Dynamical Systems 2007

The last two exercises are homework, to be handed in on 5 March.

4.1 No attractors

Show that in a Hamiltonian system no asymptotically stable equilibria or periodic solutions occur.

4.2 Rotating pendulum

Analyse the dynamics of the rotating pendulum $\ddot{x} = M - \sin x$ in dependence of M.

4.3 Kepler's 3rd law

The 3rd law of Kepler states that the ratio of the square of the period of an elliptic orbit and the cube of its semi major axis is constant. The aim of this exercise is to check the relation to the $\frac{1}{r}$–form of the gravitational potential, from which one obtaines an effective potential after reduction to one degree of freedom.

1. Check that the minimum of the effective potential leads to a circular orbit which satisfies Kepler's 3rd law.

2. Derive the $\frac{1}{r}$–form of the gravitational potential from Kepler's 3rd law.

4.4 Consequence of correction term

The correction of the gravitational law in general relativity leads in first order to the Hamiltonian funktion

$$H(x, y) \quad = \quad \frac{y_1^2 + y_2^2}{2} - \frac{1}{\sqrt{x_1^2 + x_2^2}} + \frac{\varepsilon}{x_1^2 + x_2^2} \ , \quad 0 < \varepsilon \ll 1 \ .$$

What happens to "Kepler's ellipses" under the influence of this perturbation ?