Dynamical Systems 2007

The last two exercises are homework, to be handed in on 26 March.

7.1 Reduced Euler top

Analyse the dynamics of the "reduced Euler top"

$$H(x,y,z) \;=\; \frac{x^2}{2a} + \frac{y^2}{2b} + \frac{z^2}{2c}\ , \quad 0 < a \leq b \leq c$$

on S^2 in the limiting cases $a \to b$ and $b \to c$.

7.2 A harmonic n-body problem

The particles A_i with masses m_i $(i = 1, 2, \ldots, n)$ move in three dimensional space. Any two distinct points A_i, A_j attract each other by the force $F_{ij} = k^2 m_i m_j d_{ij}$, where $k > 0$ and d_{ij} denotes the distance $\overline{A_i A_j}$. We suppose that the motions of A_i and A_j are not disturbed if they pass simultaneously through the same point. Determine the general motion of the particles.

7.3 Colombo's top

Analyse the dynamics of "Colombo's top" on S^2, the 2–parameter family with Hamiltonian functions

$$H_{\lambda,\mu}(x,y,z) \;=\; -\frac{1}{2}(z-\lambda)^2 \;+\; \mu\, y\ .$$

7.4 Steiner ellipse

A particle P of unit mass moves in the plane of a given fixed triangle $A_1A_2A_3$. The force F_i on P is directed towards A_i and is equal to $k\overline{PA_i}$ for $i = 1, 2, 3$, where k is a positive constant, and $\overline{PA_i}$ denotes the distance between P and A_i. Prove that there is a motion of P the path of which coincides with the Steiner ellipse S of $A_1A_2A_3$ (the ellipse S passes through the vertices and the tangent at each vertex is parallel to the opposite side). Show moreover that P covers the three arcs A_1A_2, A_2A_3 and A_3A_1 of S in equal time.