Conservative Dynamical Systems 2010/2011

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The last two exercises are homework, to be handed in on Tuesday 9 November.

9.1 Linear Hamiltonian systems

What are the possible types of linear Hamiltonian systems? How do the corresponding Hamiltonian functions look like? Which of the occurring equilibria are structurally stable — and in what sense? Consider in particular the case of one degree of freedom.

9.2 Perturbations of an anharmonic oscillator

Determine the bifurcation diagram of the family

\[ H_{\lambda,\mu}(x, y) = \frac{1}{2}y^2 + \frac{1}{24}x^4 + \frac{\lambda}{2}x^2 + \mu x \]

of Hamiltonian systems.

9.3 Symplectic geometry

1. Show that a nonzero vector in the symplectic vector space \( \mathbb{R}^{2n} \) can be carried into any other nonzero vector by a symplectic transformation.

2. Show that not every two-dimensional plane of \( \mathbb{R}^{2n} \) can be obtained from a given two-dimensional plane by a symplectic transformation.

3. Show that any non-isotropic two-dimensional plane in \( \mathbb{R}^{2n} \) can be carried into any other non-isotropic two-dimensional plane by a symplectic transformation.

9.4 Colombo’s top

Analyse the dynamics of “Colombo’s top” on \( S^2 \), the 2–parameter family with Hamiltonian functions

\[ H_{\lambda,\mu}(x, y, z) = -\frac{1}{2}(z - \lambda)^2 + \mu y \]

and Poisson bracket relations \( \{x, y\} = z \) plus cyclic permutations.