Conservative Dynamical Systems 2010/2011

Heinz Hanßmann and Holger Waalkens

The last two exercises are homework, to be handed in on Tuesday 9 November.

9.1 Linear Hamiltonian systems

What are the possible types of linear Hamiltonian systems? How do the corresponding Hamiltonian functions look like? Which of the occurring equilibria are structurally stable — and in what sense? Consider in particular the case of one degree of freedom.

9.2 Perturbations of an anharmonic oscillator

Determine the bifurcation diagram of the family

$$H_{\lambda,\mu}(x,y) \;=\; \frac{1}{2}y^2 \;+\; \frac{1}{24}x^4 \;+\; \frac{\lambda}{2}x^2 \;+\; \mu x$$

of Hamiltonian systems.

9.3 Symplectic geometry

1. Show that a nonzero vector in the symplectic vector space $\mathbb{R}^{2n}$ can be carried into any other nonzero vector by a symplectic transformation.
2. Show that not every two-dimensional plane of $\mathbb{R}^{2n}$ can be obtained from a given two-dimensional plane by a symplectic transformation.
3. Show that any non-isotropic two-dimensional plane in $\mathbb{R}^{2n}$ can be carried into any other non-isotropic two-dimensional plane by a symplectic transformation.

9.4 Colombo's top

Analyse the dynamics of "Colombo's top" on S^2, the 2–parameter family with Hamiltonian functions

$$H_{\lambda,\mu}(x,y,z) \;=\; -\frac{1}{2}(z-\lambda)^2 \;+\; \mu\, y$$

and Poisson bracket relations $\{x,y\} = z$ plus cyclic permutations.