Dynamical Systems 2007

The last two exercises are homework, to be handed in on 26 February.

3.1 Asymptotics of the period of the pendulum

Consider the Hamilton function $H(x, y) = \frac{1}{2}y^2 - \omega^2 \cos x$ of the pendulum. For $|z| < \omega^2$ we consider the level set $H^{-1}(z)$. What is the amplitude of oscillation in this level? If $T(z)$ denotes the period of oscillation in this level, then give an explicit integral expression for this. Determine $\lim_{z \to -\omega^2} T(z)$ and $\lim_{z \to \omega^2} T(z)$.

3.2 Isoperimetric problem

In $\mathbb{R}^3 = \{x, y, z\}$ consider the (x, y)-plane. In the (x, y)-plane a curve is given, connecting the points $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$. Revolve this curve arond the x-axis. For which curve does the corresponding surface of revolution have minimal area.

3.3 Period and area

Let $H : \mathbb{R}^2 \to \mathbb{R}$ be a smooth function and assume that for $z_0 \in \mathbb{R}$ the motion in the level $H^{-1}(z_0)$ is periodic. Then show that, for z near z_0 the motion is periodic as well. Let $A(z)$ denote the area enclosed by the level $H^{-1}(z)$, while $T(z)$ denotes the period of oscillation in this level set. Show that $T = dA/dz$.

3.4 A minimum

Compare Arnold [MMCM]. Show that the uniform motion of a free mass point in $\mathbb{R}^3$ is a minimum of the corresponding variational problem.