Singular and Algebraic Reduction

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Abstract

Regular reduction deals with the case of a constraint submanifold M of a symplectic manifold (P, ω) such that the kernel $\ker \omega_M$ of the restriction of ω to M is an integrable distribution and the space R of its integral manifolds is a quotient manifold of M. In this case ω induces a symplectic form ω_R on R. The symplectic space (R, ω_R) is called the reduced space of M, and this reduction procedure is usually called *geometric reduction*.

Of special interest is the case of *Marsden-Weinstein reduction*. In this case, there is an action of a Lie group G on P with an equivariant momentum map $J : P \to \mathfrak{g}^*$ and the constraint set given as the level set of J. In other words, $M = J^{-1}(\mu)$ for $\mu \in \mathfrak{g}^*$. If the action of G on P is free and proper, the assumptions of geometric reduction are satisfied and the reduced space is the quotient of $J^{-1}(\mu)$ by the isotropy group of $\mu \in \mathfrak{g}^*$.

In general, singular reduction deals with the cases when regularity assumptions are not satisfied. However, at present the term *singular reduction* is restricted to the reduction technique initiated in 1983 by Richard Cushman. Also in 1983, Alan Weinstein and I introduced *algebraic reduction* as an attempt to generalize the results of Guillemin and Sternberg on commutativity of quantization and reduction.

In the first part of my lecture, I shall review the highlights of the development of singular reduction and algebraic reduction. Next, I shall give a brief outline of geometric quantization and describe results of Guillemin and Sternberg. In the last part of my lecture I outline a generalization of geometric quantization to reduced Poisson algebras obtained, by singular and by algebraic reduction, and present their application to a decomposition of a quantization representation of $SU(2)$ into its irreducible components [joint work with Larry Bates, Richard Cushman and Mark Hamilton].