Greek calendars. There was great variety in the season when the year began in different calendars. But each month was kept roughly to one season of the year by the insertion of a thirteenth or intercalary month when required. In some calendars this was done by repeating the six month, in some by repeating the twelfh month; but in a

few intercalary month occupied other position, at Athens there are four instances preserved on inscriptions where an intercalation was made at an exceptional place in the year, and it is probable that the same happened elsewhere from time to time. Not only the intercalation of months, but also the regulation of the length of each month, appears to have been always in the hands of the public authorities, an it, as time advanced, they paid increasing repect to astronomical calendars there is no evidence that any astronomical calendar ever alquired legal validity. We have less definite information as to the extent to which the beginning of the civil month was permitted to depart from the New Moon, but Aristophanes in the Clouds, acted in 423 B.C., makes the Moon complain that the days are not being kept correctly according to the moon. During the fifth century B.C., the Athenians had a senatorial or financial year, which was independent of the ordinary civil year and of the Moon.

The Macedonian calendar, which was of the Greek type, became current in western Asia as a result of Alexander's conquests, and even competed with the native calendar in Egypt. But in the Roman period the Greek calendars of Asia became purely solar calendars.

The Roman Republican calendar which was of the Greek type, which is now used throughout the whole world, had its origine in the local calendar of the city of Rome. It is generally stated by the ancient anthorities that the year of Romunlus consisted of 304 days divided into 10 months beginning with March, and that Numa introduced a lunar year and added January and February. It may be refarded as certain that the Ruman months were originally lunar, and throughout the republican period the normal length of the year remained 355 days, exceeding 12 lunations by 0.63 day. This small excess could have been compensated by making the intercalary month consist sometimes of 27 and sometimes of 28 days.

The ancient Jewish calendar was of the normal lunar type with twelve months, each of which began with the first visiblity of the crescent Moon. The papyri belonging to the Jewish colony at Elephantine in Southern Egypt in the fifth century B.C. shows that at that place the beginning of the month was reckoned from the first evening when mean sunset or 6 p.m. followed mean new moon, so that we have a calendar determined by astronomical calculation, not by astronomical observations.

Before Islam, the Arabs used a system for their calendar the nature of which is not quite known. All knowledge about the ancient Arabic calendar is based on the verses and folkes. By means of used systems they tried to fit a certain lunar month to fall at the same season every year, in particular Zulheggia, the month for pilgrimage. During this month, the one after and the one before as well as during the month of Regab, fighting was prohibited. The idea was to make the journey to and from Mecca for pilgrimage secure. Different historians quote different systems for Arabs to achieve this object. It was even stated that there had been, on some occasions an abuse by the specialist who used to fix their pilgrimage month. The system whatever had been its nature was known as Al-nasea which meant addition or omission of one lunar month every now and then, go that pilgrimage month may fall at a certain season of the solar year most favourable for the journey to Mecca for pilgrimage.

The nesea system was adopted until the tenth year after immigration of the prophet Mohammed from Mecca. In that year Islam prohibited that year and enferced the adoption of the lunar month without nasea irrespective of whether the pilgrimage month should always reoccure at a certain season of the year. On account of the fact that the system of nasea is not known exactly, chronohogy of events prior the tenth year after immigration is not at all easy.

It is a fact that nearly all primitive tribes had determined time by observing the new crescent, and celebreted this occaison. Arabs greet the crescent with cries of "hilal", the ancient greeting of the arriving God, which later became the name of the new crescent.

Since the days of the Caliph Omar, the immigration of Prophet Mohammed to Medina, being the most important event of the rise of Islam, was considered the beginning of the Islamic or Higri year which consists of 12 lunar months. Each lunar month starting at least one day after new moon, at sunset of the evening of the first sighting of the lunar crescent. The mean lunar month i.e. the synodic month has 29-53 mean solar days. If there are no clouds, the new crescent can always be seen 30 days after the previous one (a complete month), but in almost half of the cases it is seen already on the 29 th day (an incomplete month). Therefore, no lunar month exceeds 30 days. Then chronologists have made a rule that each of the odd numbers comprises of 29 days while each of the even months comprises of days.

Mohammed the Prophet has advised that always " fast when see the new moon and break fasting we you see the next new moon. But if the heavens are clouded, so as to prevent your sighting, count the month of Sha'ban as 30 days". Accordingly, Mohammadens determine the beginning and end of at least 3 or 4 of the Higri months that is, Ramadan when they have to fast, al-muharram the beginning of the Higri year, Zulheggia when they go for pilgrimage and Rabia I the month in which the prophet has been born.

Poeple would gather on beaches or hills waiting for the fain sickle to appear in the pale blue edge of the yellow evening sky, and when it was suddenly sighted there would be excitement and celebretion. Following the tradition which al-Biruni ascribes to al-Sakin: "when you observe the new moon of Rajab count 59 days and then begin fasting". However, as early as the evening of the 27 th of Jamadi al-Akhir, people will be gathering on a hill to see if the moon of Rajab was visible. As soon as the crescent was seighted by at least two men of good repute, the party to the court or kadı to report the observation. Kadı may ask these men questions such as how the horns of the crescent were pointed, and how high the crescent was above the horizon. If the answers were in accord with the data obtained from astronomical tables, he would put the sighting on record and forward the news to the capital. In ancient time, it was customary to spread the news of the sighting to light fires on the hills.

The time of the first visibility of the lunar crescent may be predicted from observation and calculation. Determination of the conditions of visibility of the new crescent after the conjunction time had been, one of the subjects of study of ancient astronomers, as well as Muslim astronomers like Abul Wafa, Abul Faraz, al-Farghani, al-Battani, al-Biruni, Ulug Bey and others.

The theoretical problem of first visibility of the lunar crescent is to predict from astronomical arguments, the conditions under which the moon may be sighted for the first time after the conjuction. Accurate prediction greatly facilitated the observation of the first visibility of the new moon, and for this reason, much effort was made already by the early Islamic astronomers to master this problem theoretically. Altogether, the computation of the first visibility was a complex problem involving nearly every aspect of mathematical astronomy. Ptolemy's planetary theory, however, already contained all the elements for handling a problem of this kind, and early Islam astronomers made full use of this.

The problem of predicting the first crescent may have originated in Babylon and seems to have been transmitted to the Arabs by the Hindus. It was not given much attention in Greek astronomy. Ghiyath al-Din al-Kashi mentions, in its Khaqani zij, that a water-clock was used to measure the time of sunset. The criterion for sighting the cerescent of the new moon was a time interval between sunset and moonset of two pinkans. A pinkan is a hemispherical bowl of brass or copper having a small hole at the underside. When it is placed on water, it fills and sinks in 24 minutes. But, in the critical situation the crescent was then seen only for a short moment at about one pinkan, that is, 24 minutes after sunset.

The values of one and two pinkans seen to describe the ancient ritual of first sighting quite well, and were adopted-with further refinements-to the present day.

In various places in the Islamic literature one finds mention of the conditions under which the new crescent may be seen. One finds limits and ranges for Arc of Separation, Arc of Descent and Angular Distance between the sun and the moon called the Arc of Light and values with out denomination. Also there are criteria of successive limitation which confine areas where the crescent may definitely be seen or not seen at all.

But Islamic astronomers generally emphasized on the two criteria. The angular distance between the sun and the moon or the difference between their ecliptical longitudes and the altitude of the new crescent from the horizon at the moment of sunset or the difference between the sunset and the moonset in degrees.

THE CONDITIONS OF THE VISIBILITY OF THE NEW MOON

Let us examine the conditions of the visibility of the new mon. The visibility of the new Moon is based on the relativ brightness of the Moon and the sky as well as on the measured ability of the human eye to observe contrast. In order to see the new crescent we must wait until the moon reaches a certain brightness after the conjunction time. This brightness depends on the width of the crescent. As it is well known the width, as seen on a perfect lunar sphere, observed and illuminated from infinity can be written down immediatly using Fig. 1.

$$\omega = \frac{r}{2}(1 - \cos d) = r \cdot \sin^2 \frac{d}{2} \qquad (1)$$

where r is the lunar diameter and d is the angular distance between the Sun and Moon. According to the equation (1) the width of the crescent is the function of the angular distance of the Moon from the Sun. Thus the first condition towards the first visibility of the new Moon will be based on attaining a certain value of the angular distance.

Danjon (1936) reported that the Moon's crescent could not be seen closer to the sun for elongations less than 8°. This is no doubt due to the increase of the sky brightness in the area close to the sun and the rough character of the Moon's surface that makes the crescent fade rapidly at increasing glancing angles.

When the visibility of the thin crescent during sunset is under consideration, we must encounter the earth's atmosphere and the physical phenomena related into it. Because before reaching our eye, the lunar light traverses a rather thick layer of atmosphere in the proximity of horizon and somewhat in this fashion its intensity decreases. In this case the most important phenomena on is also the twilight following sunset. Then, the new Moon can be seen if its brightness is larger than of the twilight brightness of the sky above the western horizon.

The average brightness of the sky can be measured directly with an ordinary photogrophic exposure meters. The measurements in the neightbourhood of the sun show that brightness of the sky above the western horizon after sunset at a certain moment is taken to be same at the same altitude. Thus we assume that it does not depend on the azimuth. In order to the new crescent to be visible, the Moon's altitude from the horizon must be greater than a certain value.

Fortheringham (1910) computed the altitude of the moon at the moment of the sunset, and the difference of azimuth between the two bodies, with the result that he obtained a simple criterion for the visibility of the moon when in small phase much more difinite and satisfactory than that nonally adopted. It has been usual to adopte the interval of time from true conjunction as the measure of visibility. Adding a few more observations to those used by Fortheringham, E.W. Maunder (1911) drew the curve some what lower.

The Indian Ephemeris and Nautical Almanac (for example on page 462 of the 1971 volume) uses a slightly modified form of the Maunder criterion for visibility of the moon. Both are listed in the Table, along with Fortheringham's.

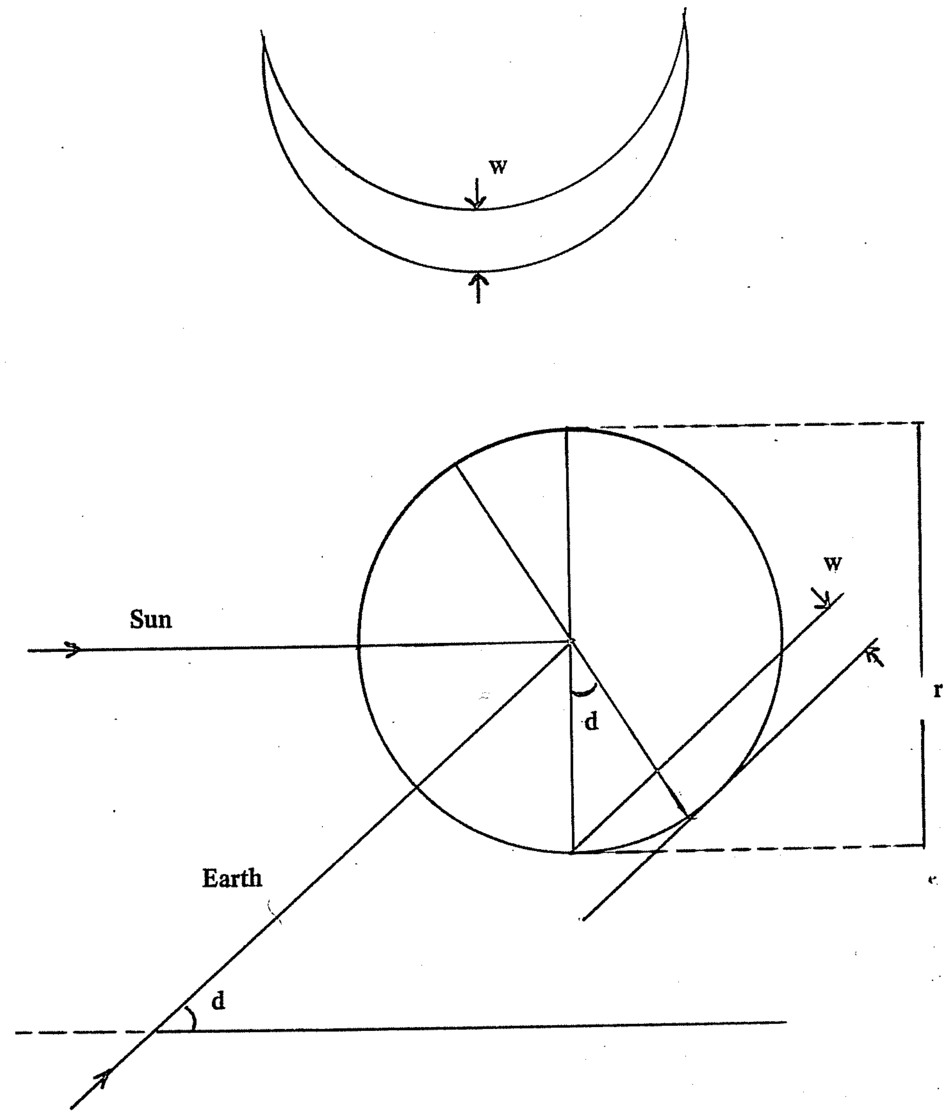

Fig. 1. The width of the crescent

CRITERIA FOR CRESCENT VISIBILITY

Azimuth Difference	Foth.	Moon's Maunder	Altitude Ind. Eph.
0°	12°.0	11°.0	10°.4
5°	11°.9	10°.5	10°.0
10°	11°.4	9°.5	9°.3
15°	11°.0	8°.0	8°.0
20°	10°.0	6°.0	6°.2
23°	7°.7		

Then two criteria generally used by the Islam astronomers correspond to the criteria deduced from modern knowledge, but only the limits obtained by the modern technique is different.

In the Conference for Determining the first day of the Lunar Months held in Istanbul, the historical capital of Islam from 28th November 1978 to 30th November 1978 / 26 Zilhicce to 29 Zilhicce 1938 A. D., two committees of inquiry were established : "The committee of Religion" and the "Committee of Astronomers". In each committee the specialists worked on the paper that came under their specializations. After a detailed and comprehensive dicussion, the comference unanimously accepted the following resolutions.

1. What is basic, is to see the moon, whether it is seen simply by naked human eye, or through modern astronomical observations makes no difference whatsoever.

2. So as to take the astronomers' account into consideration from a religion point of view, they ought to base their astronomical observations upon the fact that the Moon could be seen in the horizon by human eye after the sunset and after the elimination of all the obstacles which hinder the visibility of the Moon. This is called "Visibility of Judgment" (Hukmi ru'yet).

3. For the visibility of the Moon, there are two basic conditions which have to be fulfilled :

a. After "conjuction" , the angular distance between the Moon and the Sun should not be less than 8 degrees. As it is known, the visibility of the Moon starts between 7 and 8 degrees, but for the sake of precaution, taking 8 degree as essantial is accepted unanimously.

b. The angular distance of the Moon from the horizon should not be less than 5 degrees.

Only under this condition does it becom possible to see the crescent by naked human eye in the normal conditions.

4. For the visibility of the Moon no special place is required. When such visibility becomes possible in any part of the earth, it will be legally concluded that the lunar month has started.

COMPUTATION OF THE MOON'S POSITION

This booklet is intended primarily for finding the date of the first visibility of the new crescent. Astronomers and others, however, will find the data useful for calculations requiring the positions of the Sun and the Moon.

To facilitate chronological computations for many purposes, the astronomical days, beginning at Greenwich mean noon or 12h U.T., are numbered consecutively from an epoch sufficiently far in the past to precede the historical period. The number which denotes a day in this continuous count is the Julian Day Number (J.D.). The Julian Day reckoning begins with Julian Day Number 0 for - 4712 January 1, at 12h U.T. Dates expressed in Julian Days and fractions of a day represent time elapsed since this epoch. The Julian Day Number can be computed from prepared Tables.

The lunar ephemeris is calculated directly from Brown's theory instead of from his Tables of the Motion of the Moon (New Haven, Yale University Press, 1919); but in order to obtain a strictly gravitional ephemeris expressed in the same measure of time as defined by Newcomb's Tables of the Sun, the orbital elements upon which Brown's tables are based have been amended by removing the empirical term

$$10''.71 \sin(240°.7 + 140°.0\,T)$$

and by applying to the Moon's mean longitude the correction

$$-8''.72 - 26''.74T - 11''.22\,T^2$$

where T is the time measured in Julian centuries of 36525 ephemeris days, from the epoch 1900 January 0.5 E.T. = J.D. 2415020.0 by the following relation.

$$T = \frac{JD - 2415020.0}{36525}$$

Consequently, the fundamental orbital elements are given by the following equations :

The mean longitude of the Moon, measured in the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit and then along the orbit is given by the following equation :

$$L' = 270^\circ\ 26'\ 02''.99 + 1732\ 564\ 379''.31T - 4''.08T^2 + 0''.0068T^3$$

To the mean values of the arguments must be added some periodic variations, called "additive terms" in Improval Lunar Ephemeris 1952–1959 (Washington, 1954). The additive terms for geocentric mean longitude of the Moon are equivalent to the followings ones :

$$+0''.84 \sin(51^\circ.2 + 20^\circ.2T)$$
$$+14''.27 \sin(346^\circ.560 + 132^\circ.870T - 0^\circ.009\ 1731\ T^2)$$
$$+7''.261 \sin \Omega$$

where Ω is the mean longitude of the Moon's ascending node. This orbital element of the Moon is given by the following relation :

$$\Omega = 259^\circ\ 10'59''.79 - 6962911''.23T + 7''.48T^2 + 0''.0080T^3$$

The longitude of the mean ascending node of the lunar orbit on the ecliptic is measured from the mean equinox of date. The additive terms for the mean longitude of the Moon's ascending node is given by the following terms:

$$+0''.63 \sin(51^\circ.2 + 20^\circ.2T)$$
$$+95''.96 \sin \Omega$$
$$+15''.58 \sin(\Omega + 275^\circ.05 - 2^\circ.305)$$

Moon's mean anomaly being equal to the difference between Geocentric mean longitude of Moon and Mean longitude of the Moon's perigee is :

$$M' = 296^\circ 06'\ 16''.59 + 1717\ 915\ 856''.79T + 33''.09T^2 + 0''.0518\ T^3$$

The additive terms belonging to this M' are given as follows :

$$2''.94 \sin(51^\circ.2 + 20^\circ.2T)$$
$$14''.27 \sin(346^\circ.560 + 132^\circ.870T - 0^\circ.0091731\ T^2)$$
$$+9''.337 \sin \Omega$$

Mean elongation of the Moon from Sun i.e., the difference between geocentric mean longitude of Moon and geocentric Mean longitude of Sun is represented by the following relation :

$$D = 350^\circ\ 44'\ 14''.95 + 1602961611''.18T - 5''.17\ T^2 - 0''.0068\ T^3$$

The additive terms belonging to this D are :

$$7''.24 \sin(51^\circ.2 + 20^\circ.2T)$$
$$14''.27 \sin(346^\circ.560 + 132^\circ\ 870T - 0^\circ.009\ 1731\ T^2)$$
$$7''.261 \sin \Omega$$

Mean distance of Moon from ascending node being equal to the difference between geocentric mean longitude of Moon and mean longitude of Moon's ascending node is given by the following equation,

$$F = 11^\circ 15'\ 03''.20 + 1739\ 527\ 290''.54T - 11''.56\ T^2 - 0''.0012T^3$$

The following terms are additive to F :

$+0''.21 \sin(51^\circ.2 + 20^\circ.2T)$ is negligible
$$+14''.27 \sin(346^\circ.500 + 132^\circ.870T - 0^\circ.009\ 1731\ T^2)$$
$$-88''.690 \sin \Omega$$
$$-15''.58 \sin(\Omega + 275^\circ.05 - 2^\circ 30\ T)$$

Sun's mean anomaly i.e., difference between geocentric mean longitude of Sun and mean longitude of Sun's perigee is given by the following relation :

$$M = 358^\circ\ 28'\ 33''.00 + 129\ 596''.10T - 0''.54T^2 - 0''.0120\ T^3$$

The following term is additive to M :

$$-6''.40 \sin(51^\circ.2 + 20^\circ.2T)$$

The Moon's equation of centre : The Moon's equation of centre is computed on the basis of the formula :

$$C' = 22\ 639''.500 \sin M' + 769''.016 \sin 2M' + 36''.124 \sin 3M' + 1''.938 \sin 4M' + 0''.113 \sin 5M'$$

The adopted expressions for the solar perturbations in the coordinates of the Moon are those given by Brown. They express the perturbations as harmonic series, in which each argument is a combination of the four fundamental arguments D, F, M, M'. More precisely, the argument of each term in the series is the form

$$iD + jF + kM + lM'$$

where

$$i, j, k, l = 0 \pm 1, \pm 2 \ldots\ldots$$

Arguments of perturbations in longitude, arguments of perturbations in latitude and arguments of perturbations in parallax are represented respectively.

$$a_1, a_2, a_3, \ldots\ldots$$
$$b_1, b_2, b_3, \ldots\ldots$$
$$a_1, b_2, b_3, \ldots\ldots$$

The arguments of perturbations in parallax are the same as for the perturbations in longitude. Perturbations are given by the following relations :

$$L_i = k_i \sin a_i \quad (i = 1,2 \ldots\ldots)$$
$$B_i = k_i \sin b_i$$
$$P_i = k_i \cos a_i$$

where L_i, B_i and P_i represent the perturbations in longitude, latitude and parallax respectevely. The expression of the arguments are given in Table 1.

The periodic terms L_i are tabulated for different values of the arguments a_i. Similarly, the terms B_i and P_i are tabulated.

Some of the terms L_i, B_i, P_i are functions of the eccentricity of the Earth's orbit. More specifically, the term L3, L6, L8, L9, ...and B11, B14, B15, B16, ... are proportional to e, whereas some of the other terms are proportional to e^2.

The eccentricity e of the Earth's orbit is represented by the following equation with the eccentricity at 1900.0 (t = 0) being taken as unitly

$$e = 1 - 0.0024954T - 0.000\,007522\,T^2$$

The correction in longitude due to the additive terms (i) and (k) has a maximum value of only 0''.4, which is negligible for our purpose. But the correction in latitude may reach 10''. Then we have,

$$\omega_1 = -0.0004664 \cos \Omega$$
$$\omega_2 = -0.0000754 \cos(\Omega + N)$$

as correction factors in latitude, where

$$N = 275^\circ.05 - 2^\circ.30\,T$$

All arguments are referred to the mean equinox of date. In order to take into account the most important planetary perturbations in the Moon's longitude, the following two quantities also are needed :

V = heliocentric mean longitude of Venus - heliocentric mean longitude of Earth + 180 °.

J = heliocentric mean longitude of Earth - heliocentric mean longitude of Jupiter.

Thus, V is measured from Venu's (mean) superior coujunction , while j is measured from jupiter's (mean) opposition. We have

$$V = 63^\circ.07037 + 22518^\circ.442986\,T$$
$$J = 221^\circ.64742 + 32964^\circ.466939\,T$$

the secular terms (i.e., terms in T^2, T^3) being quite negligible for our purpose.

Finally, we can calculate the Moon's geocentric coordinates as follows :

Moon's longitude :

$$\lambda' = L' + C' + \Sigma L^0 + e\Sigma L' + e^2 \Sigma L^2$$

Moon's latitude :

$$\beta' = (1 + \omega_1 + \omega_2)(\Sigma B^0 + e\Sigma B' + e^2 \Sigma B^2)$$

Moon's parallax :

$$\pi' = P_0 + \Sigma P^0 + e\,\Sigma P' + e^2\,\Sigma P^2$$

COMPUTATION OF THE APPARENT RIGHT ASCENSION AND DECLINATION OF THE MOON

The apparent right ascension and declination are derived directly the values of the apparent longitude and latitude, using the true obliquity of ecliptic. The longitude and latitude may be converted into ascension and declination by the usual formulae :

$$\cos\delta \, . \cos\alpha = \cos\beta \, . \cos\lambda$$
$$\cos\delta \, . \sin\alpha = \cos\beta \, . \sin\lambda \, . \cos\epsilon - \sin\beta \, . \sin\epsilon$$
$$\sin\delta = \cos\beta \, . \sin\lambda \, . \sin\epsilon - \sin\beta \, . \cos\epsilon$$

the obliquity of ecliptic is to be computed using the following formulae :

$$\epsilon = 23^\circ 27'\, 08''.26 - 46''.845\,T - 0''.0059\,T^2 + 0''.00181\,T^3$$

However, befor doing so, the nutations in longitude and obliquity are to be added to λ' and ϵ , respectevely, if one wishes to find the "apparent" coordinates of the Moon. These correction for nutation are to be calculated by the following formulae :

Nutation in longitude :

$$\Delta \lambda' = -(17''.2327 + 0''.01737\,T) \sin \Omega$$
$$+ (0''.2088 + 0''.00002\,T) \sin 2\Omega$$
$$-1''.273 \sin 2L$$
$$-0''.2037 \sin 2L'$$
$$+ 0''.126 \sin M$$
$$+ 0''.0675 \sin M'$$
$$-0''.0497 \sin(2L + M)$$
$$+ 0''.0214 \sin (2L - M)$$

Nutation in obliquity :

$$\Delta \epsilon = +(9''.2100 + 0''.00091\,T) \cos \Omega$$
$$- (0''.0904 - 0''.00004\,T) \cos 2\Omega$$
$$+ 0''.552 \cos 2L$$
$$+ 0''.0884 \cos 2L'$$
$$+ 0''.0216 \cos (2L + M)$$
$$- 0''.0093 \cos (2L - M)$$

Hence,
True obliquity : Mean obliquity + Nut. in obliq.

$$\epsilon = \epsilon_0 + \Delta \epsilon$$

Moon's apparent longitude : Moon's longitude + Nut. in long.

$$\lambda_1 = \lambda' + \Delta \lambda$$

Moon's apparent latitude : Moon's latitude (unchanged by nutation)

$$\beta = \beta '$$

Using the formulae (1) given above, we then find for the Moon's apparent right ascenion

$$\mathrm{tg}\, \alpha = \mathrm{tg}\, \lambda_1 \cos \epsilon - (\sin \epsilon \;/ \cos\lambda_1).\, \mathrm{tg}\, \beta$$

and declination :

$$\sin \delta = \cos \beta \;.\; \sin \lambda_1 \;.\; \sin \epsilon + \sin\beta \;.\; \cos\epsilon$$

COMPUTATION OF THE SUN'S POSITION

The ephemerides of the sun are derived from the geometric longitude referred to the mean equinox of date, the latitude referred to the ecliptic of date and the mean obliquity of date, that are taken from Newcomb's Tables of the Sun (A. P. A. E. , 6, par 1, 1895).

The sun's geometric mean longitude, L is given by the following equation according to Newcomb. (Epoch 1900 January 0 Greenwich Mean Noon = J.D. 2415020.0)

$$L = 279^\circ\, 41'\, 48''.04 + 129\,602\,768''.13\,T + 1''.089\,T^3$$

In order to take into account the most important planetary perturbations in the sun's position, the following quantities also are needed : V, J, Q and S. For V, J see p.12.

Q = heliocentric mean longitude of Earth - heliocentric mean longitude of Mars;
S = heliocentric mean longitude of Earth - heliocentric mean longitude of Saturn;
Thus, Q and S are measured from the planet's (mean) opposition. We have

$$Q = 165^\circ.94905 + 16\,859^\circ.069667\,T$$
$$S = 193^\circ.13230 + 34777^\circ.259042\,T$$

The secular terms are quite negligible for our purposes.

Furthermore, there are inequalities of long period in the sun's mean longitude. These corrections should be applied to the sun's mean longitude and also to the sun's mean anomaly before computing other corrections. The long period perturbations are :

$$\delta L = + 6''.40 \sin (231^\circ.19 + 20^\circ.20\,T)$$
$$+ (1''.882 - 0''.016\,T) \sin (57^\circ.24 + 150^\circ.27\,T)$$
$$+ 0''.266 \sin (31^\circ.8 + 119^\circ.0\,T)$$
$$+ 0''.202 \sin (315^\circ.6 + 893^\circ.3\,T)$$

Equation of centre of the Sun. – the sun's equation of centre is :

$$C = + (6910''.057 - 17''.240\,T - 0''.052\,T^2) \sin M$$
$$+ (72''.338 - 0''.361\,T) \sin 2M$$
$$+ (1''.054 - 0''.001\,T) \sin 3M$$
$$+ 0''.018 \sin 4M.$$

The planetary perturbations in the coordinates of the sun are given by the following equations according to Newcomb : (Tables of the Sun, Astron. Papers American Ephemeris and Nut. Alm., vol. VI, PartI, Washington, 1895)

Sun, perturbations in longitude :

by Venus	by Mars
+4".838 cos (V+ 90 °.00)	+0".273 cos (Q + 90° .6)
+ 5".526 cos (2V+ 90° .12)	+2".043 cos (2 Q + 89° .76)
+ 0".666 cos (3V+ 270° .41)	+0".129 cos (3 Q + 273° .0)
+ 0".210 cos (4V+ 89 ° .8)	+1".770 cos (2 Q – M + 306° .27)
+ 0".084 cos (5V +270 ° .1)	+0".585 cos (4 Q –2M + 185° .82)
+ 2".497 cos (2V –M+ 257 ° .75)	+0".500 cos (4 Q –M + 316° .94)
+ 1".559 cos (3V –M+ 77 ° .96)	+0".425 cos (3 Q –M + 317° .70)
+ 1".024 cos (3V –2M+ 50 ° .85)	+0".204 cos (5 Q –2M + 185° .5)
+ 0".154 cos (5V –3M +34° .1)	+0".154 cos (6 Q –2M + 185° .0)
+ 0".152 cos (4V –2M+ 227° .4)	+0".106 cos (7 Q –3M + 53° .3)
+ 0".144 cos (4V –M+ 79° .0)	+0".101 cos (6 Q –3M + 53° .9)
+ 0".123 cos (5V –2M+ 229° .8)	+0".085 cos (5 Q –M + 139° .3)
+ 0".116 cos (2V +M + 90° .7)	
+ 0".075 cos (V +M + 87° .5)	
+ 0".074 cos (V –M +358 ° .8)	

by Saturn	by Jupiter
+ 0".419 cos (S + 90 °.34)	+7".208 cos (J + 91° .09)
+ 0".108 cos (2S + 270° .1)	+2".731 cos (2J + 270° .25)
+ 0".320 cos (S –M +259 ° .22)	+0".164 cos (3J + 265° .2)
+0".112 cos (2S –M + 273° .1)	+2".600 cos (J –M +174 ° .77)
	+1".610 cos (2J –M +292 ° .60)
	+0".556 cos (3J –2M +177 ° .31)
	+0".210 cos (3J –2M +193 ° .2)
	+ 0".163 cos (J+ M + 110° .2)
	+0".080 cos (4J –2M+ 83 ° .9)
	+0".073 cos (J –2M +187 ° .9)
	+0".073 cos (2J –2M +75° .7)
	+0".069 cos (2J+ M + 263° .9)

Sun, perturbations in latitude

by Venus	by Jupiter
+0".210 cos (3V – M +64 ° .5)	+0".166 cos (2J – M+ 268 ° .6)
+0".092 cos (V – M +64 ° .6)	+0".023 cos (J –2 M)
+0".067 cos (2V – M+ 244 ° .8)	+0".018 cos (3J –2 M +182 °)
+ 0".031 cos (4V – M+ 65° .4)	+0".017 cos (J + 5°)
+0".029 cos (V + M+ 116 °)	+0".016 cos (J – M +272 °)
+ 0".023 cos (2V+ M + 295°)	
+0".019 cos (5V –2 M +233 °)	
+ 0".014 cos (3V +M + 114°)	
+0".014 cos (2V –2 M +233 °)	
+ 0".012 cos (2V+ 271°)	
+ 0".012 cos (4V –2 M +244 °)	

The coordinates of the sun obtained by using the preceding formulae are the coordinates referred to the centre-of-mass of the Earth-Moon system. The action of the Moon is purely geometric in nature; it is simply the transfer of the origine to the centre of the Earth in order to obtain the geocentric coordinates of the Sun.

According to Newcomb, the perturbation in longitude and latitude are given by the following equations :

Δ λ = + 6".454 sin D	Δ β = + 0".576 sin F
+ 0".013 sin 3D	+ 0".016 sin (F + M')
+ 0".177 sin (D+ M')	– 0".047 sin (F – M')
– 0".424 sin (D – M')	
+ 0".039 sin (3D – M')	
– 0".064 sin (D + M)	
+ 0".172 sin (D – M)	

The Sun's aberration in longitude is :

Aberration in the Sun's longitude A.D. 0 = –(20".47 + 0".358 cos M)

Aberration in the Sun's longitude A.D.2000 = –(20".47 + 0".1342 cos M)

Finally we can calculate the sun's longitude and latitude as fallows computation of the sun's longitude

λ = L + C + (Perturbation in longitude = Venus + Mars + Saturn + Jupiter)+ (The perturbation produced by the Moon = Δλ) (geometric longitude, mean equinox of date)

The longitude of the sun, computed in the manner described above, is the true or geometric longitude, i.e. not affected by aberration. The Sun's apparent longitude is obtained by adding the aberations

$\lambda_1 = \lambda +$ (Sun's aberration in longitude = –(20''.47 + 0''.342 cos M))
(apparent longitude, mean equinox of date)

Computation of the Sun's latitude :

β = (Perturbation in latitude = Venus + Jupiter) + (Perturbation produced by the Moon = $\Delta\beta$)

The longitude and latitude may be converted into right ascension and declination by the equations (1). However, befor doing so, the nutations in longitude and obliquity to be added to apparent longitude of the Sun and the mean obliquity respectevely, if one wiches to refer the Sun's coordinates to the time equinox of date.

Hence,

True obliquity :

ϵ = Mean obliquity + Nut. in obliquity

Sun's apparent longitude :

λ' = apparent longitude + Nut. in longitude

Sun's apparent latitude :

$\beta = \beta$ (unchanged by nutation)

Then we have, using the formulae (1)

$$\operatorname{tg}\alpha = \operatorname{tg}\lambda' \cos\epsilon - \sec\lambda' \operatorname{tg}\beta \sin\epsilon$$
$$\sin\delta = \sin\lambda \cos\beta \sin\epsilon + \sin\beta \cos\epsilon$$

or since β is very small and $\cos\lambda' = \cos\alpha \cos\delta \sec\beta$

$$\operatorname{tg}\alpha_0 = \operatorname{tg}\lambda' . \cos\epsilon$$

Since the nutation does not affect the position of the ecliptic itself, the latitude of a celectial body is not affected by it. The effect upon the longitudes of the stars is simply to increase all of them by the same quantity $\Delta\psi$ "nutation in longitude".

The simplest and most direct method to convert positions measured from the mean equinox and the mean equator into those referred to the true equinox and the true equator is to add $\Delta\psi$ to the longitudes, the latitudes remaining unchanged. In converting the ecliptic coordinates to the equtorial coordinates, the true obliquity (i.e. mean obliquity + $\Delta\epsilon$) must be used. However, corrections to right ascension and declination may be calculated directly from :

$$\Delta\epsilon = (\cos\epsilon + \sin\epsilon \sin\alpha \operatorname{tg}\delta)\,\Delta\psi - \cos\alpha \operatorname{tg}\delta\,\Delta\epsilon$$
$$\Delta\delta = \sin\epsilon \cos\alpha\,\Delta\psi + \sin\alpha\,\Delta\epsilon$$

Hence,

$$\alpha = \alpha_0 + \Delta\alpha$$
$$\delta = \delta_0 + \Delta\delta$$

TIME,

The astronomical clock, by means of which time is measured, is the Earth whose axial rotation cause the heavenly bodies to appear to revolve round the Earth from east to west. For the hands of this clock the sun, Moon or stars may be selected, and different times will result according to the choice made. The most convenient unit of measure for time is the day, which is defined as the interval between successive transits over the same meridian of the heaventy body by which the time is measured. If the heavenly bodies were absolutely fixed, all days would be of the same length, this length corresponding exactly to the Earth's period of rotation.

The apparent solar day was formerly considered to begin and end at apparent noon, the moment when the centre of the true Sun is on the upper meridian, since 1925 January 1 it has been considered to begin and end at apparent mid-night, the moment of lower meridian passage of the true sun. It is divided into 24 hours, and the time resulting is called apparent time. Thus apparent time any instant is the westward hour angle of the the sun $+12^h$.

Owing to the non-uniform motion of the true sun in right ascension, the apparent solar day is of variable length, and is therefore not suitable as a measure of time. Hence a fictitious mean sun is conceived which moves uniformly in the equator and, in the long run, is as much ahead of the true sun as behind it. The interval between successive trasits over the same meridian of this mean constitutes the mean solar day, and gives rise to mean solar time.

The difference between the right ascentsion of the true sun and that of the mean sun is known as the equation of time. The equation of time (EQT) is also the hour angle of the true sun minus the hour angle of the mean sun. Thus it is the difference : apparent solar time – mean solar time.

If no great accuracy is required, the equation of time (EQT) may be computed in the following manner.

$$\operatorname{tg}\alpha = \cos\epsilon \operatorname{tg}(\Gamma + M + C)$$
$$EQT = \Gamma + M - \alpha$$

where Γ is given by the following relation :

$$\Gamma = 281^\circ\ 13'\ 15''.00 + 6189''.03\ T + 1''.635\ T^2 + 0''.012\ T^3$$

The values of EQT, resulting from this method, will seldom be in error by more than 2 second of time. Since the Earth rotates uniformly on its axis, and since longitudes are measured uniformly round the Earth from the meridian of Greenwich which is universally accepted as the prime meridians, it follows that the difference between Greenwich mean time and the local mean time of any place is equal to the longitude of that place. Denoting by λ the longitude, considered positive to the west, we have, since the Earth rotates from west to east

$$\text{Local mean time} = \text{GMT} - \lambda$$

TO PREDICT ASTRONOMICALLY THE CURVE OF THE FIRST VISIBILITY

According to the accepted criteria, to find the curve of the first visibility of the lunar crescent, we will resolve the following astronomical problem : when the angular distance between the Sun and the Moon after the conjunction is 8° , what is the geogrophical coordinates of the place on the Earth surface where, at the moment of the sunset, the altitude of the Moon is equal to 5° ? In order to resolve this problem, we will use the spherical trigonometric formulae.

First of all we have to compute the time when the angular distance between the Sun and the Moon is 8° after the conjunction time.

Let us suppose that the equatorial coordinates of the Sun and the Moon are α_0, δ_0 and α_c, δ_c respectevely. From the spherical triangle PSM is defined by the Sun, Moon and Northpole on the celestial sphere (Fig. 2.) The angular distance d can be written as follows :

$$\cos d = \cos(90-\delta_0)\cos(90-\delta_c) + \sin(90-\delta_0)\sin(90-\delta_c).\cos(H_0 - H_c)$$

Hence we have

$$\cos d = \sin\delta_0 \sin\delta_c + \cos\delta_0 \cos\delta_c \cos(H_0 - H_c) \qquad (1)$$

For any given time, since both the Sun's and Moon's equatorial co-ordinates i.e., the right ascension and the declination can be computed by the stated method at the beginning of this booklet. Therefore α_0, δ_0 and α_c, δ_c are known. Then the value of the $H_0 - H_c$ can be calculated by the right ascensions of the Sun and Moon. Since the co-ordinates of the Sun and Moon are calculated for the same sideral time (T), according to the definition of the sideral time, we have

$$T = H_0 + \alpha_0 = H_c + \alpha_c$$

Hence we have

$$H_0 - H_c = \alpha_c - \alpha_0 \qquad (2)$$

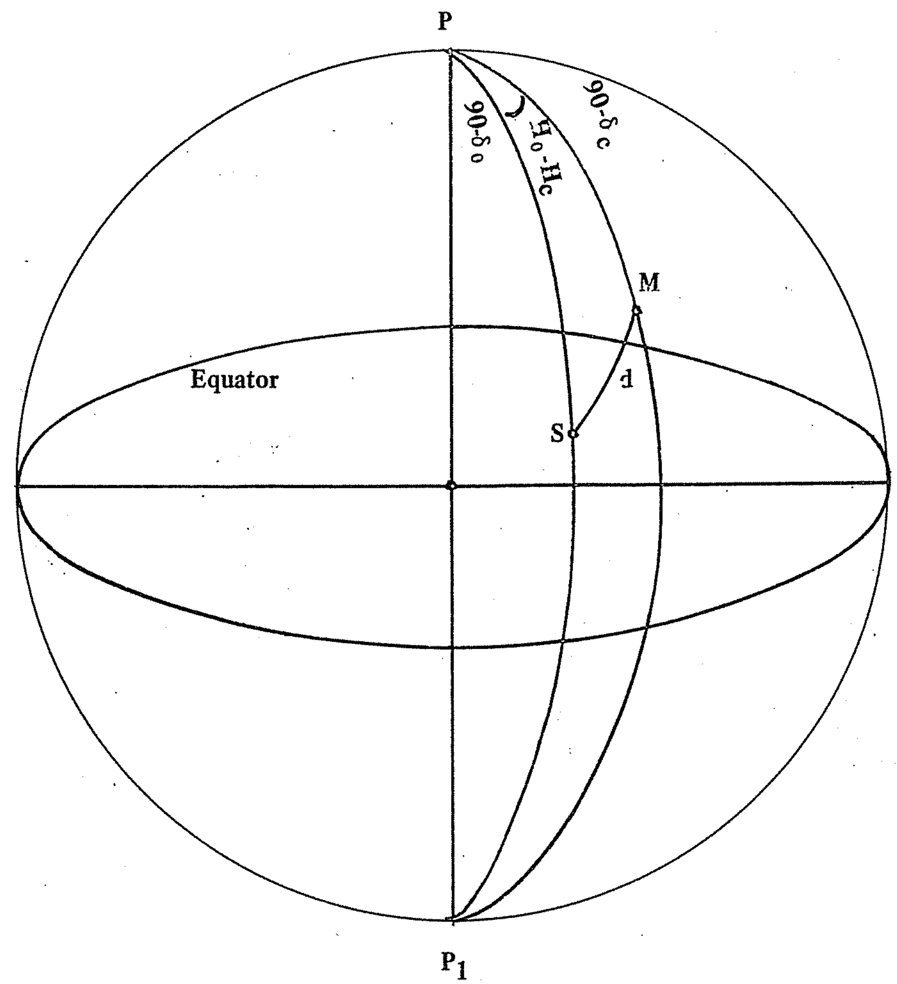

Fig. 2. Celestial sphere

and if this relation is inserted in equation (1)

$$\cos d = \sin \delta_0 \sin \delta_c + \cos \delta_0 \cos \delta_c \cos(\alpha_c - \alpha_0) \qquad (3)$$

is obtained. Since the co-ordinates of the Sun and the Moon are known, the angular distance between the Sun and the Moon can be calculated for a given time using equation (1). But the problem is to find the time when the angular distance is 8°.

Since the conjunction time is known, its angular distance can be computed by the equation (1) for conjunction time, T_0. Let its angular distance be d. As it is well known the angular distance of the Moon from the Sun increases approximately 0°.508 . per. hour, then (8 – d) /0°.508 . is defined the time passing from the conjunction time until its angular distance is equal to 8°. Let this time be t. For T_0 +t the angular distance between the Sun and the Moon will be 8°. If for T_0 + t its angular distance is not equal to 8°, the procedure is repeated so the time T is determined for angular distance 8° in desired precision. Then this is the time of the first visibility of the new Moon. For this time, T, the right ascensions and the declinations of the Sun and the Moon are calculated by the stated method at the beginning of this booklet. Now let us take the problem concerning to the second criteria, i.e, in this case, the propesed problem is that, at the moment of the sunset, when the angular distance between the Sun and the Moon is equal to 8°, the altitude of the Moon is equal to 5° from the horizon, what is the geogrophical co-ordinates on the Earth surface to satify these conditions? From the definition of the sideral time

$$T = H_0 + \alpha_0 = H_c + \alpha_c \quad \text{and} \quad H_0 - H_c = \alpha_c - \alpha_0$$

is written. From this, equating the cosines of both sides.

$$\mathrm{Cos}(H_0 - H_c) = \mathrm{Cos}(\alpha_c - \alpha_0)$$

$$\alpha_c - \alpha_0 = \Delta\alpha$$

$$\mathrm{Cos}\,H_0 \,\mathrm{Cos}\,H_c + \mathrm{Sin}\,H_0 \,\mathrm{Sin}\,H_c = \mathrm{Cos}\,\Delta\alpha$$

$$\mathrm{Cos}\,H_0 \,\mathrm{Cos}\,H_c - \mathrm{Cos}\,\Delta\alpha = -\sqrt{(1-\mathrm{Cos}^2 H_0)}\,\sqrt{(1-\mathrm{Cos}^2 H_c)}$$

$$\mathrm{Cos}^2 H_0 \mathrm{Cos}^2 H_c + \mathrm{Cos}^2\Delta\alpha - 2\mathrm{Cos}\,H_0 \mathrm{Cos}\,H_c \,\mathrm{Cos}\,\Delta\alpha = (1-\mathrm{Cos}^2 H_0)(1-\mathrm{Cos}^2 H_c)$$

$$\mathrm{Cos}^2 H_0 + \mathrm{Cos}^2 H_c - 2\,\mathrm{Cos}\,H_0 \,\mathrm{Cos}\,H_c \,\mathrm{Cos}\,\Delta\alpha + \mathrm{Cos}^2\Delta\alpha - 1 = 0 \qquad (4)$$

As H_0 and H_c are known by the following formulae :

$$\mathrm{Cos}\,H_0 = \frac{\mathrm{Cos}\,90^\circ.83 - \mathrm{Sin}\,\delta_0 \,\mathrm{Sin}\,\varphi}{\mathrm{Cos}\,\delta_0 \;\mathrm{Cos}\,\varphi}$$

and

$$\mathrm{Cos}\,H_c = \frac{\mathrm{Sin}\,5^\circ - \mathrm{Sin}\,\delta_c \,\mathrm{Sin}\,\varphi}{\mathrm{Cos}\,\delta_c \,\mathrm{Cos}\,\varphi}$$

Let us insert the value of H_0 and H_c in equation. (4)
Hence we have

$$C\,\mathrm{tg}^2\varphi + \frac{E}{\mathrm{Cos}^2\varphi} + F + 2D\frac{\mathrm{tg}\,\varphi}{\mathrm{Cos}\,\varphi} = 0$$

$$(F-C)\,\mathrm{Sin}^2\varphi - 2D\,\mathrm{Sin}\,\varphi - (E+F) = 0 \qquad (5)$$

The roots of this equation are :

$$\frac{D \pm \sqrt{D^2 - (E+F)(F-C)}}{(F-C)}$$

where

$$A = \frac{\mathrm{Sin}\,5.83}{\mathrm{Cos}\,\delta_c}, \qquad B = \frac{\mathrm{Cos}\,90^\circ 83}{\mathrm{Cos}\,\delta_0}$$

$$C = [\,\mathrm{tg}^2\delta_c + \mathrm{tg}^2\delta_0 - 2\mathrm{Cos}\,\Delta\alpha\;\mathrm{tg}\,\delta_c\,\mathrm{tg}\,\delta_0\,]$$

$$D = [\,\mathrm{tg}\,\delta_c(B\,\mathrm{Cos}\,\Delta\alpha - A) + \mathrm{tg}\,\delta_0\,(A\,\mathrm{Cos}\,\Delta\alpha - B)\,]$$

$$E = [\,A^2 + B^2 - 2\,\mathrm{Cos}\,\Delta\alpha\;A\,.\,B\,]$$

$$F = \mathrm{Cos}^2\Delta\alpha - 1$$

These roots will yield the boundary latitude where $h_c = 5°.83$ at sunset and when $d = 8°$.

The latitude alone is not enough to determine the point of the earth on where the critaria is satisfied. From the hour angle for the obtained latitudes, it is possible to calculate the local apparent time. The relation

$$T = \frac{H_0}{15} + 12 - E$$

will yield the local mean time of sunset. As the Greenwich mean time T_G when $d = 8°$ is known, the relation $T_G - T = \Delta L$ is the longitude difference between Greenwich and the point where the two criteria is satified.

ACKNOWLEDGEMENT

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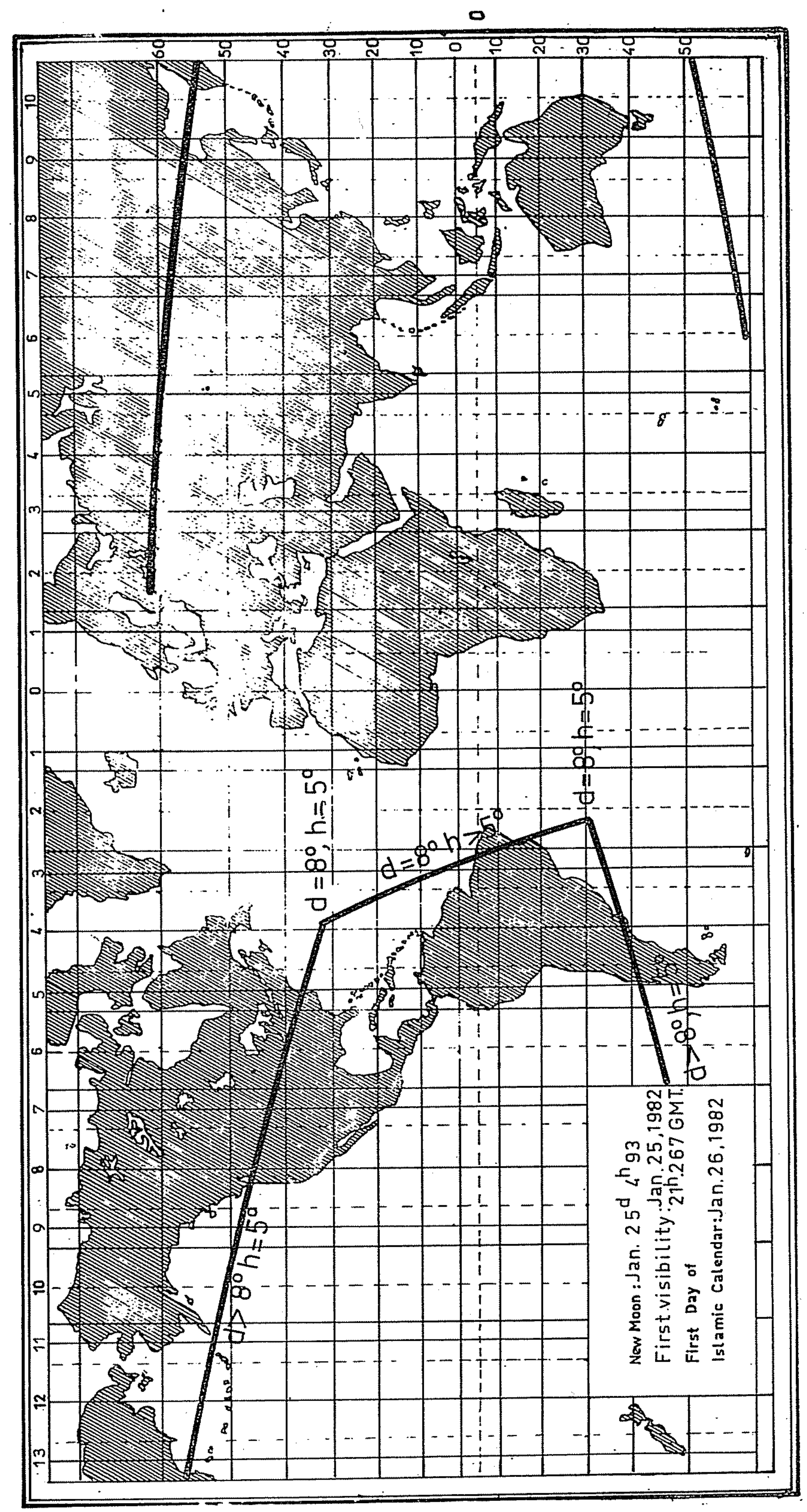

MOON, ARGUMENTS OF PERIODIC TERMS

	Arguments	Variation per hour		Arguments	Variation per hour
a_{1}	2D–M'	+0°.47152	a_{37}	3D–M' ± 180	+0,97947
a_{2}	2D	+1.01590	a_{38}	M+2D+M' ± 180	+1.60134
a_{3}	M ± 180	+0.04107	a_{39}	4D–2M'–M	+0.90198
a_{4}	2F ± 180	+1.10245	a_{40}	M'–2M	+ 0.46224
a_{5}	2M'–2D ± 180	+0.07285	a_{41}	M'–2M–2D	–0.55365
a_{6}	2D–M–M'	+0.43045	a_{42}	M+2D–2M'	–0.03179
a_{7}	2D +M'	+1.56027	a_{43}	2D–M–2F	–0.12762
a_{8}	2D–M	+0.97483	a_{44}	M' +4D	+2.57617
a_{9}	M'–M	+0.50331	a_{45}	4D–M	+1.99072
a_{10}	D ± 180	+0.50795	a_{46}	2M'–D	+0.58080
a_{11}	M' +M ± 180	+0.50544	a_{47}	2F–M–2D	+0.04548
a_{12}	2F–2D ± 180	+ 0.08655	a_{48}	2F–2M'	+ 0.01370
a_{13}	2F +M' ± 180	+1.64682	a_{49}	M+ D +M'	+1.09339
a_{14}	2F–M' ± 180	+0.55807	a_{50}	2M'–3D	–0.43509
a_{15}	4D–M'	+1.48742	a_{51}	4D–3M'	+0.39867
a_{16}	4D–2M'	+ 0.94304	a_{52}	2D+ 2M'–M	+ 2.06358
a_{17}	M +2D–M' ± 180	+0.51259	a_{53}	2M +M' ± 180	+0.62651
a_{18}	2D+M ± 180	+1.05696	a_{54}	2D–M' +2J +180.3	+0.54673
a_{19}	M'–D	+0.03643	a_{55}	M +M–M'	+0.00464
a_{20}	M+D	+0.54901	a_{56}	2D +3M'	+ 2.64902
a_{21}	M'–M+2D	+1.51920	a_{57}	M' +2D +2F ± 180	+ 2.66272
a_{22}	2D+2M'	+2.10465	a_{58}	2D–4M'	–1.16160
a_{23}	4D	+2.03179	a_{59}	V	+0.02569
a_{24}	3M'–2D ± 180	+0.61723	a_{60}	M'–2M+2D	+1.47814
a_{25}	2M'–M	+1.04768	a_{61}	3M'–M	+1.59206
a_{26}	M'–2F–2D	–1.57397	a_{62}	3V+2D–M'	+0.54859
a_{27}	2D–M–2M'	–0.11392	a_{63}	J+ 1°.2	+ 0.03760
a_{28}	M'+D ± 180	+1.05232	a_{64}	M'–M–4D	–1.52848
a_{29}	2D–2M	+0.93376	a_{65}	2M'+ D ± 180	+1.59670
a_{30}	2M'+M ± 180	+1.12982	a_{66}	2F–D	+0.59450
a_{31}	2M ± 180	+0.08213	a_{67}	6D–2M'	+1.95894
a_{32}	2D–M'–2M	+ 0.38939	a_{68}	M–D	–0.46688
a_{33}	M'+ 2D–2F ± 180	+0.45782	a_{69}	2M'–2D+ 2F	+1.17530
a_{34}	2F +2D ± 180	+2.11834	a_{70}	3M'+ M ± 180	+1.67419
a_{35}	4D–M–M'	+ 1.44635	a_{71}	2M'–2D–2F	–1.02959
a_{36}	2M' +2F ± 180	+2.19120	a_{72}	2F–2D+M' ± 180	+0.63092

MOON, ARGUMENTS OF PERIODIC TERMS

	Arguments	Variation per hour		Arguments	Variation per hour
b_1	F	+0°.55122	b_{44}	M'+3F±180	+2.19804
b_2	M'+F	+1.09560	b_{45}	F−2D−M−M'	−1.05011
b_3	M'−F	−0.00685	b_{46}	F−M+2M'	+1.59891
b_4	2D−F	+0.46467	b_{47}	D+M−F	−0.00221
b_5	2D−M'+F	+1.02274	b_{48}	M+F+D	+1.10024
b_6	2D−M'−F	−0.07970	b_{49}	M'+F−2D−M	+0.03864
b_7	2D+F	+1.56712	b_{50}	M'+F+D±180	+1.60355
b_8	2M'+F	+1.63997	b_{51}	F+2M'+M−2D±180	+0.66514
b_9	2D−F+M'	+1.00905	b_{52}	M+F+2M'±180	+1.68104
b_{10}	2M'−F	+0.53753	b_{53}	4D−F−2M'	+0.39182
b_{11}	2D−M−F	+0.42361	b_{54}	4D−F−M−M'	+0.89513
b_{12}	2D−F−2M'	−0.62408	b_{55}	F−M'−D	−0.50110
b_{13}	M'+F+2D	+2.11149	b_{56}	M'+4D−F	+2.02494
b_{14}	F−2D−M	−0.50574	b_{57}	M'+F−D	+0.58765
b_{15}	2D−M'+F−M	+0.08168	b_{58}	4D−F−M	+1.43950
b_{16}	2D−M+F	+1.52605	b_{59}	2D−2M+F	+1.48499
b_{17}	M'+F−2D+M±180	+0.12077	b_{60}	F−3D	−0.97062
b_{18}	M'−M+F	+1.05453	b_{61}	F+4D−M−M'	+1.99757
b_{19}	4D−F−M'	+0.93619	b_{62}	2D−M'−3F	−1.18215
b_{20}	M+F±180	+0.59229	b_{63}	2D−M'+F−2M	+0.94061
b_{21}	M'−M−F	−0.04791	b_{64}	F−M−2M'	−0.57859
b_{22}	F+D±180	+1.05917	b_{65}	M'+F−3D	−0.42825
b_{23}	M+F+M'±180	+1.13666	b_{66}	2M'−M−F	+0.49646
b_{24}	F−M−M'	−0.03422	b_{67}	3F−M'−2D	+0.09340
b_{25}	F−M	+0.51016	b_{68}	M'+F−2D+2M±180	+0.16184
b_{26}	F−D	+0.04327	b_{69}	F+4M'	+2.72872
b_{27}	3M'+F	+2.18435	b_{70}	F+2D−3M'	−0.06601
b_{28}	4D−F	+1.48057	b_{71}	2D−M'+3F±180	+2.12519
b_{29}	F−M'+4D	+2.03864	b_{72}	M+F+M'+2D±180	+2.15256
b_{30}	M'−3F	−1.10929	b_{73}	F+4D−M−2M'	+1.45320
b_{31}	F+4D−2M'	+1.49427	b_{74}	M'+F+4D	+3.12739
b_{32}	2D−3F	−0.63777	b_{75}	F−M'+3D±180	+1.53069
b_{33}	2D−F+2M'	+1.55342	b_{76}	4D−F+M−M'	+0.97726
b_{34}	M'−M−F+2D	+0.96798	b_{77}	F+4D−M	+2.54195
b_{35}	2M'−F−2D	−0.47837	b_{78}	2D−F+3M'	+2.09780
b_{36}	3M'−F	+1.08190	b_{79}	2D+3F±180	+2.66956
b_{37}	2M'+2D+F	+2.65587	b_{80}	F−M'+D	+0.51480
b_{38}	2D−F−3M'	−1.16845	b_{81}	F+2D+3M'	+3.20024
b_{39}	M+F+2D−M'±180	+1.06381	b_{82}	F−2D−2M	−0.54681
b_{40}	M+F+2D±180	+1.60819	b_{83}	2D−F−4M'	−1.71283
b_{41}	F+4D	+2.58301	b_{84}	3F−2M'	+0.56492
b_{42}	2D−M+F+M'	+2.07043	b_{85}	2M'−F−M+2D	+1.51236
b_{43}	2D−2M−F	+0.38254			

MOON, PERIODIC TERMS IN LATITUDE

	Terms Not Depending on e	Terms Proportional to e	Terms Proportional to e^2		Terms Not Depending on e	Terms Proportional to e	Terms Proportional to e^2
B_1	18467.78	–	–	B_{44}	1.02	–	–
B_2	1010.18	–	–	B_{45}	–	0.83	–
B_3	999.69	–	–	B_{46}	–	0.81	–
B_4	623.66	–	–	B_{47}	–	0.81	–
B_5	199.48	–	–	B_{48}	–	0.80	–
B_6	166.58	–	–	B_{49}	–	0.70	–
B_7	117.26	–	–	B_{50}	0.67	–	–
B_8	61.91	–	–	B_{51}	–	0.66	–
B_9	33.36	–	–	B_{52}	–	0.64	–
B_{10}	31.76	–	–	B_{53}	0.63	–	–
B_{11}	29.69	–	–	B_{54}	–	0.60	–
B_{12}	15.56	–	–	B_{55}	0.59	–	–
B_{13}	15.12	–	–	B_{56}	0.47	–	–
B_{14}	–	12.14	–	B_{57}	0.43	–	–
B_{15}	–	8.90	–	B_{58}	–	0.42	–
B_{16}	–	8.00	–	B_{59}	–	–	0.39
B_{17}	–	7.46	–	B_{60}	0.35	–	–
B_{18}	–	6.76	–	B_{61}	–	0.34	–
B_{19}	6.58	–	–	B_{62}	0.33	–	–
B_{20}	–	6.49	–	B_{63}	–	–	0.32
B_{21}	–	5.65	–	B_{64}	–	0.31	–
B_{22}	5.36	–	–	B_{65}	0.31	–	–
B_{23}	–	5.33	–	B_{66}	–	0.30	–
B_{24}	–	5.10	–	B_{67}	0.29	–	–
B_{25}	–	4.86	–	B_{68}	–	–	0.27
B_{26}	4.79	–	–	B_{69}	0.27	–	–
B_{27}	3.98	–	–	B_{70}	0.25	–	–
B_{28}	3.67	–	–	B_{71}	0.24	–	–
B_{29}	3.00	–	–	B_{72}	–	0.24	–
B_{30}	2.81	–	–	B_{73}	–	0.22	–
B_{31}	2.41	–	–	B_{74}	0.21	–	–
B_{32}	2.18	–	–	B_{75}	0.21	–	–
B_{33}	2.15	–	–	B_{76}	–	0.17	–
B_{34}	–	1.77	–	B_{77}	–	0.15	–
B_{35}	1.62	–	–	B_{78}	0.15	–	–
B_{36}	1.58	–	–	B_{79}	0.14	–	–
B_{37}	1.52	–	–	B_{80}	0.14	–	–
B_{38}	1.52	–	–	B_{81}	0.14	–	–
B_{39}	–	1.32	–	B_{82}	–	–	0.14
B_{40}	–	1.27	–	B_{83}	0.13	–	–
B_{41}	1.19	–	–	B_{84}	0.13	–	–
B_{42}	–	1.14	–	B_{85}	–	0.13	–
B_{43}	–	–	1.10				

MOON, PERIODIC TERMS IN LONGITUDE

	Terms Not Depending one	Terms Proportional to e	Terms Proportional to e^2		Terms Not Depending one	Terms Proportional to e	Terms Proportional to e^2
L_1	4586.46	—	—	L_{37}	3.21	—	—
L_2	2369.91	—	—	L_{38}	—	2.92	—
L_3	—	668.15	—	L_{39}	—	2.74	—
L_4	411.61	—	—	L_{40}	—	—	2.58
L_5	211.66	—	—	L_{41}	—	—	2.53
L_6	—	205.96	—	L_{42}	—	2.49	—
L_7	191.95	—	—	L_{43}	—	2.15	—
L_8	—	165.14	—	L_{44}	1.98	—	—
L_9	—	147.69	—	L_{45}	—	1.88	—
L_{10}	125.15	—	—	L_{46}	1.75	—	—
L_{11}	—	109.67	—	L_{47}	—	1.44	—
L_{12}	55.17	—	—	L_{48}	1.30	—	—
L_{13}	45.10	—	—	L_{49}	—	1.27	—
L_{14}	39.53	—	—	L_{50}	1.22	—	—
L_{15}	38.43	—	—	L_{51}	1.19	—	—
L_{16}	30.77	—	—	L_{52}	—	1.18	—
L_{17}	—	28.47	—	L_{53}	—	—	1.17
L_{18}	—	24.42	—	L_{54}	1.14	—	—
L_{19}	18.61	—	—	L_{55}	—	1.09	—
L_{20}	—	18.02	—	L_{56}	1.06	—	—
L_{21}	—	14.58	—	L_{57}	0.99	—	—
L_{22}	14.39	—	—	L_{58}	0.95	—	—
L_{23}	13.90	—	—	L_{59}	0.82	—	—
L_{24}	13.19	—	—	L_{60}	—	—	0.76
L_{25}	—	9.70	—	L_{61}	—	0.68	—
L_{26}	9.37	—	—	L_{62}	0.66	—	—
L_{27}	—	8.63	—	L_{63}	0.64	—	—
L_{28}	8.47	—	—	L_{64}	—	0.64	—
L_{29}	—	—	8.10	L_{65}	0.59	—	—
L_{30}	—	7.65	—	L_{66}	0.58	—	—
L_{31}	—	—	7.49	L_{67}	0.57	—	—
L_{32}	—	—	7.41	L_{68}	—	0.56	—
L_{33}	6.38	—	—	L_{69}	0.56	—	—
L_{34}	5.74	—	—	L_{70}	—	0.55	—
L_{35}	—	4.39	—	L_{71}	0.54	—	—
L_{36}	4.00	—	—				

FORTRAN IV PROGRAM TO PREDICT THE CURVE OF THE FIRST VISIBILITY

```
         DIMENSION
         REAL JD,J,NUTLON,JD1,NUTOB,KATS,J1,M1,J2
         INTEGER YEAR,DAY,HOUR
         IND=0
         D=60.
         S=3600.
         PI=3.14159265358979
         RAD=PI/180.
         READ(5,100) YEAR,MONTH,DAY,HOUR,MINUTE
  100    FORMAT(5I5)
         UT=FLOAT(HOUR)+(FLOAT(MINUTE)/60.)
C        CALCULATION OF THE JULIAN DAY

         FX=(7*(YEAR+(MONTH+9)/12))/4
         FY=(275*MONTH)/9

         JD=367.*FLOAT(YEAR)-FX+FY+FLOAT(DAY)+1721013.5
         UTD=UT/24.
         JD=JD+UTD
         JD1=JD
         WRITE(6,222) JD
  222    FORMAT(10X,'THE CALCULATION OF THE FIRST VISIBILITY OF THE  NEW
        1 MOON',10X)
C
         WRITE(6,1000)
 1000    FORMAT(1X,'DAY',4X,'MONTH',3X,'YEAR',5X,'UT',8X,'GDELTA',9X,
        1 'AYDEL',9X,'GALFA',9X,'AYALF',6X,'EQT',4X,'DIS')
   10    T=(JD -2415020.0)/36525.0
C
         AI = 51.2 + 20.2 *T
         SIN AI = SIN (AI *RAD)
C
         BI = 346.560 + 132.870 *T - 0.009 1731 *T **2
         SIN BI = SIN(BI *RAD)
C
C        MEAN LONGITUDE OF THE MOON.S ASCENDING NODE = OMEGA
C
         OMEGA = 933059.79 - 6962911.23 *T + 7.48 *T **2 + 0.0080 *T **3
         OMEGA = OMEGA /S
         OMEGA = OMEGA / 360.
         I OMEGA = OMEGA
C
         OMEGA = (OMEGA - FLOAT (IOMEGA)) *360.
C
         SOMEGA = SIN (OMEGA *RAD)
C
C        GEOCENTRIC MEAN LONGITUDE OF THE MOON = AYLON
         AYLON = 973562.99 + 1732564379.3 *T - 4.08 *T **2 + 0.0068 *T **3
        1 + 0.84 *SINAI + 14.27 *SINBI + 7.261 *SOMEGA
C
         AYLON = AYLON / S
         AYLON = AYLON / 360.
         IAYLON = AYLON
         AYLON = (AYLON - FLOAT (I AYLON)) *360.
C
C        MEAN ANOMALY OF THE MOON = AYAN
         AYAN = 1065976.59 + 1717915 856.79 *T + 33.09 *T **2 + 0.0518 *T **3
        1 +2.94 *SIN AI + 14.27 *SIN BI + 9.337 *SOMEGA
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      AYAN = AYAN / S
      AYAN = AYAN / 360.
      IAYAN = AYAN
      AYAN = (AYAN - FLOAT(IAYAN)) *360.
C
C     MEAN ANOMALY OF THE SUN = GUNAN
C
      GUNAN = 1290513.00 + 129596 579.10 *T - 0.54 *T **2 - 0.0120 *T **3
     1 -6.40 *SIN AI
      GUNAN = GUNAN / S
      GUNAN = GUNAN / 360.
      IGUNAN = GUNAN
      GUNAN = (GUNAN - FLOAT(IGUNAN)) *360.
C
C     MEAN ELONGATION OF THE MOON FROM SUN = AYEL
C
      AYEL = 126 2654.95 + 1602 961 611.18 *T - 5.17 *T **2 + 0.0068 *T **3
     1 +7.24 *SIN AI + 14.27 *SIN BI + 7.261 *SOMEGA
      AYEL = AYEL / S
      AYEL = AYEL / 360.
      IAYEL = AYEL
      AYEL = (AYEL - FLOAT (IAYEL)) *360.
C
C     MEAN DISTANCE OF THE MOON FROM ASCENDING NODE = AYDIS
C
      AYDIS = 40503.2 + 1739 527 290.54 *T - 11.56 *T **2 - 0.0012 *T **3
     1       + 0.21 *SINAI + 14.27 *SIN BI - 88.699 *SOMEGA
     2       - 15.58 SIN((OMEGA + 275.05 - 2.30 *T) *RAD)
             AYDIS = AYDIS / S
             AYDIS = AYDIS / 360.
             IAYDIS = AYDIS
             AYDIS = (AYDIS - FLOAT (IAYDIS)) *360.
C
C             EQUATION OF THE MOON.S CENTRE = AYMER
C
              AYMER = 22639.500 *SIN (AYAN *RAD) + 769.016 *SIN (2.*AYAN *RAD)
     1 +36.124 *SIN ( 3. *AYAN *RAD) + 1.938 *SIN (4.*AYAN *RAD)
     2 +0.113 *SIN ( 5. *AYAN *RAD)
      AYMER = AYMER / S
C
C     THE PLANETARY PERTURBATIONS IN THE MOON.S LONGITUDE = J,V.
C

C
C     JUPITER = J
C
      J = 221.64 742 + 32964.466 939 *T
      J = J / 360.
      IJ = J
      J = (J - FLOAT ( IJ )) *360.

C
C     VENUS = V.
C

      V = 63.07037 + 22 518.442986 *T
      V = V / 360.
      IV = V
      V = ( V - FLOAT ( IV )) *360.

C
C     MOON / AUXILARY QUANTITY = AN
C
      AN = 275.05 - 2.30 *T
C
C     OBLIQUITY OF THE ECLIPTIC = OROB
C
      OROB = 84428.26 - 46.845 *T - 0.0059 *T **2 + 0.00181 *T **3

C
C     THE ECCENTRICITY OF THE EART.S ORBIT = ECC
C
      ECC = 1. - 0.0024954 *T - 0.000007522 *T **2

C
C     CORRECTION FACTORS IN LATITUDE = OMEGI , OMEG2.
C
      OMEGI = - 0.0004664 *COS(OMEGA *RAD)
C
      OMEG2 = - 0.0000754 *COS ( ( OMEGA + AN ) *RAD)

C
C     COMPUTATION OF THE PERIODIC TERMS IN MOON'S LONGITUDE
C

      L1= SIN ( ( 2. *AYEL - AYAN ) *RAD)
      L2= SIN ( ( 2. *AYEL) *RAD)
      L4= -( SIN ( 2. *AYDIS *RAD) )
      L5= - ( SIN ( ( 2. *AYAN - 2. *AYEL) *RAD ) )
      L7= SIN ( ( 2. *AYEL + AYAN) *RAD)
      L10= -( SIN ( AYEL *RAD ) )
      L12= - ( SIN ( ( 2. *AYDIS - 2. *AYEL ) *RAD ) )
      L13= - (SIN ( ( 2. *AYDIS + AYAN ) *RAD ) )
      L14= -( SIN ( ( 2. *AYDIS - AYAN ) *RAD ) )
      L15= SIN ( ( 4. *AYEL - AYAN ) *RAD )
      L16= SIN ( ( 4. *AYEL - 2. *AYAN) *RAD)
      L19= SIN ( ( AYAN - AYEL) *RAD)
      L22= SIN ( ( 2. *AYEL + 2. *AYAN ) *RAD)
      L23= SIN ( ( 4. *AYEL ) *RAD)
      L24= - ( SIN ( ( 3. *AYAN - 2. *AYEL) *RAD ) )
      L26= SIN ( ( AYAN- 2. *AYDIS - 2. *AYEL) *RAD)
      L28= -( SIN ( ( AYAN + AYEL) *RAD ) )
      L33= - (SIN ( ( AYAN + 2. *AYEL - 2. *AYDIS ) *RAD ) )
      L34= - ( SIN ( ( 2. *AYDIS + 2. *AYEL) *RAD ) )
      L36= - ( SIN ( ( 2. *AYAN + 2. *AYDIS ) *RAD ) )
      L37= - ( SIN ( ( 3. *AYEL - AYAN) *RAD ) )
      L44= SIN ( ( AYAN + 4. *AYEL) *RAD )
      L46= SIN ( ( 2. *AYAN - AYEL) *RAD)
      L48= SIN ( ( 2. *AYDIS - 2. *AYAN) *RAD)
      L50= SIN ( ( 2. *AYAN - 3. *AYEL) *RAD )
      L51= SIN ( ( 4. *AYEL - 3. *AYAN) *RAD)
      L54= SIN ( ( 2. *AYEL - AYAN + 2. *J + 180.3) *RAD )
      L56= SIN ( ( 2. *AYEL + 3. *AYAN) *RAD )
      L57= - ( SIN ( ( AYAN + 2. *AYEL + 2. *AYDIS ) *RAD ) )
      L58 = SIN ( ( 2. *AYEL - 4. *AYAN ) *RAD)
      L59= SIN ( V *RAD )
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      L62 = SIN ( ( 3. *V + 2. *AYEL – AYAN ) *RAD )
      L63 = SIN ( ( J + 1.2 ) *RAD )
      L65 = – ( SIN ( ( 2. *AYAN + AYEL ) *RAD ) )
      L66 = SIN ( ( 2. *AYDIS – AYEL ) *RAD )
      L67 = SIN ( ( 6. *AYEL – 2. AYAN ) *RAD )
      L69 = SIN ( ( 2. *AYAN – 2. *AYEL + 2. *AYDIS ) *RAD )
      L71 = SIN ( ( 2. *AYAN – 2. *AYEL – 2. *AYDIS ) *RAD )
C
      TOPLO=4586.46*L1 + 2369.91*L2 + 411.61*L4 + 211.66*L5
     1 +191.95*L7 + 125.15*L10 + 55.17*L12 + 45.10*L13
     2 +39.53*L14 + 38.43*L15 + 30.77*L16 + 18.61*L19
     3 +14.39*L22 + 13.90*L23 + 13.19*L24 + 9.37 *L26
     4 +8.47*L28 + 6.38*L33 + 5.74 *L34 + 4.00*L36 + 3.21 *L37
     5 +1.98*L44 + 1.75*L46 + 1.30 *L48 + 1.22*L50 + 1.19*L51
     6 +1.14*L54 + 1.06*L56 + 0.99*L57 + 0.95*L58 + 0.82*L59
     7 +0.66*L62 + 0.64*L63 + 0.59*L65 + 0.58*L66
     8 +0.57*L67 + 0.56*L69 + 0.54*L71

      L3 = – ( SIN ( GUNAN *RAD ) )
      L6 = SIN ( ( 2. *AYEL – GUNAN – AYAN ) *RAD )
      L8 = SIN ( ( 2. *AYEL – GUNAN ) *RAD )
      L9 = SIN ( ( AYAN – GUNAN ) *RAD)
      L11 = – ( SIN ( ( AYAN + GUNAN ) *RAD ) )
      L17 = – ( SIN ( ( GUNAN + 2. *AYEL – AYAN ) *RAD ) )
      L18 = – ( SIN ( ( 2. *AYEL + GUNAN ) *RAD ) )
      L20 = SIN ( ( GUNAN + AYEL ) *RAD )
      L21 = SIN ( ( AYAN – GUNAN + 2. *AYEL ) *RAD )
      L25 = SIN ( ( 2. *AYAN – GUNAN ) *RAD)
      L27 = SIN ( ( 2. *AYEL – GUNAN – 2. *AYAN ) *RAD )
      L30 = – ( SIN ( ( 2. *AYAN + GUNAN ) *RAD ) )

      L35 = SIN ( ( 4. *AYEL – GUNAN – AYAN ) *RAD )
      L38 = – ( SIN ( ( GUNAN + 2. *AYEL + AYAN ) *RAD ) )
      L39 = SIN ( ( 4. *AYEL – 2. *AYAN – GUNAN ) *RAD )
      L42 = SIN ( ( GUNAN + 2. *AYEL – 2. *AYAN ) *RAD )
      L43 = SIN ( ( 2. *AYEL – GUNAN – 2. *AYDIS ) *RAD )
      L45 = SIN ( ( 4. *AYEL – GUNAN ) *RAD )
      L47 = SIN ( ( 2. *AYDIS – GUNAN – 2. *AYEL ) *RAD )
      L49 = SIN ( ( GUNAN + AYEL + AYAN ) *RAD )
      L52 = SIN ( ( 2. *AYEL + 2. *AYAN – GUNAN ) *RAD )
      L55 = SIN ( ( GUNAN + AYEL – AYAN ) *RAD )
      L61 = SIN ( ( 3. *AYAN – GUNAN ) *RAD )
      L64 = SIN ( ( AYAN – GUNAN – 4. *AYEL ) *RAD )
      L68 = SIN ( ( GUNAN – AYEL ) *RAD )
      L70 = – (SIN ( ( 3. *AYAN + GUNAN ) *RAD ) )
C
      TOPL1=668.15*L3 + 205.96*L6 + 165.14*L8 + 147.69*L9
     1 +109.67*L11 + 28.47*L17 + 24.42*L18 + 18.02 *L20
     2 +14.58*L21 + 9.70*L25 + 8.63*L27 + 7.65*L30 + 4.39*L35
     3 +2.92*L38 + 2.74*L39 + 2.49*L42 + 2.15*L43 + 1.88 *L45
     4 +1.44*L47 + 1.27*L49 + 1.18*L52 + 1.09*L55 + 0.68*L61
     5 +0.64*L64 + 0.56*L68 + 0.55*L70
C
      L29 = SIN ( ( 2. *AYEL – 2. *GUNAN ) *RAD )
      L31 = – ( SIN ( ( 2. *GUNAN ) *RAD ) )
      L32 = SIN ( ( 2. *AYEL – AYAN – 2. *GUNAN )*RAD )
      L40 = SIN ( ( AYAN – 2. *GUNAN ) *RAD)
      L41 = SIN( ( AYAN – 2. *GUNAN – 2. *AYEL ) *RAD)
      L53 = – ( SIN ( ( 2. *GUNAN + AYAN ) *RAD ) )
      L60 = SIN ( ( AYAN – 2. *GUNAN + 2. * AYEL ) *RAD )
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C
C     COMPUTATION OF THE PERIODIC TERMS IN MOON'S LATITUDE
C
      B1 = 18467.78 *SIN ( AYDIS *RAD )
      B2 = 1010.18 *SIN ( ( AYAN + AYDIS ) *RAD )
      B3 = 999.69 *SIN ( ( AYAN – AYDIS ) *RAD )
      B4 = 623.66 *SIN ( ( 2. *AYEL – AYDIS ) *RAD )
      B5 = 199.48 *SIN ( ( 2. *AYEL – AYAN + AYDIS ) *RAD )
      B6 = 166.58 *SIN ( ( 2. *AYEL – AYAN – AYDIS ) *RAD )
      B7 = 117. 26 *SIN ( ( 2. *AYEL + AYDIS ) *RAD )
      B8 = 61. 91 *SIN ( ( 2. *AYAN + AYDIS ) *RAD )
      B9 = 33.36 *SIN ( ( 2. *AYEL – AYDIS + AYAN ) *RAD )
      B10 = 31.76 *SIN ( ( 2. *AYAN – AYDIS ) *RAD )
      B12 = 15.56 *SIN ( ( 2. *AYEL – AYDIS – 2. * AYAN ) *RAD )
      B13 = 15. 12 *SIN ( ( AYAN + AYDIS + 2. *AYEL ) *RAD)
      B19 = 6.58 *SIN ( ( 4. * AYEL – AYDIS – AYAN ) *RAD )
      B22 = – 5.36 *SIN ( ( AYDIS + AYEL ) *RAD )
      B26 = 4.79 *SIN ( ( AYDIS – AYEL ) *RAD )
      B27 = 3.98 *SIN ( ( 3. *AYAN + AYDIS ) *RAD )
      B28 = 3.67 *SIN ( ( 4. *AYEL – AYDIS ) *RAD )
      B29 = 3.00 *SIN ( ( AYDIS – AYAN + 4. *AYEL ) *RAD )
      B30 = 2.81 *SIN ( ( AYAN – 3. AYDIS ) *RAD )
      B31 = 2.41 *SIN ( ( AYDIS + 4. *AYEL – 2. *AYAN ) *RAD )
      B32 = 2.18 *SIN ( ( 2. *AYEL – 3. *AYDIS ) *RAD )
      B33 = 2.15 *SIN ( ( 2. *AYEL – AYDIS + 2. *AYAN ) *RAD )
      B35 = 1.62 *SIN ( ( 2. *AYAN – AYDIS – 2. *AYEL ) *RAD )
      B36 = 1.58 *SIN ( ( 3. *AYAN – AYDIS ) *RAD )
      B37 = 1.52 *SIN ( ( 2. AYAN + 2. *AYEL + AYDIS ) *RAD )
      B38 = 1.52 *SIN ( ( 2. *AYEL – AYDIS – 3. AYAN ) *RAD )
      B41 = 1.19 *SIN ( ( AYDIS + 4. *AYEL ) *RAD )
      B44 = – 1.02 *SIN ( ( AYAN + 3. *AYDIS ) *RAD )
      B50 = –0.67 *SIN ( ( AYAN + AYDIS + AYEL ) *RAD )
      B53 = 0.63 *SIN ( ( 4. *AYEL – AYDIS – 2. *AYAN ) *RAD )

      B55 = 0.59 *SIN ( ( AYDIS – AYAN – AYEL ) *RAD )
      B56 = 0.47 *SIN ( ( AYAN + 4. *AYEL – AYDIS ) *RAD )
      B57 = 0.43 *SIN ( ( AYAN + AYDIS – AYEL ) *RAD )
      B60 = 0.35 *SIN ( ( AYDIS – 3. *AYEL ) *RAD )
      B62 = 0.33 *SIN ( ( 2. *AYEL – AYAN – 3. *AYDIS ) *RAD )
      B65 = 0.31 *SIN ( ( AYAN + AYDIS – 3. *AYEL ) *RAD )
      B67 = 0.29 *SIN ( ( 3. *AYDIS – AYAN – 2. *AYEL ) *RAD )
      B69 = 0.27 *SIN ( ( AYDIS + 4. *AYAN ) *RAD )
      B70 = 0.25 *SIN ( ( AYDIS + 2. *AYEL – 3. *AYAN ) *RAD )
      B71 = – 0.24 *SIN ( ( 2. *AYEL – AYAN + 3. *AYDIS ) *RAD )
      B74 = 0.21 *SIN ( ( AYAN + AYDIS + 4. *AYEL ) *RAD )
      B75 = – 0.21 *SIN ( ( AYDIS – AYAN + 3. *AYEL ) *RAD )
      B78 = 0.15 *SIN ( ( 2. *AYEL – AYDIS + 3. *AYAN ) *RAD )
      B79 = – 0.14 *SIN ( ( 2. *AYEL + 3. *AYDIS ) *RAD )
      B80 = 0.14 *SIN ( ( AYDIS – AYAN + AYEL ) *RAD )
      B81 = 0.14 *SIN ( ( AYDIS + 2. *AYEL + 3. * AYAN ) *RAD )
      B83 = 0.13 *SIN ( ( 2. *AYEL – AYDIS – 4. *AYAN ) *RAD )
      B84 = 0.13 *SIN ( ( 3. *AYDIS –2. *AYAN ) *RAD )
      TOPB0 = B1 + B2 + B3 + B4 + B5 + B6 + B7 + B8 + B9 + B10 + B12 + B13 + B19 + B22 + B26 + B27
     1+ B28 + B29 + B30 + B31 + B32 + B33 + B35 + B36 + B37 + B38 + B41 + B44 + B50 + B53 + B55
     2+ B56 + B57 + B60 + B62 + B65 + B67 + B69 + B70 + B71 + B74 + B75 + B78 + B79 +B80
     3+ B81 + B83 + B84
C
      B11 = SIN ( ( 2. *AYEL – GUNAN – AYDIS ) *RAD )
      B14 = SIN ( ( AYDIS – 2. * AYEL – GUNAN ) *RAD )
      B15 = SIN ( ( 2. *AYEL – AYAN + AYDIS – GUNAN ) *RAD )
      B16 = SIN ( ( 2. *AYEL – GUNAN + AYDIS ) *RAD )
      B17 = – ( SIN ( ( AYAN + AYDIS – 2. *AYEL + GUNAN ) *RAD ) )
      B18 = SIN ( ( AYAN – GUNAN + AYDIS ) *RAD )
      B20 = – ( SIN ( ( GUNAN + AYDIS ) *RAD ) )
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      B21 = SIN (( AYAN - GUNAN -- AYDIS ) *RAD )
      B23 = - SIN ( ( GUNAN + AYDIS + AYAN ) *RAD )
      B24 = SIN ( ( AYDIS - GUNAN - AYAN ) *RAD )
      B25 = SIN ( ( AYDIS - GUNAN ) *RAD )
      B34 = SIN ( ( AYAN - GUNAN - AYDIS + 2. *AYEL ) *RAD )
      B39 = - ( SIN ( ( GUNAN + AYDIS + 2. *AYEL - AYAN ) *RAD ) )
      B40 = - ( SIN ( ( GUNAN + AYDIS + 2. *AYEL ) *RAD ))
      B42 = SIN ( ( 2. *AYEL - GUNAN + AYDIS + AYAN ) *RAD )
      B45 = SIN ( ( AYDIS - 2. *AYEL - GUNAN - AYAN ) *RAD )
      B46 = SIN ( ( AYDIS - GUNAN + 2. *AYAN ) *RAD )
      B47 = SIN ( ( AYEL + GUNAN - AYDIS ) *RAD )
      B48 = SIN ( ( GUNAN + AYDIS + AYEL ) *RAD )
      B49 = SIN ( ( AYAN + AYDIS - 2. *AYEL - GUNAN ) *RAD )
      B51 = - ( SIN ( ( AYDIS + 2. *AYAN + GUNAN - 2. *AYEL ) *RAD ) )
      B52 = - ( SIN ( ( GUNAN + AYDIS + 2. *AYAN ) *RAD ) )
      B54 = SIN ( ( 4. *AYEL - AYDIS - GUNAN - AYAN ) *RAD )
      B58 = SIN ( ( 4. *AYEL - AYDIS - GUNAN ) *RAD )
      B61 = SIN ( ( AYDIS + 4. *AYEL - GUNAN - AYAN ) *RAD )
      B64 = SIN ( ( AYDIS - GUNAN - 2. *AYAN ) *RAD )
      B66 = SIN ( ( 2. *AYAN - GUNAN - AYDIS ) *RAD )
      B72 = - ( SIN ( ( GUNAN + AYDIS + AYAN + 2. *AYEL ) *RAD ) )
      B73 = SIN ( ( AYDIS + 4. *AYEL - GUNAN - 2. *AYAN ) *RAD )
      B76 = SIN ( ( 4. *AYEL - AYDIS + GUNAN - AYAN ) *RAD )
      B77 = SIN ( ( AYDIS + 4. *AYEL - GUNAN ) *RAD )
      B85 = SIN ( ( 2. *AYAN -- AYDIS - GUNAN + 2. *AYEL ) *RAD )

      TOPB1=29.69*B11 + 12.14*B14 + 8.90*B15 + 8.00*B16
     1      +7.46*B17 + 6.76*B18 + 6.49*B20 + 5.65*B21 + 5.33*B23
     2      +5.10*B24 + 4.86*B25 + 1.77*B34 + 1.32*B39 + 1.27*B40
     3      +1.14*B42 + 0.83*B45 + 0.81*B46 + 0.81*B47 + 0.80*B48
     4        + 0.79 *SINB49 + 0.66 *SINB51 + 0.64 *SINB52 + 0.60 *SINB54 + 0.42 *SINB58
     5        + 0.34 *SINB61 + 0.31 *SINB64 + 0.30 *SINB66 + 0.24 *SINB72 + 0.22 *SINB73
     6        + 0.17 *SINB76 + 0.15 *SINB77 + 0.13 *SINB85
C
C
      B43 = SIN ( ( 2. *AYEL - 2. *GUNAN - AYDIS ) *RAD )
      B59 = SIN ( ( 2. *AYEL - 2. *GUNAN + AYDIS ) *RAD )
      B63 = SIN ( ( 2. *AYEL - AYAN + AYDIS - 2. *GUNAN ) *RAD )
      B68 = - ( SIN ( ( AYAN + AYDIS - 2. *AYEL + 2. * GUNAN ) *RAD ) )
      B82 = SIN ( ( AYDIS - 2. *AYEL - 2. *GUNAN ) *RAD )
C
      TOPB2= 1.10*B43 + 0.39*B59 + 0.32*B63 + 0.27*B68 + 0.14*B82

C
C     GEOCENTRIC MEAN LONGITUDE OF THE SUN = GLON
C
      GLON = 1006908.04 + 129 602 768.13 *T + 1.089 *T **2
      GLON = GLON / S
      GLON = GLON / 360.
      IGLON = GLON
      GLON = ( GLON - FLOAT ( IGLON ) ) *360.
C
C     PLANETARY PERTURBATIONS IN SUN.S POSITION
C
C     MARS = OPMAR
C
      OPMAR = 165.94905 + 16859.069 667 *T
      OPMAR = OPMAR / 360.
      IOPMAR = OPMAR
      OPMAR = ( OPMAR - FLOAT ( IOPMAR ) ) *360.

C
C     SATURN = OPSAT
C
      OPSAT = 193.13230 +34777.259 042 *T
      OPSAT = OPSAT / 360.
      IOPSAT = OPSAT
      OPSAT = ( OPSAT - FLOAT ( IOPSAT ) ) *360.

C
C     SUN, EQUATION OF CENTRE = GUNMER
C
      GUNMER = ( 6910.057 - 17.240 *T - 0.052 * T **2 ) *SIN ( GUNAN *RAD)
     1 +( 72.338 - 0.361 *T ) *SIN ( 2. *GUNAN *RAD )
     2 +( 1.054 -- 0.001 *T ) *SIN ( 3. *GUNAN *RAD )
     3 + 0.018 *SIN ( 4. *GUNAN *RAD )
      GUNMER = GUNMER / S

C
C     NUTATION
C

C     NUTATION IN LONGITUDE = NUTLON
C
      NUTLON = -(17.2327 + 0.01737 *T) *SIN ( OMEGA *RAD )
     1 +(0.2088 + 0.00002 *T) *SIN (2. *OMEGA *RAD) - 1.273 *SIN (2. *GLON *RAD)
     2 -0.2037 *SIN (2. *AYLON *RAD) + 0.126 *SIN ( GUNAN *RAD)
     3 + 0.0675 *SIN (AYAN *RAD) - 0.0497 SIN ( ( 2. *GLON + GUNAN ) *RAD)
     4 + 0.0214 *SIN ( ( 2. *GLON - GUNAN ) *RAD )
C
      NUTLON = NUTLON / S

C
C     NUTATION IN OBLIQUITY = NUTOB
C
      NUTOB = (9.2100 + 0.00091 *T) *COS (OMEGA *RAD)
     1 - (0.0904 - 0.00004 *T) *COS (2. *OMEGA *RAD)
     2 + 0.5520 *COS ( 2. *GLON *RAD) + 0.0884 *COS (2. *AYLON *RAD)
     3 + 0.0216 *COS ( ( 2. *GLON + GUNAN ) *RAD )
     4 - 0.0093 COS ( ( 2. *GLON - GUNAN ) *RAD )
C
C
C     SUN, PERTURBATION IN LONGITUDE
C
C     BY VENUS = VI
C
      VI = 4.838 *COS ( ( V + 90.00) *RAD) + 5.526 *COS ( ( 2. *V + 90.12) *RAD)
     1 + 0.666 *COS ( ( 3. *V + 270.41) *RAD) + 0.210 *COS ( ( 4. *V + 89.80 ) *RAD )
     2 + 0.084 *COS ( ( 5. *V + 270.10) *RAD) + 2.497 *COS ( ( 2. *V - GUNAN + 257.75) *RAD)
     3 + 1.559 *COS ( ( 3. *V - GUNAN + 77.96) *RAD) + 1.024 *COS ( ( 3. *V - 2. *GUNAN + 50.85) *RAD)
     4 + 0.154 *COS ( ( 5. *V - 3. *GUNAN + 34.10) *RAD) + 0.152 *COS ( ( 4. *V - 2. *GUNAN + 227.40) *RAD)
     5 + 0.144 *COS ( ( 4. *V - GUNAN + 79.0) *RAD) + 0.123 *COS ( ( 5. *V - 2. *GUNAN + 229.8) *RAD)
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     6   + 0.116 *COS ( ( 2. *V + GUNAN  + 90.70 ) *RAD ) + 0.075 *COS ( ( V + GUNAN + 87.50 ) *RAD )
     7   + 0.074 *COS ( ( V – GUNAN + 358.80 ) *RAD
         VI = VI / S
C
C        BY  SATURN = SI
C

         SI = 0.419 *COS ( ( OPSAT + 90.34 ) *RAD ) + 0.108 *COS ( ( 2. *OPSAT + 270.10) *RAD )
     1      + 0.320 *COS ( ( OPSAT – GUNAN + 259.22) *RAD ) + 0.112 *COS ( ( 2. *OPSAT – GUNAN + 273.1) *RAD)
         SI = SI / S
C
C        BY  MARS = MI
C

         MI = 0.273 *COS ( ( OPMAR + 90.6) *RAD) + 2.043 *COS ( ( 2. *OPMAR + 89.76 ) *RAD)
     1   + 0.129 *COS ( ( 3. *OPMAR + 273.0) *RAD) + 1.770 *COS ( ( 2. *OPMAR – GUNAN + 306.27) *RAD)
     2   + 0.585 *COS ( ( 4. *OPMAR – 2. *GUNAN + 185.82) *RAD)
     3   + 0.500 *COS ( ( 4. *OPMAR – GUNAN + 316.94) *RAD)
     4   + 0.425 *COS ( ( 3. *OPMAR – GUNAN + 317.70) *RAD)
     5   + 0.204 *COS ( ( 5. *OPMAR – 2. *GUNAN + 185.5) *RAD)
     6   + 0.154 *COS ( ( 6. *OPMAR – 2. *GUNAN + 185.0) *RAD)
     7   + 0.106 *COS ( ( 7. *OPMAR – 3. *GUNAN + 53.3) *RAD)
     8   + 0.101 *COS ( ( 6. *OPMAR – 3. *GUNAN + 53.9) *RAD)
     9   + 0.085 *COS ( ( 5. *OPMAR – GUNAN + 139.3) *RAD )
         MI = MI / S.
C
C        BY  JUPITER = JI
C

         JI = 7.208 *COS ( ( J + 91.09 ) *RAD ) + 2.731 *COS ( ( 2. *J + 270.25) *RAD )
     1   + 0.164 *COS ( ( 3. *J + 265.2) *RAD) + 2.600 *COS ( ( J – GUNAN + 174.77) *RAD)
     2   + 1.610 *COS ( ( 2. *J – GUNAN + 292.60) *RAD) + 0.556 *COS ( ( 3. *J – GUNAN + 177.31) *RAD)
     3   + 0.210 *COS ( ( 3.*J – 2. *GUNAN + 193.2) *RAD) + 0.163 *COS ( ( J + GUNAN + 110.2) *RAD)
     4   + 0.080 *COS ( ( 4. *J – 2. *GUNAN + 83.9) *RAD) + 0.073 *COS ( ( J – 2. GUNAN + 187.9) *RAD)
     5   + 0.073 *COS ( ( 2. *J – 2. *GUNAN + 75.7) *RAD) + 0.069 *COS ( ( 2. *J + GUNAN + 263.9) *RAD)
         JI = JI / S
C
C        PERTURBATION  PRODUCED  BY  THE  MOON (IN  LONGITUDE) = DLAM
C

         DLAM = 6.454 *SIN ( AYEL *RAD) + 0.013 *SIN ( 3. *AYEL *RAD)
     1   + 0.177 *SIN ( ( AYEL + AYAN) *RAD) – 0.424 *SIN ( ( AYEL – AYAN) *RAD)
     2   + 0.039 *SIN ( ( 3. *AYEL – AYAN) *RAD) – 0.064 *SIN ( ( AYEL + GUNAN) *RAD)
     3   + 0.172 *SIN ( ( AYEL – GUNAN) *RAD)
         DLAM = DLAM / S
C
C        THE  SUN.S  ABERRATION  IN LONGITUD = ABLON
C
         ABLON = – ( 20.47 + 0.342 *COS(GUNAN *RAD)
C
C        SUN, PERTURBATIONS  IN  LATITUDE
C
C        BY  VENUS = V2
C
         V2 = 0.210 *COS ( ( 3. *V – GUNAN + 64.5)*RAD) + 0.092 *COS ((V – GUNAN + 64.6) *RAD)
     1   + 0.067 *COS ( ( 2. *V – GUNAN + 244.8) *RAD) + 0.031 *COS ( ( 4. *V– GUNAN + 65.4) *RAD)
     2   + 0.029 *COS ( ( V + GUNAN + 116) *RAD) + 0.023 *COS ( ( 2. *V + GUNAN + 295.0) *RAD)
     3   + 0.019 *COS ( ( 5. *V – 2. *GUNAN + 233.0) *RAD) + 0.014 *COS (( 3. *V + GUNAN + 114.0) *RAD)
     4   + 0.014 *COS ( ( 2. *V – 2. *GUNAN + 233.0) *RAD) + 0.012 *COS ( ( 2. *V + 271.0) *RAD)
     5   + 0.012 *COS ( ( 4. *V – 2. *GUNAN + 244.0) *RAD)
```

```
C
C        BY  JUPITER = J2
C
         J2 = 0.166 *COS ( ( 2. *J – GUNAN + 268.0) *RAD) + 0.023 *COS ( ( J – 2. *GUNAN) *RAD)
     1   + 0.018 *COS ( ( 3. *J – 2. *GUNAN + 182.0) *RAD) + 0.017 *COS ( ( J + 5.0 ) *RAD)
     2   + 0.016 *COS ( ( J – GUNAN + 272.0) *RAD)
C
C        PERTURBATION  PRODUCED  BY  THE  MOON  (IN  LATITUDE) = DBETA
C
         DBETA = 0.576 *SIN ( AYDIS *RAD) + 0.016 *SIN ( ( AYDIS + AYAN) *RAD)
     1   – 0.047 *SIN ( ( AYDIS – AYAN) *RAD)
C
C        COMPUTATION  OF  THE  MOON.S  LONGITUDE = ALONG
C
         SIGML=TOPL0 + ECC *TOPL1 + ECC **2 *TOPL2
         SIGML= SIGML/S
         ALONG=AYLON + AYMER + SIGML
C
C        MOON.S  APPARENT  LONGITUDE = APLON
C
         APLON = ALONG + NUTLON
C
C        TRUE  OBLIQUITY  OF  THE  ECLIPTIC = TROB
C
         TROB = OROB + NUTOB
C
         TROB = TROB / S
C
C        COMPUTATION  OF  THE  MOON.S  LATITUDE = APPARENT
C        BY  NUTATION = APLAT
         SIGMB= TOPB0 + ECC *TOB1 + ECC **2 *TOPB2
C
         KATS = 1. + OMEGI + OMEG2

         APLAT = KATS *SIGMB
C
         APLAT = APLAT / S
C
C    o   CALCULATION  OF  THE  RIGHT  ASCENTION  AND  DECLINATION  OF  MOON
C
C        AYALF = RIGHT  ASCENTION
C
C        AYDEL = DECLINATION
C
         TAYALF = TAN ( APLON *RAD) *COS (TROB *RAD) – ( TAN (APLAT *RAD)
     1               *SIN ( TROB *RAD ) / COS ( APLON *RAD ) )
C
         AYALF = ATAN ( TAYALF) / RAD
         IF(AYALF . LT . O) AYALF=360 + AYALF
C
         SAYDEL = COS ( APLAT *RAD) *SIN (APLON *RAD) *SIN (TROB *RAD) +
     1       + SIN (APLAT *RAD) *COS (TROB *RAD)
C
         AYDEL = ASIN (SAYDEL) / RAB
C
C        COMPUTATION  OF  THE  SUN.S  LONGITUDE = GLONG
C
         GLONG = GLON + GUNMER + VI + MI + JI + SI + DLAM
C
C        SUN.S  APPARENT  LONGITUDE = GAPLON
C
C
         GAPLON = GLON – ( 20.47 + 0.342 *COS ( GUNAN *RAD ) ) / S + NUTLON
C
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```
C
C      COMPUTATION OF THE SUN.S LATITUDE = GLAT
C
       GLAT = V2 + J2 + DBETA
C
       GLAT = GLAT / S
C
C
C      CALCULATION OF THE RIGHT ASCENSION AND DECLINATION OF SUN
C
C      GALFA = RIGHT ASCENSION
C
C      GDELTA= DECLINATION
C
       GALFA = TAN ( GAPLON *RAD) *COS (TROB *RAD)
C
       GALFA = ATAN ( GALFA) / RAD
       IF(GALFA . LT . O) GALFA=360 + GALFA
       GDELTA = SIN ( GAPLON *RAD) *SIN (TROB *RAD)
C
       GDELTA = ASIN (GDELTA) / RAD
C
C      CORRECTIONS
C
       DALFA= -(GLAT *0.250)
       DALFA= DALFA / S
       D DELTA = (GLAT *0.971)
       D DELTA = D DELTA / S
C
       GALFA = GALFA + DALTA
C
       GDELTA = GDELTA + DDELTA
C
C      CALCULATION OF THE ANGULAR DISTANCE BETWEEN SUN AND MOOM = DIS
C
       DIS = (SIN ( GDELTA *RAD) *SIN ( AYDEL *RAD ) )
     1     + (COS ( GDELTA *RAD) *COS ( AYDEL *RAD ) *COS ( ( AYALF - GALFA ) *RAD )
C
       DIS = ACOS (DIS) / RAD
  40   IF(DIS. GT. 7.9) GO TO 20
       FARK= (29.5*(8.0-DIS))/360.0
       JD=JD + FARK
       GO TO 10
  20   IND= IND + 1
       UT= (JD-JD1) *24.0
       UT=UT + HOUR + (MINUTE) / 60.0
       IF (UT. EQ. 24.0 . OR . UT . GT . 24.0)  UT= UT-24.0
C
C      APPARENT SIDERAL TIME= APST
C
       APST= 6.64606556 +(8640184.542*T)/3600.0+(0.0929*T **2)/3600.0
     1       + (NUTLON *COS(TROB *RAD) )/15.0
       APST= APST/24.0
       IAPST= APST
       APST= (APST   FLOAT(IAPST) ) *24.0
C
C      EQUATION OF TIME (IN MINUTE) = EQT
       EQT= (APST-(GALFA/15.0) ) *60.0
C
       WRITE(6,  3000)  DAY,MONTH,YEAR,UT,GDELTA,AYDEL,GALFA,AYALF,EQT,DIS
 3000  FORMAT (2X,I2,5X,I2,3X,I4,3X,F12.8,2X,F12.8,2X,F12.8,2X,F12.8,2X,
     1 F12.8, 2X, F6.3, 2X, F5.2)
```

```
       ALFA= AYALF-GALFA
       T1=12.-EQT
       ALFA=AYALF-GALFA
       DG=GDELTA
       DA=AYDEL
       TG=UT
       WRITE (6, 333)
 333   FORMAT (1H1)
       SINDG=SIN (DG *RAD)
       COSDG=COS (DG *RAD)
       SINDA=SIN(DA *RAD)
       COSDA=COS(DA *RAD)
       TANDG=TAN(DG *RAD)
       TANDA=TAN(DA *RAD)
       COSAL=COS(ALFA *RAD)
       SINS1=SIN(5.83 *RAD)
       COSC=COS(90.83 *RAD)
       FK1=SINS1/GOSDA
       FK2=COSC/COSDG
       FK3=(TANDA **2+TANDG **2)-(2.*COSAL *TANDA *TANDG)
       D1=TANDA *(COSAL *FK2-FK1)+ TANDG *(COSAL *FK1-FK2)
       E=FK1*FK1+FK2*FK2-2.*COSAL *FK1*FK2
       F=COSAL *COSAL--1.
       C1=-(E+F)
       B=-2.*D1
       A=F--FK3
       DISK=B *B  4.*A *C1
       IF(DISK) 22, 33, 44
  22   WRITE (6, 13)
  13   FORMAT(1X,'THERE ARE NO REAL ROOTS')
       GO TO 50
  33   ROOT=-B/(2.0 *B)
       WRITE (6, 14) ROOT, ROOT
  14   FORMAT(1X,'THERE ARE TWO IDENTICAL ROOTS',2(3X,F10.2))
       GO TO 50
  44   ROOT1= (-B+SQRT(DISK))/(2.0 *A)
       ROOT2=(-B-SQRT(DISK))/(2.0*A)
       IF(ABS(ROOT1). LE . 1 .) GO TO 77
       IF(ABS(ROOT2) . LE . 1 .) GO TO 77
       WRITE(6, 13)
       GO TO 50
  77   FI1= (ASIN (ROOT1))/RAD
       FI2= (ASIN (ROOT2))/RAD
       SINFI1= SIN(FI1 *RAD)
       COSFI1= COS(FI1 *RAD)
       SINFI2= SIN(FI2 *RAD)
       COSFI2=COS(FI2 *RAD)
       HG1= ACOS((COSC- SINFI1*SINDG)/(COSFI1*COSDG))
       HG1= HG1/RAD
       AMB1=TG-(HG1/15.+T1)
       HG2=ACOS (( COSC-SINFI2*SINDG)/(COSFI2*COSDG))
       HG2=HG2/RAD
       AMB2=TG-(HG2/15.+ T1)
       WRITE (6, 333)
       WRITE (6, 999)
 999   FORMAT(1H/ / / )
       WRITE (6, 444)
 444   FORMAT (20X,'KANDILLI OBSERVATORY',/
     1         15X,'THE FIRST VISIBILITY OF THE
     2         NEW MOON', / / )
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```
      WRITE (6, 555) YEAR, MONTH,DAY,TG,DG,DA,ALFA,T1,FI1,AMB1,FI2,AMB2
555   FORMAT (5X,'DATE',23X,I6,2I3,/
   1          5X,'TIME',30X,F9.3,/
   2          5X,'DECLINATION OF THE SUN',12X,F9.3,/
   3          5X,'DECLINATION OF THE MOON',11X,F9.3,/
   4          5X,'DIFFERENCE OF THE RIGHT ASSENTION',F10.3,/
   5          5X,'EPHEMERIS TRANSIT TIME' ,12X,F9.3,/
   6          5X,'FIRST LATITUDE',20X,F9.3,/
   7          5X,'FIRST LONGITUDE DIFFERENCE',8X,F9.3,/
   8          5X,'SECOND LATITUDE',19X,F9.3,/
   9          5X,'SECOND.LONGITUDE DIFFERENCE',7X,F9.3)
      WRITE (6, 999)
      WRITE (6, 888)
888   FORMAT (15X,'H MOON',8X,'LAT',8X,'D:F LONG')
      FI=FI1
 99   IF(FI . GT . FI2) GO TO 11
      SINFI= SIN(FI *RAD)
      COSFI= COS(FI*RAD)
      HG= ACOS( ( COSC-SINFI-SINDG) / (COSFI *COSDG) )
      HG= HG/RAD
      AMB= TG-(HG/15.0+T1)
      FARK= (HG-ALFA) *RAD
      HC=ASIN (COS(FARK)*(COSFI*COSDA)+SINFI*SINDA)
      HCD=HC/RAD
      WRITE (6, 600)HCD,FI,AMB
600   FORMAT (10X, 2F10.3, F12.3)
      FI=FI+2.
      GO TO 99
 11   CONTINUE
      JD=JD+(2./24.)
      IF(IND . LT . 10 .) GO TO 10
 50   STOP
      END
```

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