Conservative Dynamical Systems

The last two exercises are homework, to be handed in on 3 March.

4.1 No attractors

Show that in a Hamiltonian system no asymptotically stable equilibria or periodic solutions occur.

4.2 Recurrence

Give an explicit proof of the Poincaré Recurrence Theorem.

4.3 Dense Lissajous figure

Show that for real $\omega \notin \mathbb{Q}$ the Lissajous figure

$$x = \cos t\,, \quad y = \cos \omega t$$

densely fills the square $\{(x, y) \in \mathbb{R}^2 \mid |x| \leq 1, |y| \leq 1\}$.

4.4 Decimals

Consider the sequence $\{2^n\}_{n=1}^{\infty}$ in the decimal notation system and also consider the corresponding sequence of first digits: $1, 2, 4, 8, 1, 3, 6, 1, 2, 5, 1, 2, 4, \ldots$ Does the digit 7 occur in the latter sequence? Which number occurs more often, 7 or 8? How much more often?